

Causality and singularities in relativistic spacetime

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Introduction

Relativity (both *special* and *general*) has changed dramatically our views on each of the three concepts in the theme

Causes, Foundations, Origins

of the HiPhiS seminar.

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- **Causes** : the **causality structure** of relativistic spacetime is radically different from that pervading in Newtonian physics.

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- **Causes** : the **causality structure** of relativistic spacetime is radically different from that pervading in Newtonian physics.
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In this talk : focus on causality structure

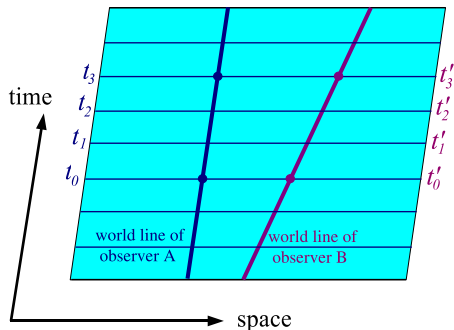
Outline

- 1 The relativistic spacetime
- 2 Causality in Minkowski spacetime
- 3 An illustration with tachyons
- 4 Causality in curved spacetime : the Schwarzschild black hole
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Newtonian and relativistic spacetimes



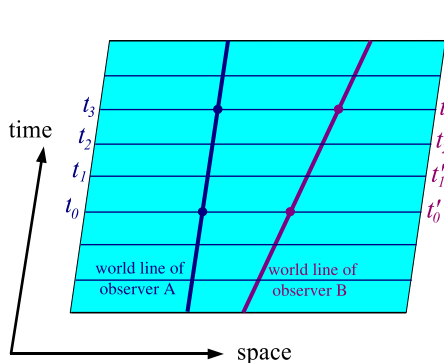
Newtonian spacetime :

math. description : affine space \mathbb{R}^4

absolute structure : **universal time**

All observers measure the same time

Newtonian and relativistic spacetimes

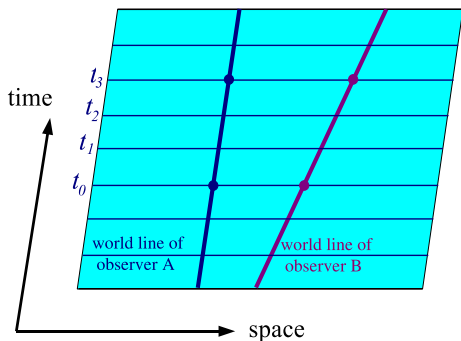


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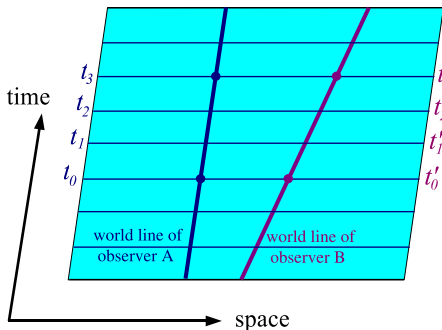


Special relativity spacetime :

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Newtonian and relativistic spacetimes

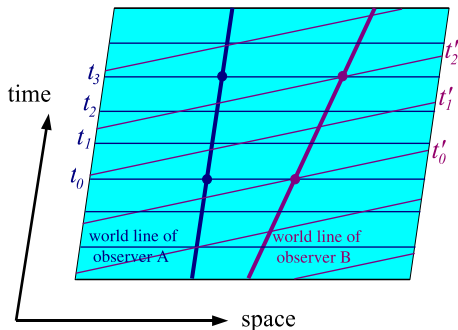


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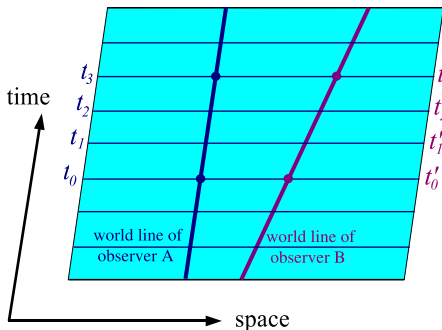
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\implies simultaneity is relative

\implies *time dilation* phenomenon

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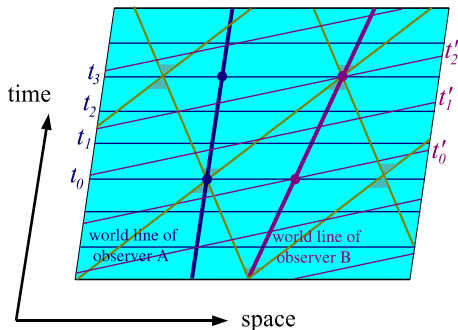


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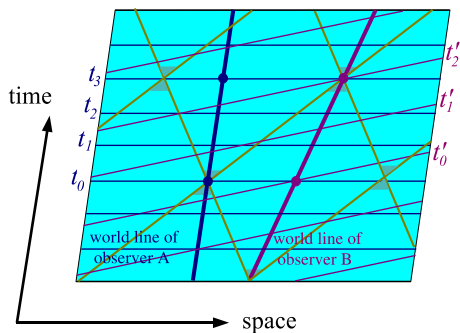
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absolute structure : **light cones**

\implies simultaneity is relative

\implies *time dilation* phenomenon

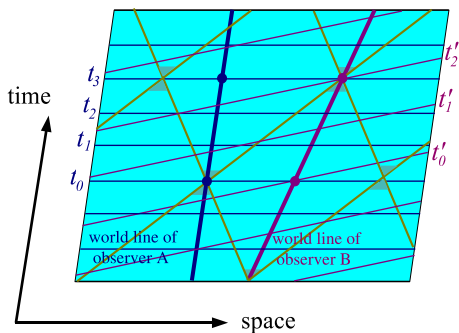
Relativistic spacetime



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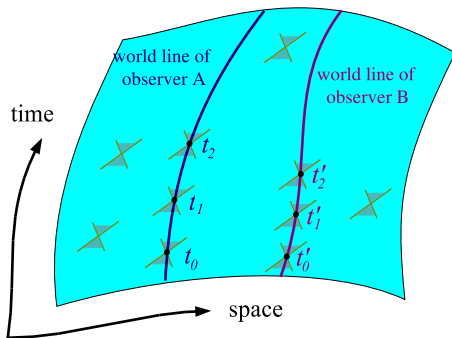
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Relativistic spacetime



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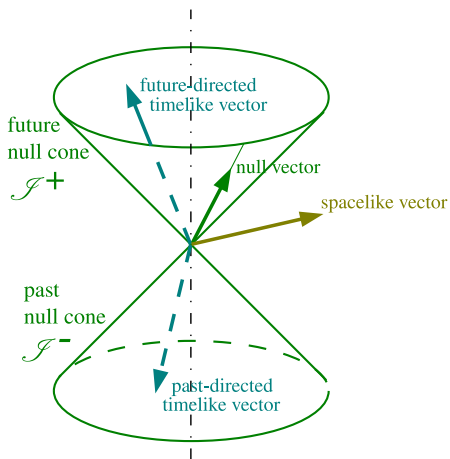
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General relativity spacetime :
math. description : 4-dimensional curved space (manifold)

- OK for **electromagnetism**
- OK for **gravitation**

The light cone

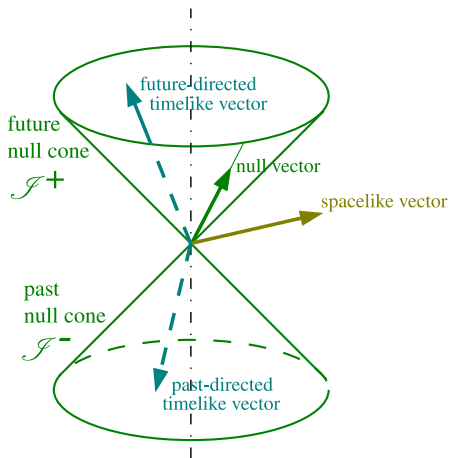


Einstein has based special relativity on two postulates :

Einstein's first postulate

The laws of physics are the same in all inertial frames.

The light cone



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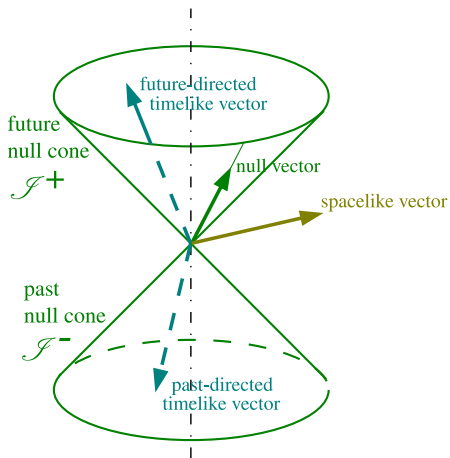
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The speed of light in free space has the same value c in all inertial frames.

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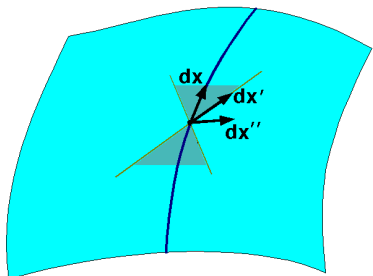
Einstein's second postulate

The speed of light in free space has the same value c in all inertial frames.

Geometrical translation :

Light rays converging to or emitted from an event form an **absolute structure** in spacetime, i.e. independent of any inertial frame : the **light cone**

The metric tensor



Algebraic translation of the absolute structure provided by the light cones :

Metric tensor g

At each point of spacetime :

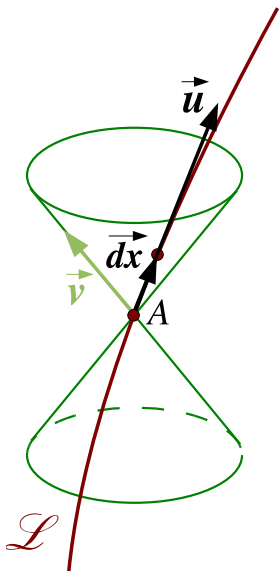
$g : (\text{vector}_1, \text{vector}_2) \mapsto \text{a number}$

Mathematically :

$g =$ symmetric bilinear form of signature $(-, +, +, +)$

- $g(d\vec{x}, d\vec{x}) < 0$ along a timelike displacement
- $g(d\vec{x}', d\vec{x}') = 0$ along the light cone
- $g(d\vec{x}'', d\vec{x}'') > 0$ along a spacelike displacement

Worldlines and the metric tensor

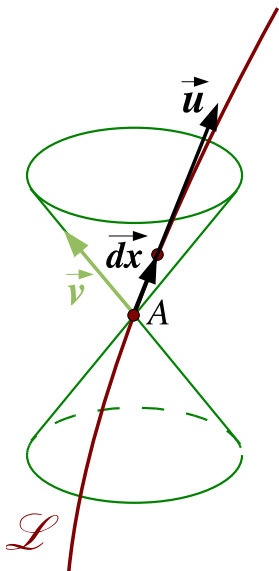


Physical interpretation of the metric tensor :

Length given by the metric tensor = **proper time** elapsed along a (massive) particle worldline

$$d\tau = \frac{1}{c} \sqrt{-g(d\vec{x}, d\vec{x})}$$

Worldlines and the metric tensor



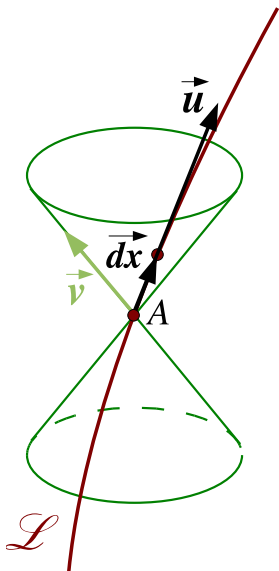
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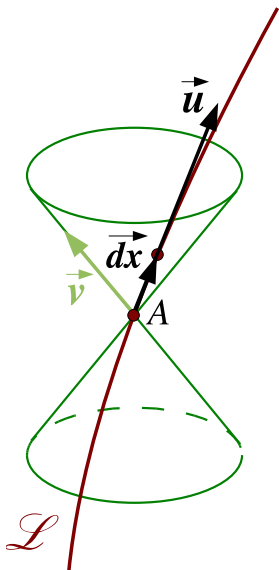
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4-velocity \vec{u} = unit timelike future-directed tangent to the worldline : $\vec{u} := \frac{1}{c} \frac{d\vec{x}}{d\tau}$, $g(\vec{u}, \vec{u}) = -1$

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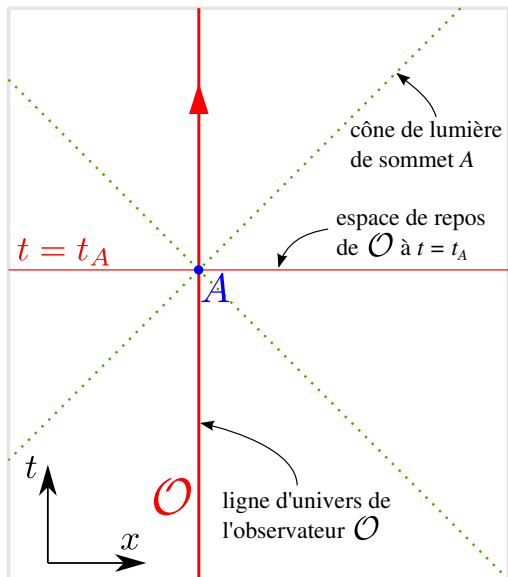
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Physical interpretation of the metric tensor (con't) :

The worldlines of photons are null geodesics of g

Diagrammes d'espace-temps

Minkowski (1909)



Intérêt des diag. d'espace-temps :

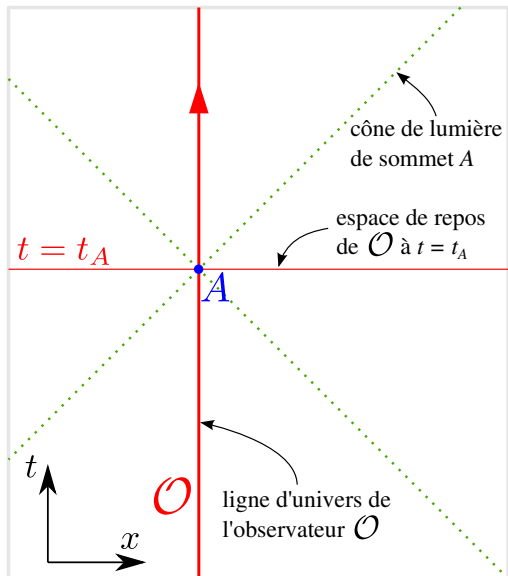
- description d'une particule dans sa globalité spatio-temporelle : **ligne d'univers**
- mise en exergue de la structure fondamentale de la relativité : les **cônes de lumière**

En un point A , le cône de lumière est formé par les lignes d'univers des photons arrivant ou issus de A .

Les cônes de lumière sont des structures *absolues*, indépendantes de tout observateur (traduction géométrique de l'invariance de c). Ils correspondent aux directions isotropes de la métrique d'espace-temps g .

Diagrammes d'espace-temps

Minkowski (1909)

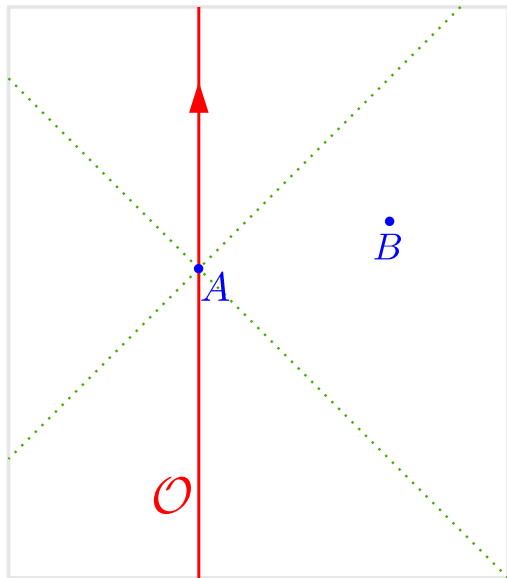


Observateur inertiel

La ligne d'univers de tout observateur inertiel est une droite située à l'intérieur de tout cône de lumière ayant son sommet sur elle.

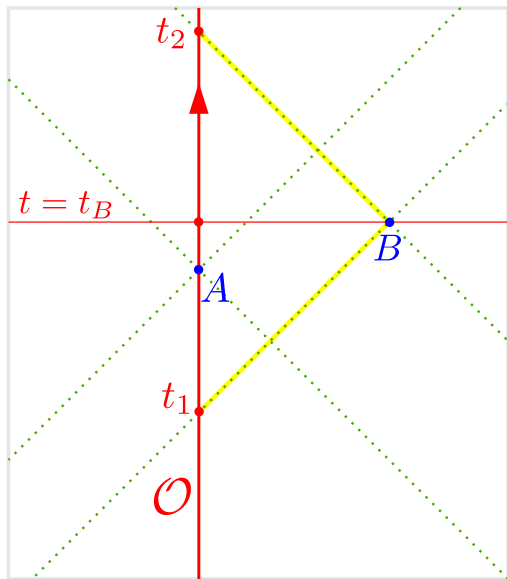
L'espace de repos de \mathcal{O} à $t = t_A$ est l'ensemble des événements simultanés à A du point de vue de \mathcal{O}

Simultanéité et espace de repos



Datation d'un événement distant B
par l'observateur \mathcal{O} ?

Simultanéité et espace de repos



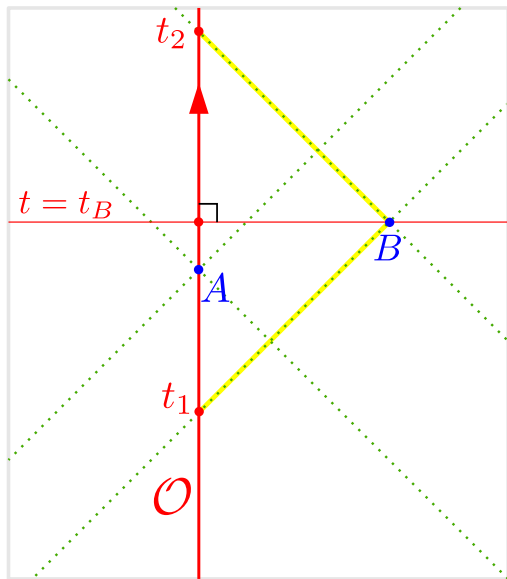
Datation d'un événement distant B
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Simultanéité d'Einstein-Poincaré

Aller-retour d'un signal lumineux de
 \mathcal{O} vers B : mesure de t_1 (départ) et
de t_2 (retour)

$$\Rightarrow t_B := \frac{1}{2} (t_1 + t_2)$$

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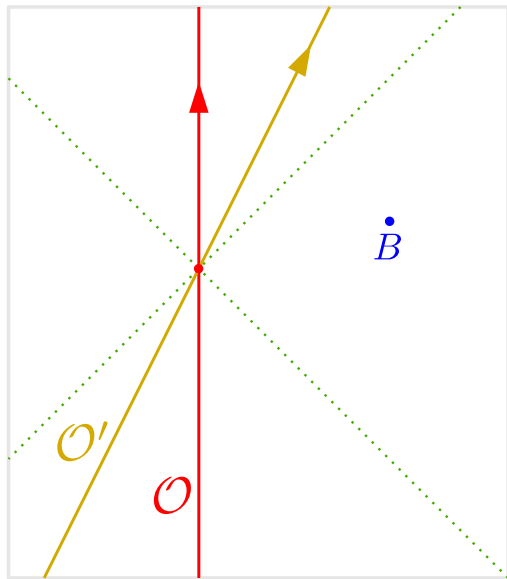
$$\Rightarrow t_B := \frac{1}{2} (t_1 + t_2)$$

Orthogonalité vis-à-vis du tenseur métrique

L'ensemble des événements simultanés à B pour \mathcal{O} est un espace de dimension 3 **orthogonal** (au sens du tenseur métrique g) à la ligne d'univers de \mathcal{O}

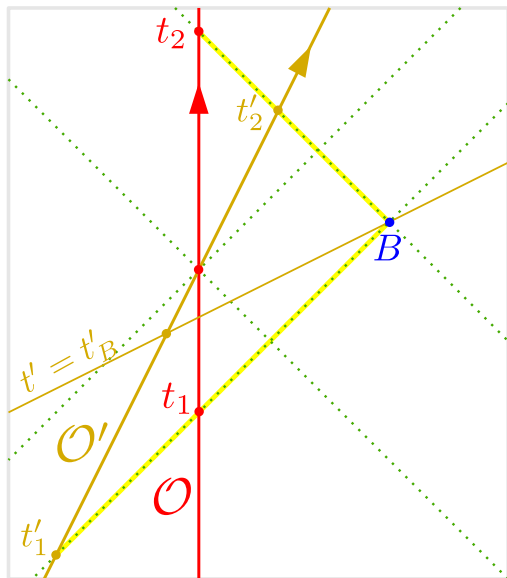
\Rightarrow **espace de repos** de \mathcal{O}

Simultanéité et espace de repos



Datation de B par un deuxième observateur O'

Simultanéité et espace de repos

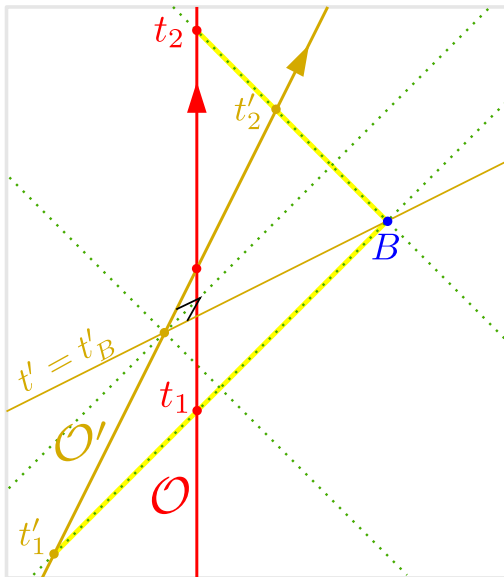


Datation de B par un deuxième observateur O'

Critère d'Einstein-Poincaré

$$\Rightarrow t'_B = \frac{1}{2} (t'_1 + t'_2)$$

Simultanéité et espace de repos



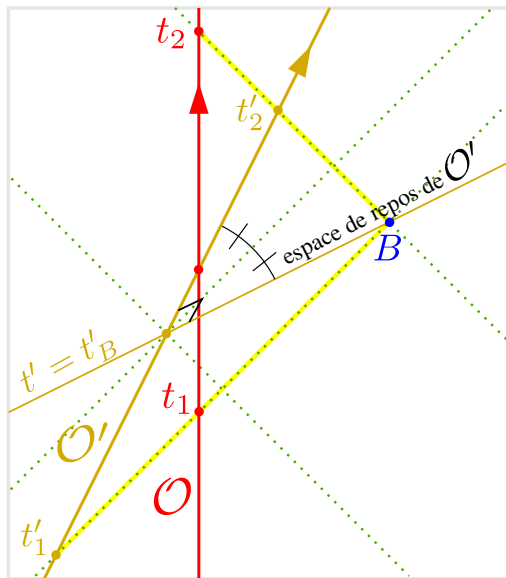
Datation de B par un deuxième observateur \mathcal{O}'

Critère d'Einstein-Poincaré

$$\Rightarrow t'_B = \frac{1}{2} (t'_1 + t'_2)$$

L'ensemble des événements simultanés à B pour \mathcal{O}' (espace de repos de \mathcal{O}') est un espace **orthogonal** (au sens de la métrique g) à la ligne d'univers de \mathcal{O}'

Simultanéité et espace de repos



Datation de B par un deuxième observateur O'

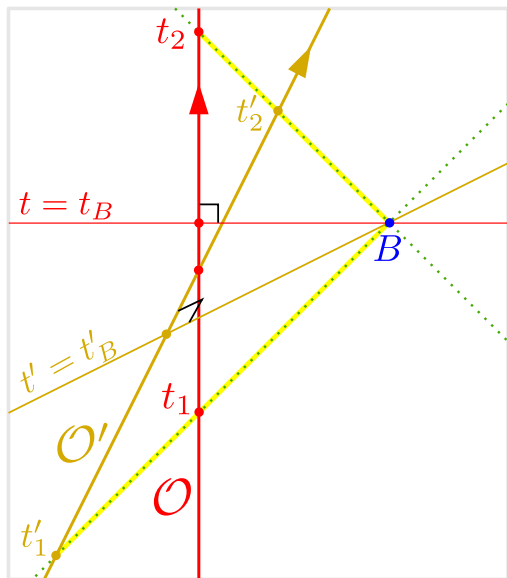
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L'ensemble des événements simultanés à B pour O' (espace de repos de O') est un espace **orthogonal** (au sens de la métrique g) à la ligne d'univers de O'

Graphiquement il s'obtient par symétrie de la ligne d'univers de O' par rapport à la bissectrice du quadrant ($t > 0, x > 0$)

Simultanéité et espace de repos

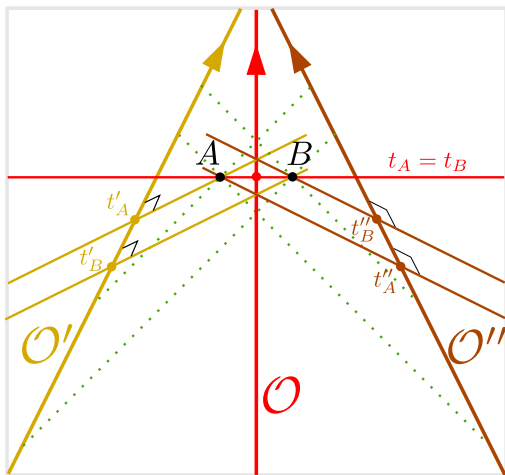


Les espaces de repos de \mathcal{O} et \mathcal{O}' ne coïncident pas :
relativité de la notion de simultanéité

Outline

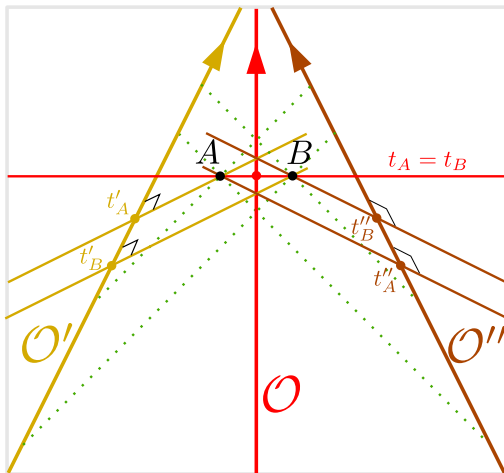
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Causality for spacelike-separated events



- for \mathcal{O} : $t_A = t_B$
- for \mathcal{O}' : $t'_A > t'_B$
- for \mathcal{O}'' : $t''_A < t''_B$

Causality for spacelike-separated events

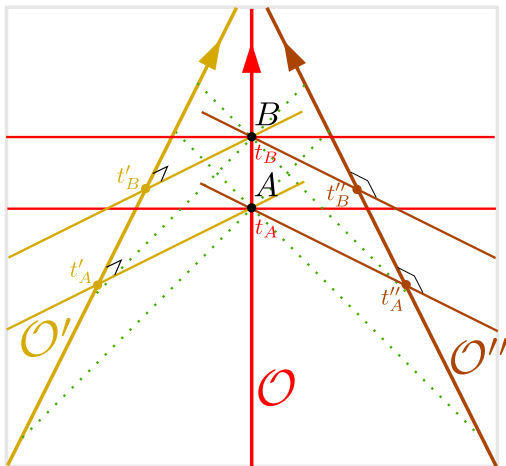


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\implies the order of occurrence of two spacelike-separated events depends upon the observer

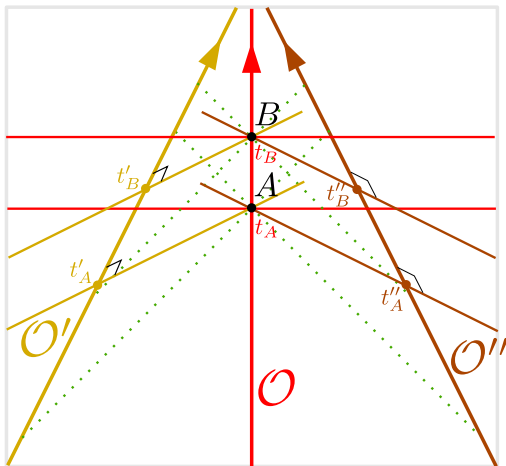
No causal relation possible between spacelike-separated events

Causality for timelike-separated events



- for $\mathcal{O} : t_A < t_B$
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Causality for timelike-separated events



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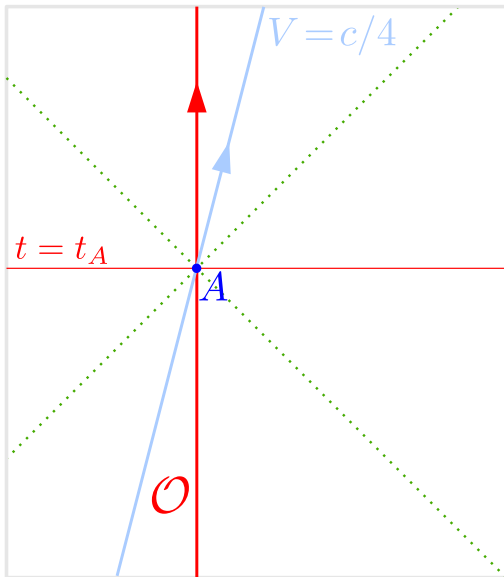
\implies all observers agree about the order of occurrence of two timelike-separated events

Causality well defined between timelike-separated events

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Définition des tachyons

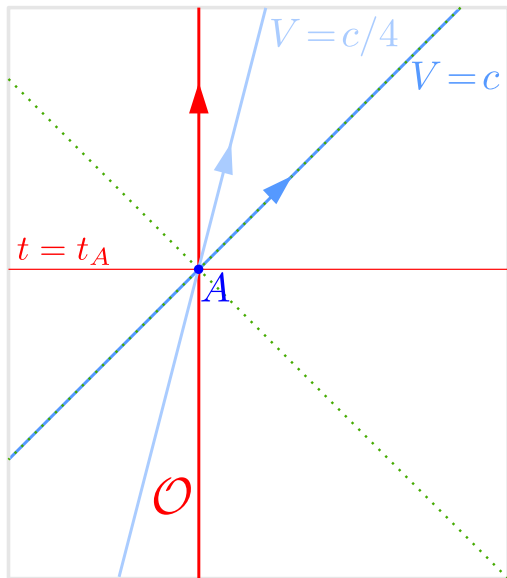


Classification des particules

- **bradyon** \equiv ligne d'univers du genre *temps*

← ci-contre, V désigne la vitesse par rapport à l'observateur \mathcal{O}

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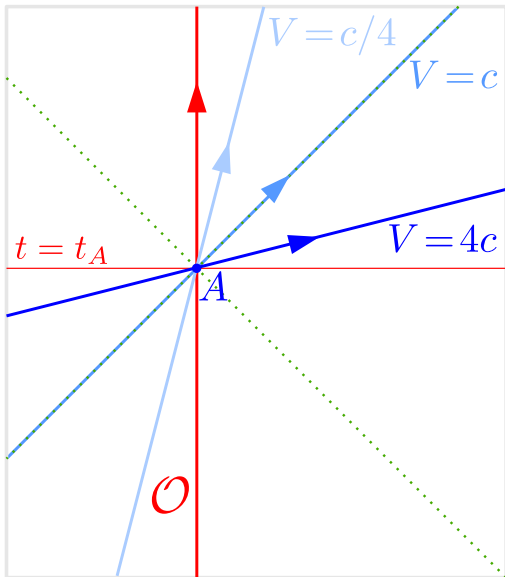


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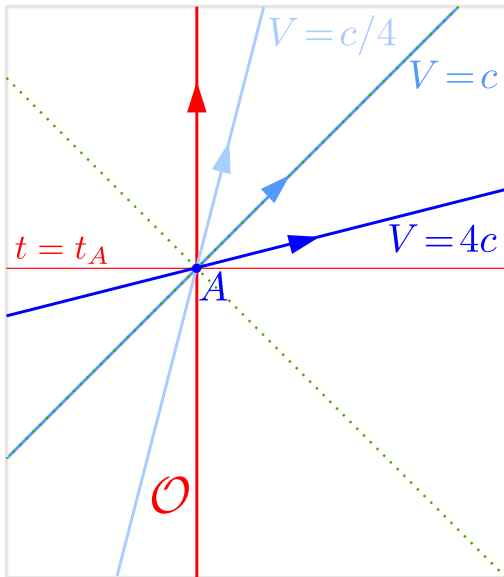


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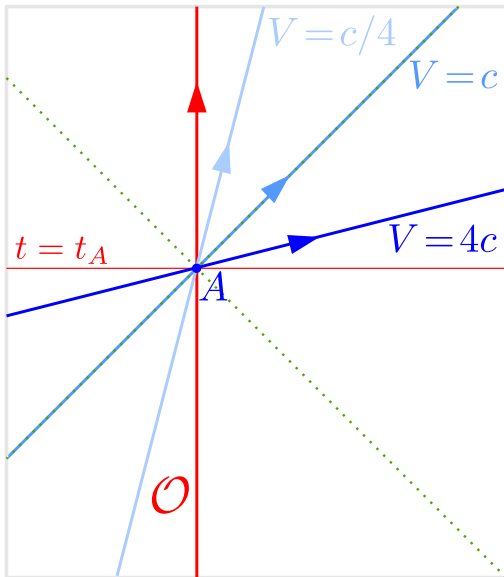


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- **bradyon** = particule massive ordinaire (proton, neutron, électron, quark, etc.)
 - **luxon** = particule de masse nulle (photon, graviton)

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Définition des tachyons



Classification des particules

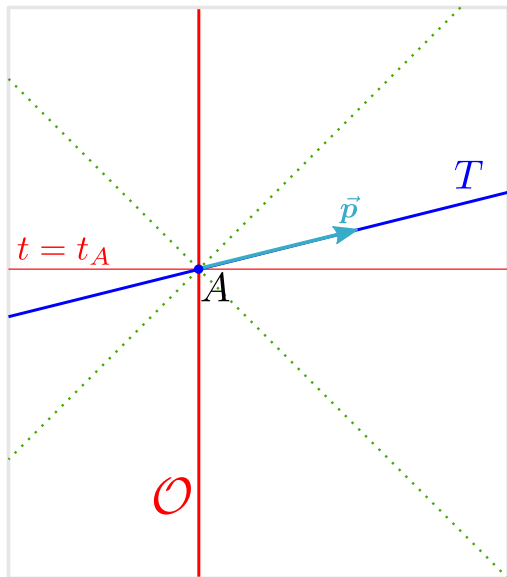
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Remarques :

- 1 Cette classification est indépendante de tout observateur
- 2 La ligne d'univers d'un tachyon est *partout* du genre espace

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Dynamique d'un tachyon

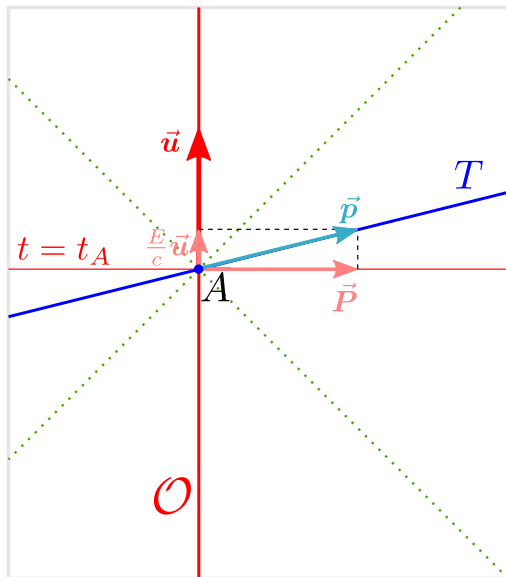


4-impulsion

Comme pour toute particule, la dynamique d'un tachyon T peut être décrite par un vecteur ${}^a \vec{p}$, appelé **4-impulsion**, défini en tout point de sa ligne d'univers et tangent à elle.

a. vecteur d'espace-temps (*quadrivecteur*)

Dynamique d'un tachyon



Énergie et quantité de mouvement par rapport à un observateur

L'énergie E et la quantité de mouvement \vec{P} mesurées par \mathcal{O} sont données par la décomposition orthogonale de la 4-impulsion :

$$\vec{p} = \frac{E}{c} \vec{u} + \vec{P}$$

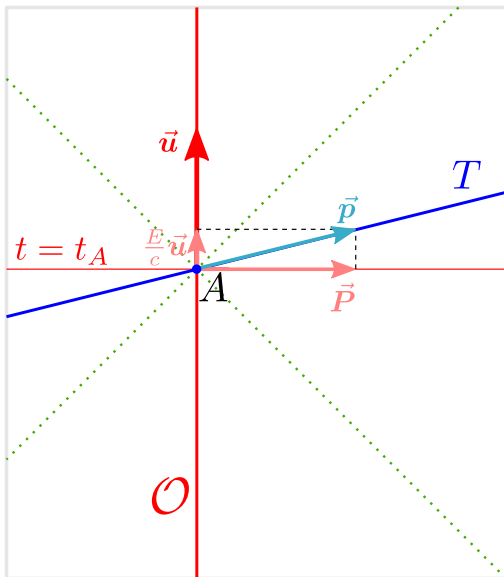
où \vec{u} est la 4-vitesse de \mathcal{O} .

Rappel : 4-vitesse : vecteur unitaire tangent à la ligne d'univers et dirigé vers le futur

Remarque : écriture équivalente :

$$p^\alpha = (E/c, P^x, P^y, P^z)$$

Masse d'un tachyon

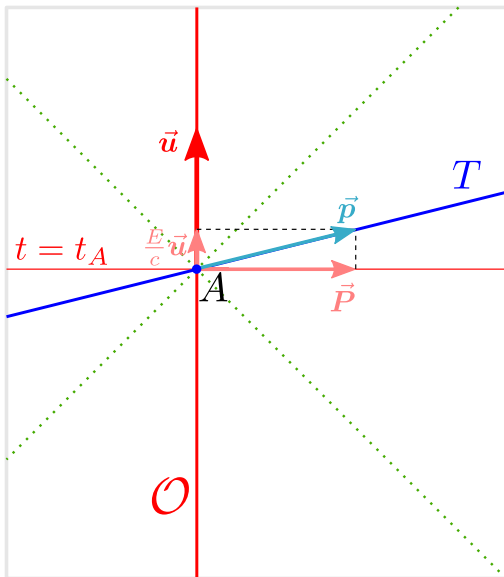


Masse

La masse m d'une particule est donnée par la norme de la 4-impulsion vis-à-vis du tenseur métrique^a :

$$m^2 c^2 = -\vec{p} \cdot \vec{p}$$

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Masse

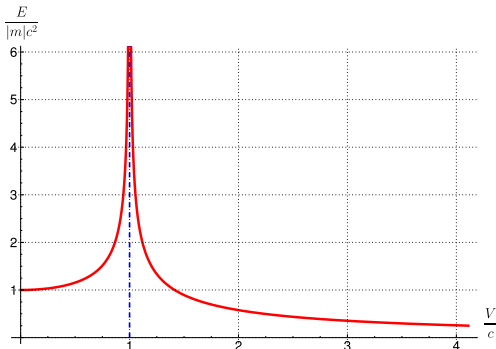
La masse m d'une particule est donnée par la norme de la 4-impulsion vis-à-vis du tenseur métrique^a :

$$m^2 c^2 = -\vec{p} \cdot \vec{p}$$

- bradyon $\iff \vec{p}$ genre temps
 $\iff m > 0$
- luxon $\iff \vec{p}$ genre lumière
 $\iff m = 0$
- tachyon $\iff \vec{p}$ genre espace
 $\iff m^2 < 0$
 $\iff m$ imaginaire

a. on utilise la signature $(-, +, +, +)$

Relation énergie-vitesse



Énergie d'un tachyon

La relation précédente se met sous la forme

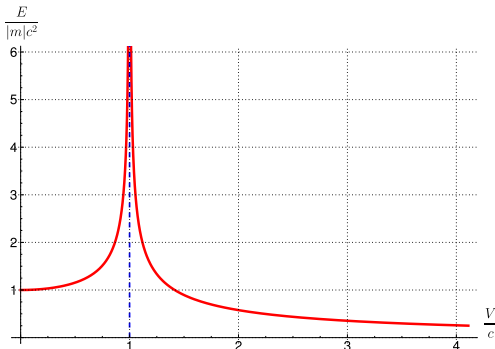
$$E = \frac{|m|c^2}{\sqrt{V^2/c^2 - 1}}$$

Remarque : On peut réécrire cette formule sous la forme usuelle :

$$E = \Gamma mc^2$$

avec $\Gamma := 1/\sqrt{1 - V^2/c^2}$ et m tous deux imaginaires.

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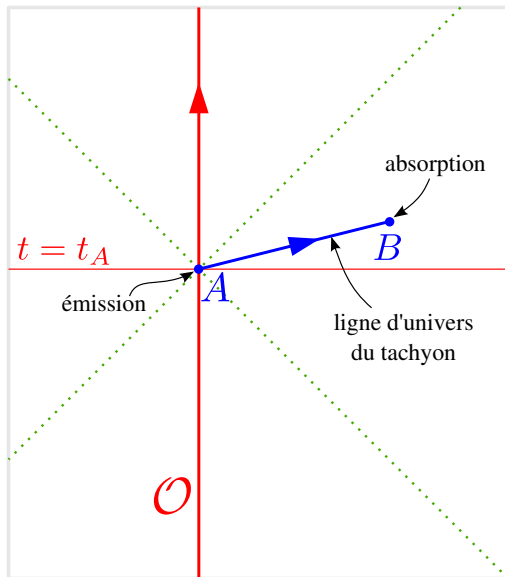
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L'énergie des tachyons est une fonction décroissante de la vitesse, avec $E \rightarrow 0$ lorsque $V \rightarrow +\infty$.

Tachyons et chronologie



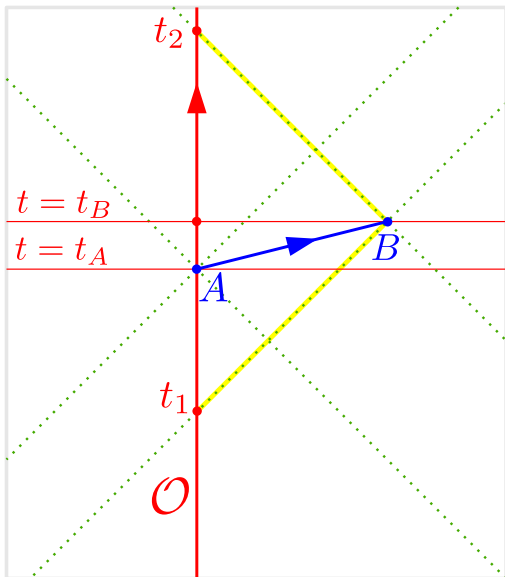
Émission et absorption d'un tachyon

- émission en A
- absorption en B

La ligne d'univers du tachyon est à l'extérieur du cône de lumière.

Dans le cas présent, la vitesse du tachyon par rapport à \mathcal{O} est $V = 4c$.

Tachyons et chronologie

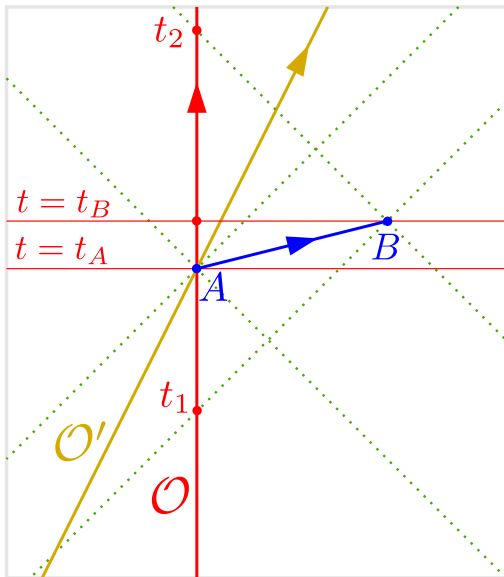


Instant de l'absorption mesuré par \mathcal{O}

$t_B = \frac{1}{2}(t_1 + t_2)$
(simultanéité d'Einstein-Poincaré)

$t_B > t_A$: l'absorption se produit
après l'émission

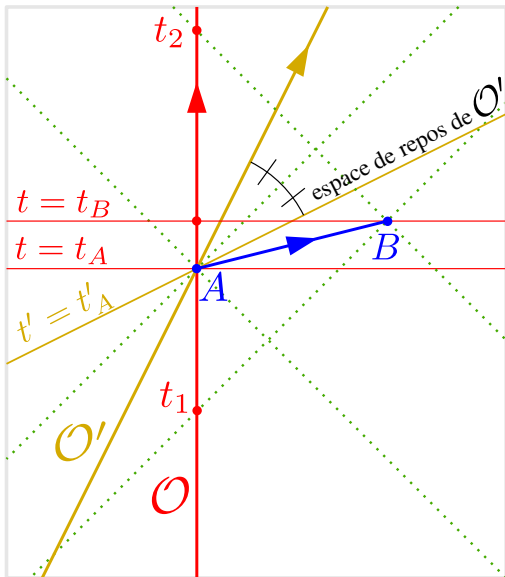
Tachyons et chronologie

Deuxième observateur \mathcal{O}'

\mathcal{O}' est en mouvement (subluminique) par rapport à \mathcal{O} .

Dans le cas présent, la vitesse de \mathcal{O}' par rapport à \mathcal{O} est $U = c/2$ et \mathcal{O}' rencontre \mathcal{O} au moment de l'émission du tachyon.

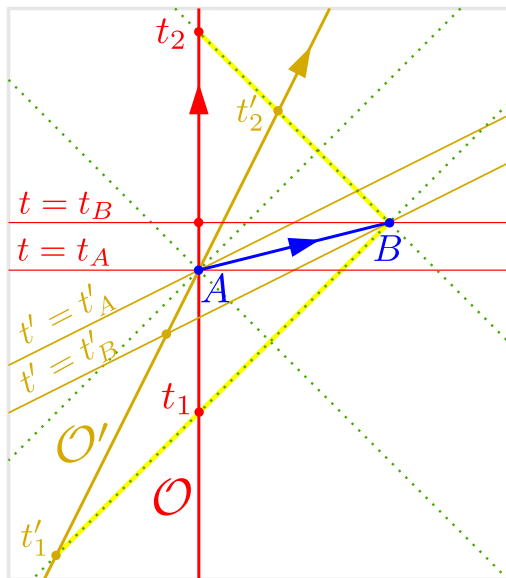
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Tachyons et chronologie



Le paradoxe (Einstein, 1907)

Pour O' , la date t'_B de l'événement B est obtenue en considérant l'espace de repos de O' passant par B .

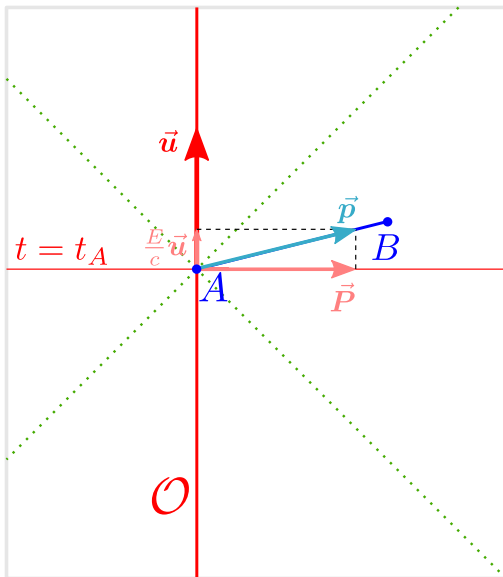
Numériquement, on l'obtient à partir de l'aller-retour d'un signal lumineux (critère d'Einstein-Poincaré) :

$$t'_B = \frac{1}{2}(t'_1 + t'_2)$$

On constate que $t'_B < t'_A$

Pour l'observateur O' , l'absorption du tachyon se produit avant son émission !

Tachyons et chronologie



Résolution du paradoxe (1/4)

Introduisons la 4-impulsion \vec{p} du tachyon.

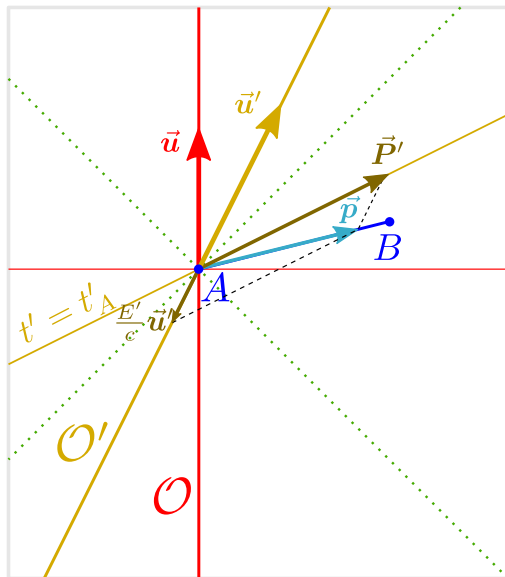
L'énergie E et la quantité de mouvement \vec{P} mesurées par \mathcal{O} sont données par la décomposition orthogonale de \vec{p} :

$$\vec{p} = \frac{E}{c} \vec{u} + \vec{P},$$

où \vec{u} est la 4-vitesse de \mathcal{O} .

On constate que $E > 0$.

Tachyons et chronologie



Résolution du paradoxe (2/4)

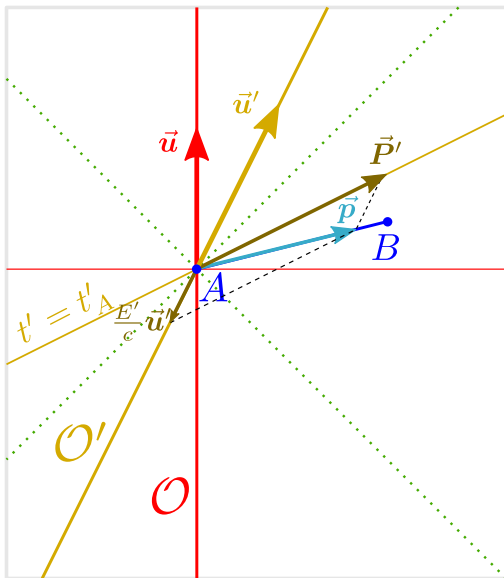
Point de vue de l'observateur O' :
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$$\vec{p} = \frac{E'}{c} \vec{u}' + \vec{P}',$$

où \vec{u}' est la 4-vitesse de O' .

On constate que $E' < 0$.

Tachyons et chronologie



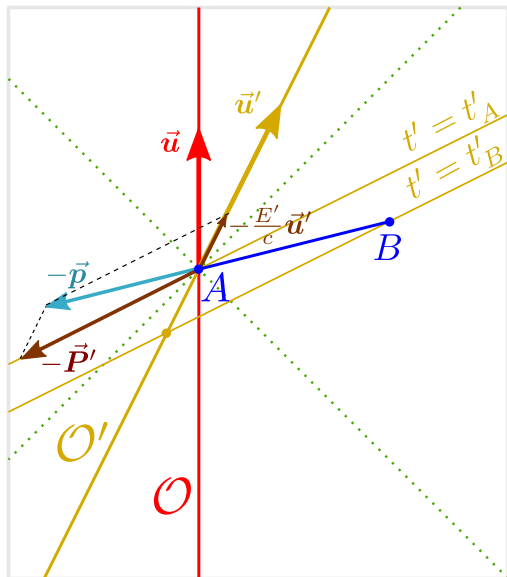
Résolution du paradoxe (3/4)

Point de vue de l'observateur O' :
 Le tachyon apparaît comme une particule d'énergie *négative* qui remonte le temps (puisque $t'_B < t'_A$)

Interprétation de Stueckelberg-Feynman : une telle particule est équivalente à une particule d'énergie *positive* qui va de B vers A (antiparticule).

[Bilaniuk, Deshpande, & Sudarshan, Amer. J. Phys. 30, 718 (1962)]

Tachyons et chronologie

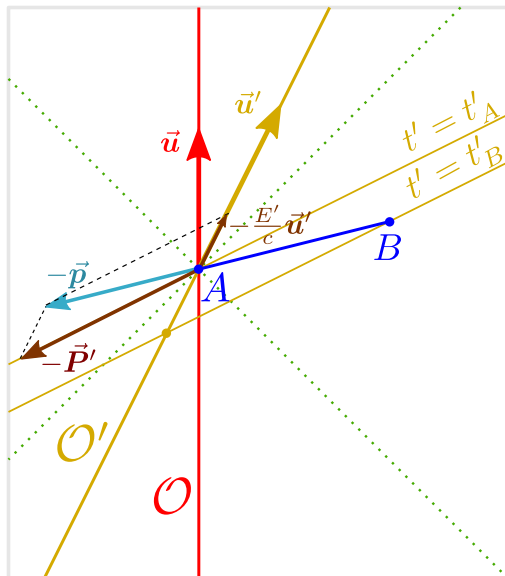


Résolution du paradoxe (4/4)

	\mathcal{O}	\mathcal{O}'
A	émission ↓	absorption ↑
B	absorption	émission

Pour chaque observateur, l'émission précède l'absorption : la causalité est préservée !

Tachyons et chronologie



Résolution du paradoxe (4/4)

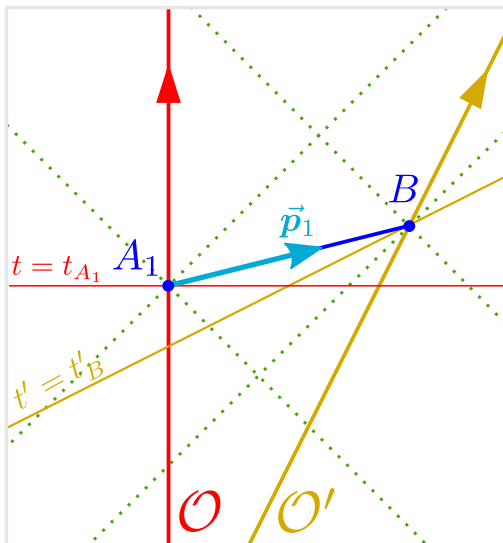
	O	O'
A	émission	absorption
	↓	↑
B	absorption	émission

Pour chaque observateur, l'émission précède l'absorption : la **causalité est préservée!**

Les concepts d'émission et d'absorption d'un tachyon sont donc relatifs à l'observateur.

Il n'y a que les **lois physiques** qui doivent être invariantes par changement d'observateur inertiel,

Anti-téléphone

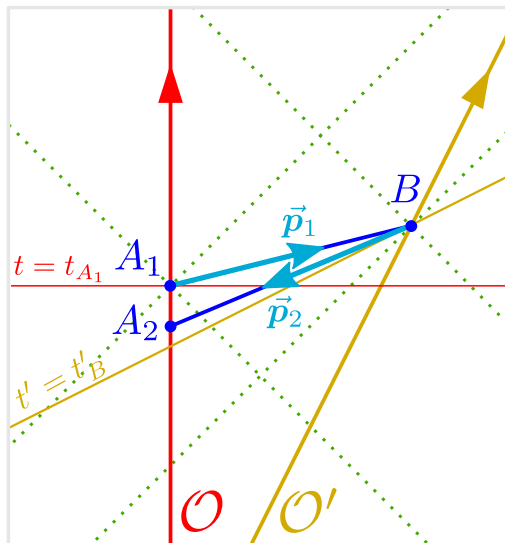


Version proposée par Feinberg (1967) :

Principe

L'observateur inertiel \mathcal{O} émet un tachyon (d'énergie positive) en A_1 qui atteint l'observateur \mathcal{O}' en B .

Anti-téléphone



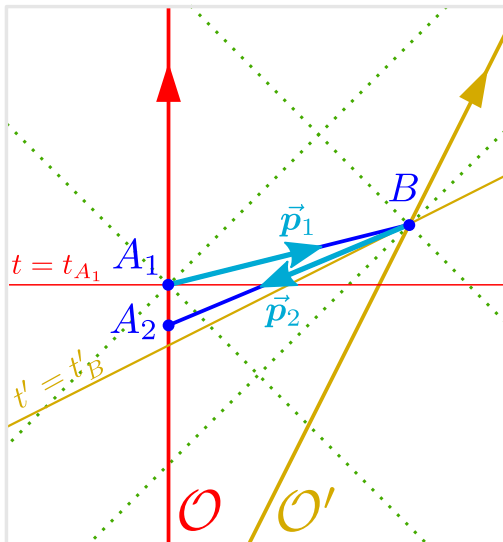
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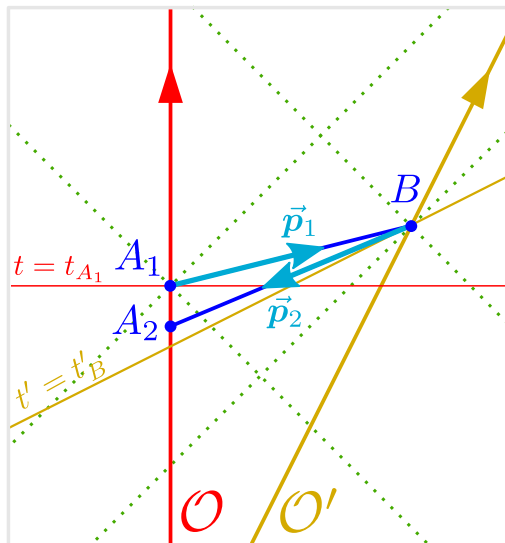
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Le paradoxe

$t_{A_2} < t_{A_1}$: la réponse de \mathcal{O}' arrive avant l'appel de \mathcal{O} !

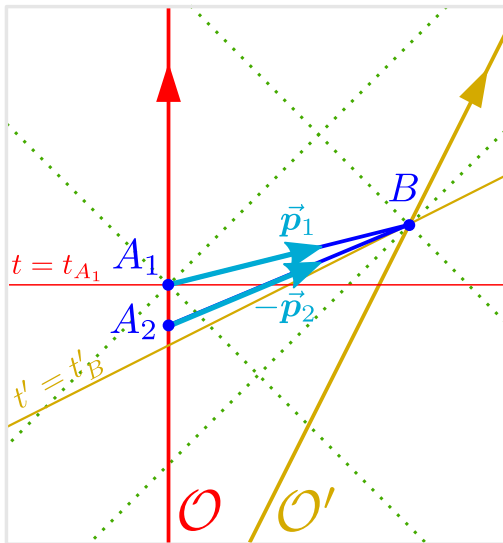
Anti-téléphone



Résolution du paradoxe

Pour O le tachyon qui arrive en A_2 a une énergie négative et remonte le temps

Anti-téléphone



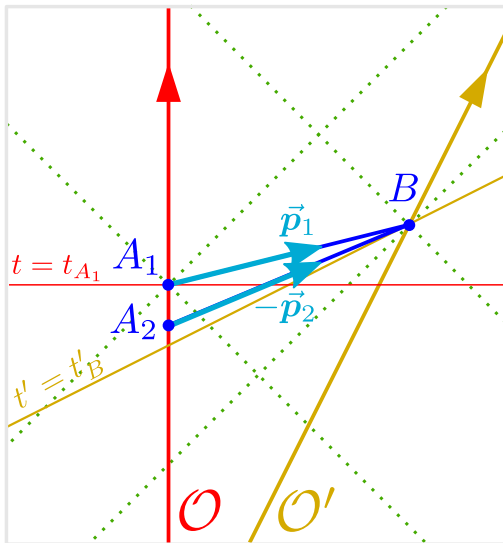
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$\Rightarrow \mathcal{O}$ considère donc plutôt qu'il émet un tachyon en A_2

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Anti-téléphone



Résolution du paradoxe

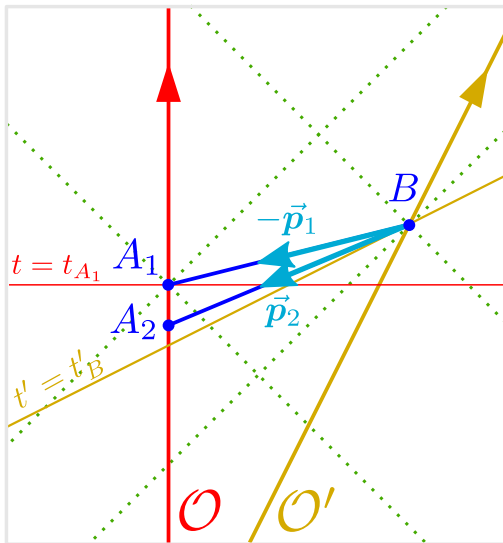
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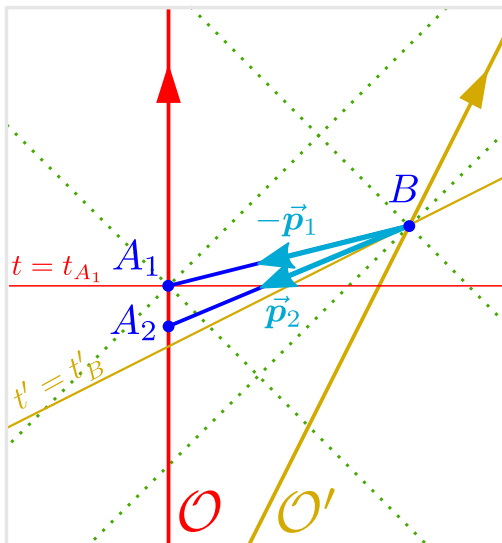
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Chaque observateur ne fait qu'émettre des tachyons \Rightarrow **pas d'échange d'information**

Outline

- 1 The relativistic spacetime
- 2 Causality in Minkowski spacetime
- 3 An illustration with tachyons
- 4 Causality in curved spacetime : the Schwarzschild black hole**
- 5 More causality features in spacetime : the Kerr black hole
- 6 Conclusions

Spacetime dynamics

- **Special relativity** : metric tensor $g = \text{fixed}$ bilinear form on the vector space $\sim \mathbb{R}^4$ associated with the spacetime affine space
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$$\text{Einstein equation : } \mathbf{R} - \frac{1}{2}Rg = \frac{8\pi G}{c^4} \mathbf{T}$$

- \mathbf{R} = Ricci tensor = symmetric bilinear form = trace of *curvature tensor* (Riemann tensor) : " $\mathbf{R} \sim g \partial^2 g + g \partial g \partial g$ "
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 - $E = \mathbf{T}(\vec{u}, \vec{u})$ is the energy density of matter as measured by an observer \mathcal{O} of 4-velocity \vec{u}
 - $p_i = -\mathbf{T}(\vec{u}, \vec{e}_i)$ component i of the matter momentum density as measured by \mathcal{O} in the direction \vec{e}_i
 - $S_{ij} = \mathbf{T}(\vec{e}_i, \vec{e}_j)$ component i of the force exerted by matter on the unit surface normal to \vec{e}_j

Birth of general relativity : Nov. 1915

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

VON A. EINSTEIN.

In zwei vor kurzem erschienenen Mitteilungen¹ habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariablen gegenüber kovariant sind.

$$R - \frac{1}{2}Rg = \frac{8\pi G}{c^4} T$$

[A. Einstein, Sitz. Preuss. Akad. Wissenschaften Berlin, 844 (1915)]

The Schwarzschild solution (1915)

Karl Schwarzschild (letter to Einstein 22 Dec. 1915; publ. submitted 13 Jan 1916)
Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie,
 Sitz. Preuss. Akad. Wiss., Phys. Math. Kl. 1916, 189 (1916)

⇒ First exact non-trivial solution of Einstein equation :

$$ds^2 = - \left(1 - \frac{2m}{r}\right) c^2 dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (1)$$

with

- coordinates¹ $(t, \bar{r}, \theta, \varphi)$
- “auxiliary quantity” : $r := (\bar{r}^3 + 8m^3)^{1/3}$
- parameter $m = GM/c^2$, with M gravitational mass of the “mass point”

1. Schwarzschild's notations : $r = \bar{r}$, $R = r$, $\alpha = 2m$

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The “center”

Origin of coordinates : $\bar{r} = 0 \iff r = 2m$

1. Schwarzschild's notations : $r = \bar{r}$, $R = r$, $\alpha = 2m$

Droste contribution (1916)

Johannes Droste (communication 27 May 1916)

The Field of a Single Centre in Einstein's Theory of Gravitation, and the Motion of a Particle in that Field, Kon. Neder. Akad. Weten. Proc. **19**, 197 (1917)

⇒ derives the Schwarzschild solution (independently of Schwarzschild) via some coordinates (t, r', θ, φ) such that $g_{r'r'} = 1$; presents the result in the standard form (1) via a change of coordinates leading to the areal radius r

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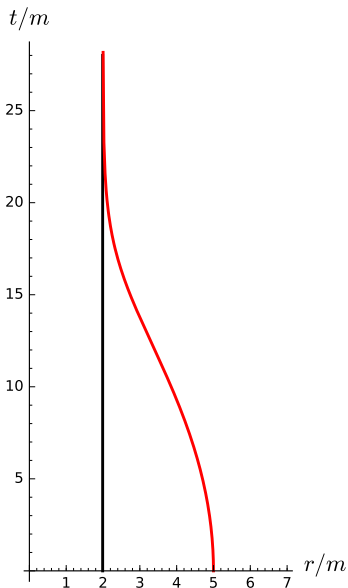
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Apparent barrier at $r = 2m$

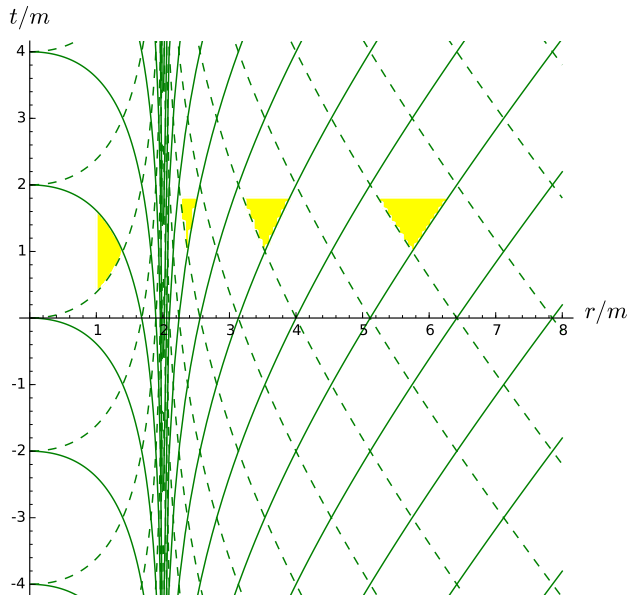
A particle falling from infinity never reaches $r = 2m$ within a finite amount of "time" t .

The Schwarzschild radius : $R_S := 2m = \frac{2GM}{c^2}$

The “barrier” at $r = R_S$ 

Free fall trajectory (timelike geodesic) starting from rest at $r = 5m$ in Schwarzschild spacetime, in term of **Schwarzschild-Droste coordinates** (t, r)

$$\text{Schwarzschild radius : } R_S := 2m = \frac{2GM}{c^2}$$

The “barrier” at $r = R_S$ 

Radial null geodesics of Schwarzschild spacetime in term of **Schwarzschild-Droste coordinates** (t, r) . Solid (resp. dashed) lines correspond to outgoing (resp. ingoing) geodesics. The interiors of some future light cones are depicted in yellow.

The Schwarzschild solution : early discussions

- 1920 : Alexander Anderson : light cannot emerge from the region $r < R_S := 2m = \frac{2GM}{c^2}$ (region “shrouded in darkness”)

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- **1923 : George Birkhoff** : outside any *spherical* body, the metric is Schwarzschild metric
- **1924 : Arthur Eddington** introduced the coord. $t' := t - \frac{2m}{c} \ln\left(\frac{r}{2m} - 1\right)$, leading to

$$ds^2 = -c^2 dt'^2 + dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) + \frac{2m}{r} (cdt' - dr)^2 \quad (2)$$

but did not noticed that the metric components w.r.t. coordinates (t', r, θ, φ) are regular at $r = 2m$!

Actually, Eddington's aim was elsewhere : comparing Whitehead theory (1922) to general relativity

The Schwarzschild solution : Lemaître breakthrough

Georges Lemaître (1932)

L'univers en expansion, Publ. Lab. Astron. Géodésie Univ. Louvain **9**, 171 (1932);
reprinted in Ann. Soc. Scient. Bruxelles A **53**, 51 (1933)

et la nouvelle forme du champ s'écrit sans singularité

$$(11.12) \quad ds^2 = -2m \frac{d\chi^2}{r} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) + dt^2,$$

où

$$(11.13) \quad r = \left[\frac{3}{2} \sqrt{2m} (t - \chi) \right]^{\frac{2}{3}}$$

La singularité du champ de Schwarzschild est donc une singularité fictive, analogue à celle qui se présentait à l'horizon du centre dans la forme originale de l'univers de de Sitter.

The singularity at $r = R_S$ is a mere **coordinate singularity** : the metric components are regular in Lemaître coordinates $(\tau, \chi, \theta, \varphi)$:

$$ds^2 = -c^2 d\tau^2 + \frac{R_S}{r} d\chi^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (3)$$

$$r = r(\tau, \chi) := \left[\frac{3}{2} \sqrt{R_S} (c\tau - \chi) \right]^{2/3} \quad (4)$$

Coordinate and curvature singularities

Coordinate singularity

- pathology (divergence) of the metric components $g_{\alpha\beta}$ in some coordinate system (x^α) , which can be removed by a change of coordinates
- no physical meaning
- not a singularity of spacetime
- **example** : singularity at $r = R_S$ of the Schwarzschild-Droste coordinates

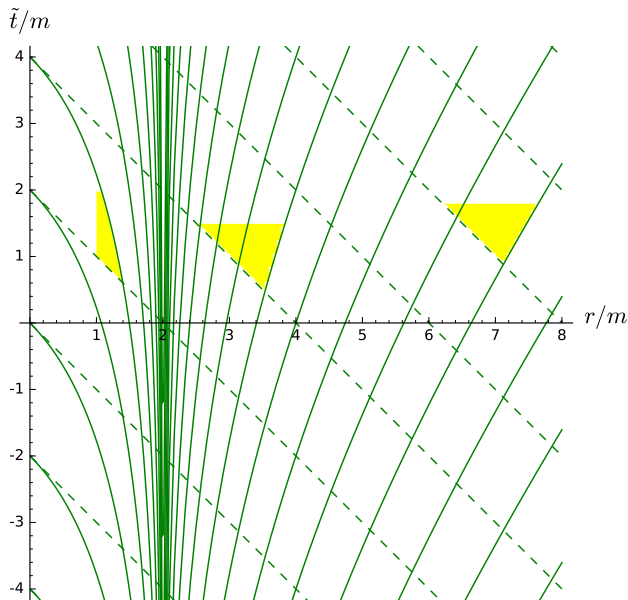
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Curvature singularity

- divergence of the Riemann curvature tensor
- can be monitored by the unbounded growth of scalar quantities, like the Kretschmann scalar $\mathcal{K} := R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, which are independent of any coordinate system
- represents a true singularity of *classical* spacetime (should be removed by a *quantum* theory of gravity)
- **example** : singularity at $r = 0$ in Schwarzschild spacetime : $\mathcal{K} = \frac{48m^2}{r^6}$

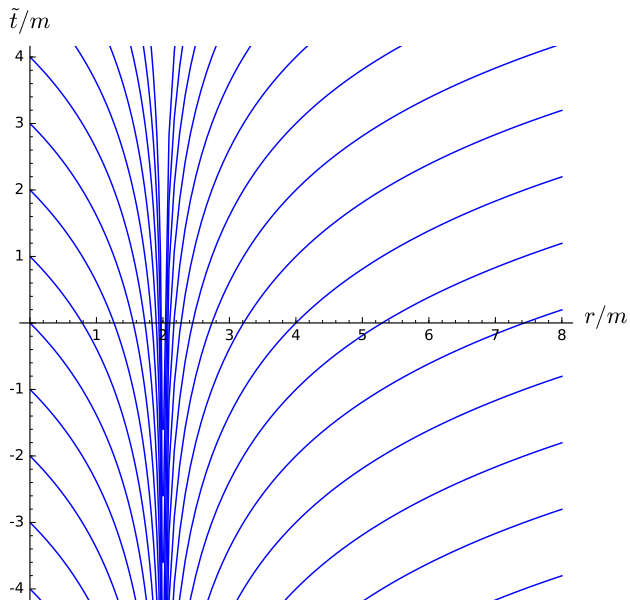
No longer any barrier at $r = R_S$ 

Radial null geodesics of Schwarzschild spacetime in term of **ingoing Eddington-Finkelstein coordinates** (\tilde{t}, r)

$$\tilde{t} = t + \frac{2m}{c} \ln \left| \frac{r}{2m} - 1 \right|$$

The ingoing null geodesics (dashed lines) do enter the region $r < R_S$.

Pathology of Schwarzschild-Droste coordinates



Hypersurfaces of constant Schwarzschild-Droste coordinate t in term of the ingoing Eddington-Finkelstein coordinates (\tilde{t}, r)

Gravitational collapse : Lemaître-Tolman solutions

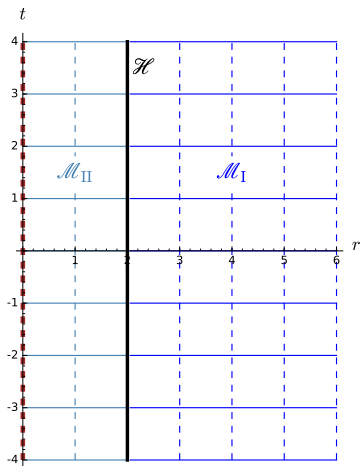
- 1932 : Georges Lemaître : general solution of Einstein equation for a spherically symmetric pressureless fluid \implies gravitational collapse

Gravitational collapse : Lemaître-Tolman solutions

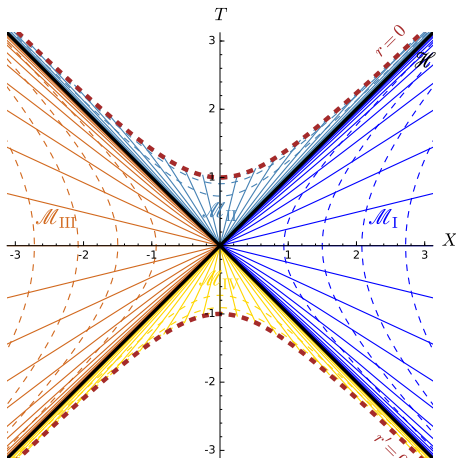
- 1932 : Georges Lemaître : general solution of Einstein equation for a spherically symmetric pressureless fluid \implies gravitational collapse
- 1939 : Robert Oppenheimer & Hartland Snyder : gravitational collapse of a homogeneous dust ball of radius R (special case of Lemaître's general solution)
 - \implies for an external observer, $R \rightarrow R_S$ as $t \rightarrow +\infty$
 - \implies "frozen star"

The Schwarzschild solution : the complete picture

John L. Synge (1950), Martin Kruskal (1960), George Szekeres (1960) : complete mathematical description of Schwarzschild spacetime ($\mathbb{R}^2 \times \mathbb{S}^2$ manifold)



Schwarzschild-Droste coordinates (t, r)



Carter-Penrose diagram of Schwarzschild spacetime

conformal diagram : light rays as $\pm 45^\circ$ lines \implies makes causal structure clear

compactified diagram : all spacetime is depicted

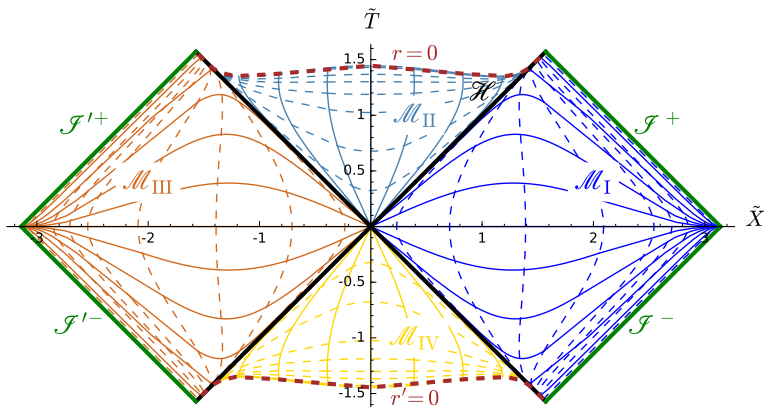
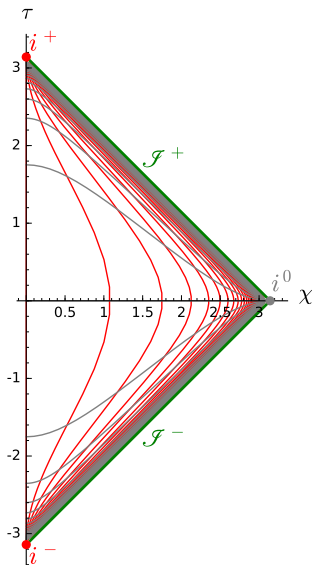


Figure drawn with SageMath : <http://sagemanifolds.obspm.fr>

solid curves : $t = \text{const}$; dashed curves : $r = \text{const}$

\mathcal{M}_I : asymptotically flat region; \mathcal{M}_{II} : **black hole** region

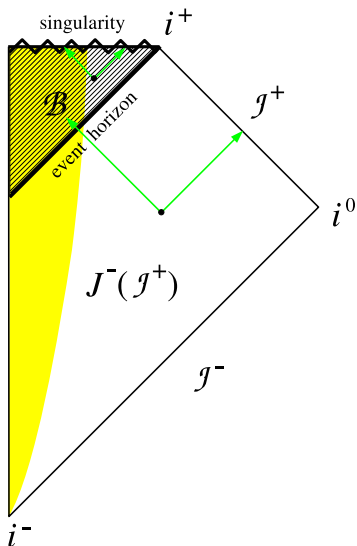
Same conformal diagram for Minkowski spacetime



Minkowski spacetime : flat spacetime of special relativity

- grey curves : $t = \text{const}$
- red curves : $r = \text{const}$
- \mathcal{I}^+ : future null infinity
- \mathcal{I}^- : past null infinity
- i^+ : future timelike infinity
- i^- : past timelike infinity
- i^0 : spacelike infinity

General definition of a black hole



The textbook definition

[Hawking & Ellis (1973)]

black hole : $\mathcal{B} := \mathcal{M} - J^-(\mathcal{I}^+)$

where

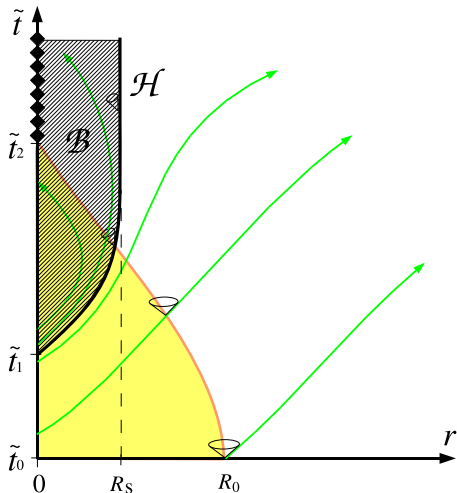
- (\mathcal{M}, g) = asymptotically flat manifold
- \mathcal{I}^+ = future null infinity
- $J^-(\mathcal{I}^+)$ = causal past of \mathcal{I}^+

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : $\mathcal{H} := \partial J^-(\mathcal{I}^+)$
(boundary of $J^-(\mathcal{I}^+)$)

\mathcal{H} smooth $\implies \mathcal{H}$ null hypersurface

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Rotation enters the game : the Kerr solution

Almost 50 years after Schwarzschild : **Roy Kerr (1963)**

$$ds^2 = - \left(1 - \frac{2mr}{\rho^2} \right) dv^2 + 2dv dr - \frac{4amr \sin^2 \theta}{\rho^2} dv d\tilde{\varphi} \\ - 2a \sin^2 \theta dr d\tilde{\varphi} + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2mr \sin^2 \theta}{\rho^2} \right) \sin^2 \theta d\tilde{\varphi}^2.$$

Boyer & Lindquist (1967) coordinate change $(v, r, \theta, \tilde{\varphi}) \rightarrow (t, r, \theta, \varphi)$:

$$ds^2 = - \left(1 - \frac{2mr}{\rho^2} \right) dt^2 - \frac{4amr \sin^2 \theta}{\rho^2} dt d\varphi + \frac{\rho^2}{\Delta} dr^2 \\ + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2mr \sin^2 \theta}{\rho^2} \right) \sin^2 \theta d\varphi^2,$$

where $\rho^2 := r^2 + a^2 \cos^2 \theta$, $\Delta := r^2 - 2mr + a^2$ and $r \in (-\infty, \infty)$

→ spacetime manifold $\mathcal{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \{r = 0 \ \& \ \theta = \pi/2\}$

→ 2 parameters : $m = \frac{GM}{c^2}$ and $a = \frac{J}{cM}$; black hole $\iff 0 \leq a \leq m$

→ Schwarzschild metric for $a = 0$

Physical meaning of the parameters M and J

- **mass M** : *not* a measure of the “amount of matter” inside the black hole, but rather a *characteristic of the external gravitational field*
→ measurable from the orbital period of a test particle in far circular orbit around the black hole (*Kepler's third law*)

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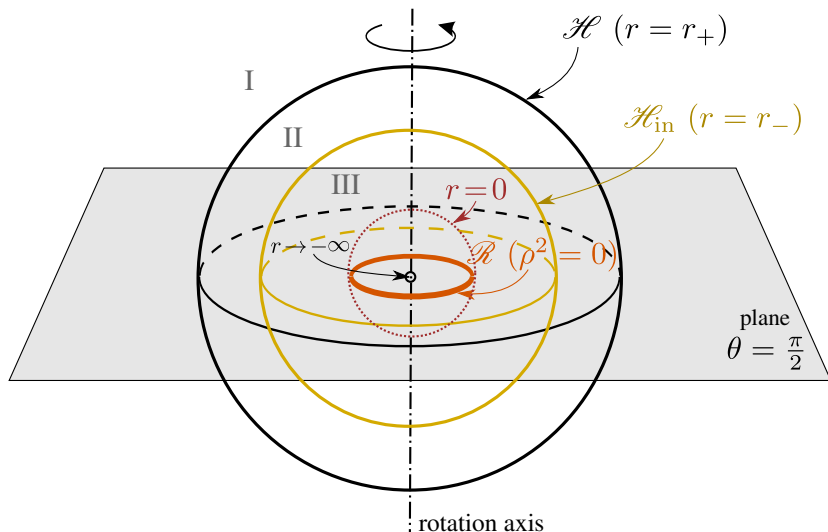
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Remark : the **radius** of a black hole is not a well defined concept : it *does not* correspond to some distance between any black hole “centre” and the event horizon. A well defined quantity is the **area** of the event horizon, A .
 The “radius” can be defined from it : for a Schwarzschild black hole :

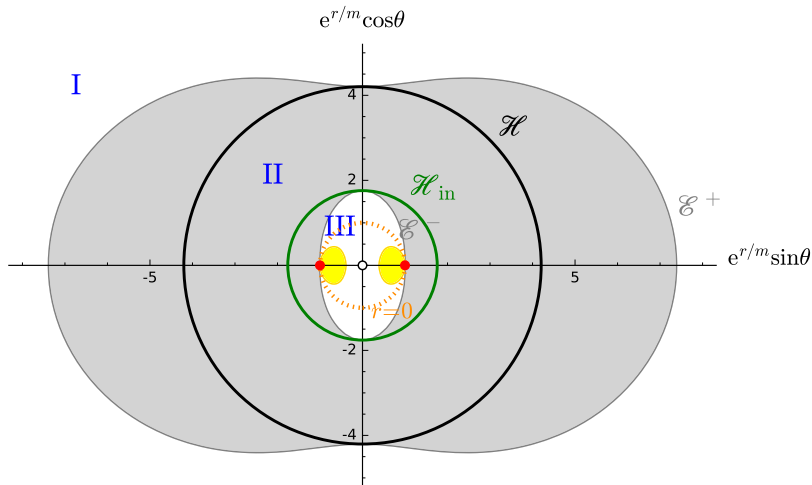
$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left(\frac{M}{M_{\odot}} \right) \text{ km}$$

Kerr spacetime



Slice $t = \text{const}$ of the Kerr spacetime viewed in O'Neill coordinates (R, θ, φ) , with $R := e^r$, $r \in (-\infty, +\infty)$.

Kerr spacetime : Carter time machine

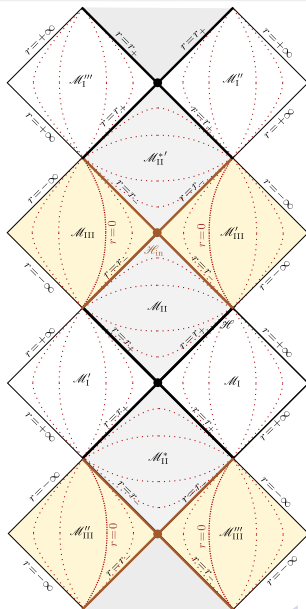


Meridional view of a section $t = \text{const}$ of Kerr spacetime with $a/m = 0.90$

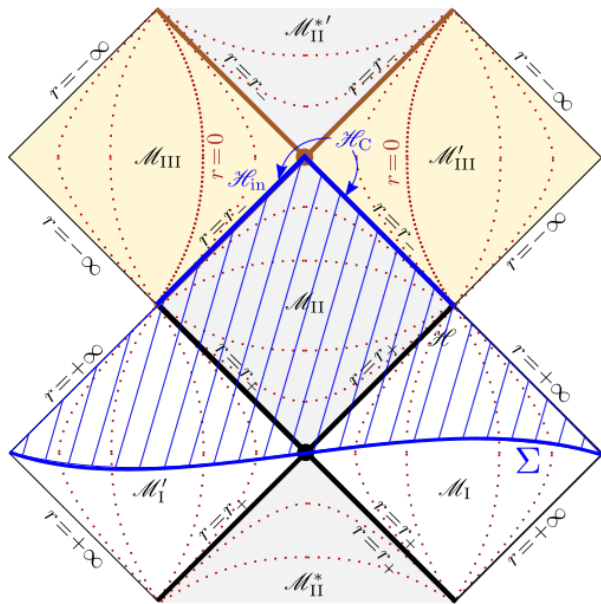
Carter time machine : yellow region, contains **closed timelike curves**

\Rightarrow **causality violation in all region III**

Carter-Penrose diagram of the maximal analytic extension

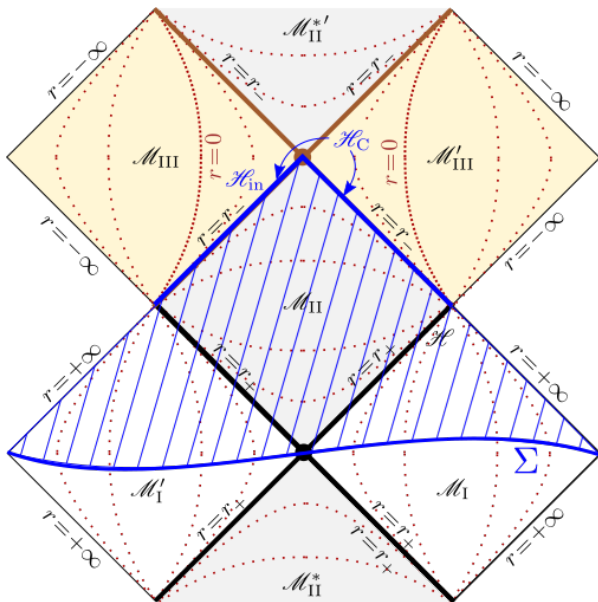


Cauchy horizon in Kerr spacetime



Cauchy problem
 Compute evolution of a physical system from initial data

Cauchy horizon in Kerr spacetime



Cauchy problem

Compute evolution of a physical system from initial data

The **Cauchy horizon** \mathcal{H}_C is a **predictability boundary** : physical phenomena taking place in regions \mathcal{M}_{III} and \mathcal{M}'_{III} cannot be predicted from only initial data on the spacelike hypersurface Σ (**Cauchy surface** for regions \mathcal{M}_I , \mathcal{M}'_I and \mathcal{M}_{II} only)

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Causal boundaries are null hypersurfaces (3-dimensional subspaces ruled by null geodesics), which are 1-way *immaterial* membranes :

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As shown by the example of the *Carter time machine* in Kerr spacetime, general relativity by itself does not forbid the existence of **closed timelike curves**.

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