# A short introduction to SageMath

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## The mission

*Create a viable free open source alternative to Magma, Maple, Mathematica and Matlab.*

# Some advantages of SageMath

## SageMath is free (GPL v2)

Freedom means

1. everybody can use it, by downloading the software from http://sagemath.org
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- no need to learn any specific syntax to use it
- easy access for students
- Python is a very powerful *object oriented language*, with a neat syntax

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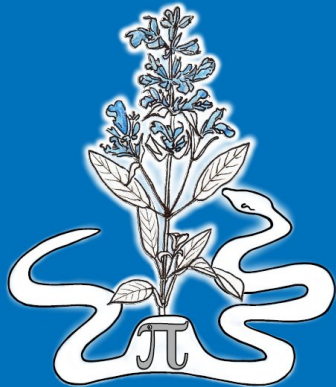- Python is a very powerful *object oriented language*, with a neat syntax

## SageMath is developing and spreading fast

...sustained by an enthusiastic community of developers

# The Sage book



by A. Casamayou, N. Cohen, G. Connan,
T. Dumont, L. Fousse, F. Maltey, M. Meulien,
M. Mezzarobba, C. Pernet, N.M. Thiéry &
P. Zimmermann (2013)

Released under *Creative Commons* license:

- freely downloadable from
  http://sagebook.gforge.inria.fr/
- printed copies can be ordered at moderate
  price ($10 \, €$)
- English version in preparation:
  https://members.loria.fr/
  PZimmermann/sagebook/english.html

# Object-oriented notation in Python

As an object-oriented language, Python (and hence SageMath) makes use of the following **postfix notation** (same in C++, Java, etc.):

$$result = object.function(arguments)$$

In a procedural language, this would be written as

$$result = function(object, arguments)$$

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**Examples**

1. `list_divisors = n.divisors()`
2. `test = n.divides(189)`

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NB: no argument in example 1

# Let us try it!

- First contact with SageMath:
  http://nbviewer.jupyter.org/github/egourgoulhon/SageMathTour/blob/master/Notebooks/first_contact.ipynb
- 2D plots:
  http://nbviewer.jupyter.org/github/egourgoulhon/SageMathTour/blob/master/Notebooks/plot_tour_2D.ipynb
- 3D plots:
  http://nbviewer.jupyter.org/github/egourgoulhon/SageMathTour/blob/master/Notebooks/plot_tour_3D.ipynb
- Solving equations:
  http://nbviewer.jupyter.org/github/egourgoulhon/SageMathTour/blob/master/Notebooks/solve_tour.ipynb
- Solving differential equations:
  http://nbviewer.jupyter.org/github/egourgoulhon/SageMathTour/blob/master/Notebooks/diff_solve_tour.ipynb

# An example of advanced mathematics with SageMath

`SageManifolds`: extends `SageMath` towards **differential geometry** and **tensor calculus**
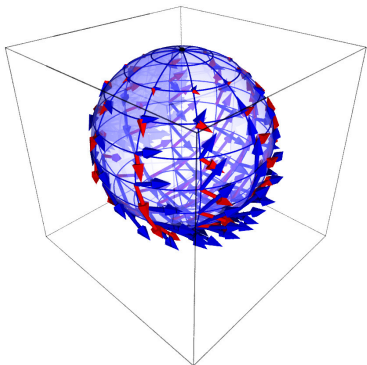


Stereographic-coordinates frame on $\mathbb{S}^2$

- <http://sagemanifolds.obspm.fr/>
- $\sim$ 75,000 lines of Python code (including comments and doctests)
- submitted to `SageMath` community as a sequence of 31 tickets
  cf. list at <https://trac.sagemath.org/ticket/18528>
  - $\rightarrow$ first ticket accepted in March 2015, the 31th one in Jan 2018
- a dozen of contributors (developers and reviewers)
  cf. <http://sagemanifolds.obspm.fr/authors.html>

`SageManifolds 1.1` released on 7 Dec. 2017 and fully included in `SageMath 8.1`