Detecting bodies orbiting the Galactic Center black hole Sgr A* with LISA

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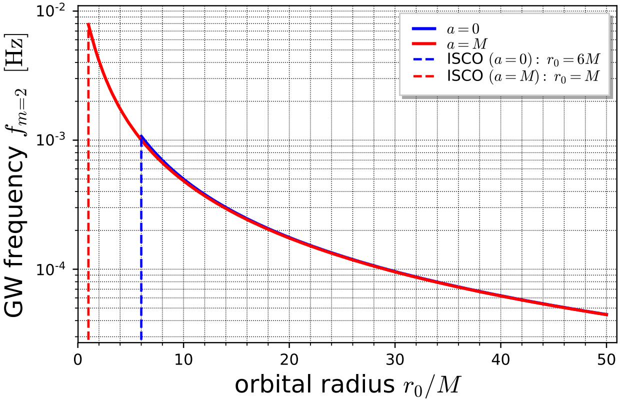
based on A&A **627**, A92 (2019) [arXiv:1903.02049]

Journée LISA de l'Observatoire de Paris
10 December 2021

GW frequencies from circular orbits around Sgr A*



Angular velocity of circular equatorial orbits around a Kerr BH

$$\omega_0 = \frac{M^{1/2}}{r_0^{3/2} + aM^{1/2}}$$

Dominant GW frequency

$$f_{m=2} = 2f_0 = \frac{\omega_0}{\pi}$$

Sgr A* mass

$$\begin{aligned} M &= 4.10 \times 10^6\, M_\odot \\ &= 20.2\ \mathrm{s} \end{aligned}$$

[Gravity team, A&A **615**, L15 (2018)]

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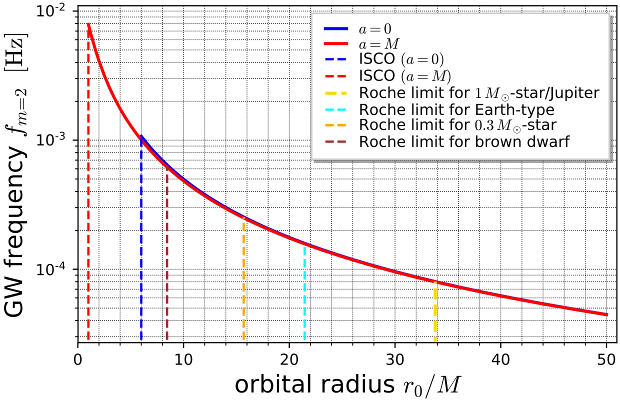

Roche radius: $r_{\rm R} \simeq 1.14 \left(\frac{M}{\rho}\right)^{1/3}$

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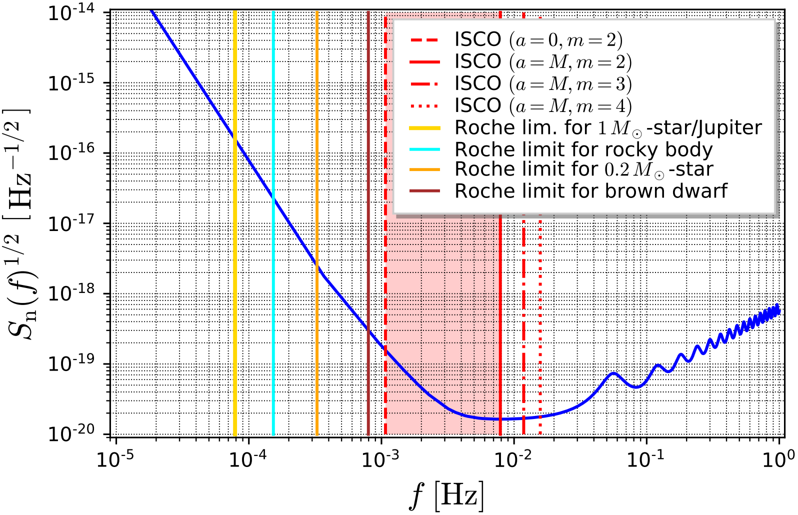
Frequencies of Sgr A* close orbits are in LISA band



ISCO for $a = M$: $f_{m=2} = 7.9$ mHz

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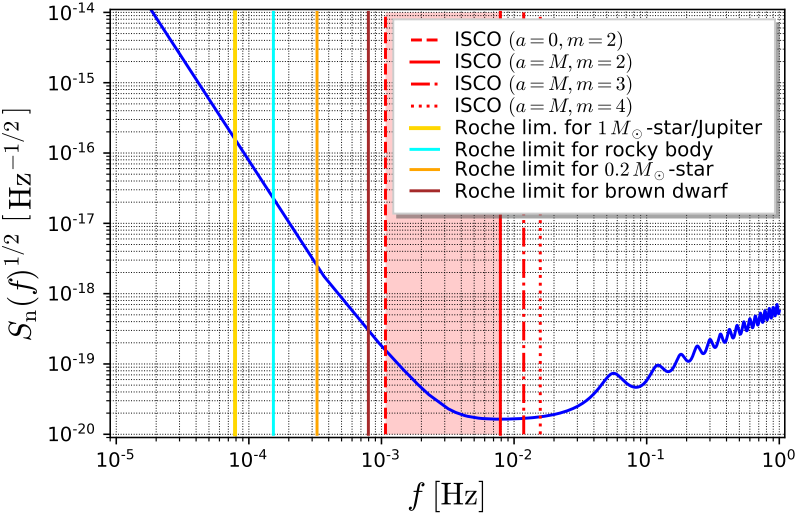

ISCO for $a = M$: $f_{m=2} = 7.9$ mHz ← coincides with LISA max. sensitivity!

Previous studies of Sgr A* as a source for LISA

- Freitag (2003) [ApJ **583**, L21]: GW from orbiting stars at quadrupole order; low-mass main-sequence (MS) stars are good candidates for LISA
- Barack & Cutler (2004) [PRD **69**, 082005]: $0.06 M_\odot$ MS star observed $10^6 \mathrm{yr}$ before plunge $\Longrightarrow$ SNR = 11 in 2 yr of LISA data $\Longrightarrow$ Sgr A*'s spin within 0.5% accuracy
- Berry & Gair (2013) [MNRAS **429**, 589]: extreme-mass-ratio burst (single periastron passage on a highly eccentric orbit) $\Longrightarrow$ GW burst $\Longrightarrow$ LISA detection of $10 M_\odot$ for periastron $< 65M$; event rate could be $\sim 1\,\mathrm{yr}^{-1}$
- Linial & Sari (2017) [MNRAS **469**, 2441]: GW from orbiting MS stars undergoing Roche lobe overflow $\Longrightarrow$ detectability by LISA; possibility of a *reverse chirp signal (outspiral)*
- Amaro-Seoane (2019) [PRD **99**, 123025]: *Extremely Large Mass-Ratio Inspirals (X-MRI)* $\Longrightarrow$ brown dwarfs orbiting Sgr A* should be detected in great numbers by LISA: ~ 20 in band at any time
- Kühnel et al. (2020) [EPJ C **80**, 627]: GW from an ensemble of macroscopic dark matter candidates orbiting Sgr A*, such as primordial BHs, with masses in the range $10^{-13} - 10^3\ M_\odot$

Our study

All previous studies have been performed in a Newtonian framework (quadrupole formula). Now, for orbits close to the ISCO, relativistic effects are expected to be important.

$\Longrightarrow$ we have adopted a fully relativistic framework:

- Sgr A* is modeled as a Kerr BH and GW are computed via the theory of perturbations of the Kerr metric
- tidal effects are evaluated via the theory of Roche potential in the Kerr metric developed by Dai & Blandford (2013) [MNRAS **434**, 2948]

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Limitation: circular equatorial orbits; valid for

- inspiralling compact objects arising from the tidal disruption of a binary *(zero-eccentricity EMRI)*
- main-sequence stars formed in an accretion disk
- compact objects resulting from the most massive of such stars
- $\sim 1/4$ of the population of brown dwarfs studied by Amaro-Seoane (2019)

Waveforms from circular orbits

computed as linear perturbations of Kerr metric (Teukolsky 1973)

Detweiler (1978)

$$h_+ - \mathrm{i} h_\times = \frac{2\mu}{r} \sum_{\ell=2}^{\infty} \sum_{\substack{m=-\ell \\ m\neq 0}}^{\ell} \frac{Z^{\infty}_{\ell m}(r_0)}{(m\omega_0)^2} \, {}_{-2}S^{am\omega_0}_{\ell m}(\theta,\varphi) \, \mathrm{e}^{-\mathrm{i}m(\omega_0(t-r_*)+\varphi_0)}$$

μ: mass of orbiting object; (t, r, θ, φ): Boyer-Lindquist coordinates of the observer

${}_{-2}S^{am\omega_0}_{\ell m}(\theta,\varphi)$: spheroidal harmonics of spin weight -2

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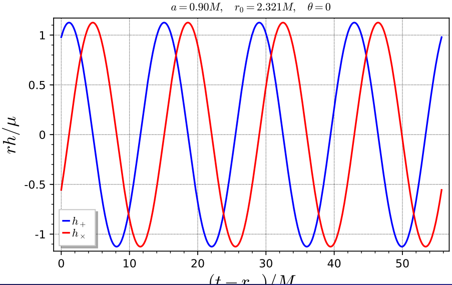
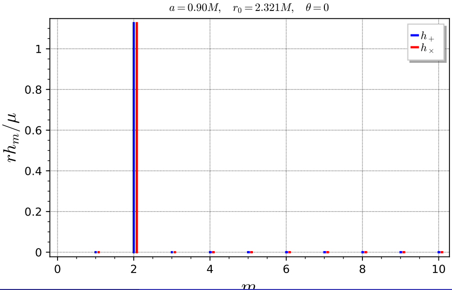
Example for $a = 0.9\,M$, $r_0 = r_{\rm ISCO}(a)$ and viewing angle $\theta = 0$ (face-on)





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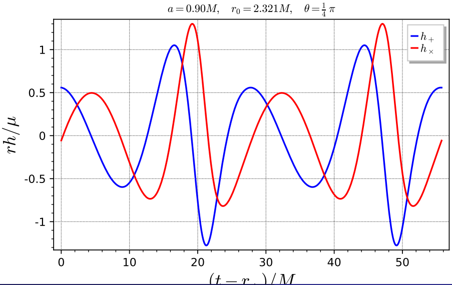
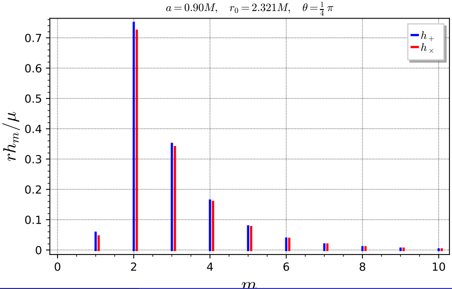
Example for $a = 0.9\,M$, $r_0 = r_{\rm ISCO}(a)$ and viewing angle $\theta = \pi/4$





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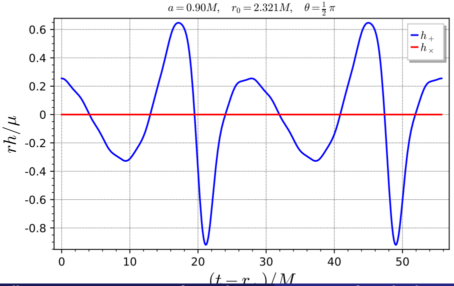
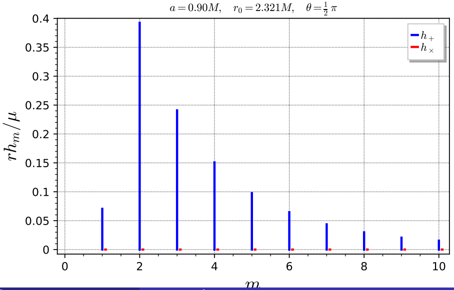
Example for $a = 0.9\,M$, $r_0 = r_{\rm ISCO}(a)$ and viewing angle $\theta = \pi/2$ (edge-on)





Implementation: the kerrgeodesic_gw package

All computations (GW waveforms, SNR in LISA, energy fluxes, inspiralling time, etc.) have been implemented as a Python package for the open-source mathematics software system SageMath:

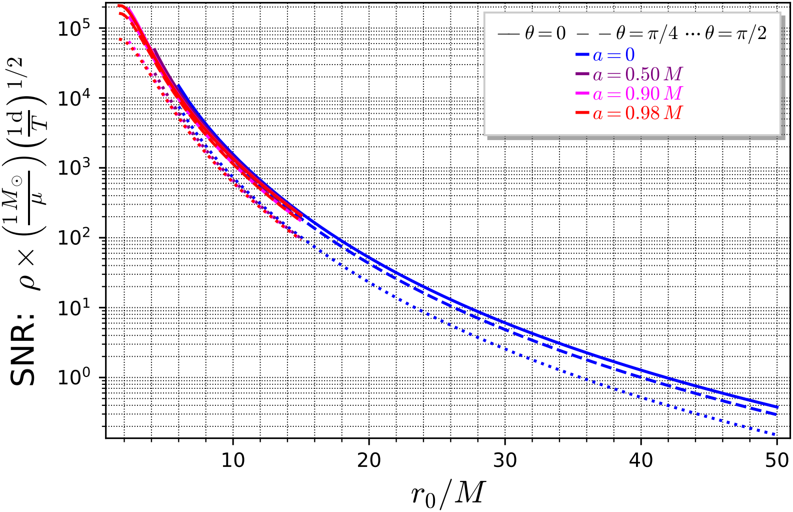
`kerrgeodesic_gw`

`kerrgeodesic_gw` is

- entirely open-source:
  https://github.com/BlackHolePerturbationToolkit/kerrgeodesic_gw
- is distributed via PyPi (Python Package Index):
  https://pypi.org/project/kerrgeodesic-gw/
- is part of the *Black Hole Perturbation Toolkit*:
  http://bhptoolkit.org/

Signal-to-noise ratio in the LISA detector

as a function of the circular orbit radius r_0



[Gourgoulhon, Le Tiec, Vincent & Warburton, A&A **627**, A92 (2019)]

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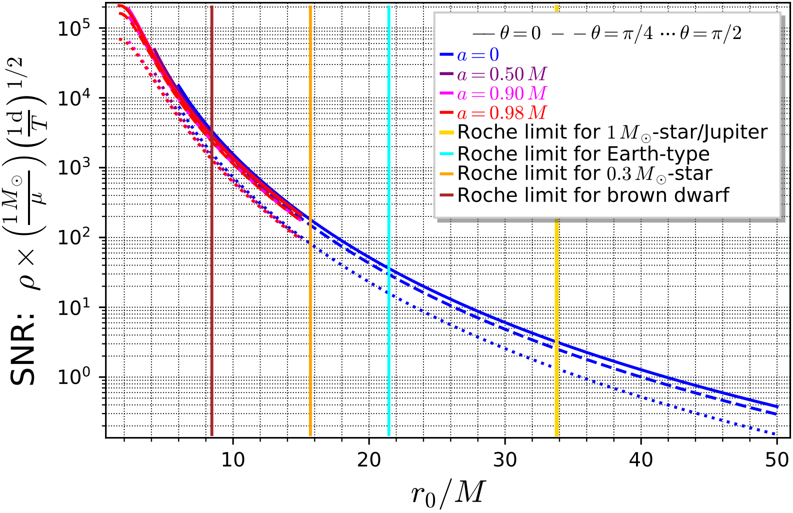

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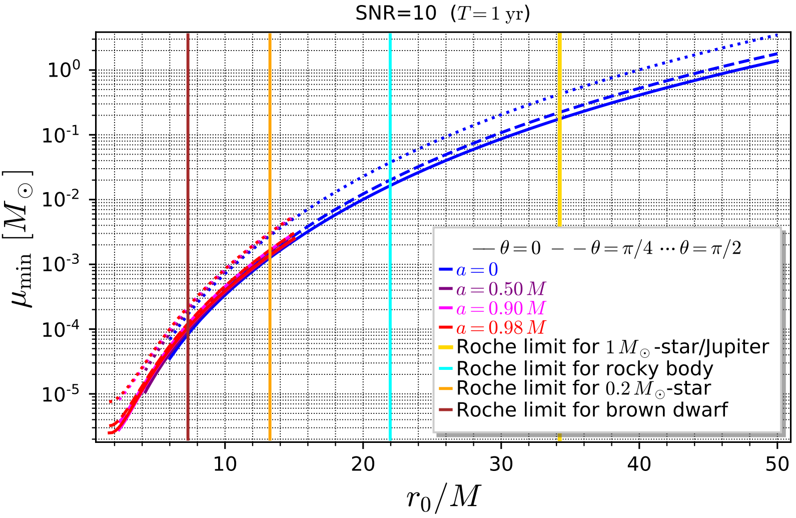
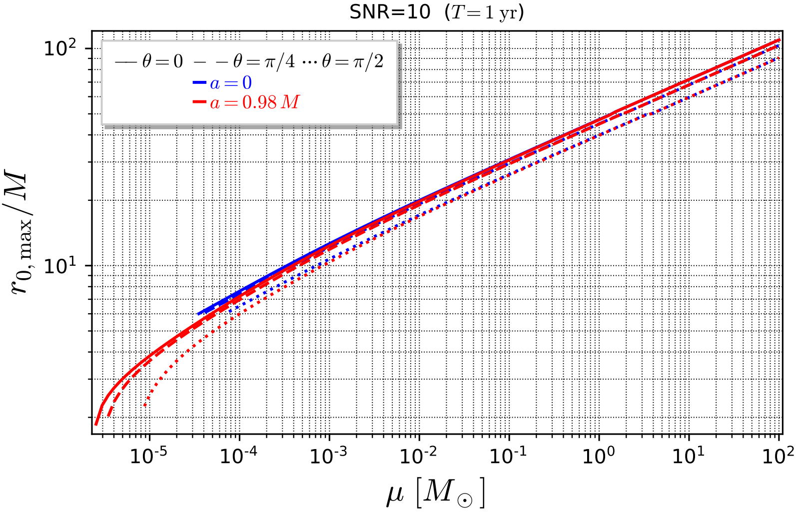
Minimal detectable mass by LISA

Detection criteria: SNR $\geq$ 10
Observation time: $T = 1$ yr



Maximum orbital radius for LISA detection

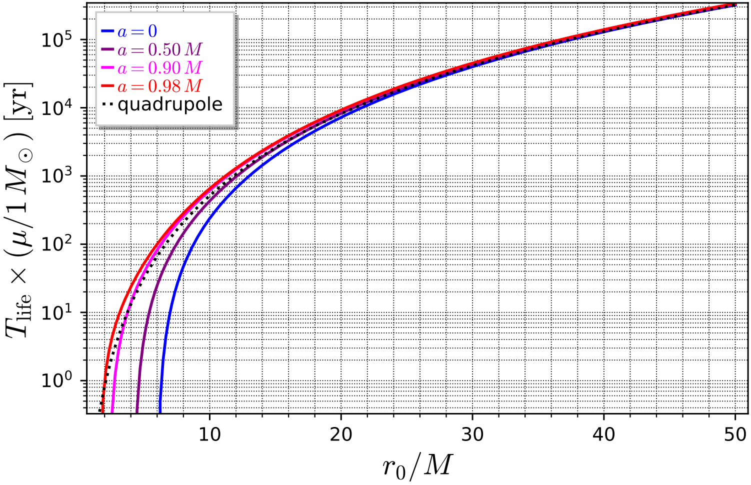
Maximum orbital radius $r_{0,\max}$ for a SNR $= 10$ detection by LISA in one year of data, as a function of the mass μ of the object orbiting around Sgr A*:



Detection probability governed by the life time of orbits

gravitational radiation reaction $\Longrightarrow$ slow inspiral motion

$T_{\rm life}(r_0)$: time for a compact object to reach the ISCO starting from circular orbit of radius r_0



Time spent in LISA band

Inspiral time from orbit r_0 to orbit r_1 due to reaction to gravitational radiation:

$$T_{\rm ins}(r_0, r_1) = \frac{M^2}{2\mu} \int_{r_1/M}^{r_0/M} \frac{1 - 6/x + 8\bar{a}/x^{3/2} - 3\bar{a}^2/x^2}{\left(1 - 3/x + 2\bar{a}/x^{3/2}\right)^{3/2}} \frac{{\rm d}x}{x^2(\tilde{L}_\infty(x) + \tilde{L}_{\rm H}(x))}$$

where $\tilde{L}_{\infty,{\rm H}}(x) := (M/\mu)^2 L_{\infty,{\rm H}}(xM)$ and L_∞ (resp. $L_{\rm H}$) is the total GW power emitted at infinity (resp. through the BH event horizon) by a particle of mass μ orbiting at $r = xM$

Compact object

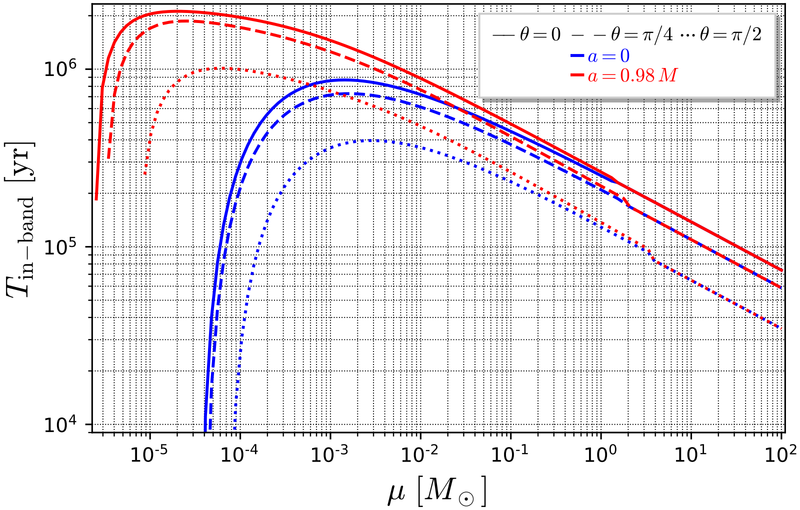
$T_{\text{in-band}} = T_{\rm ins}(r_{0,\rm max}, r_{\rm ISCO}) = T_{\rm life}(r_{0,\rm max})$

Main-sequence stars and brown dwarfs

$T_{\text{in-band}} \geq T^{\rm ins}_{\text{in-band}} = T_{\rm ins}(r_{0,\rm max}, r_{\rm Roche})$

Time in LISA band for an inspiralling compact object

as a function of the compact object mass μ



Time in LISA band for brown dwarfs and main-sequence stars

Results for

- inclination angle $\theta = 0$
- BH spin $a = 0$ (outside parentheses) and $a = 0.98M$ (inside parentheses)

	brown dwarf	red dwarf	Sun-type	$2.4\,M_\odot$-star
$\mu/M_\odot$	0.062	0.20	1	2.40
$\rho/\rho_\odot$	131.	18.8	1	0.367
$r_{0,\max}/M$	28.2 (28.0)	35.0 (34.9)	47.1 (47.0)	55.6 (55.6)
$f_{m=2}(r_{0,\max})$ [mHz]	0.105 (0.106)	0.076 (0.076)	0.049 (0.049)	0.038 (0.038)
$r_{\rm Roche}/M$	7.31 (6.93)	13.3 (13.0)	34.2 (34.1)	47.6 (47.5)
$T^{\rm ins}_{\text{in-band}}$ [10^5 yr]	4.98 (5.55)	3.72 (3.99)	1.83 (1.89)	0.938 (0.945)

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Brown dwarfs stay for $\sim 5 \times 10^5$ yr in LISA band

Conclusions

- GW emission and SNR in LISA for close circular orbits around Sgr A* has been computed in full general relativity.
- The time spent in LISA band (SNR ≥ 10) during the slow inspiral has been evaluated.
- All computations have been implemented in the open-source SageMath package `kerrgeodesic_gw`, as part of the Black Hole Perturbation Toolkit.
- LISA has the capability to detect orbiting masses close to the ISCO as small as $\sim 10 M_{\rm Earth}$ or even $\sim 1 M_{\rm Earth}$ if Sgr A* is a fast rotator ($a \geq 0.9M$); this could involve primordial BHs or (hypothetical) very dense artificial objects.
- The longest times in-band, of the order of 10^6 years, are achieved for primordial black holes of mass $\sim 10^{-3} M_\odot$ down to $10^{-5} M_\odot$ (depending on Sgr A*'s spin), as well as for brown dwarfs, just followed by white dwarfs and low mass main-sequence stars.