Gravitational waves and black holes

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Outline

1. The theoretical framework
    - A brief overview of general relativity
    - Gravitational waves
    - Black holes
2. The current observational status
    - Gravitational waves
    - Black holes
3. The near-future projects
    - Gravitational waves
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4. Tests of gravity
    - The framework
    - The Gyoto tool

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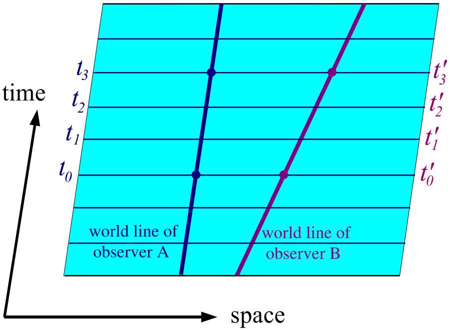
Spacetime



Newtonian spacetime:

math. description: affine space $\mathbb{R}^4$

absolute structure: universal time

All observers measure the same time

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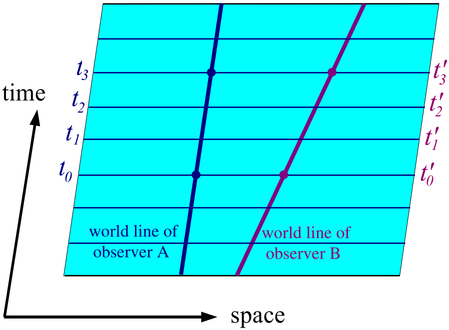

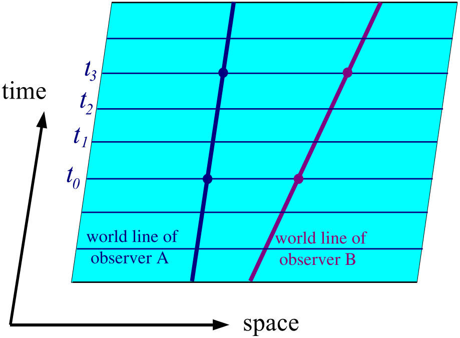

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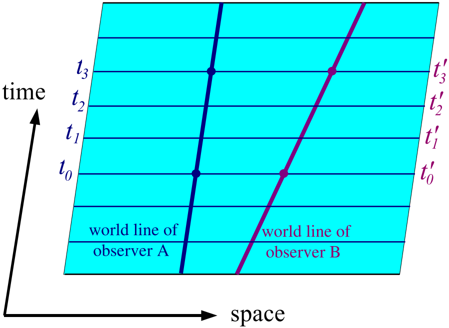

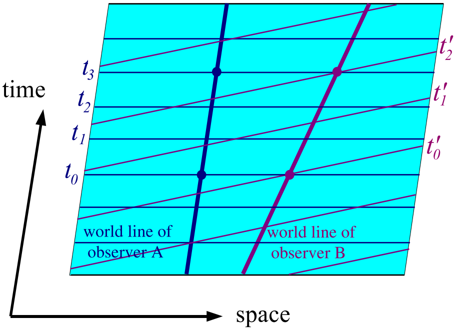

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$\Longrightarrow$ simultaneity is relative
$\Longrightarrow$ *time dilation* phenomenon

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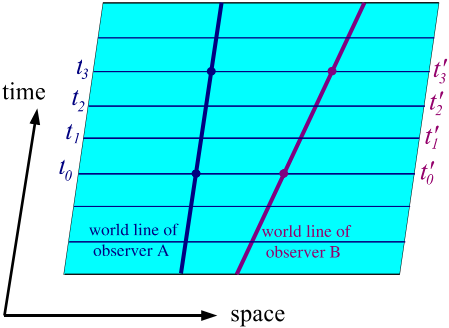

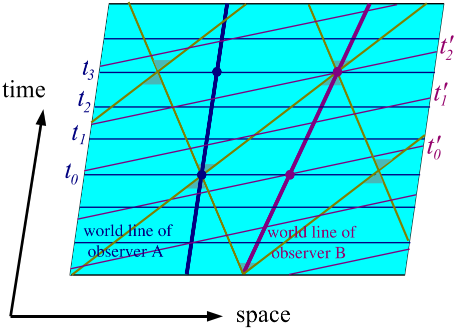

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Special relativity spacetime:

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absolute structure: light cones

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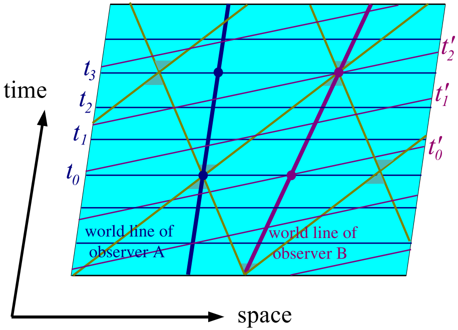
Relativistic spacetime



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- OK for **electromagnetism**
- not satisfactory for **gravitation**

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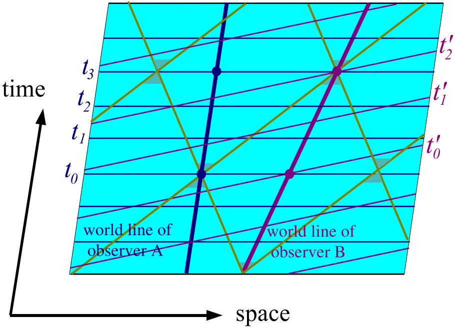

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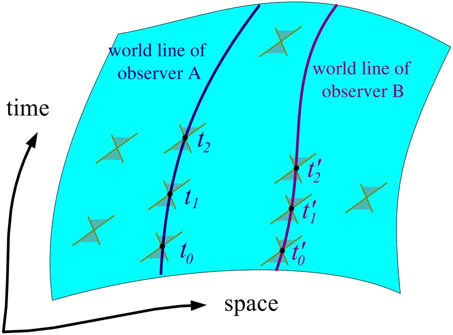


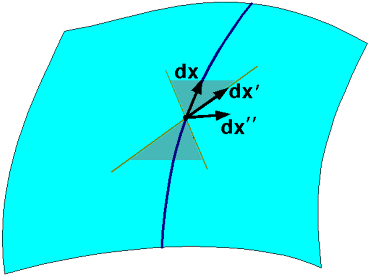
General relativity spacetime:
math. description: 4-dimensional curved space (manifold)

- OK for **electromagnetism**
- OK for **gravitation**

The metric tensor

Algebraic translation of the absolute structure provided by the light cones:

metric tensor $\boldsymbol{g}$



$\boldsymbol{g}$ = symmetric bilinear form of signature $(-,+,+,+)$ such that:

- proper time $d\tau$ for a displacement $d\vec{\boldsymbol{x}}$: $c^2 d\tau^2 = -\boldsymbol{g}(d\vec{\boldsymbol{x}}, d\vec{\boldsymbol{x}}) = -d\vec{\boldsymbol{x}} \cdot d\vec{\boldsymbol{x}}$
- along a light cone : $\boldsymbol{g}(d\vec{\boldsymbol{x}'}, d\vec{\boldsymbol{x}'}) = 0$
- proper distance along a displacement $d\vec{\boldsymbol{x}''}$: $dl^2 = \boldsymbol{g}(d\vec{\boldsymbol{x}''}, d\vec{\boldsymbol{x}''})$

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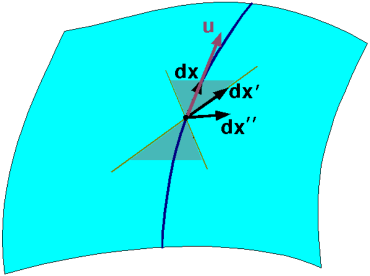

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4-velocity along a world line: $\vec{\boldsymbol{u}} = \dfrac{d\vec{\boldsymbol{x}}}{d\tau}$

NB: $\boldsymbol{g}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{u}}) = -c^2$

Spacetime dynamics

- Special relativity: metric tensor $\boldsymbol{g}$ = fixed bilinear form on the vector space $\sim \mathbb{R}^4$ associated with the spacetime affine space
- General relativity: metric tensor $\boldsymbol{g}$ = field of bilinear forms: $\boldsymbol{g} = \boldsymbol{g}(p)$

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Einstein equation : $\boxed{\boldsymbol{R} - \frac{1}{2} R\, \boldsymbol{g} = \frac{8\pi G}{c^4} \boldsymbol{T}}$

- $\boldsymbol{R}$ = Ricci tensor = symmetric bilinear form = trace of *curvature tensor* (Riemann tensor) : "$\boldsymbol{R} \sim \boldsymbol{g}\, \partial^2 \boldsymbol{g} + \boldsymbol{g}\, \partial \boldsymbol{g}\, \partial \boldsymbol{g}$"
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  - $E = \boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{u}})$ is the energy density of matter as measured by an observer $\mathcal{O}$ of 4-velocity $\vec{\boldsymbol{u}}$
  - $p_i = -\boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{e}}_i)$ component i of the matter momentum density as measured by $\mathcal{O}$ in the direction $\vec{\boldsymbol{e}}_i$
  - $S_{ij} = \boldsymbol{T}(\vec{\boldsymbol{e}}_i, \vec{\boldsymbol{e}}_j)$ component i of the force exerted by matter on the unit surface normal to $\vec{\boldsymbol{e}}_j$

Comparing Newtonian and relativistic gravitation theories

Newtonian gravitation:

fundamental equation: Poisson equation for the gravitational potential Φ:

$$\Delta\Phi = 4\pi G\rho$$

- scalar equation
- linear equation
- elliptic equation
  ($\Rightarrow$ instantaneous propagation)
- only source: mass density ρ

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- tensorial equation (10 scalar equations)
- non-linear equation
- propagation at finite speed (c)
- source: energy-momentum of matter and electromagnetic field

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Remark: for a weak gravitational field, one of the 10 components of Einstein equation reduces to the Poisson equation (and the other 9 reduced to $0 = 0$).

What is a strong gravitational field ?

Relativity parameter or compactness parameter of a self-gravitating body of mass M and mean radius R:

$$\Xi = \frac{GM}{c^2 R} \sim \frac{|E_{\rm grav}|}{Mc^2} \sim \frac{|\Phi_{\rm surf}|}{c^2} \sim \frac{v_{\rm esc}^2}{c^2}$$

- $E_{\rm grav}$: gravitational potential energy[1]
- $\Phi_{\rm surf}$: gravitational potential at the surface of the body
- $v_{\rm esc}$: escape velocity from the body's surface[2]

	Earth	Sun	white dwarf	neutron star	black hole
Ξ	10^{-10}	10^{-6}	10^{-3}	0.2	1

if $\Xi \gtrsim 0.1$, general relativity must be employed to describe the body (**compact object**)

---

[1] for a homogeneous ball: $E_{\rm grav} = -\frac{3}{5}\frac{GM^2}{R}$

[2] for a spherically symmetric body: $v_{\rm esc} = \sqrt{\frac{2GM}{R}}$

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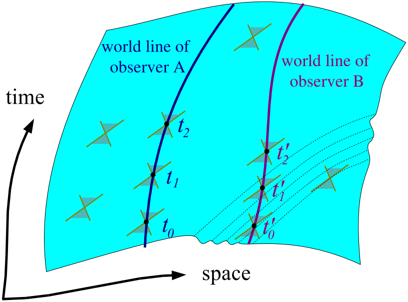
Gravitational waves

Linearization of Einstein equation in weak field: $\boldsymbol{g} = \boldsymbol{\eta} + \boldsymbol{h}$,
$\boldsymbol{\eta}$ = Minkowski metric[3]

$$\Longrightarrow \text{wave equation: } \boxed{\Box \bar{\boldsymbol{h}} = -\frac{16\pi G}{c^4}\boldsymbol{T}} \qquad \text{(Lorenz gauge)}$$

with $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\bar{\boldsymbol{h}} = \boldsymbol{h} - \frac{1}{2} h\, \boldsymbol{\eta}$ and $h = \mathrm{Trace}(\boldsymbol{h})$.



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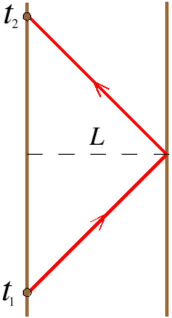
[3] $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ en Cartesian coordinates

Measurable effects of a gravitational wave passage

Measure of the distance L between two free masses by a “radar” method:

$$L = \frac{1}{2}\, c(t_2 - t_1)$$



Variation of length L when a gravitational wave passes by:

$$\boxed{\delta L \simeq h\, L}$$

h = amplitude of the gravitational wave

In practice, h is so small that our senses are not sensitive to it:
for the most important **astrophysical sources:** $h \sim 10^{-21}$!!!

Generation of gravitational waves in the lab

In the 19th Century, Hertz has demonstrated the existence of electromagnetic waves by producing them in his laboratory.
Is the same experiment possible for gravitational waves ?

- electromagnetic waves: produced by the acceleration of *electric charges*
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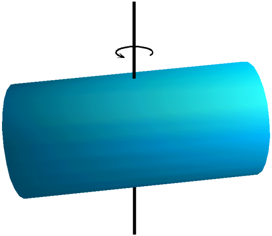
A simple mean of providing a constant acceleration to some material: make it *rotate*



Steel cylinder: diameter = 1 m, length = 20 m, mass = 490 t, rotating at 28 rad/s (break-up limit)
$\Longrightarrow$ emitted energy in gravitational waves per unit of time: 2×10^{-29} W !

$\Longrightarrow$ No hope of detection !

Generation of gravitational waves by astrophysical sources

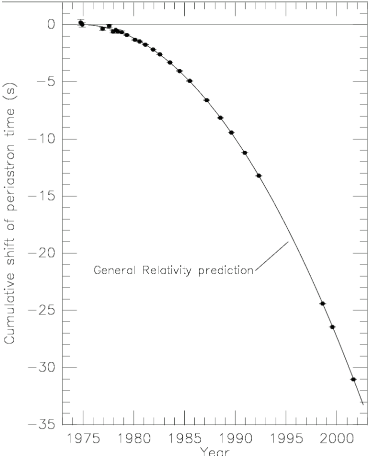
Emitted energy per unit of time in the form of gravitational waves:

$$\boxed{\mathcal{L} \sim \frac{c^5}{G}\, s^2 \left(\frac{v}{c}\right)^6 \boldsymbol{\Xi}^2}$$ gravitational luminosity

- G : Newton's constant → *gravitation*
- c : velocity of light → *relativity*
- s : asymmetry factor: $s = 0$ if spherical symmetry
- v : characteristic speed of motions inside the source
- $\boldsymbol{\Xi}$: *compactness parameter* ◂ reminder or *relativity parameter*

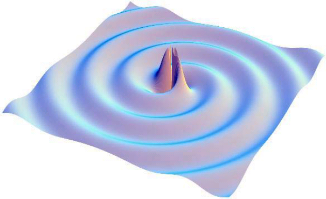
Only compact objects are good emitters of gravitational waves

Gravitational waves do exist !



[Weisber & Taylor (2002)]

Emission of gravitational waves by the neutron star binary system PSR B1913+16 (*binary pulsar*)



← Observed decay of the orbital period $P = 7\,\mathrm{h}\,45\,\mathrm{min}$ of the binary pulsar PSR B1913+16 produced by the *reaction to gravitational radiation*

$\Longrightarrow$ coalescence in 140 millions year.

Nobel Prize in Physics to R. Hulse & J. Taylor (1993)

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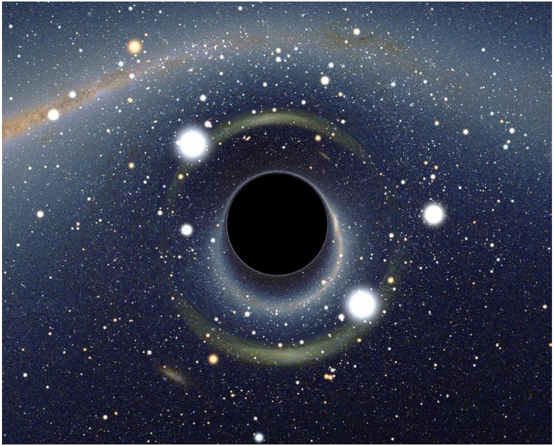

[Alain Riazuelo, 2007]

... for the layman:

A **black hole** is a region of spacetime from which nothing, not even light, can escape.

The (immaterial) boundary between the black hole interior and the rest of the Universe is called the **event horizon**.

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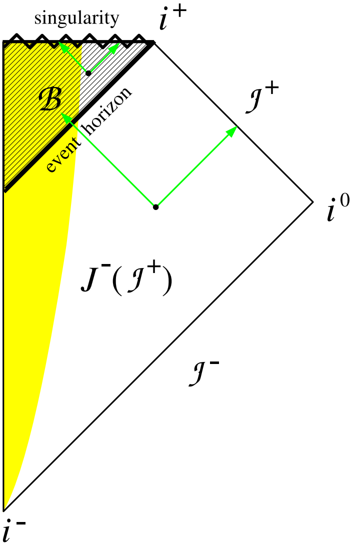

... for the mathematical physicist:

black hole: $\boxed{\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)}$

i.e. the region of spacetime where light rays cannot escape to infinity

- $(\mathscr{M}, \boldsymbol{g})$ = asymptotically flat manifold
- $\mathscr{I}^+$ = future null infinity
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event horizon: $\mathcal{H} := \partial J^-(\mathscr{I}^+)$
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$\mathcal{H}$ smooth $\Longrightarrow$ $\mathcal{H}$ null hypersurface

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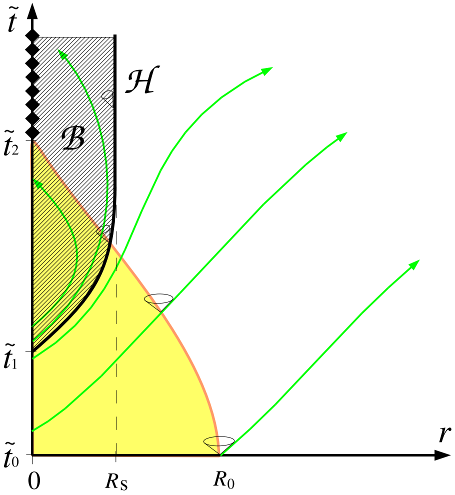

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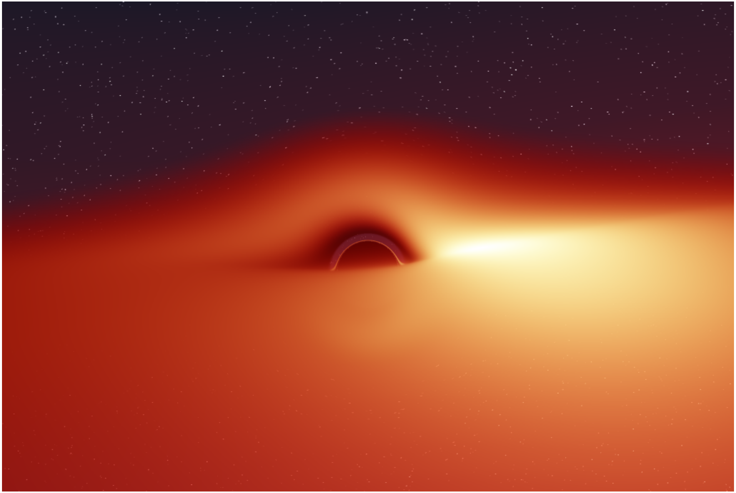
... for the astrophysicist: a very deep gravitational potential well



[J.A. Marck, CQG **13**, 393 (1996)]

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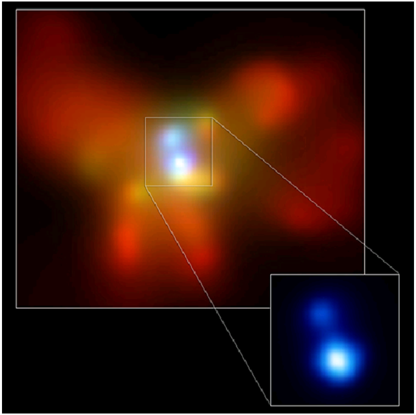
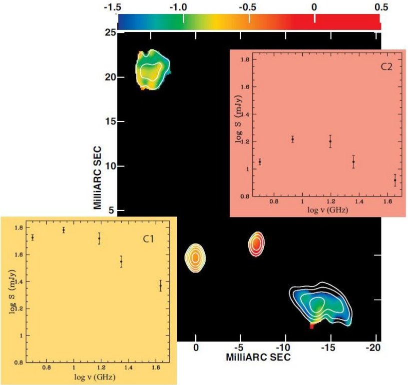


Binary BH in galaxy NGC 6240

$d = 1.4 \text{ kpc}$

[Komossa et al., ApJ **582**, L15 (2003)]



Binary BH in radio galaxy 0402+379

$d = 7.3 \text{ pc}$

[Rodriguez et al., ApJ **646**, 49 (2006)]

Main properties of black holes (1/3)

- In general relativity, a black hole contains a region where the spacetime curvature diverges: the singularity *(NB: this is not the primary definition of a black hole)*. The singularity is inaccessible to observations, being hidden by the event horizon.

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- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation
- A black hole is not an infinitely dense object: on the contrary it is made of vacuum (except maybe at the singularity); black holes can form in spacetimes empty of any matter, by collapse of gravitational wave packets.

Main properties of black holes (2/3)

Uniqueness theorem

(Dorochkevitch, Novikov & Zeldovitch 1965, Israel 1967, Carter 1971, Hawking 1972) :
A black hole in equilibrium is necessarily a Kerr-Newmann black hole, which is a vacuum solution of Einstein described by only three parameters:

- the total mass M
- the total angular momentum J
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

- $Q = 0$ and $J = 0$: Schwarzschild solution (1916)
- $Q = 0$: Kerr solution (1963)

Main properties of black holes (3/3)

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- The radius of a black hole is not a well defined concept: it *does not* correspond to some distance between the black hole "centre" (the singularity) and the event horizon. A well defined quantity is the area of the event horizon, A.

  The radius can be then defined from it: for a Schwarzschild black hole:

$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left(\frac{M}{M_\odot} \right) \text{ km}$$

Other theoretical aspects

- The four laws of black hole dynamics
- Quantum properties (Bekenstein entropy, Hawking radiation)
- Black holes and gravitational waves
- Quasi-local approaches: trapping horizons, dynamical horizons, isolated horizons
- Black holes in higher dimensions

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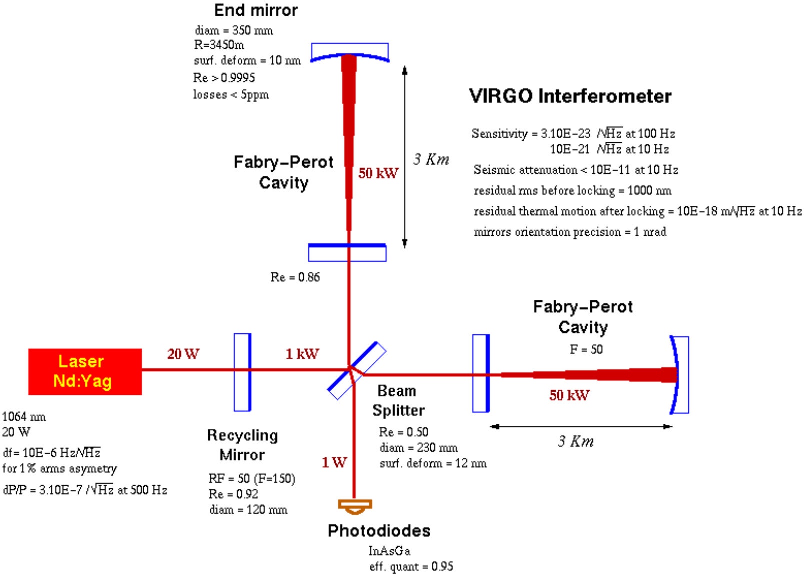
Road map of Pisa neighbourhood (Italy)...

VIRGO: a giant Michelson interferometer...

Gravitational wave detector VIRGO in Cascina, near Pisa (Italy) [CNRS/INFN]

Optical scheme of the VIRGO interferometer



End mirror

diam = 350 mm
R=3450m
surf. deform = 10 nm
Re > 0.9995
losses < 5ppm

VIRGO Interferometer

Sensitivity = 3.10E−23 /√Hz at 100 Hz
10E−21 /√Hz at 10 Hz
Seismic attenuation < 10E−11 at 10 Hz
residual rms before locking = 1000 nm
residual thermal motion after locking = 10E−18 m/√Hz at 10 Hz
mirrors orientation precision = 1 nrad

Fabry−Perot Cavity

50 kW

3 Km

Re = 0.86

Fabry−Perot Cavity

F = 50

50 kW

3 Km

Laser Nd:Yag

20 W

1 kW

1064 nm
20 W
df = 10E−6 Hz/√Hz
for 1% arms asymetry
dP/P = 3.10E−7 /√Hz at 500 Hz

Recycling Mirror

RF = 50 (F=150)
Re = 0.92
diam = 120 mm

1 W

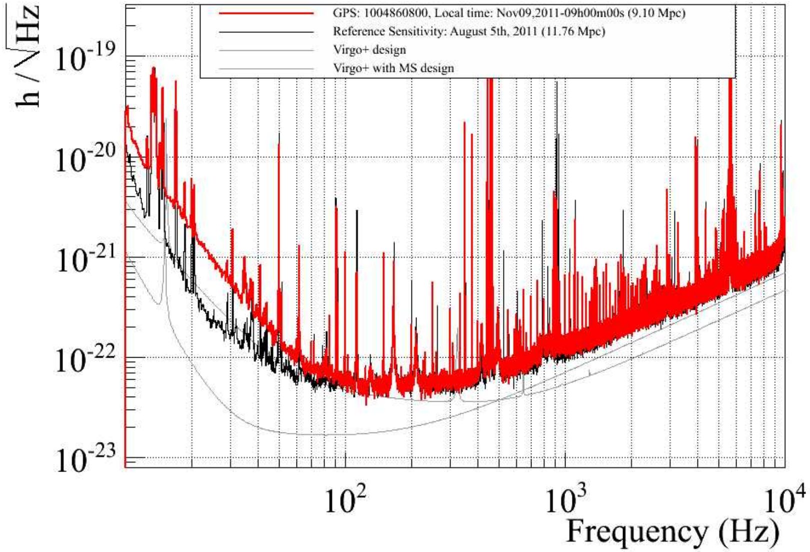
Beam Splitter

Re = 0.50
diam = 230 mm
surf. deform = 12 nm

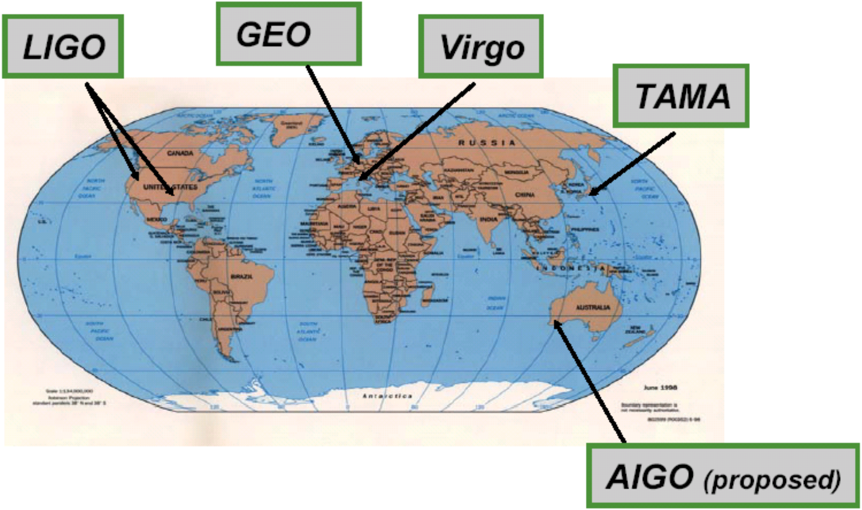
Photodiodes

InAsGa
eff. quant = 0.95

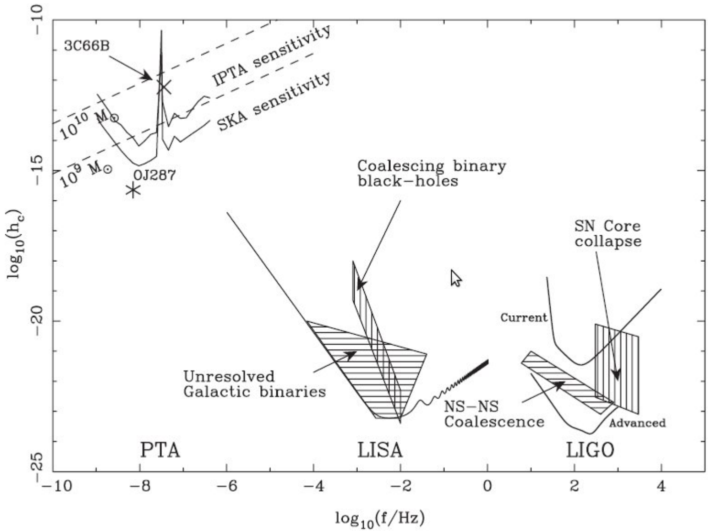
VIRGO sensitivity curve



Other interferometric detectors operating in the world



International Pulsar Timing Array (IPTA)



[Hobbs et al., CQG **27**, 084013 (2010)]

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Astrophysical black holes

Three kinds of black holes are known in the Universe:

- **Stellar black holes**: supernova remnants:
  $M \sim 10 - 30\ M_\odot$ and $R \sim 30 - 90\ \mathrm{km}$

  example: Cyg X-1 : $M = 15\ M_\odot$ and $R = 45\ \mathrm{km}$

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- **Supermassive black holes**, in galactic nuclei:
  $M \sim 10^5 - 10^{10}\ M_\odot$ and $R \sim 3 \times 10^5\ \mathrm{km} - 200\ \mathrm{UA}$

  example: Sgr A* : $M = 4,3 \times 10^6\ M_\odot$ and
  $R = 13 \times 10^6\ \mathrm{km} = 18\, R_\odot = 0,09\ \mathrm{UA} = \frac{1}{4}$radius of Mercury's orbit

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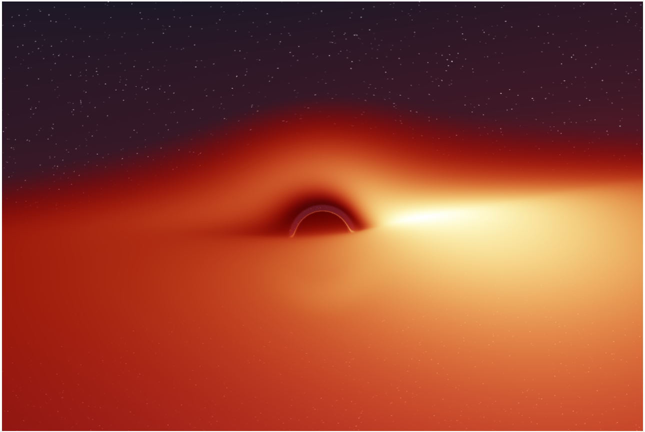
  $R = 13 \times 10^6\ \mathrm{km} = 18\, R_\odot = 0,09\ \mathrm{UA} = \frac{1}{4}$radius of Mercury's orbit
- **Intermediate mass black holes**, as ultra-luminous X-ray sources (?):
  $M \sim 10^2 - 10^4\ M_\odot$ and $R \sim 300\ \mathrm{km} - 3 \times 10^4\ \mathrm{km}$

  example: ESO 243-49 HLX-1 : $M > 500\ M_\odot$ and $R > 1500\ \mathrm{km}$

The black hole: a fantastic source of energy !

Release of potential gravitational energy by **accretion** on a black hole: up to 42% of the mass-energy mc^2 of accreted matter !

NB: thermonuclear reaction release less than 1% mc^2

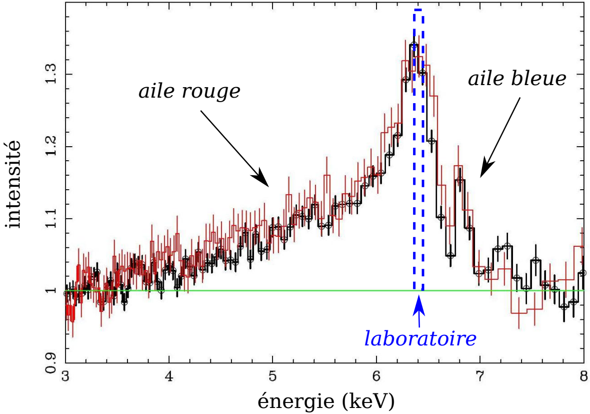


Matter falling in a black hole forms an **accretion disk** [Donald Lynden-Bell (1969), Nicolaï Shakura & Rachid Sunayev (1973)]

[J.-A. Marck (1996)]

The accretion disk as a spacetime probe

X-ray spectrum of the accretion disk around the supermassive black hole in the nucleus of the galaxy MCG-6-30-15 :

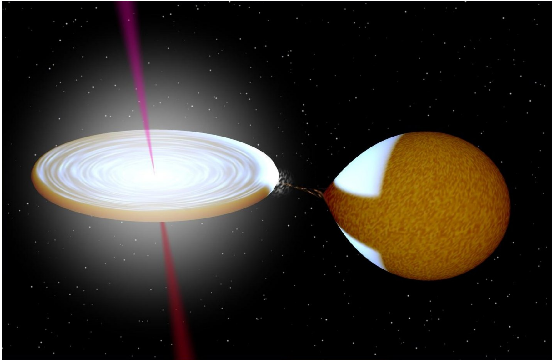


Kα line observed by the satellites XMM-Newton (red) and Suzaku (black) (adapted from [Miller (2007)])

Kα line: X fluorescence line of Fe atoms in the accretion disk (the Fe atoms are excited by the X-ray emitted from the plasma corona surrounding the disk).

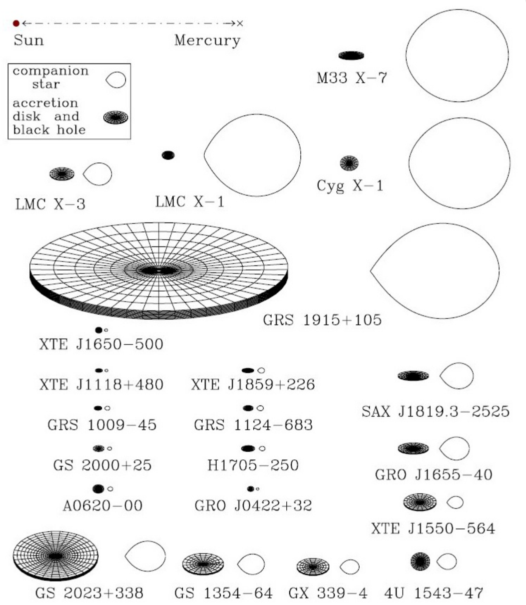
Redshift ⇒ time dilatation

Black holes in X-ray binaries



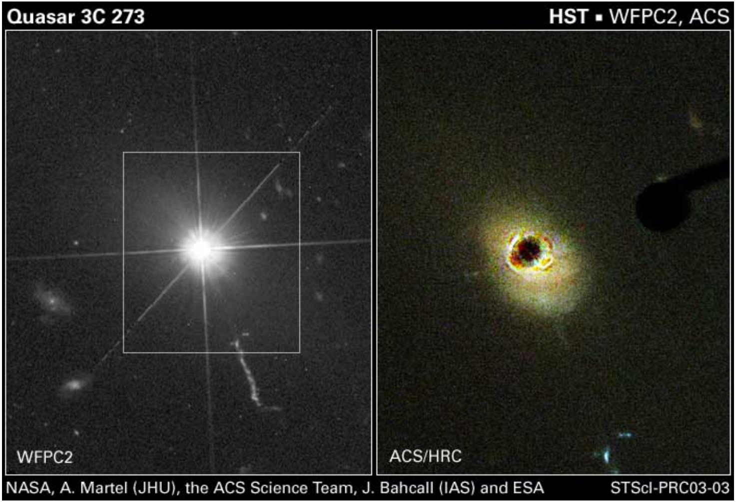
$\sim$ 20 identified stellar black holes in our galaxy

Black holes in X-ray binaries



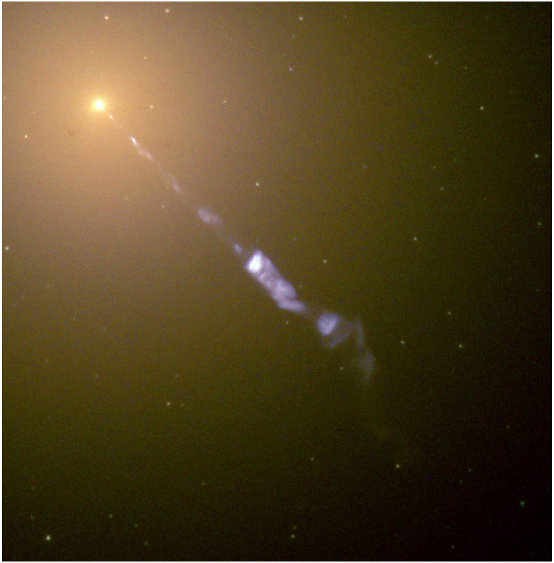
[McClintock et al. (2011)]

Black holes in the core of quasars



NASA, A. Martel (JHU), the ACS Science Team, J. Bahcall (IAS) and ESA STScI-PRC03-03

Black holes in active galactic nuclei (AGN)

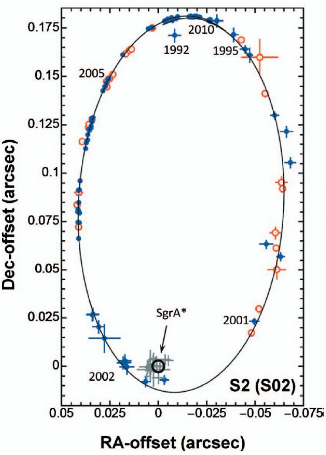


Jet emitted by the nucleus of the giant elliptic galaxy M87, at the centre of Virgo cluster [HST]

$M_{\rm BH} = 3 \times 10^9 \, M_\odot$

$V_{\rm jet} \simeq 0.99 \, c$

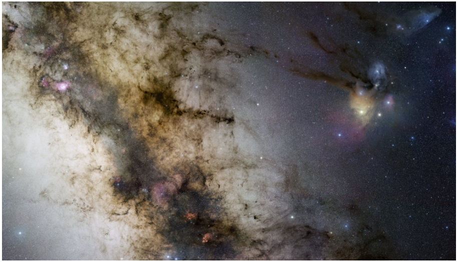
The black hole at the centre of our galaxy



Orbit of the star S2 around the black hole Sgr A*

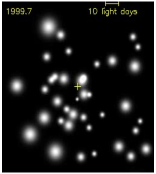
[Genzel et al. (2010)]

$$M_{\rm BH} = 4.3 \times 10^6 \, M_\odot$$



[ESO (2009)]

Detection via the stellar dynamics



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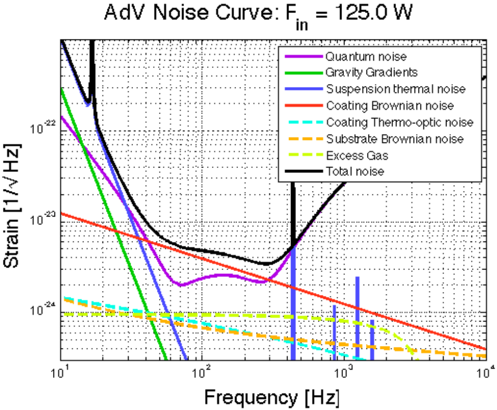
Advanced VIRGO

Advanced VIRGO: dual recycled (power + signal) interferometer with laser power ~ 125 W

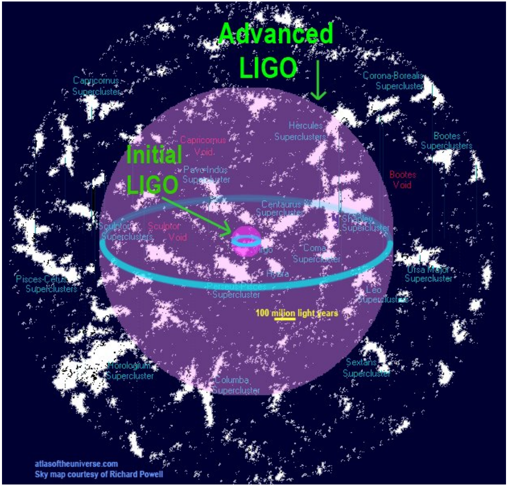


[CNRS/INFN/NIKHEF]

- VIRGO+ decommissioned in Nov. 2011
- Construction of Advanced VIRGO underway
- First lock in 2015
- Sensitivity $\sim 10 \times$ VIRGO
- $\Longrightarrow$ explored Universe volume 10^3 times larger !

Advanced LIGO

Advanced LIGO: dual recycled (power + signal) interferometer with laser power ∼ 200 W and better seismic insulation



[Advanced LIGO, NSF]

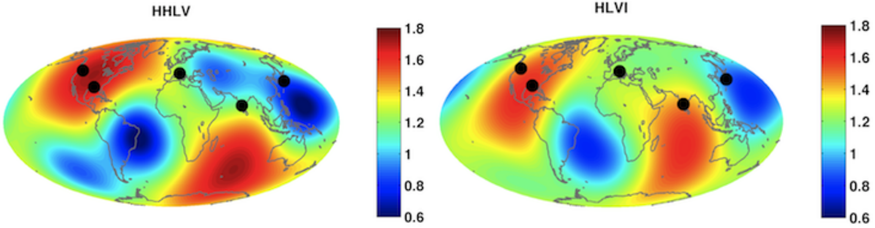
- LIGO Livingston decommissioned in 2011
- LIGO Hanford decommissioned in 2012
- Advanced LIGO optical cavities locked in summer 2012
- Advanced LIGO in operation by 2014
- Sensitivity ∼ 10 × LIGO
- $\Longrightarrow$ explored Universe volume 10^3 times larger !

IndIGO / LIGO-India

Project under consideration of the science funding agencies in India and USA:

Move one Advanced LIGO detector from Hanford to India

$\Longrightarrow$ better sky coverage:

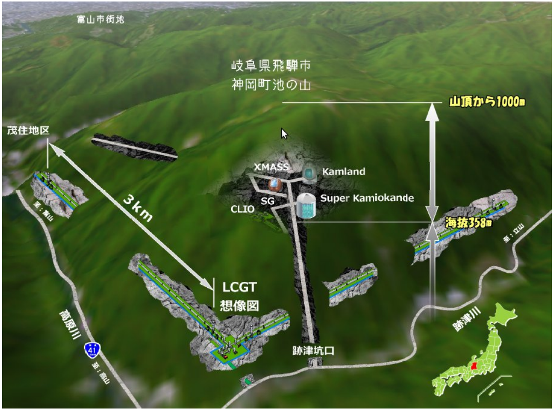


[IndIGO]

Schedule: start of LIGO-India science run: 2020

KAGRA

3-km cryogenic interferometric detector at Kamioka (Japan)

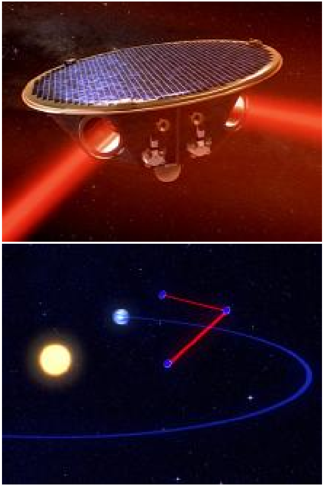


[ICRR GW group, U. Tokyo]

- Construction started in 2012 (tunnel excavation)
- Start of observations: ∼ 2019

eLISA

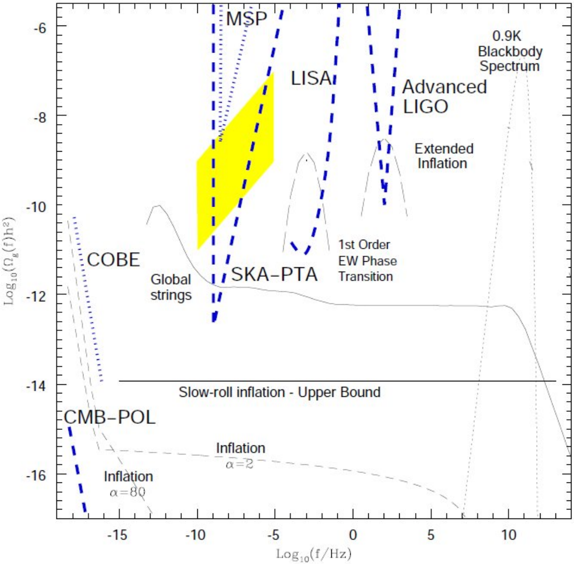
Gravitational wave detector in space



[eLISA / NGO]

- Selection in 2013 ? (ESA L2 mission)
- LISA Pathfinder launched in 2015

International Pulsar Timing Array with SKA



[Lorimer, LLR **11**, 8 (2008)]

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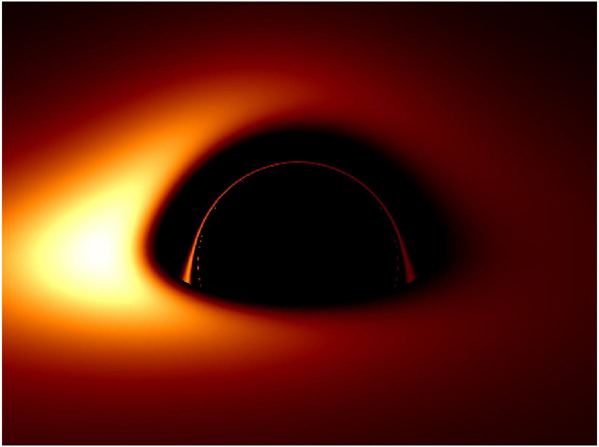
Seeing the black hole shadow



Thin accretion disk

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Largest black-hole apparent sizes in the Earth's sky:

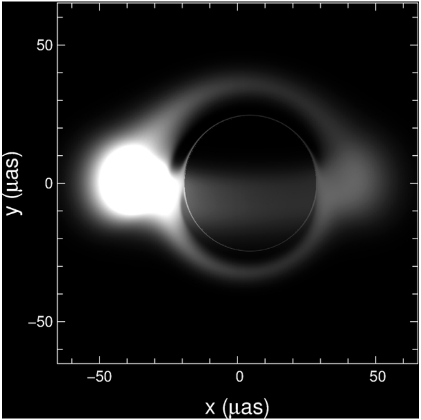
Sgr A* : $D = 53\ \mu\mathrm{as}$
M87 : $D = 21\ \mu\mathrm{as}$
M31 : $D = 20\ \mu\mathrm{as}$

Rem. 1: black holes in X-ray binaries are $\sim 10^5$ times smaller, for $D \propto M/d$

Rem. 2: HST angular resolution:

$$D_{\rm min} \sim 10^5\ \mu\mathrm{as}\ !$$

Seeing the black hole shadow



Ion torus

[Straub, Vincent, Abramowicz, Gourgoulhon & Paumard, A&A **543**, A83 (2012)]

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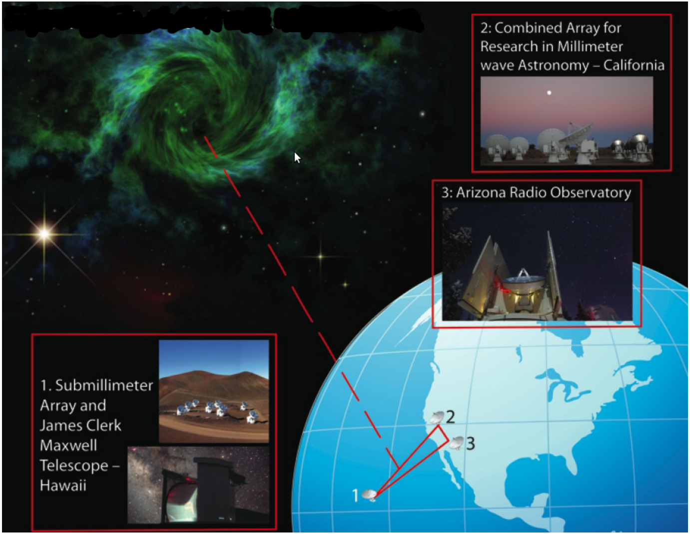
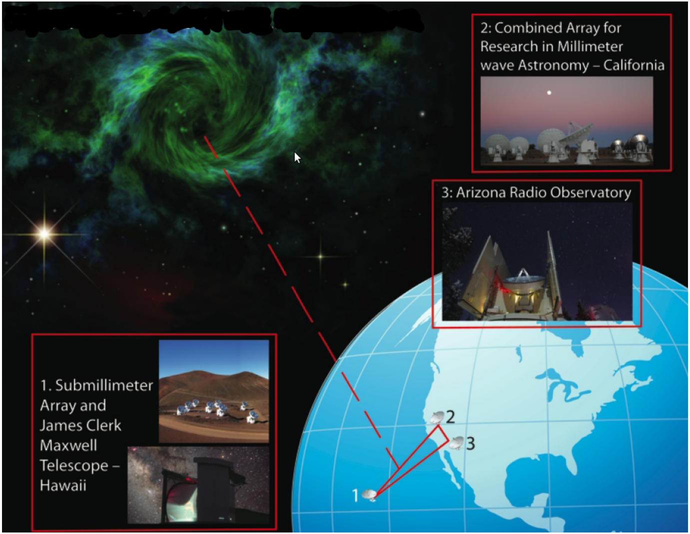
The solution to reach the μas regime: interferometry !


[redacted]

Existing American VLBI network [Doeleman et al. 2011]

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The solution to reach the μas regime: interferometry !



Existing American VLBI network [Doeleman et al. 2011]

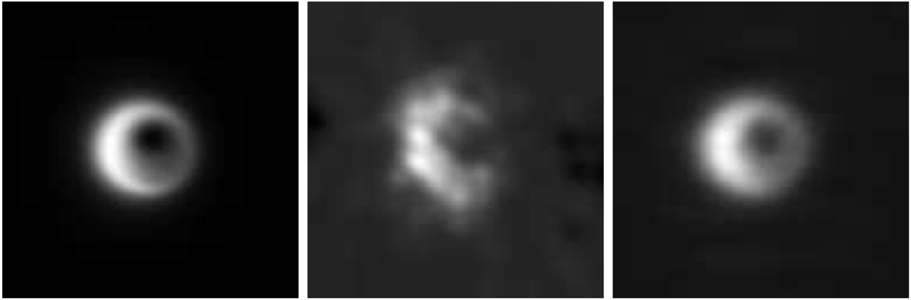
Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far: VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only $37\ \mu\mathrm{as}$.

The near future: the Event Horizon Telescope

Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020

The near future: the Event Horizon Telescope



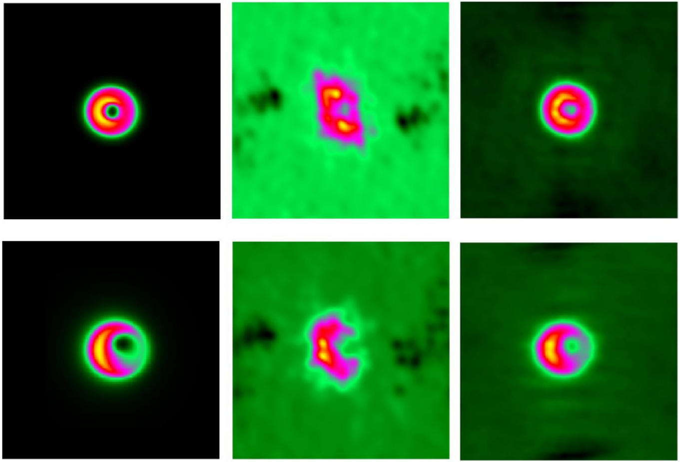
Simulations of VLBI observations of Sgr A* at $\lambda = 0.8$ mm

left: perfect image, *centre:* 7 stations (~ 2015), *right:* 13 stations (~ 2020)

$a = 0$, $i = 30°$

[Fish & Doeleman, arXiv:0906.4040 (2009)]

The near future: the Event Horizon Telescope



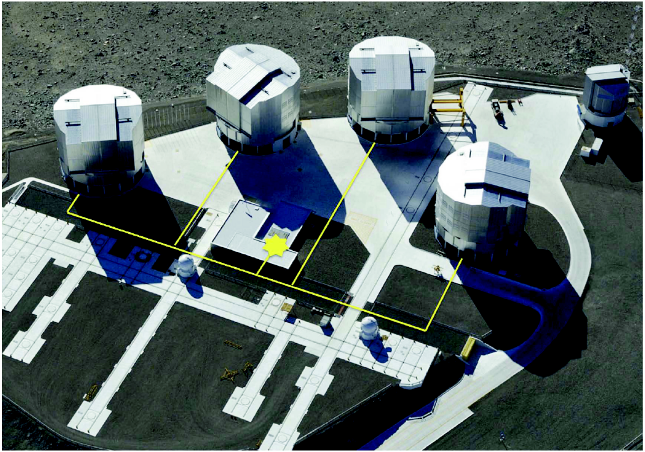
Simulations of VLBI observations of Sgr A* at $\lambda = 0.8$ mm

left: perfect image, *centre:* 7 stations (~ 2015), *right:* 13 stations (~ 2020)

top: $a = 0.5$, $i = 85°$; *bottom:* $a = 0$, $i = 60°$

[Doeleman et al. (2009)]

Near-infrared optical interferometry

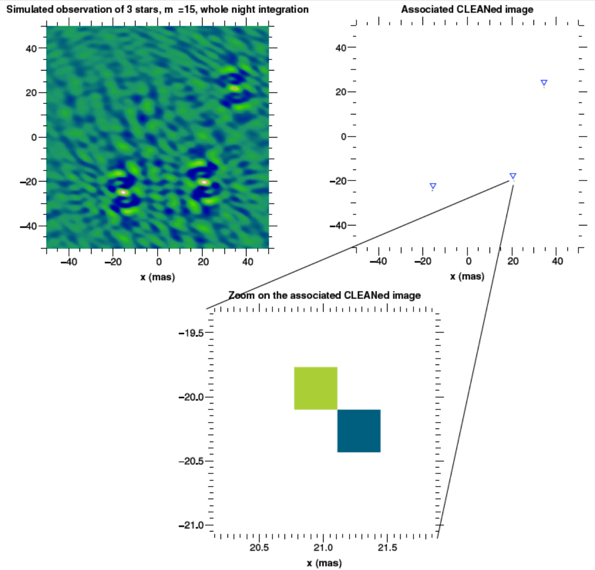


[Gillessen et al. 2010]

GRAVITY instrument at VLT (2014)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes) $\Longrightarrow$ astrometric precision of $10\ \mu$as

Simulations of GRAVITY observations



Observation of 3 stars of magnitude 15 during a whole night.

[Vincent et al., MNRAS **412**, 2653 (2011)]

Testing the no-hair theorem

GRAVITY is expected to observe stars on relativistic orbites (closer than S2)
Measure of relativistic effects:

- periastron advance
- Lense-Thirring precession

$\Longrightarrow$ constraints on the spacetime metric in the vicinity of the central object
$\Longrightarrow$ is it really the Kerr metric (a, M) ?

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Theoretical alternatives to the Kerr black hole

Within general relativity

- boson stars
- gravastar
- Q-star
- dark stars
- ...

Beyond general relativity

“hairy” black holes

- in Einstein-Yang-Mills
- in Einstein-Gauss-Bonnet with dilaton
- in Chern-Simons gravity
- ...

How to test the alternatives ?

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories)
- images of the black hole shadow different from that of a Kerr black hole (EHT)

How to test the alternatives ?

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Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric

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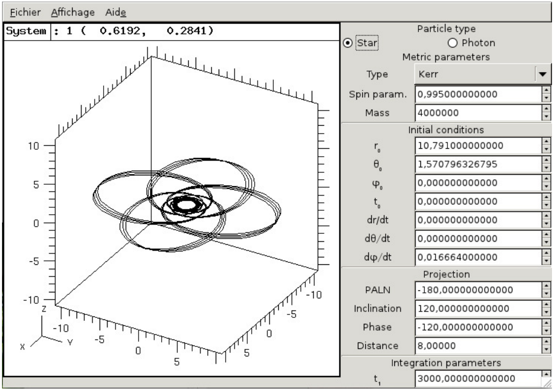
Gyoto code



System : 1 (0.6192, 0.2841)

Particle type

● Star ○ Photon

Metric parameters

Type: Kerr
Spin param.: 0,995000000000
Mass: 4000000

Initial conditions

r_0: 10,791000000000
θ_0: 1,570796326795
φ_0: 0,000000000000
t_0: 0,000000000000
dr/dt: 0,000000000000
dθ/dt: 0,000000000000
dφ/dt: 0,016664000000

Projection

PALN: -180,000000000000
Inclination: 120,000000000000
Phase: -120,000000000000
Distance: 8,00000

Integration parameters

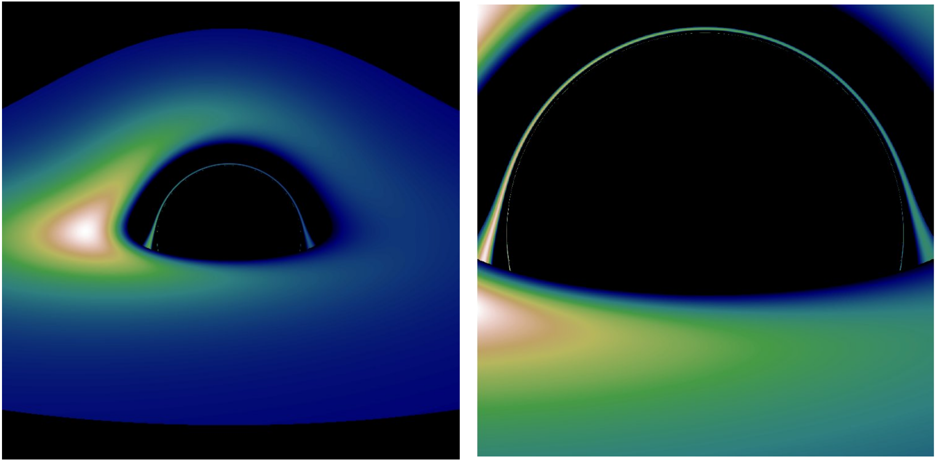
t_1: 3000,000000000000

- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick interface
- Free software (GPL) : `http://gyoto.obspm.fr/`

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

Gyoto code

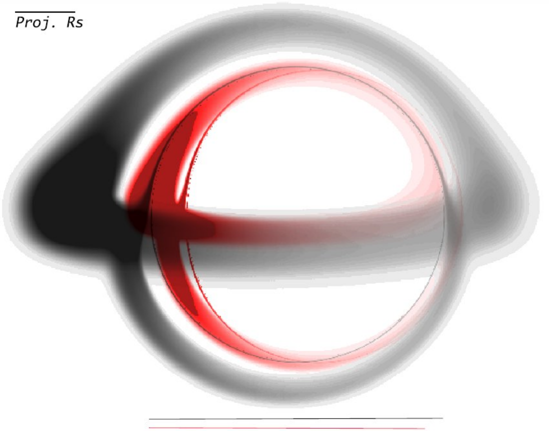


Computed images of a thin accretion disk around a Schwarzschild black hole

Measuring the spin from the black hole silhouette

Spin parameter of a Kerr black hole : $a = \frac{J}{M}$

Accretion structure around Sgr A* modeled as a ion torus, derived from the *polish doughnut* class [Abramowicz, Jaroszynski & Sikora (1978)]



Radiative transfer included (thermal synchrotron, bremsstrahlung, inverse Compton)

← Image of an ion torus computed with Gyoto for the inclination angle $i = 80°$:

- black: $a = 0.5M$
- red: $a = 0.9M$

[Straub, Vincent, Abramowicz, Gourgoulhon & Paumard, A&A **543**, A83 (2012)]