Numerical approach to the Cauchy problem: recent developments

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Introduction

Historical context

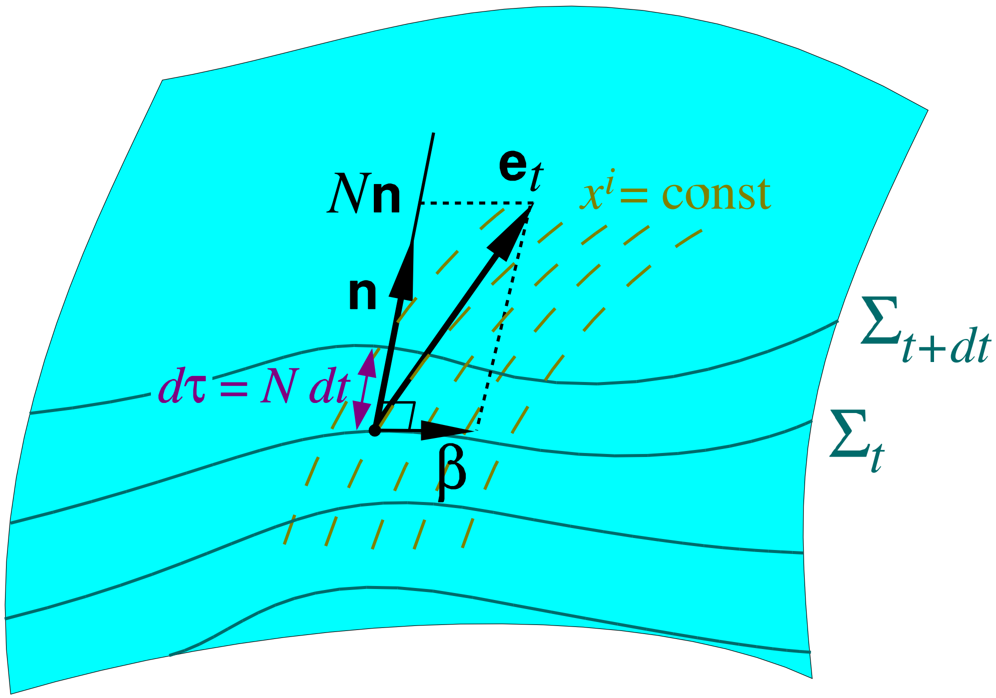
- **Darmois (1927), Lichnerowicz (1939):** Cauchy problem for *analytic* initial data
- **Lichnerowicz (1944):** First 3+1 formalism, conformal decomposition of spatial metric
- **Fourès-Bruhat (1952):** Cauchy problem for C^5 initial data: local existence and uniqueness in harmonic coordinates
- **Fourès-Bruhat (1956):** 3+1 formalism (moving frame)
- **Arnowitt, Deser & Misner (1962):** 3+1 formalism (Hamiltonian analysis of GR)
- **York (1972):** gravitational dynamical degrees of freedom carried by the conformal spatial metric
- **Ó Murchadha & York (1974):** Conformal transverse-traceless (CTT) method for solving the constraint equations
- **Smarr & York (1978):** Radiation gauge for numerical relativity: elliptic-hyperbolic system with asymptotic TT behavior
- **York (1999):** Conformal thin-sandwich (CTS) method for solving the constraint equations

Historical context (numerical relativity)

- **Smarr (1977):** 2-D (axisymmetric) head-on collision of two black holes: first numerical solution beyond spherical symmetry of the Cauchy problem for asymptotically flat spacetimes
- **Nakamura (1983), Stark & Piran (1985):** 2-D (axisymmetric) gravitational collapse to a black hole
- **Bona & Masso (1989), Choquet-Bruhat & York (1995), Kidder, Scheel & Teukolsky (2001), and many others:** (First-order) (symmetric) hyperbolic formulations of Einstein equations within the 3+1 formalism
- **Shibata & Nakamura (1995), Baumgarte & Shapiro (1999):** BSSN formulation: conformal decomposition of the 3+1 equations and promotion of some connection function as an independent variable
- **Shibata (2000):** 3-D full computation of binary neutron star merger: first full GR 3-D solution of the Cauchy problem of astrophysical interest

3+1 formalism

Foliation of spacetime by a family of spacelike hypersurfaces $(\Sigma_t)_{t\in\mathbb{R}}$; on each hypersurface, pick a coordinate system $(x^i)_{i\in\{1,2,3\}}$
$\Longrightarrow (x^\mu)_{\mu\in\{0,1,2,3\}} = (t, x^1, x^2, x^3)$ = coordinate system on spacetime



$\mathbf{n}$: future directed unit normal to Σ_t :
$\mathbf{n} = -N\,\mathbf{d}t$, N : lapse function
$\mathbf{e}_t = \partial/\partial t$: time vector of the natural basis associated with the coordinates (x^μ)

$$\left.\begin{array}{l} N \text{ : lapse function} \\ \boldsymbol{\beta} \text{ : shift vector} \end{array}\right\} \mathbf{e}_t = N\mathbf{n} + \boldsymbol{\beta}$$

Geometry of the hypersurfaces Σ_t:
– induced metric $\boldsymbol{\gamma} = \mathbf{g} + \mathbf{n}\otimes\mathbf{n}$
– extrinsic curvature : $\mathbf{K} = -\frac{1}{2}\pounds_{\mathbf{n}}\boldsymbol{\gamma}$

$$g_{\mu\nu}\,dx^\mu\,dx^\nu = -N^2\,dt^2 + \gamma_{ij}\,(dx^i + \beta^i dt)\,(dx^j + \beta^j dt)$$

3+1 decomposition of Einstein equation

Orthogonal projection of Einstein equation onto Σ_t and along the normal to Σ_t :

- Hamiltonian constraint: $$R + K^2 - K_{ij}K^{ij} = 16\pi E$$
- Momentum constraint : $$D_j K^{ij} - D^i K = 8\pi J^i$$
- Dynamical equations :

$$\frac{\partial K_{ij}}{\partial t} - \pounds_{\boldsymbol{\beta}} K_{ij} = -D_i D_j N + N \left[R_{ij} - 2K_{ik}K^k{}_j + K K_{ij} + 4\pi((S-E)\gamma_{ij} - 2S_{ij}) \right]$$

$E := \mathbf{T}(\mathbf{n}, \mathbf{n}) = T_{\mu\nu} n^\mu n^\nu$, $\quad J_i := -\gamma_i{}^\mu T_{\mu\nu} n^\nu$, $\quad S_{ij} := \gamma_i{}^\mu \gamma_j{}^\nu T_{\mu\nu}$, $\quad S := S_i{}^i$

D_i : covariant derivative associated with γ, $\quad R_{ij}$: Ricci tensor of D_i, $\quad R := R_i{}^i$

Kinematical relation between γ and $\mathbf{K}$: $$\frac{\partial \gamma^{ij}}{\partial t} + D^i \beta^j + D^j \beta^i = 2N K^{ij}$$

Resolution of Einstein equation $\equiv$ Cauchy problem

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Conformal 3+1 formalism

Conformal metric

York (1972) : **Dynamical degrees of freedom** of the gravitational field carried by the conformal "metric"

$$\hat{\gamma}_{ij} := \gamma^{-1/3} \gamma_{ij} \qquad \text{with } \gamma := \det \gamma_{ij}$$

$$\hat{\gamma}_{ij} = \text{tensor density of weight } -2/3$$

To work with tensor fields only, introduce an *extra structure* on Σ_t: a flat metric **f** such that $\frac{\partial f_{ij}}{\partial t} = 0$ and $\gamma_{ij} \sim f_{ij}$ at spatial infinity (asymptotic flatness)

Define $\boxed{\tilde{\gamma}_{ij} := \Psi^{-4} \gamma_{ij}}$ or $\boxed{\gamma_{ij} =: \Psi^4 \tilde{\gamma}_{ij}}$ with $\boxed{\Psi := \left(\frac{\gamma}{f}\right)^{1/12}}$, $f := \det f_{ij}$

$\tilde{\gamma}_{ij}$ is invariant under any conformal transformation of γ_{ij} and verifies $\det \tilde{\gamma}_{ij} = f$

Notations:
- $\tilde{\gamma}^{ij}$: inverse conformal metric : $\tilde{\gamma}_{ik} \tilde{\gamma}^{kj} = \delta_i^{\ j}$
- $\tilde{D}_i$: covariant derivative associated with $\tilde{\gamma}_{ij}$, $\tilde{D}^i := \tilde{\gamma}^{ij} \tilde{D}_j$
- $\mathcal{D}_i$: covariant derivative associated with f_{ij}, $\mathcal{D}^i := f^{ij} \mathcal{D}_j$

Conformal decomposition

Relation between the Ricci tensor **R** of γ at the Ricci tensor $\tilde{\mathbf{R}}$ of $\tilde{\gamma}$:

$$R_{ij} = \tilde{R}_{ij} - 2\tilde{D}_i\tilde{D}_j \ln\Psi + 4\tilde{D}_i \ln\Psi \, \tilde{D}_j \ln\Psi - 2\left(\tilde{D}^k\tilde{D}_k \ln\Psi + 2\tilde{D}_k \ln\Psi \, \tilde{D}^k \ln\Psi\right) \tilde{\gamma}_{ij}$$

Trace : $R = \Psi^{-4}\left(\tilde{R} - 8\tilde{D}_k\tilde{D}^k \ln\Psi - 8\tilde{D}_k \ln\Psi \, \tilde{D}^k \ln\Psi\right)$

Conformal representation of the traceless part of the extrinsic curvature:

$$A^{ij} := \Psi^4\left(K^{ij} - \frac{1}{3}K\gamma^{ij}\right)$$

Indices lowered with the conformal metric: $A_{ij} := \tilde{\gamma}_{ik}\tilde{\gamma}_{jl}A^{kl} = \Psi^{-4}\left(K_{ij} - \frac{1}{3}K\gamma_{ij}\right)$

Conformal decomposition of Einstein equations

Hamiltonian constraint $\longrightarrow$ $\tilde{D}_i\tilde{D}^i\Psi = \frac{\Psi}{8}\tilde{R} - \Psi^5\left(2\pi E + \frac{1}{8}A_{ij}A^{ij} - \frac{K^2}{12}\right)$

Momentum constraint $\longrightarrow$ $\tilde{D}_j A^{ij} + 6A^{ij}\tilde{D}_j \ln\Psi - \frac{2}{3}\tilde{D}^i K = 8\pi\Psi^4 J^i$

Trace of the evolution equation for **K** $\longrightarrow$

$$\frac{\partial K}{\partial t} - \beta^i\tilde{D}_i K = -\Psi^{-4}\left(\tilde{D}_i\tilde{D}^i N + 2\tilde{D}_i \ln\Psi\, \tilde{D}^i N\right) + N\left[4\pi(E+S) + A_{ij}A^{ij} + \frac{K^2}{3}\right],$$

combined with the Hamiltonian constr. $\longrightarrow$ equation for $\boxed{Q := \Psi^2 N}$:

$$\begin{aligned}\tilde{D}_i\tilde{D}^i Q \quad &= \quad \Psi^6\left[N\left(4\pi S + \frac{3}{4}A_{ij}A^{ij} + \frac{K^2}{2}\right) - \frac{\partial K}{\partial t} + \beta^i\tilde{D}_i K\right] \\ &\quad +\Psi^2\left[N\left(\frac{1}{4}\tilde{R} + 2\tilde{D}_i \ln\Psi\, \tilde{D}^i \ln\Psi\right) + 2\tilde{D}_i \ln\Psi\, \tilde{D}^i N\right]\end{aligned}$$

Conformal decomposition of Einstein equations (con't)

Traceless part of the evolution equation for **K** $\longrightarrow$

$$
\begin{aligned}
\frac{\partial A^{ij}}{\partial t} \quad - \quad & \pounds_{\boldsymbol{\beta}} A^{ij} - \frac{2}{3}\tilde{D}_k\beta^k\, A^{ij} = -\Psi^{-6}\left(\tilde{D}^i\tilde{D}^j Q - \frac{1}{3}\tilde{D}_k\tilde{D}^k Q\,\tilde{\gamma}^{ij}\right) \\
& +\Psi^{-4}\Bigg\{ N\left(\tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\tilde{R}_{kl} + 8\tilde{D}^i\ln\Psi\,\tilde{D}^j\ln\Psi\right) + 4\left(\tilde{D}^i\ln\Psi\,\tilde{D}^j N + \tilde{D}^j\ln\Psi\,\tilde{D}^i N\right) \\
& -\frac{1}{3}\left[N\left(\tilde{R} + 8\tilde{D}_k\ln\Psi\tilde{D}^k\ln\Psi\right) + 8\tilde{D}_k\ln\Psi\tilde{D}^k N\right]\,\tilde{\gamma}^{ij}\Bigg\} \\
& +N\left[K A^{ij} + 2\tilde{\gamma}_{kl}A^{ik}A^{jl} - 8\pi\left(\Psi^4 S^{ij} - \frac{1}{3}S\tilde{\gamma}^{ij}\right)\right] \qquad (1)
\end{aligned}
$$

Introduction of the metric potential h

Define **h** as the twice contravariant tensor such that $\boxed{\tilde{\gamma}^{ij} = f^{ij} + h^{ij}}$

h measures the failure of the spatial metric γ from being conformally flat:
$\gamma^{ij} = \Psi^{-4}(f^{ij} + h^{ij})$

Relation between the extrinsic curvature and the time derivative of the metric:

$$\frac{\partial \gamma^{ij}}{\partial t} + D^i \beta^j + D^j \beta^i = 2NK^{ij}$$

- trace part $\longrightarrow$ $\displaystyle \frac{\partial \Psi}{\partial t} = \beta^i \tilde{D}_i \Psi + \frac{\Psi}{6}\left(\tilde{D}_i \beta^i - NK\right)$
- traceless part $\longrightarrow$ $\displaystyle \frac{\partial h^{ij}}{\partial t} = 2NA^{ij} - (\tilde{L}\beta)^{ij}$

with the conformal Killing operator acting on the shift vector being defined as

$$(\tilde{L}\beta)^{ij} := \tilde{D}^j \beta^i + \tilde{D}^i \beta^j - \frac{2}{3} \tilde{D}_k \beta^k \, \tilde{\gamma}^{ij}$$

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Dirac gauge

Expressing the Ricci tensor of conformal metric as a second order operator

Investigate the second order derivatives of conformal metric $\tilde{\gamma}$ in Equation (1):

in terms of the covariant derivative $\mathcal{D}_i$ associated with the flat metric $\mathbf{f}$:

$$\tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\tilde{R}_{kl} = \frac{1}{2}\left(\tilde{\gamma}^{kl}\mathcal{D}_k\mathcal{D}_l h^{ij} - \tilde{\gamma}^{ik}\mathcal{D}_k H^j - \tilde{\gamma}^{jk}\mathcal{D}_k H^i\right) + \mathcal{Q}(\tilde{\boldsymbol{\gamma}}, \boldsymbol{\mathcal{D}}\tilde{\boldsymbol{\gamma}})$$

with $\boxed{H^i := \mathcal{D}_j h^{ij} = \mathcal{D}_j\tilde{\gamma}^{ij} = -\tilde{\gamma}^{kl}\Delta^i{}_{kl} = -\tilde{\gamma}^{kl}(\tilde{\Gamma}^i{}_{kl} - \bar{\Gamma}^i{}_{kl})}$

and $\mathcal{Q}(\mathbf{h}, \boldsymbol{\mathcal{D}}\mathbf{h})$ is quadratic in first order derivatives $\boldsymbol{\mathcal{D}}\mathbf{h}$

Dirac gauge: $\boxed{H^i = 0}$ $\Longrightarrow$ Ricci tensor becomes an elliptic operator for h^{ij}

Similar property as harmonic coordinates for the 4-dimensional Ricci tensor:

$${}^4R_{\alpha\beta} = -\frac{1}{2}g^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}g_{\alpha\beta} + \text{quadratic terms}$$

Dirac gauge: discussion

- introduced by Dirac (1959) in order to fix the coordinates in some Hamiltonian formulation of general relativity

- originally defined for Cartesian type coordinates only: $\frac{\partial}{\partial x^j}\left(\gamma^{1/3}\gamma^{ij}\right) = 0$
  but has been extended by us to more general type of coordinates (e.g. spherical) thanks to the introduction of the flat metric **f**: $\mathcal{D}_j\left[\left(\frac{\gamma}{f}\right)^{1/3}\gamma^{ij}\right] = 0$

- first discussed in the context of numerical relativity by Smarr & York (1978), as a candidate for a radiation gauge, but disregarded for not being covariant under coordinate transformation $(x^i) \mapsto (x^{i'})$ in the hypersurface Σ_t, contrary to the *minimal distortion gauge* proposed by them

Dirac gauge: discussion (con't)

- fully specifies (up to some boundary conditions) the coordinates in $\Sigma_t \Rightarrow$ allows for the search for stationary solutions

- leads asymptotically to transverse-traceless (TT) coordinates (same as minimal distortion gauge). Both gauges are analogous to Coulomb gauge in electrodynamics

- makes the principal linear part of the space-space part of Einstein equations a wave operator for h^{ij}

- is equivalent to choose the connection function Γ^i of BSSN formalism to vanish identically (since $\Gamma^i = -H^i$)

- results in a vector elliptic equation for the shift vector $\boldsymbol{\beta}$

Maximal slicing + Dirac gauge

Our choice of coordinates to solve numerically the Cauchy problem:

- choice of Σ_t foliation: **maximal slicing:** $K := \mathrm{tr}\,\mathbf{K} = 0$
- choice of (x^i) coordinates within Σ_t: **Dirac gauge:** $\mathcal{D}_j h^{ij} = 0$

Note: the Cauchy problem has been shown to be locally strongly well posed for a similar coordinate system, namely *constant mean curvature ($K = t$)* and *spatial harmonic coordinates* ($\mathcal{D}_j \left[(\gamma/f)^{1/2} \gamma^{ij} \right] = 0$) [Andersson & Moncrief, Ann. Henri Poincaré **4**, 1 (2003)]

3+1 equations in maximal slicing + Dirac gauge

[Bonazzola, Gourgoulhon, Grandclément & Novak, gr-qc/0307082]

- 5 elliptic equations (4 constraints + $K = 0$ condition) ($\underline{\Delta} := \mathcal{D}_k \mathcal{D}^k$ = flat Laplacian):

$$\underline{\Delta} N = \Psi^4 N \left[4\pi(E + S) + A_{kl} A^{kl} \right] - h^{kl} \mathcal{D}_k \mathcal{D}_l N - 2 \tilde{D}_k \ln \Psi \, \tilde{D}^k N$$

$$\begin{aligned} \underline{\Delta} Q \quad = \quad & \Psi^4 Q \left(4\pi S + \frac{3}{4} A_{kl} A^{kl} \right) - h^{kl} \mathcal{D}_k \mathcal{D}_l Q + \Psi^2 \left[N \left(\frac{1}{16} \tilde{\gamma}^{kl} \mathcal{D}_k h^{ij} \mathcal{D}_l \tilde{\gamma}_{ij} \right. \right. \\ & \left. \left. - \frac{1}{8} \tilde{\gamma}^{kl} \mathcal{D}_k h^{ij} \mathcal{D}_j \tilde{\gamma}_{il} + 2 \tilde{D}_k \ln \Psi \, \tilde{D}^k \ln \Psi \right) + 2 \tilde{D}_k \ln \Psi \, \tilde{D}^k N \right] . \end{aligned}$$

$$\begin{aligned} \underline{\Delta} \beta^i + \frac{1}{3} \mathcal{D}^i \left(\mathcal{D}_j \beta^j \right) \quad = \quad & 2 A^{ij} \mathcal{D}_j N + 16\pi N \Psi^4 J^i - 12 N A^{ij} \mathcal{D}_j \ln \Psi - 2 \Delta^i{}_{kl} N A^{kl} \\ & - h^{kl} \mathcal{D}_k \mathcal{D}_l \beta^i - \frac{1}{3} h^{ik} \mathcal{D}_k \mathcal{D}_l \beta^l \end{aligned}$$

3+1 equations in maximal slicing + Dirac gauge (con't)

- Evolution equation for h^{ij}:

$$\frac{\partial^2 h^{ij}}{\partial t^2} - \frac{N^2}{\Psi^4}\underline{\Delta} h^{ij} - 2\pounds_{\boldsymbol{\beta}}\frac{\partial h^{ij}}{\partial t} + \pounds_{\boldsymbol{\beta}}\pounds_{\boldsymbol{\beta}} h^{ij} = 2N\mathcal{S}^{ij} + \pounds_{\dot{\boldsymbol{\beta}}} h^{ij} + \frac{4}{3}\mathcal{D}_k\beta^k\left(\frac{\partial}{\partial t} - \pounds_{\beta}\right)h^{ij}$$

$$+ 2\frac{A^{ij}}{N}\left(\frac{\partial}{\partial t} - \pounds_{\beta}\right)N - N\Psi^{-6}\left(\mathcal{D}^i h^{kj} + \mathcal{D}^j h^{ik} - \mathcal{D}^k h^{ij}\right)\mathcal{D}_k Q$$

$$+ \frac{2}{3}\left[\left(\frac{\partial}{\partial t} - \pounds_{\beta}\right)\mathcal{D}_k\beta^k - \frac{2}{3}(\mathcal{D}_k\beta^k)^2\right]h^{ij} - \left(\frac{\partial}{\partial t} - \pounds_{\beta}\right)(L\beta)^{ij} + \frac{2}{3}\mathcal{D}_k\beta^k(L\beta)^{ij}$$

with $\mathcal{S}^{ij}$ containing only quadratic terms in $\mathcal{D}_k h^{ij}$, thanks to Dirac gauge and maximal slicing.

Resolution of the tensor wave equation

Rewrite the evolution equation for h^{ij} as

$$\frac{\partial^2 h^{ij}}{\partial t^2} - \underline{\Delta} h^{ij} = \sigma^{ij}$$

Split h^{ij} into its trace $h := f_{ij} h^{ij}$ and its traceless-transverse (TT) part:

$\bar{h}^{ij} := h^{ij} - \frac{1}{2}\left(h\, f^{ij} - \mathcal{D}^i \mathcal{D}^j \Phi\right)$, with $\underline{\Delta}\Phi = h$.

The TT part of the wave equation is

$$\frac{\partial^2 \bar{h}^{ij}}{\partial t^2} - \underline{\Delta} \bar{h}^{ij} = \bar{\sigma}^{ij}$$

Taking advantage of spherical components

In spherical components, the TT tensor wave equation is reduced to two scalar wave equations:

$$\frac{\partial^2 \chi}{\partial t^2} - \underline{\Delta} \chi = \sigma_\chi$$

$$\frac{\partial^2 \mu}{\partial t^2} - \underline{\Delta} \mu = \sigma_\mu$$

Thanks to its TT character, all the components of $\bar{h}^{ij}$ can be deduced from χ and μ quasi-algebraically. For instance:

$$\begin{aligned}
\bar{h}^{\hat{r}\hat{r}} &= \frac{\chi}{r^2} \\
\bar{h}^{\hat{r}\hat{\theta}} &= \frac{1}{r}\left(\frac{\partial \eta}{\partial \theta} - \frac{1}{\sin\theta}\frac{\partial \mu}{\partial \phi}\right) \\
\bar{h}^{\hat{r}\hat{\phi}} &= \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial \eta}{\partial \phi} + \frac{\partial \mu}{\partial \theta}\right)
\end{aligned}$$

with $\Delta_{\theta\phi}\eta = -r\left(r\frac{\partial \bar{h}^{\hat{r}\hat{r}}}{\partial r} + 3\bar{h}^{\hat{r}\hat{r}}\right)$

Taking advantage of spherical components (con't)

Once $\bar{h}^{ij}$ and the trace h have been obtained[1], h^{ij} is computed according to

$$h^{ij} = \bar{h}^{ij} + \frac{1}{2}\left(h\, f^{ij} - \mathcal{D}^i\mathcal{D}^j\Phi\right)$$

and automatically fulfills the Dirac gauge condition:

$$\mathcal{D}_j h^{ij} = 0$$

[1] the trace h can be get iteratively from the unit determinant condition of $\tilde{\gamma}^{ij}$

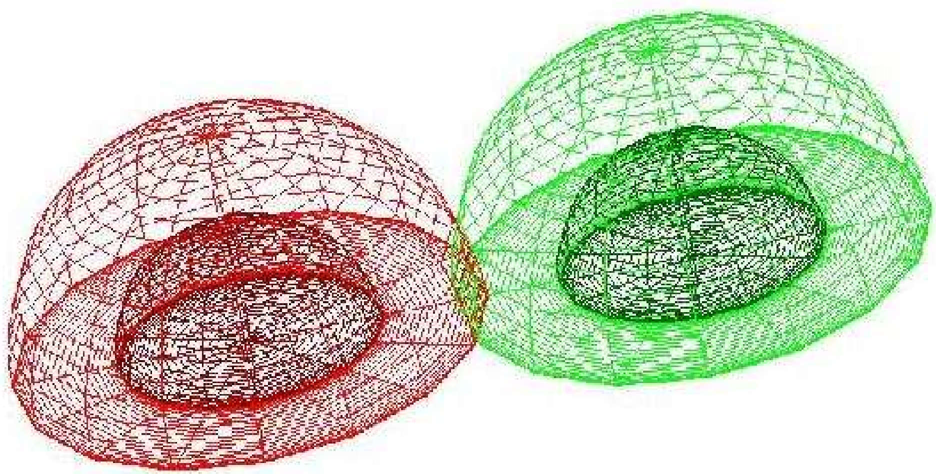
4

Numerical implementation

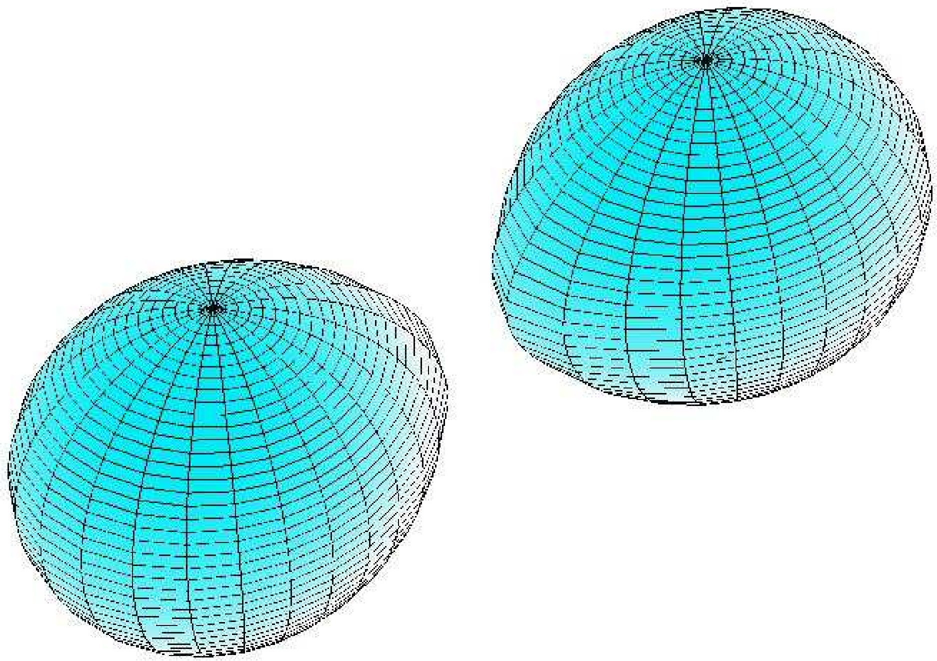
An overview of the numerical techniques employed in Meudon

- Multidomain three-dimensional spectral method
- Spherical-type coordinates (r, θ, φ)
- Expansion functions: r : Chebyshev; θ : cosine/sine or associated Legendre functions; φ : Fourier
- Domains = spherical shells + 1 nucleus (contains $r = 0$)
- Entire space ($\mathbb{R}^3$) covered: compactification of the outermost shell
- Adaptative coordinates : domain decomposition with spherical topology
- Multidomain PDEs: patching method (strong formulation)
- Numerical implementation: C++ codes based on the object oriented library LORENE (http://www.lorene.obspm.fr)

Domain decomposition



Double domain decomposition



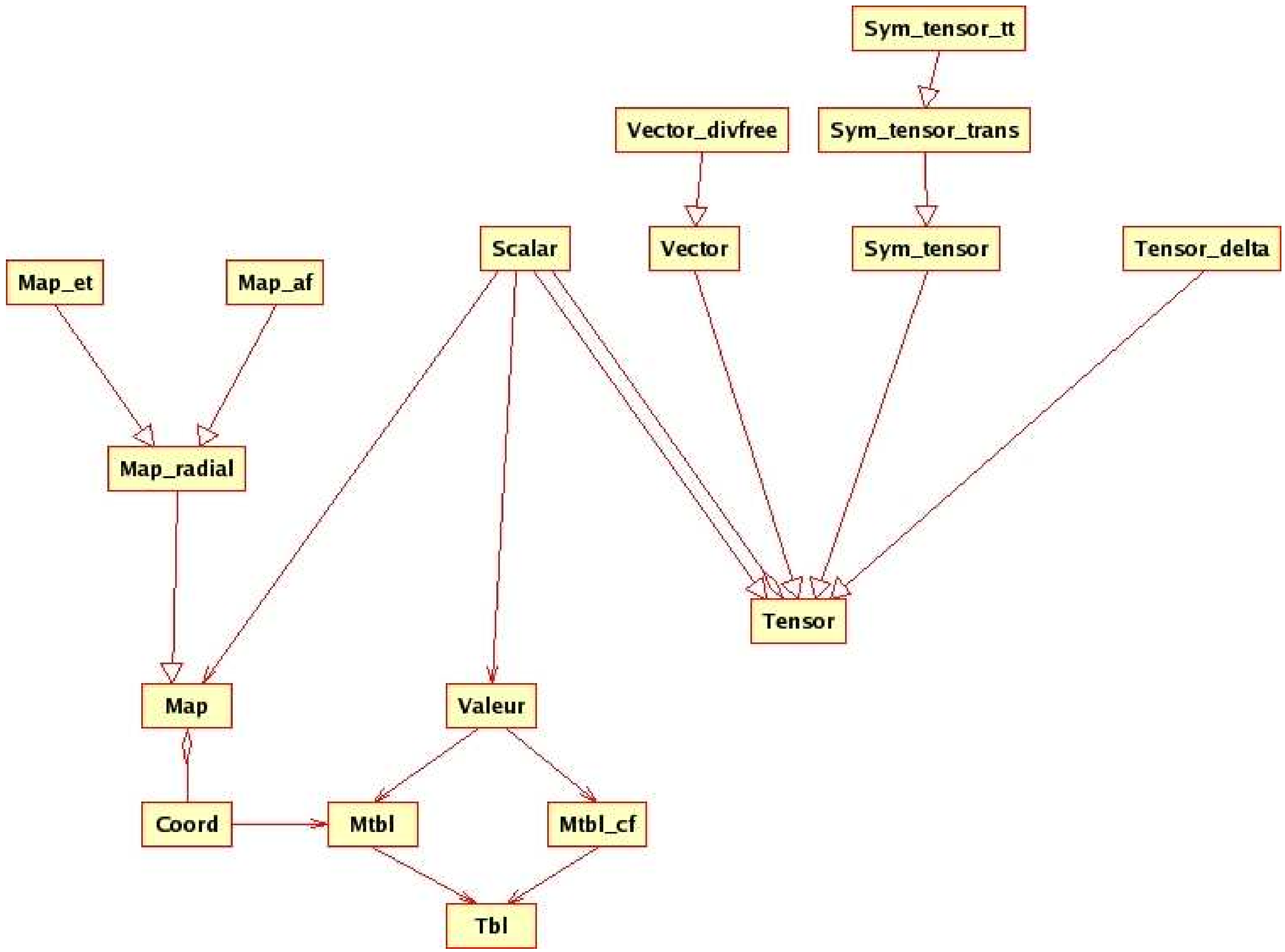
[Taniguchi, Gourgoulhon & Bonazzola, Phys. Rev. D **64**, 064012 (2001)]

Surface fitted coordinates:

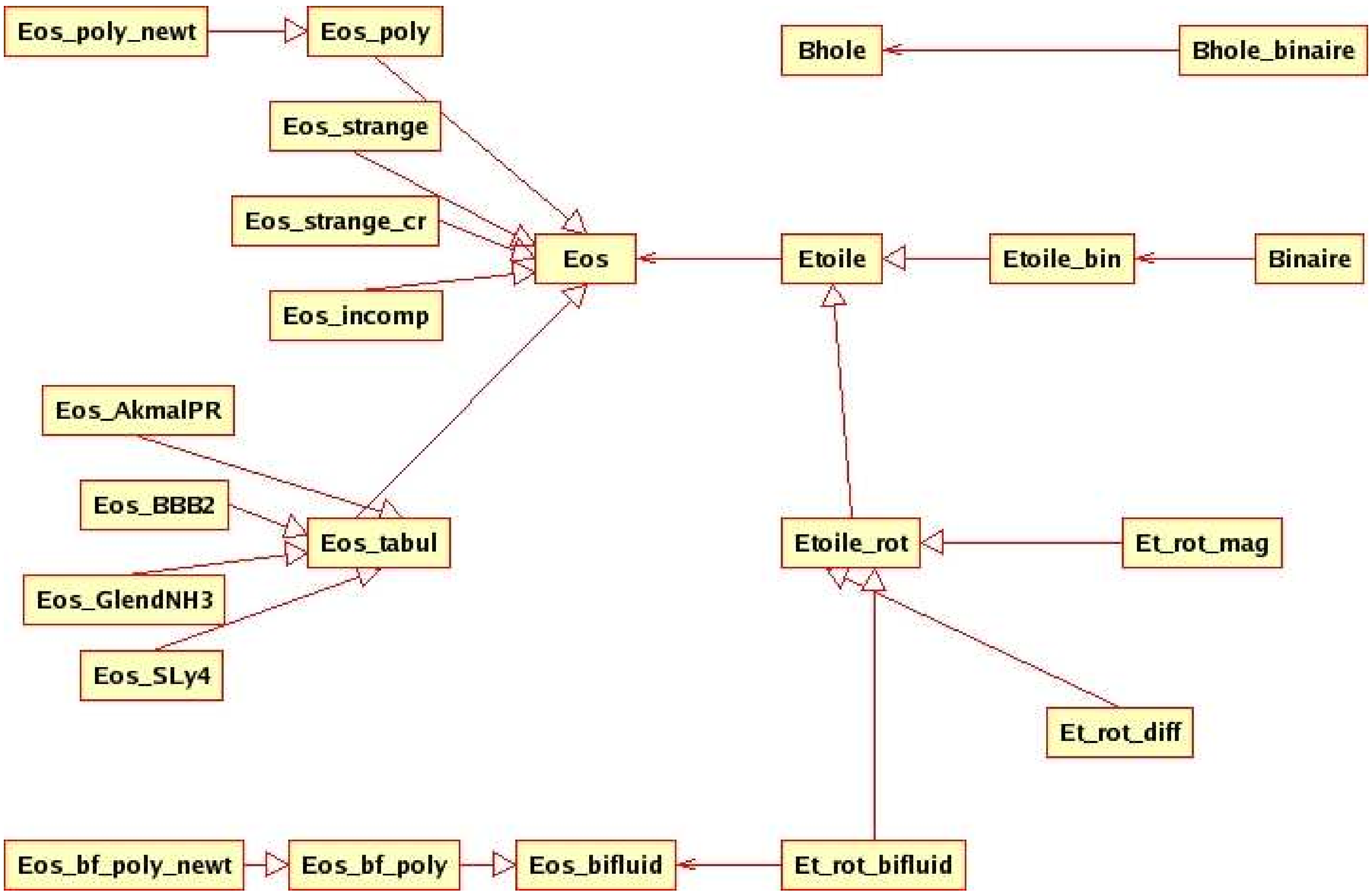
$F_0(\theta, \varphi)$ and $G_0(\theta, \varphi)$ chosen so that

$\xi = 1 \Leftrightarrow$ surface of the star

LORENE C++ classes

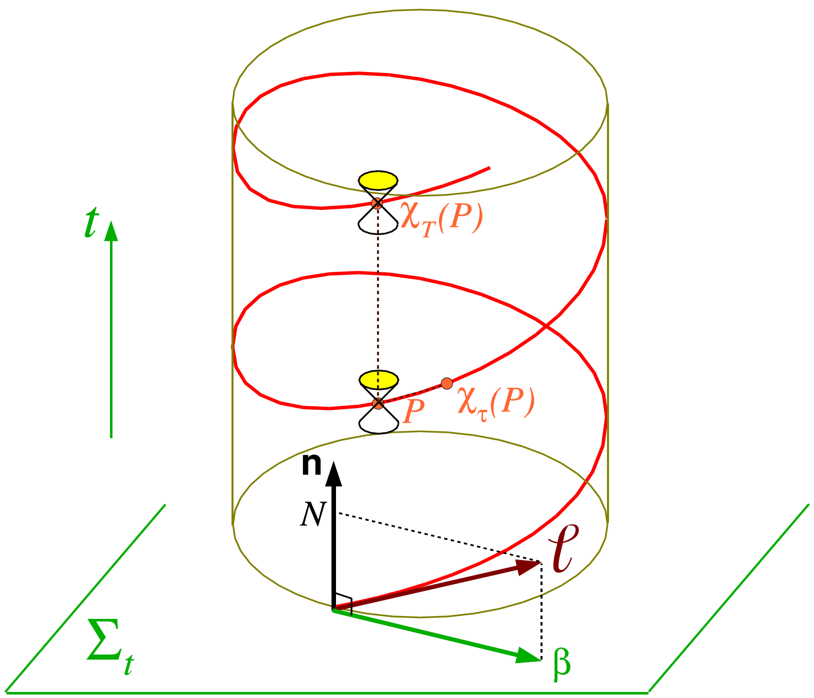


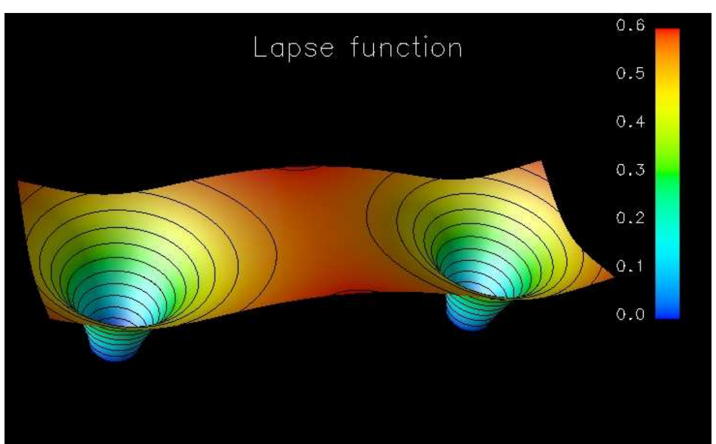
Astrophysical classes



Initial data (Cauchy data)

Quasi-equilibrium sequences of orbiting binary black holes and neutrons stars





Numerical results obtained under the assumption of **helical Killing vector**

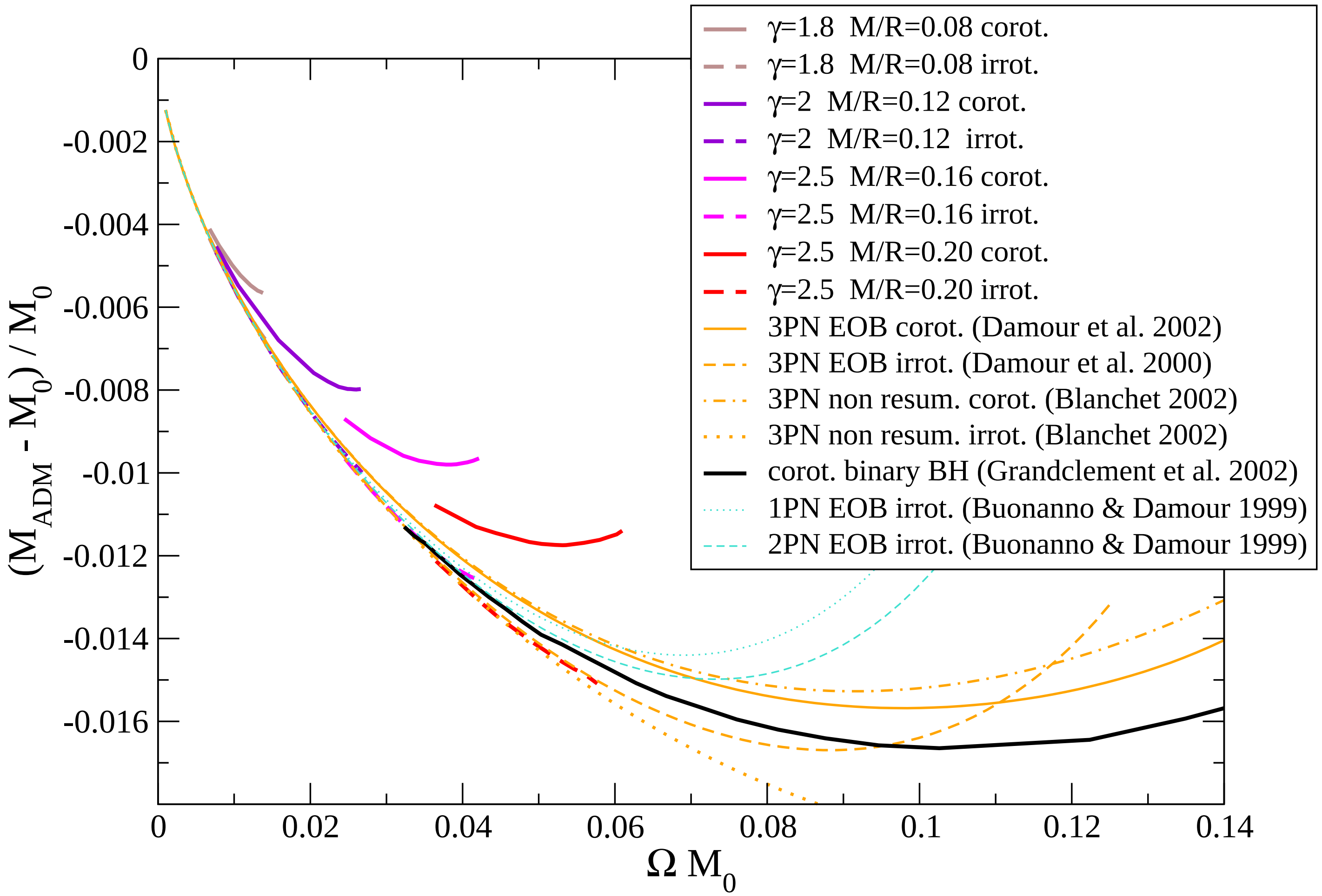
Status:

- **as 4-D spacetimes:** approximate solutions within the Isenberg-Wilson-Mathews waveless approximation of GR [Isenberg (1978), Wilson & Mathews (1989)]

- **as 3-D Cauchy data:** exact (for binary NS) or approximate (within 10^{-3}) (for binary BH) solutions of the constraints

⟵ [Grandclément, Gourgoulhon, Bonazzola, PRD **65**, 044021 (2002)]

Initial data: quasi-equilibrium sequences of binary NS and BH



← First good agreement between numerical orbiting binary black holes sequences and post-Newtonian ones

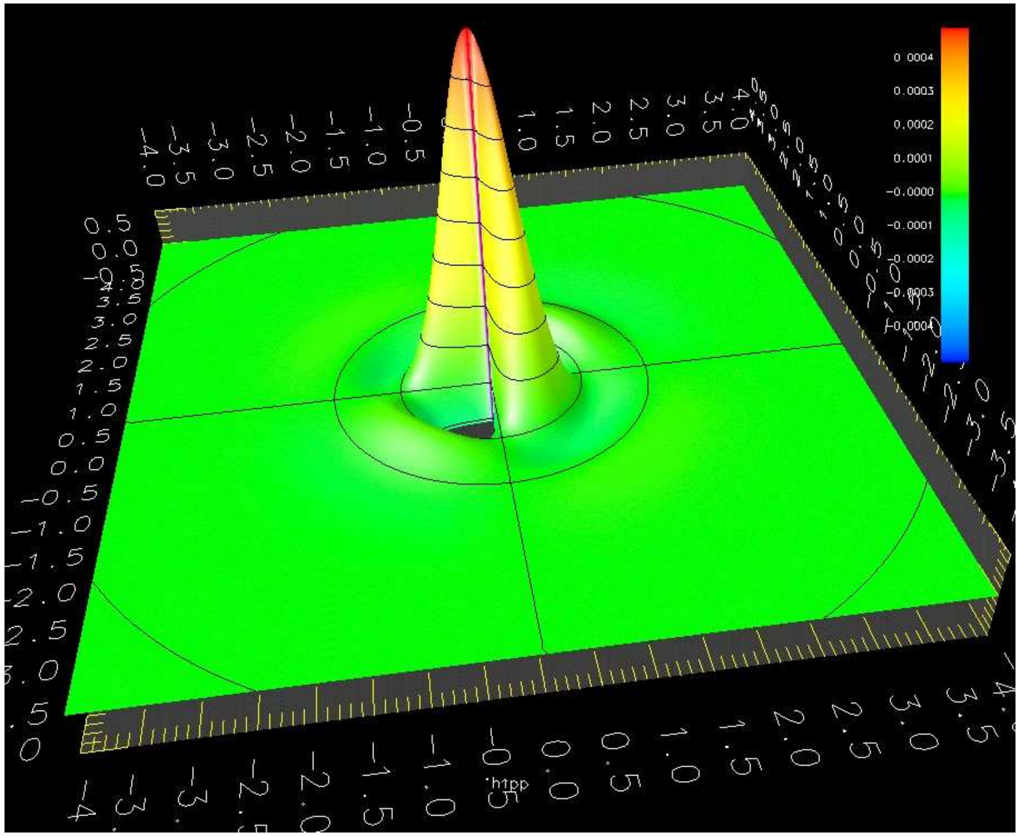
[Grandclément, Gourgoulhon, Bonazzola, PRD **65**, 044021 (2002)]

[Damour, Gourgoulhon & Grandclément PRD **66**, 024007 (2002)]

[Taniguchi & Gourgoulhon, PRD **68**, 124025 (2003)]

Time evolution within Maximal slicing and Dirac gauge

Preliminary results: pure gravitational wave evolution



Evolution of $h^{\hat{\phi}\hat{\phi}}$ in the plane $z = 0$ starting from a small wave packet $\ell = 2,\ m = 2$.

Performant outgoing wave boundary condition developed by [Novak & Bonazzola, JCP, in press, gr-qc/0203102]

Conclusions

- 3+1 Einstein equations within maximal slicing and Dirac gauge leads to an elliptic-hyperbolic system

- Thanks to the use of spherical coordinates (and tensor components), the integration scheme is a fully constrained one $\Longrightarrow$ better stability expected than with free evolution schemes

- Only two wave equations are to be solved, corresponding to the two degrees of freedom of the gravitational field

- The scalar and vector elliptic equations can be solved efficiently by means of spectral methods

- Maximal slicing + Dirac gauge coordinates = asymptotically TT coordinates $\Longrightarrow$ promising capabilities w.r.t. gravitational wave extraction