Numerical relativity and sources of gravitational waves

Eric Gourgoulhon

Laboratoire de l'Univers et de ses Théories (LUTH)
CNRS / Observatoire de Paris
F-92195 Meudon, France

eric.gourgoulhon@obspm.fr

based on collaboration with
Michal Bejger, Silvano Bonazzola, Dorota Gondek-Rosińska, Philippe Grandclément,
Pawel Haensel, José Luis Jaramillo, François Limousin
Jérôme Novak & J. Leszek Zdunik

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Outline

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Historical context: Cauchy problem of GR

- Darmois (1927), Lichnerowicz (1939): Cauchy problem for *analytic* initial data

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- Smarr & York (1978): Radiation gauge for numerical relativity: elliptic-hyperbolic system with asymptotic TT behavior
- York (1999): Conformal thin-sandwich (CTS) method for solving the constraint equations

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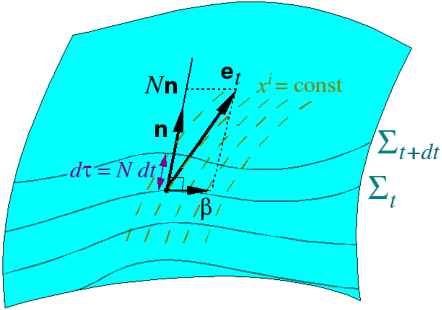
- Shibata (2000): 3-D full computation of binary neutron star merger: *first full GR 3-D solution of the Cauchy problem of astrophysical interest*

Outline

3+1 decomposition of spacetime

Foliation of spacetime by a family of spacelike hypersurfaces $(\Sigma_t)_{t\in\mathbb{R}}$; on each hypersurface, pick a coordinate system $(x^i)_{i\in\{1,2,3\}} \Longrightarrow$
$(x^\mu)_{\mu\in\{0,1,2,3\}} = (t, x^1, x^2, x^3)$ = coordinate system on spacetime



$\boldsymbol{n}$: future directed unit normal to Σ_t :
$\boldsymbol{n} = -N\,\mathbf{d}t$, N : lapse function
$\boldsymbol{e}_t = \partial/\partial t$: time vector of the natural basis associated with the coordinates (x^μ)

N : lapse function
$\boldsymbol{\beta}$: shift vector
$\Big\}\ \boldsymbol{e}_t = N\boldsymbol{n} + \boldsymbol{\beta}$

Geometry of the hypersurfaces Σ_t:

- induced metric $\boldsymbol{\gamma} = \boldsymbol{g} + \boldsymbol{n}\otimes\boldsymbol{n}$
- extrinsic curvature : $\boldsymbol{K} = -\frac{1}{2}\mathcal{L}_{\boldsymbol{n}}\boldsymbol{\gamma}$

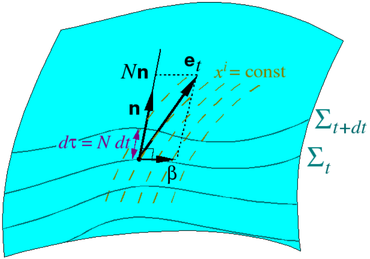
$$g_{\mu\nu}\,dx^\mu\,dx^\nu = -N^2\,dt^2 + \gamma_{ij}\,(dx^i + \beta^i dt)\,(dx^j + \beta^j\,dt)$$

Choice of coordinates within the 3+1 formalism

$$(x^\mu) = (t, x^i) = (t, x^1, x^2, x^3)$$

Choice of the lapse function N $\Longleftrightarrow$ choice of the slicing (Σ_t)

Choice of the shift vector $\boldsymbol{\beta}$ $\Longleftrightarrow$ choice of the spatial coordinates (x^i) on each hypersurface Σ_t



A well-spread choice of slicing: *maximal slicing*: $K := \mathrm{tr}\,\boldsymbol{K} = 0$

[Lichnerowicz 1944]

3+1 decomposition of Einstein equation

Orthogonal projection of Einstein equation onto Σ_t and along the normal to Σ_t :

- Hamiltonian constraint: $R + K^2 - K_{ij}K^{ij} = 16\pi E$
- Momentum constraint : $D_j K^{ij} - D^i K = 8\pi J^i$
- Dynamical equations :
$$\frac{\partial K_{ij}}{\partial t} - \mathcal{L}_{\boldsymbol{\beta}} K_{ij} =$$
$$-D_i D_j N + N \left[R_{ij} - 2K_{ik}K^k_{\ j} + K K_{ij} + 4\pi((S-E)\gamma_{ij} - 2S_{ij}) \right]$$

$E := \boldsymbol{T}(\boldsymbol{n},\boldsymbol{n}) = T_{\mu\nu}\, n^\mu n^\nu$, $J_i := -\gamma_i^{\ \mu}\, T_{\mu\nu}\, n^\nu$, $S_{ij} := \gamma_i^{\ \mu}\, \gamma_j^{\ \nu}\, T_{\mu\nu}$, $S := S_i^{\ i}$
D_i : covariant derivative associated with $\boldsymbol{\gamma}$, R_{ij} : Ricci tensor of D_i, $R := R_i^{\ i}$

Kinematical relation between $\boldsymbol{\gamma}$ and $\boldsymbol{K}$: $\frac{\partial \gamma^{ij}}{\partial t} + D^i \beta^j + D^j \beta^i = 2N K^{ij}$

Resolution of Einstein equation $\equiv$ Cauchy problem

Free vs. constrained evolution in 3+1 numerical relativity

Einstein equations split into

$$\begin{cases} \text{dynamical equations} & \frac{\partial}{\partial t} K_{ij} = \ldots \\ \text{Hamiltonian constraint} & R + K^2 - K_{ij}K^{ij} = 16\pi E \\ \text{momentum constraint} & D_j K_i^{\ j} - D_i K = 8\pi J_i \end{cases}$$

- **2-D computations(80's and 90's):**
  - partially constrained schemes: Bardeen & Piran (1983), Stark & Piran (1985), Evans (1986)
  - fully constrained schemes: Evans (1989), Shapiro & Teukolsky (1992), Abrahams et al. (1994)
- **3-D computations (from mid 90's):** Almost all based on free evolution schemes: BSSN, symmetric hyperbolic formulations, etc...
  $\Longrightarrow$ problem: exponential growth of *constraint violating modes*

"Standard issue" 1 :

The constraints usually involve elliptic equations and 3-D elliptic solvers are CPU-time expensive !

Cartesian vs. spherical coordinates in 3+1 numerical relativity

- **1-D and 2-D computations:** massive usage of spherical coordinates (r, θ, φ)
- **3-D computations:** almost all based on Cartesian coordinates (x, y, z), although spherical coordinates are better suited to study objects with spherical topology (black holes, neutron stars). Two exceptions:
  – *Nakamura et al. (1987):* evolution of pure gravitational wave spacetimes in spherical coordinates (but with Cartesian components of tensor fields)
  – *Stark (1989):* attempt to compute 3D stellar collapse in spherical coordinates

"Standard issue" **2** :

Spherical coordinates are singular at $r = 0$ and $\theta = 0$ or π !

"Standard issues" 1 and 2 can be overcome

"Standard issues" 1 and 2 are neither *mathematical* nor *physical*

they are *technical* ones

$\Longrightarrow$ they can be overcome with appropriate techniques

Spectral methods allow for

- an automatic treatment of the singularities of spherical coordinates (issue 2)
- fast 3-D elliptic solvers in spherical coordinates: 3-D Poisson equation reduced to a system of 1-D algebraic equations with banded matrices [Grandclément, Bonazzola, Gourgoulhon & Marck, J. Comp. Phys. **170**, 231 (2001)] (issue 1)

Outline

A new scheme for 3+1 numerical relativity

Constrained scheme built upon **maximal slicing** and **Dirac gauge**

[Bonazzola, Gourgoulhon, Grandclément & Novak, PRD **70**, 104007 (2004)]

Conformal metric and dynamics of the gravitational field

Dynamical degrees of freedom of the gravitational field:

York (1972) : they are carried by the conformal "metric"

$$\hat{\gamma}_{ij} := \gamma^{-1/3}\,\gamma_{ij} \qquad \text{with } \gamma := \det\gamma_{ij}$$

$\hat{\gamma}_{ij}$ = *tensor density* of weight $-2/3$

To work with *tensor fields* only, introduce an *extra structure* on Σ_t: a *flat metric* $\boldsymbol{f}$ such that $\dfrac{\partial f_{ij}}{\partial t} = 0$ and $\gamma_{ij} \sim f_{ij}$ at spatial infinity *(asymptotic flatness)*

Define $\tilde{\gamma}_{ij} := \Psi^{-4}\,\gamma_{ij}$ or $\gamma_{ij} =: \Psi^4\,\tilde{\gamma}_{ij}$ with $\Psi := \left(\frac{\gamma}{f}\right)^{1/12}$, $f := \det f_{ij}$

$\tilde{\gamma}_{ij}$ is invariant under any conformal transformation of γ_{ij} and verifies $\det\tilde{\gamma}_{ij} = f$

Notations: $\tilde{\gamma}^{ij}$: inverse conformal metric : $\tilde{\gamma}_{ik}\,\tilde{\gamma}^{kj} = \delta_i^{\ j}$

$\tilde{D}_i$: covariant derivative associated with $\tilde{\gamma}_{ij}$, $\tilde{D}^i := \tilde{\gamma}^{ij}\,\tilde{D}_j$

$\mathcal{D}_i$: covariant derivative associated with f_{ij}, $\mathcal{D}^i := f^{ij}\,\mathcal{D}_j$

Dirac gauge: definition

Conformal decomposition of the metric γ_{ij} of the spacelike hypersurfaces Σ_t:

$$\gamma_{ij} =: \Psi^4 \, \tilde\gamma_{ij} \qquad \text{with} \qquad \tilde\gamma^{ij} =: f^{ij} + h^{ij}$$

where f_{ij} is a flat metric on Σ_t, h^{ij} a symmetric tensor and Ψ a scalar field defined by $\Psi := \left(\frac{\det \gamma_{ij}}{\det f_{ij}} \right)^{1/12}$

Dirac gauge (Dirac, 1959) = *divergence-free* condition on $\tilde\gamma^{ij}$:

$$\mathcal{D}_j \tilde\gamma^{ij} = \mathcal{D}_j h^{ij} = 0$$

where $\mathcal{D}_j$ denotes the covariant derivative with respect to the flat metric f_{ij}.
Compare

- minimal distortion (Smarr & York 1978) : $D_j \left(\partial \tilde\gamma^{ij} / \partial t \right) = 0$
- pseudo-minimal distortion (Nakamura 1994) : $\mathcal{D}^j \left(\partial \tilde\gamma^{ij} / \partial t \right) = 0$

Notice: Dirac gauge $\Longleftrightarrow$ BSSN connection functions vanish: $\tilde\Gamma^i = 0$

Dirac gauge: motivation

Expressing the Ricci tensor of conformal metric as a second order operator:
In terms of the covariant derivative $\mathcal{D}_i$ associated with the flat metric $\boldsymbol{f}$:

$$\tilde{\gamma}^{ik}\tilde{\gamma}^{jl}\tilde{R}_{kl} = \frac{1}{2}\left(\tilde{\gamma}^{kl}\mathcal{D}_k\mathcal{D}_l h^{ij} - \tilde{\gamma}^{ik}\mathcal{D}_k H^j - \tilde{\gamma}^{jk}\mathcal{D}_k H^i\right) + \mathcal{Q}(\tilde{\gamma}, \boldsymbol{\mathcal{D}}\tilde{\gamma})$$

with $H^i := \mathcal{D}_j h^{ij} = \mathcal{D}_j \tilde{\gamma}^{ij} = -\tilde{\gamma}^{kl}\Delta^i{}_{kl} = -\tilde{\gamma}^{kl}(\tilde{\Gamma}^i{}_{kl} - \bar{\Gamma}^i{}_{kl})$

and $\mathcal{Q}(\tilde{\gamma}, \boldsymbol{\mathcal{D}}\tilde{\gamma})$ is quadratic in first order derivatives $\boldsymbol{\mathcal{D}h}$

Dirac gauge: $H^i = 0 \Longrightarrow$ Ricci tensor becomes an elliptic operator for h^{ij}

Similar property as harmonic coordinates for the 4-dimensional Ricci tensor:

$$^4R_{\alpha\beta} = -\frac{1}{2}g^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\nu}g_{\alpha\beta} + \text{quadratic terms}$$

Dirac gauge: discussion

- introduced by Dirac (1959) in order to fix the coordinates in some *Hamiltonian formulation* of general relativity; originally defined for Cartesian coordinates only: $\frac{\partial}{\partial x^j}\left(\gamma^{1/3}\,\gamma^{ij}\right) = 0$

  but trivially extended by us to more general type of coordinates (e.g. spherical) thanks to the introduction of the flat metric f_{ij}:
  $\mathcal{D}_j\left((\gamma/f)^{1/3}\gamma^{ij}\right) = 0$
- first discussed in the context of numerical relativity by Smarr & York (1978), as a candidate for a radiation gauge, but disregarded for not being covariant under coordinate transformation $(x^i) \mapsto (x^{i'})$ in the hypersurface Σ_t, contrary to the *minimal distortion gauge* proposed by them
- fully specifies (up to some boundary conditions) the coordinates in each hypersurface Σ_t, including the initial one $\Rightarrow$ allows for the search for *stationary solutions*

Dirac gauge: discussion (con't)

- leads asymptotically to transverse-traceless (TT) coordinates (same as minimal distortion gauge). Both gauges are analogous to *Coulomb gauge* in electrodynamics
- turns the Ricci tensor of conformal metric $\tilde{\gamma}_{ij}$ into an elliptic operator for h^{ij} $\Longrightarrow$ the dynamical Einstein equations become a *wave equation* for h^{ij}
- results in a *vector elliptic equation* for the shift vector β^i

Maximal slicing + Dirac gauge

Our choice of coordinates to solve numerically the Cauchy problem:

- choice of Σ_t foliation: maximal slicing: $K := \mathrm{tr}\, \boldsymbol{K} = 0$
- choice of (x^i) coordinates within Σ_t: Dirac gauge: $\mathcal{D}_j h^{ij} = 0$

Note: the Cauchy problem has been shown to be locally strongly well posed for a similar coordinate system, namely *constant mean curvature* $(K = t)$ and *spatial harmonic coordinates* $\left(\mathcal{D}_j \left[(\gamma/f)^{1/2} \gamma^{ij} \right] = 0 \right)$
[Andersson & Moncrief, Ann. Henri Poincaré **4**, 1 (2003)]

3+1 Einstein equations in maximal slicing + Dirac gauge

[Bonazzola, Gourgoulhon, Grandclément & Novak, PRD **70**, 104007 (2004)]

- 5 elliptic equations (4 constraints + $K = 0$ condition) ($\Delta := \mathcal{D}_k \mathcal{D}^k$):

$$\Delta N = \Psi^4 N \left[4\pi(E+S) + \tilde{A}_{kl} A^{kl} \right] - h^{kl} \mathcal{D}_k \mathcal{D}_l N - 2 \tilde{D}_k \ln \Psi \, \tilde{D}^k N$$

$$\begin{aligned} \Delta(\Psi^2 N) &= \Psi^6 N \left(4\pi S + \frac{3}{4} \tilde{A}_{kl} A^{kl} \right) - h^{kl} \mathcal{D}_k \mathcal{D}_l (\Psi^2 N) \\ &+ \Psi^2 \left[N \left(\frac{1}{16} \tilde{\gamma}^{kl} \mathcal{D}_k h^{ij} \mathcal{D}_l \tilde{\gamma}_{ij} - \frac{1}{8} \tilde{\gamma}^{kl} \mathcal{D}_k h^{ij} \mathcal{D}_j \tilde{\gamma}_{il} \right. \right. \\ &\left. \left. + 2 \tilde{D}_k \ln \Psi \, \tilde{D}^k \ln \Psi \right) + 2 \tilde{D}_k \ln \Psi \, \tilde{D}^k N \right]. \end{aligned}$$

$$\begin{aligned} \Delta \beta^i + \frac{1}{3} \mathcal{D}^i \left(\mathcal{D}_j \beta^j \right) &= 2 A^{ij} \mathcal{D}_j N + 16\pi N \Psi^4 J^i - 12 N A^{ij} \mathcal{D}_j \ln \Psi \\ &- 2 \Delta^i{}_{kl} N A^{kl} - h^{kl} \mathcal{D}_k \mathcal{D}_l \beta^i - \frac{1}{3} h^{ik} \mathcal{D}_k \mathcal{D}_l \beta^l \end{aligned}$$

3+1 equations in maximal slicing + Dirac gauge (cont'd)

- 2 scalar wave equations for two scalar potentials χ and μ :

$$-\frac{\partial^2 \chi}{\partial t^2} + \Delta \chi = S_\chi$$
$$-\frac{\partial^2 \mu}{\partial t^2} + \Delta \mu = S_\mu$$

The remaining 3 degrees of freedom are fixed by the Dirac gauge:

From the two potentials χ and μ, construct a TT tensor $\bar{h}^{ij}$ according to the formulas (components with respect to a spherical $\boldsymbol{f}$-orthonormal frame)

$$\bar{h}^{rr} = \frac{\chi}{r^2}, \quad \bar{h}^{r\theta} = \frac{1}{r}\left(\frac{\partial \eta}{\partial \theta} - \frac{1}{\sin\theta}\frac{\partial \mu}{\partial \phi}\right), \quad \bar{h}^{r\phi} = \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial \eta}{\partial \phi} + \frac{\partial \mu}{\partial \theta}\right), \text{ etc...}$$

with $\Delta_{\theta\phi}\eta = -\partial\chi/\partial r - \chi/r$

Numerical implementation

Numerical code based on the C++ library LORENE (http://www.lorene.obspm.fr) with the following main features:

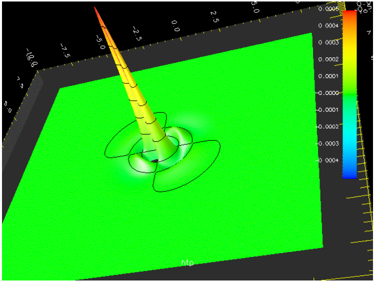
- multidomain spectral methods based on spherical coordinates (r, θ, φ), with compactified external domain ($\Longrightarrow$ spatial infinity included in the computational domain for elliptic equations)
- very efficient outgoing-wave boundary conditions, ensuring that all modes with spherical harmonics indices $\ell = 0$, $\ell = 1$ and $\ell = 2$ are perfectly outgoing
  [Novak & Bonazzola, J. Comp. Phys. **197**, 186 (2004)]
  (*recall:* Sommerfeld boundary condition works only for $\ell = 0$, which is too low for gravitational waves)

Results on a pure gravitational wave spacetime

Initial data: similar to [Baumgarte & Shapiro, PRD **59**, 024007 (1998)], namely a momentarily static ($\partial\tilde{\gamma}^{ij}/\partial t = 0$) Teukolsky wave $\ell = 2$, $m = 2$:

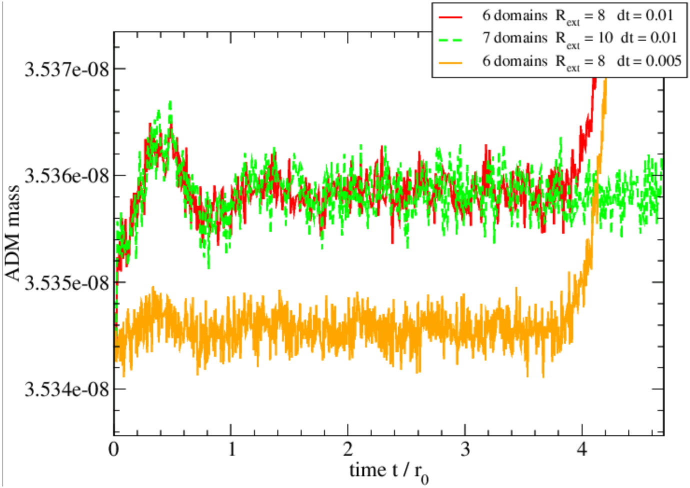
$$\begin{cases} \chi(t=0) = \frac{\chi_0}{2} r^2 \exp\left(-\frac{r^2}{r_0^2}\right) \sin^2\theta \, \sin 2\varphi \\ \mu(t=0) = 0 \end{cases} \qquad \text{with} \quad \chi_0 = 10^{-3}$$

Preparation of the initial data by means of the *conformal thin sandwich* procedure



Evolution of $h^{\phi\phi}$ in the plane $\theta = \frac{\pi}{2}$

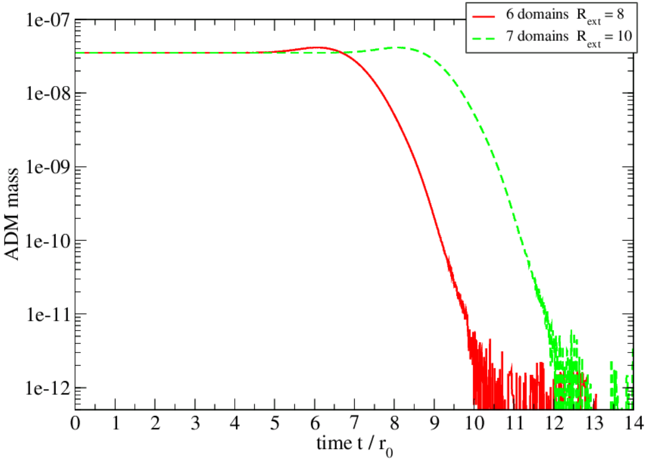
Test: conservation of the ADM mass



Number of coefficients in each domain: $N_r = 17$, $N_\theta = 9$, $N_\varphi = 8$

For $dt = 5\,10^{-3} r_0$, the ADM mass is conserved within a relative error lower than 10^{-4}

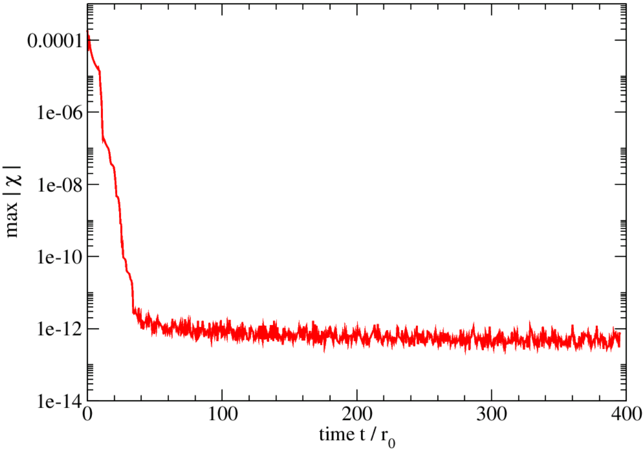
Late time evolution of the ADM mass



At $t > 10\, r_0$, the wave has completely left the computation domain

$\Longrightarrow$ Minkowski spacetime

Long term stability



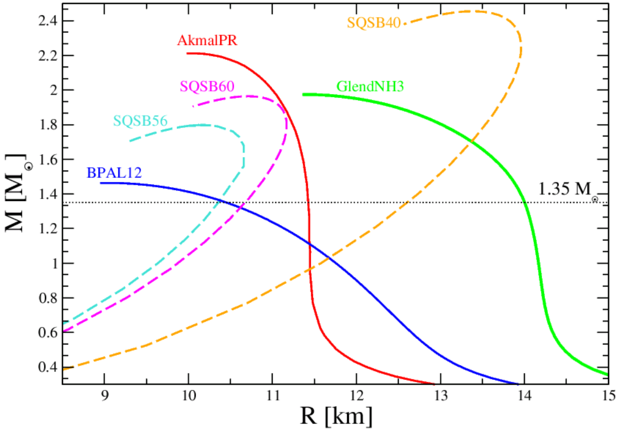
Nothing happens until the run is switched off at $t = 400\, r_0$!

Summary

- Dirac gauge + maximal slicing reduces the Einstein equations into a system of
  – two scalar elliptic equations (including the Hamiltonian constraint)
  – one vector elliptic equations (the momentum constraint)
  – two scalar wave equations (evolving the two dynamical degrees of freedom of the gravitational field)
- The usage of spherical coordinates and spherical components of tensor fields is crucial in reducing the dynamical Einstein equations to two scalar wave equations
- The unimodular character of the conformal metric ($\det \tilde{\gamma}_{ij} = \det f_{ij}$) is ensured in our scheme
- First numerical results show that Dirac gauge + maximal slicing seems a promising choice for stable evolutions of 3+1 Einstein equations and gravitational wave extraction
- It remains to be tested on black hole spacetimes !

Outline

Our current poor knowledge of nuclear matter EOS



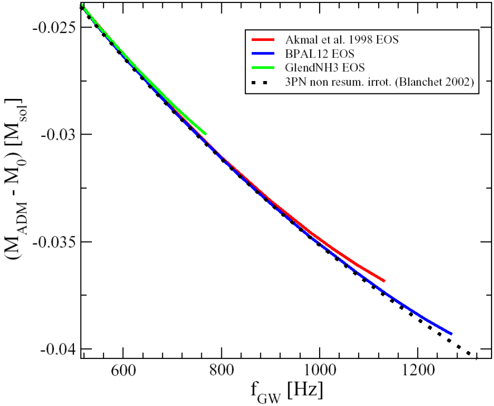
Constraining the nuclear matter EOS from GW observations of binary coalescence

Methods based on the merger or post-merger signal:

- Measure of the radius from the shape of the GW spectrum in a coalescing BH-NS system [Saijo & Nakamura, PRL **85**, 2665 (2000)]
- Constraining the EOS softness from the post-merger signal in binary NS coalescence (prompt black formation vs. supramassive NS remnant) [Shibata, Taniguchi & Uryu, PRD **71**, 084021 (2005)] [Shibata, PRL **94**, 201101 (2005)]

Constraining the nuclear matter EOS from GW observations of the inspiral phase

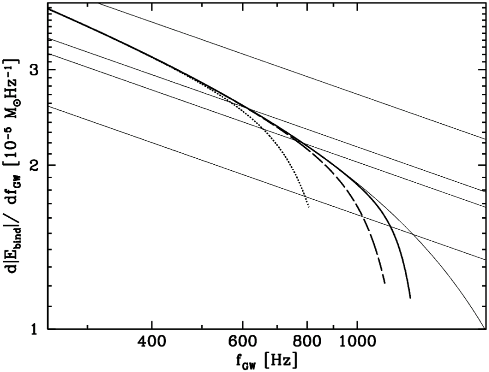
Evolutionary sequences of irrotational binary NS:



[Bejger, Gondek-Rosińska, Gourgoulhon, Haensel, Taniguchi & Zdunik, A&A **431**, 297 (2005)]

Constraining the nuclear matter EOS from GW observations of the inspiral phase

GW energy spectrum

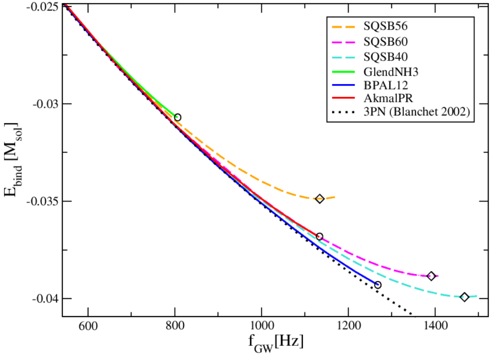


……. GNH3, – – – APR, — BPAL12

[Bejger, Gondek-Rosińska, Gourgoulhon, Haensel, Taniguchi & Zdunik, A&A **431**, 297 (2005)]

Determining the nuclear matter EOS from GW observations

Evolutionary sequences of irrotational binary strange stars:



[Limousin, Gondek-Rosińska & Gourgoulhon, PRD **71**, 064012 (2005)]

[Gondek-Rosińska, Bejger, Bulik, Gourgoulhon, Haensel, Limousin & Zdunik, preprint: gr-qc/0412010)]