

Black holes, a centenary after the birth of general relativity

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- 1 Black holes : a century-old history
- 2 Some recent developments
- 3 Black holes in the sky
- 4 Observing black holes via gravitational waves : a dream come true
- 5 Testing general relativity with black holes

Outline

- 1 Black holes : a century-old history
- 2 Some recent developments
- 3 Black holes in the sky
- 4 Observing black holes via gravitational waves : a dream come true
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A two centuries-old prehistory...

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John Michell (1784)

"If there should really exist in nature any bodies, whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us, ..., we could have no information from sight"

[Phil. Trans. R. Soc. Lond. 74, 35 (1784)]

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Pierre Simon de Laplace (1796)

"Un astre lumineux, de la même densité que la Terre, et dont le diamètre serait 250 fois plus grand que le Soleil, ne permettrait, en vertu de son attraction, à aucun de ses rayons de parvenir jusqu'à nous. Il est dès lors possible que les plus grands corps lumineux de l'univers puissent, par cette cause, être invisibles."

[Exposition du système du monde (1796)]

Limits of the Newtonian concept of a black hole

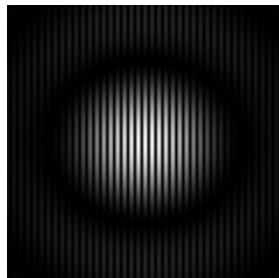
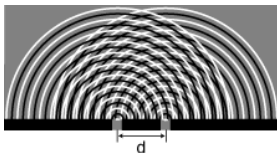
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 \implies a *relativistic* theory of gravitation is necessary !

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- $V_{\text{esc}} \sim c \implies$ gravitational potential energy \sim mass energy Mc^2
 \implies a *relativistic* theory of gravitation is necessary !
- No clear action of the gravitation field on electromagnetic waves in Newtonian gravity



[R. Taillet]

101 years ago : a relativistic theory of gravitation

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

Die Feldgleichungen der Gravitation.

VON A. EINSTEIN.

In zwei vor kurzem erschienenen Mitteilungen¹ habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen Fassung beliebigen Substitutionen der Raumzeitvariablen gegenüber kovariant sind.

$$R - \frac{1}{2}Rg = \frac{8\pi G}{c^4} T$$

[A. Einstein, Sitz. Preuss. Akad. Wissenschaften Berlin, 844 (1915)]

The Schwarzschild solution (1915)

Karl Schwarzschild (letter to Einstein 22 Dec. 1915; publ. submitted 13 Jan 1916)
Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie,
 Sitz. Preuss. Akad. Wiss., Phys. Math. Kl. 1916, 189 (1916)

⇒ First exact non-trivial solution of Einstein equation :

$$ds^2 = - \left(1 - \frac{2m}{r}\right) c^2 dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (1)$$

with

- coordinates¹ $(t, \bar{r}, \theta, \varphi)$
- “auxiliary quantity” : $r := (\bar{r}^3 + 8m^3)^{1/3}$
- parameter $m = GM/c^2$, with M gravitational mass of the “mass point”

1. Schwarzschild's notations : $r = \bar{r}$, $R = r$, $\alpha = 2m$

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The “center”

Origin of coordinates : $\bar{r} = 0 \iff r = 2m$

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Droste contribution (1916)

Johannes Droste (communication 27 May 1916)

The Field of a Single Centre in Einstein's Theory of Gravitation, and the Motion of a Particle in that Field, Kon. Neder. Akad. Weten. Proc. **19**, 197 (1917)

⇒ derives the Schwarzschild solution (independently of Schwarzschild) via some coordinates (t, r', θ, φ) such that $g_{r'r'} = 1$; presents the result in the standard form (1) via a change of coordinates leading to the areal radius r

⇒ makes a detailed study of timelike geodesics in the obtained geometry

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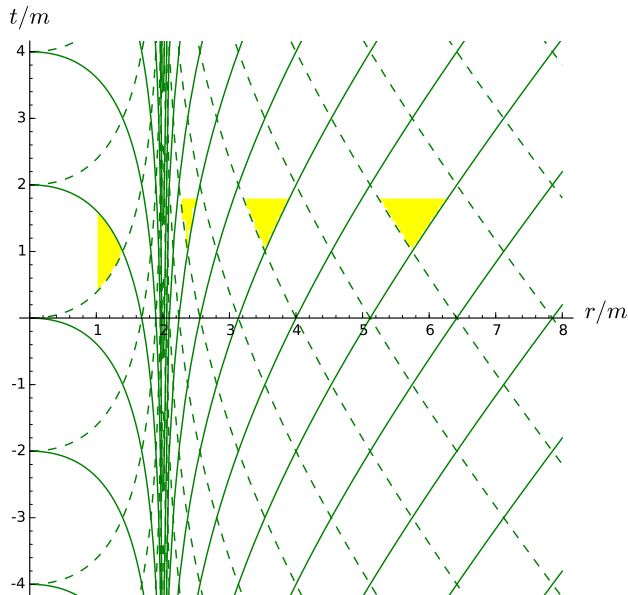
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Apparent barrier at $r = 2m$

A particle falling from infinity never reaches $r = 2m$ within a finite amount of "time" t .

The Schwarzschild radius : $R_S := 2m = \frac{2GM}{c^2}$

The “barrier” at $r = R_S$ 

Radial null geodesics of Schwarzschild spacetime in term of Schwarzschild-Droste coordinates (t, r) . Solid (resp. dashed) lines correspond to outgoing (resp. ingoing) geodesics. The interiors of some future light cones are depicted in yellow.

The Schwarzschild solution : early discussions

- 1920 : Alexander Anderson : light cannot emerge from the region $r < R_S := 2m = \frac{2GM}{c^2}$ (region “shrouded in darkness”)

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- **1923 : George Birkhoff** : outside any *spherical* body, the metric is Schwarzschild metric
- **1924 : Arthur Eddington** introduced the coord. $t' := t - \frac{2m}{c} \ln\left(\frac{r}{2m} - 1\right)$, leading to

$$ds^2 = -c^2 dt'^2 + dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) + \frac{2m}{r} (cdt' - dr)^2 \quad (2)$$

but did not noticed that the metric components w.r.t. coordinates (t', r, θ, φ) are regular at $r = 2m$!

Actually, Eddington's aim was elsewhere : comparing Whitehead theory (1922) to general relativity

The Schwarzschild solution : Lemaître breakthrough

Georges Lemaître (1932)

L'univers en expansion, Publ. Lab. Astron. Géodésie Univ. Louvain **9**, 171 (1932);
reprinted in Ann. Soc. Scient. Bruxelles A **53**, 51 (1933)

et la nouvelle forme du champ s'écrit sans singularité

$$(11.12) \quad ds^2 = -2m \frac{d\chi^2}{r} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) + dt^2,$$

où

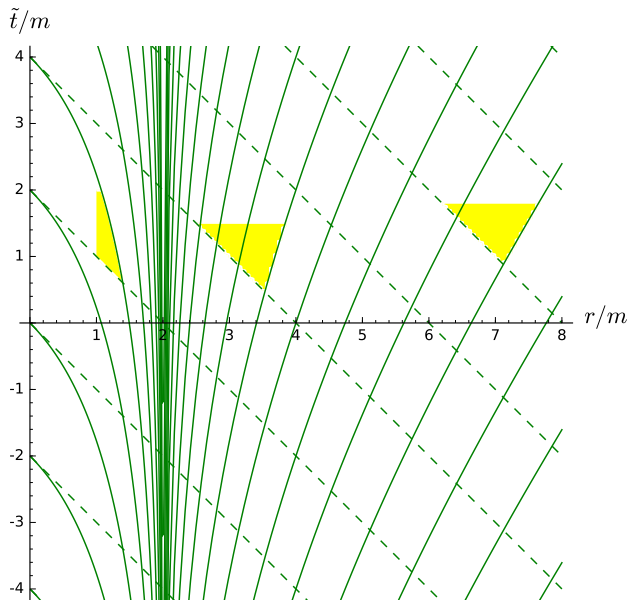
$$(11.13) \quad r = \left[\frac{3}{2} \sqrt{2m} (t - \chi) \right]^{\frac{2}{3}}$$

La singularité du champ de Schwarzschild est donc une singularité fictive, analogue à celle qui se présentait à l'horizon du centre dans la forme originale de l'univers de de Sitter.

The singularity at $r = R_S$ is a mere **coordinate singularity** : the metric components are regular in Lemaître coordinates $(\tau, \chi, \theta, \varphi)$:

$$ds^2 = -c^2 d\tau^2 + \frac{R_S}{r} d\chi^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (3)$$

$$r = r(\tau, \chi) := \left[\frac{3}{2} \sqrt{R_S} (c\tau - \chi) \right]^{2/3} \quad (4)$$

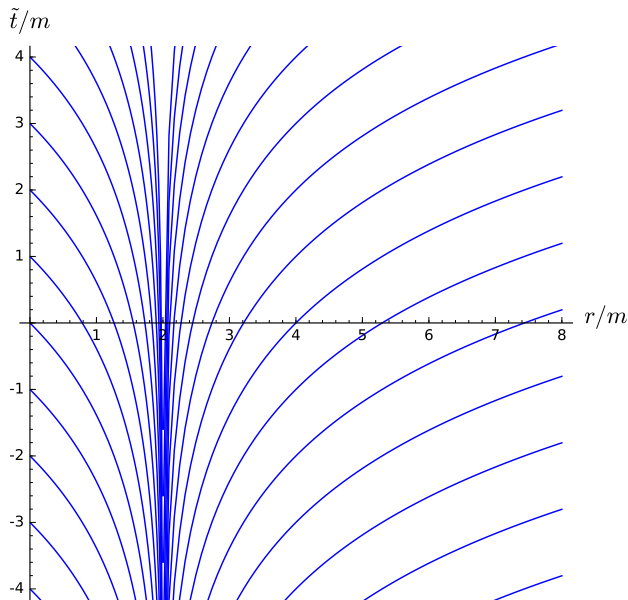
No longer any barrier at $r = R_S$ 

Radial null geodesics of Schwarzschild spacetime in term of **ingoing Eddington-Finkelstein coordinates** (\tilde{t}, r)

$$\tilde{t} = t + \frac{2m}{c} \ln \left| \frac{r}{2m} - 1 \right|$$

The ingoing null geodesics (dashed lines) do enter the region $r < R_S$.

Pathology of Schwarzschild-Droste coordinates



Hypersurfaces of constant Schwarzschild-Droste coordinate t in term of the ingoing Eddington-Finkelstein coordinates (\tilde{t}, r)

Gravitational collapse : Lemaître-Tolman solutions

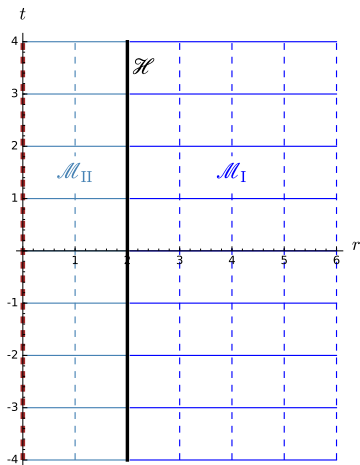
- 1932 : Georges Lemaître : general solutions of Einstein equation for spherically symmetric pressureless fluids (dust) \implies gravitational collapse

Gravitational collapse : Lemaître-Tolman solutions

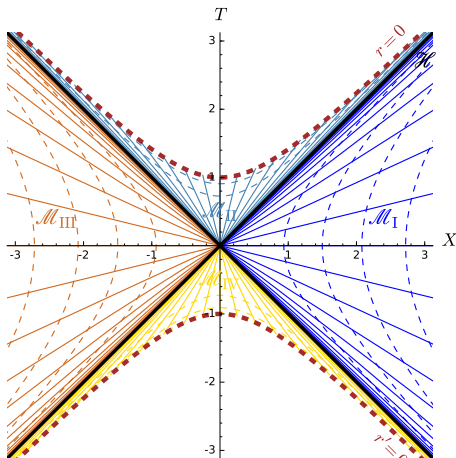
- 1932 : Georges Lemaître : general solutions of Einstein equation for spherically symmetric pressureless fluids (dust) \implies gravitational collapse
- 1939 : Robert Oppenheimer & Hartland Snyder : gravitational collapse of a homogeneous dust ball of radius R (special case of Lemaître's general solution)
 - \implies for an external observer, $R \rightarrow R_S$ as $t \rightarrow +\infty$
 - \implies "frozen star"

The Schwarzschild solution : the complete picture

John L. Synge (1950), Martin Kruskal (1960), George Szekeres (1960) : complete mathematical description of Schwarzschild spacetime ($\mathbb{R}^2 \times \mathbb{S}^2$ manifold)



Schwarzschild-Droste coordinates (t, r)



Carter-Penrose diagram of Schwarzschild spacetime

based on Frolov-Novikov coordinates

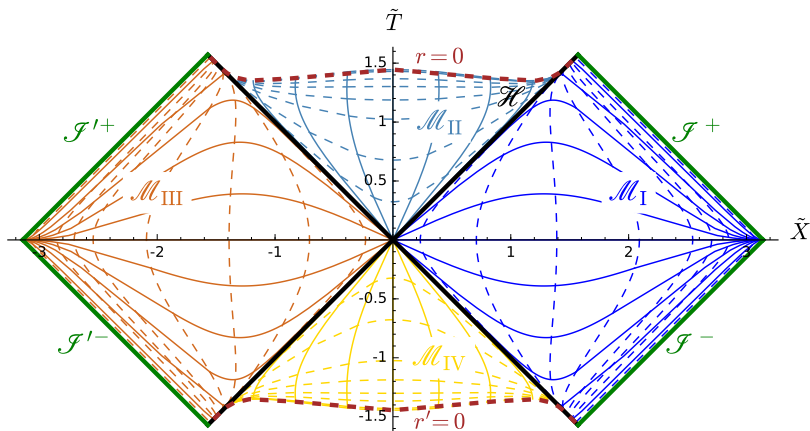
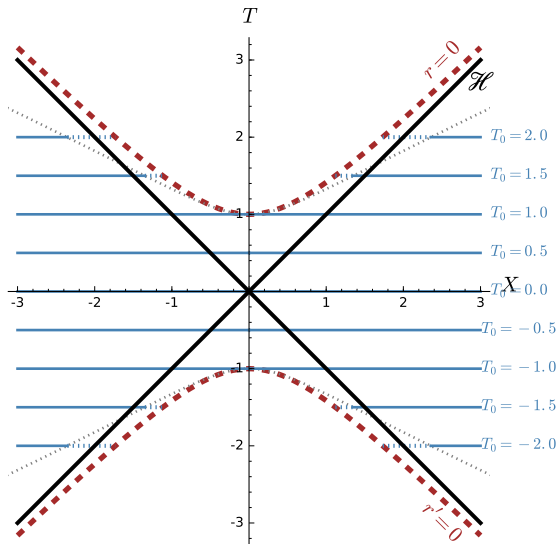


Figure drawn with SageMath : <http://sagemanifolds.obspm.fr>

Einstein-Rosen bridge

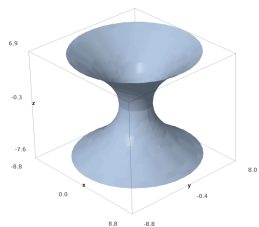
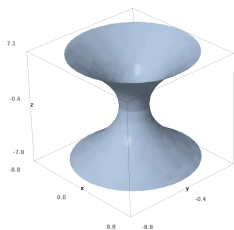
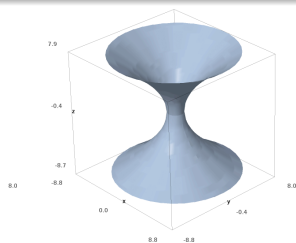
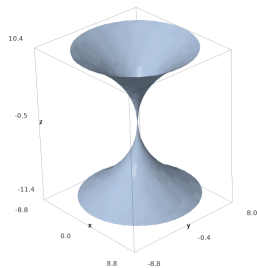
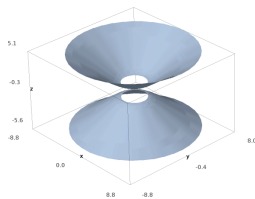
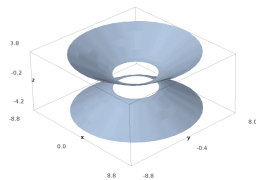


Connecting the asymptotically flat regions \mathcal{M}_I and \mathcal{M}_{III} by hypersurfaces $T = T_0 = \text{const}$ (blue horizontal lines).

\Rightarrow isometric embedding of equatorial sections ($T = T_0, \theta = \pi/2$) in the Euclidean 3-space

Rem : for $|T_0| > 1$, the dotted parts cannot be embedded isometrically in Euclidean space.

Evolving Einstein-Rosen bridge

 $T_0 = 0$ (Flamm paraboloid) $T_0 = 0.5$  $T_0 = 0.9$  $T_0 = 1$  $T_0 = 1.5$  $T_0 = 2$

Rotation enters the game : the Kerr solution

Almost 50 years after Schwarzschild : **Roy Kerr (1963)**

$$ds^2 = - \left(1 - \frac{2mr}{\rho^2} \right) dv^2 + 2dv dr - \frac{4amr \sin^2 \theta}{\rho^2} dv d\tilde{\varphi} \\ - 2a \sin^2 \theta dr d\tilde{\varphi} + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2mr \sin^2 \theta}{\rho^2} \right) \sin^2 \theta d\tilde{\varphi}^2.$$

Boyer & Lindquist (1967) coordinate change $(v, r, \theta, \tilde{\varphi}) \rightarrow (t, r, \theta, \varphi)$:

$$ds^2 = - \left(1 - \frac{2mr}{\rho^2} \right) dt^2 - \frac{4amr \sin^2 \theta}{\rho^2} dt d\varphi + \frac{\rho^2}{\Delta} dr^2 \\ + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2a^2mr \sin^2 \theta}{\rho^2} \right) \sin^2 \theta d\varphi^2,$$

where $\rho^2 := r^2 + a^2 \cos^2 \theta$, $\Delta := r^2 - 2mr + a^2$ and $r \in (-\infty, \infty)$

→ spacetime manifold $\mathcal{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \{r = 0 \ \& \ \theta = \pi/2\}$

→ 2 parameters : $m = \frac{GM}{c^2}$ and $a = \frac{J}{cM}$; black hole $\iff 0 \leq a \leq m$

→ Schwarzschild metric for $a = 0$

Physical meaning of the parameters M and J

- **mass M** : *not* a measure of the “amount of matter” inside the black hole, but rather a *characteristic of the external gravitational field*
→ measurable from the orbital period of a test particle in far circular orbit around the black hole (*Kepler's third law*)

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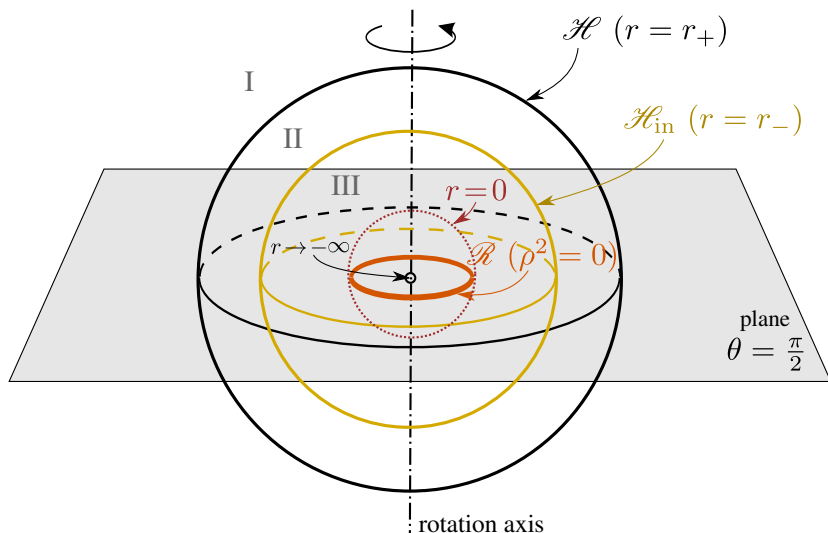
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Remark : the **radius** of a black hole is not a well defined concept : it *does not* correspond to some distance between the black hole “centre” and the event horizon. A well defined quantity is the **area** of the event horizon, A .

The “radius” can be defined from it : for a Schwarzschild black hole :

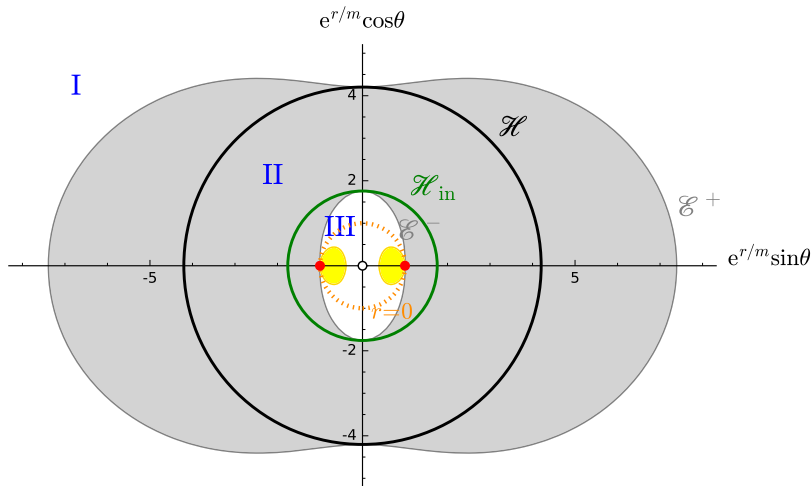
$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left(\frac{M}{M_{\odot}} \right) \text{ km}$$

Kerr spacetime

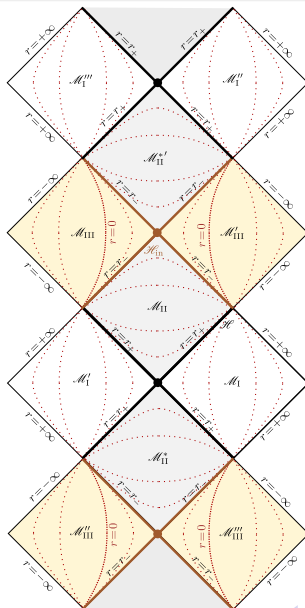


Slice $t = \text{const}$ of the Kerr spacetime viewed in O'Neill coordinates (R, θ, φ) , with $R := e^r$, $r \in (-\infty, +\infty)$.

Kerr spacetime : ergoregion and Carter time machine

Meridional view of a section $t = \text{const}$ of Kerr spacetime with $a/m = 0.90$

Maximal analytic extension of Kerr spacetime



The Golden Age of black hole theory

- 1964 : Edwin Salpeter, Yakov Zeldovich : quasars (just discovered !) shine thanks to accretion onto a supermassive black hole

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- 1965-1972 : **the no-hair theorem**

The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

*Within 4-dimensional general relativity, a stationary black hole in an otherwise empty universe is necessarily a **Kerr-Newmann black hole**, which is an **electro-vacuum solution** of Einstein equation described by only 3 parameters :*

- the total mass M
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge Q

⇒ *“a black hole has no hair”* (John A. Wheeler)

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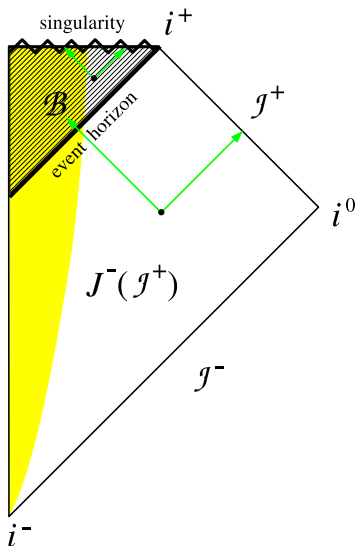
Astrophysical black holes have to be electrically neutral :

- $Q = 0$: **Kerr solution (1963)**

Other special cases :

- $a = 0$: **Reissner-Nordström solution (1916, 1918)**
- $a = 0$ and $Q = 0$: **Schwarzschild solution (1916)**
- $a = 0$, $Q = 0$ and $M = 0$: **Minkowski metric (1907)**

General definition of a black hole



The textbook definition

[Hawking & Ellis (1973)]

black hole : $\mathcal{B} := \mathcal{M} - J^-(\mathcal{I}^+)$

where

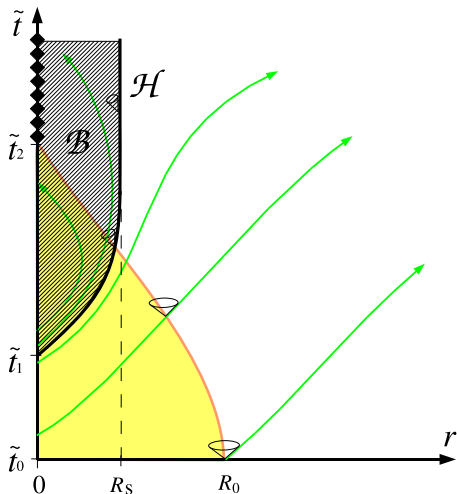
- (\mathcal{M}, g) = asymptotically flat manifold
- \mathcal{I}^+ = future null infinity
- $J^-(\mathcal{I}^+)$ = causal past of \mathcal{I}^+

i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : $\mathcal{H} := \partial J^-(\mathcal{I}^+)$
(boundary of $J^-(\mathcal{I}^+)$)

\mathcal{H} smooth $\implies \mathcal{H}$ null hypersurface

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i.e. black hole = region of spacetime from which light rays cannot escape to infinity

event horizon : $\mathcal{H} := \partial J^-(\mathcal{I}^+)$
(boundary of $J^-(\mathcal{I}^+)$)

\mathcal{H} smooth $\implies \mathcal{H}$ null hypersurface

Main properties of black holes (1/2)

- In general relativity, a black hole contains a region where the spacetime curvature diverges : **the singularity** (*NB : this is not the primary definition of a black hole*). The singularity is inaccessible to observations, being hidden by the event horizon.

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- The singularity marks the **limit of validity of general relativity** : to describe it, a quantum theory of gravitation would be required.
- The event horizon \mathcal{H} is a **global structure** of spacetime : no physical experiment whatsoever can detect the crossing of \mathcal{H} .

Main properties of black holes (2/2)

- Viewed by a distant observer, the horizon approach is perceived with an **infinite redshift**, or equivalently, by an **infinite time dilation**
- A black hole **is not an infinitely dense object** : on the contrary it is made of vacuum (except maybe at the singularity) ; if one defines its “mean density” by $\bar{\rho} = M/(4/3\pi R^3)$, then
 - for the Galactic centre BH (Sgr A*) : $\bar{\rho} \sim 10^6 \text{ kg m}^{-3} \sim 2 \cdot 10^{-4} \rho_{\text{white dwarf}}$
 - for the BH at the centre of M87 : $\bar{\rho} \sim 2 \text{ kg m}^{-3} \sim 2 \cdot 10^{-3} \rho_{\text{water}} !$

\implies a black hole is a **compact object** : $\frac{M}{R}$ large, not $\frac{M}{R^3} !$
- Due to the non-linearity of general relativity, **black holes can form in spacetimes without any matter**, by collapse of gravitational wave packets.

Outline

- 1 Black holes : a century-old history
- 2 **Some recent developments**
- 3 Black holes in the sky
- 4 Observing black holes via gravitational waves : a dream come true
- 5 Testing general relativity with black holes

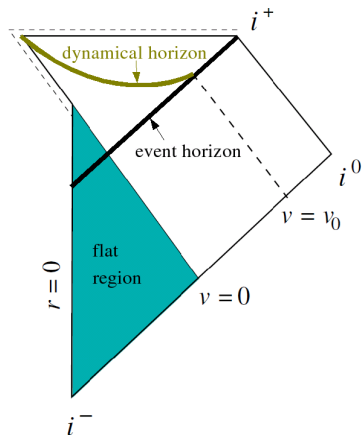
The quasi-local approach : motivation

The standard definition of a black hole is **highly non-local** : determination of $\partial J^-(\mathcal{I}^+)$ requires the knowledge of the entire future null infinity. Moreover this is *not locally linked with the notion of strong gravitational field*.

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Example of event horizon in a flat region of spacetime : Vaidya metric, describing incoming radiation from infinity :



$$ds^2 = - \left(1 - \frac{2m(v)}{r} \right) dv^2 + 2dv dr + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

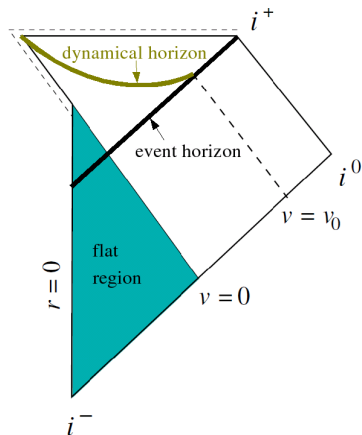
$$\begin{aligned} \text{with } m(v) &= 0 & \text{for } v < 0 \\ dm/dv &> 0 & \text{for } 0 \leq v \leq v_0 \\ m(v) &= M_0 & \text{for } v > v_0 \end{aligned}$$

[Ashtekar & Krishnan, LRR 7, 10 (2004)]

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\Rightarrow no local physical experiment can locate the event horizon

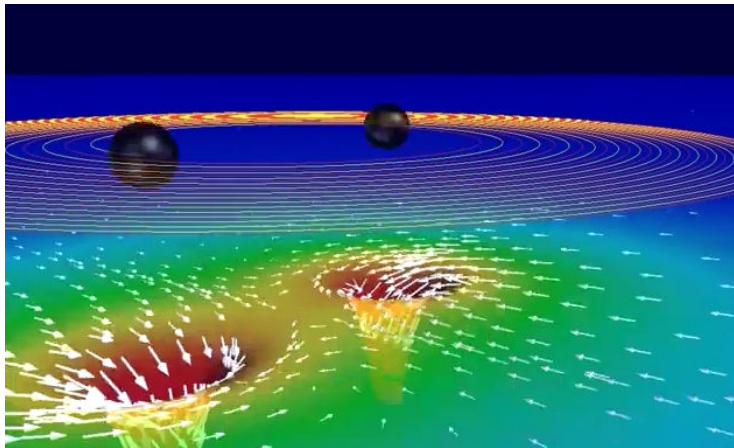
Quasi-local approaches to black holes

New paradigm for the theoretical approach to black holes, motivated by **quantum gravity** and **numerical relativity** : instead of *event horizons*, black holes are described by

- **trapping horizons** (Hayward 1994)
- **isolated horizons** (Ashtekar et al. 1999)
- **dynamical horizons** (Ashtekar and Krishnan 2002)
- **slowly evolving horizons** (Booth and Fairhurst 2004)

All these concepts are **quasi-local** and are based on hypersurfaces foliated by **marginally trapped surfaces**

The 2000's : the triumph of numerical relativity

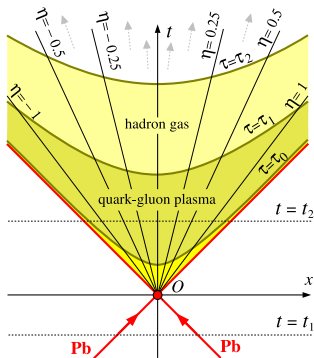


[Caltech/Cornell SXS]

[Scheel et al., PRD **79**, 024003 (2009)]

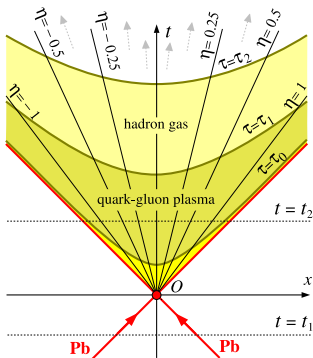
A recent hot topic : black holes and gauge/gravity duality

Gauge/gravity duality (“holographic principle”)

4D strongly-coupled gauge theory \equiv 5D gravitation*Prototype* : AdS/CFT correspondence

Spacetime diagram of a
heavy-ion collision (LHC)
 $\tau_0 \simeq 0.2 \text{ fm}/c = 6 \cdot 10^{-25} \text{ s}$
 $\tau_1 \sim 10\tau_0$

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Gauge/gravity duality (“holographic principle”)

4D strongly-coupled gauge theory \equiv 5D gravitation

Prototype : AdS/CFT correspondence

Example : Quark-gluon plasma (QGP) in heavy-ion collisions : low-viscosity fluid with *anisotropic* pressure ($p_x < p_y$)

[Aref'eva, Golubtsova & Gourgoulhon, *J. High Ener. Phys.* **09**(2016), 142 (2016)], [Ageev, Aref'eva, Golubtsova & Gourgoulhon, arXiv:1606.03995]

Thermalization of QGP \equiv 5D black hole formation

Gauge theory : QCD

Gravity : 5D Lifshitz-like spacetime (*anisotropic* generalization of AdS₅) with formation of a black brane (Vaidya-type collapse)

Results : faster thermalization in the transversal direction : evolution of the entanglement entropy

Outline

- 1 Black holes : a century-old history
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Known black holes

Three kinds of black holes are known in the Universe :

- **Stellar black holes** : supernova remnants :

$$M \sim 10 - 30 M_{\odot} \text{ and } R \sim 30 - 90 \text{ km}$$

$$\text{example : Cyg X-1 : } M = 15 M_{\odot} \text{ and } R = 45 \text{ km}$$

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- **Supermassive black holes**, in galactic nuclei :

$$M \sim 10^5 - 10^{10} M_{\odot} \text{ and } R \sim 3 \times 10^5 \text{ km} - 200 \text{ UA}$$

$$\text{example : Sgr A* : } M = 4.3 \times 10^6 M_{\odot} \text{ and } R = 13 \times 10^6 \text{ km} = 18 R_{\odot} = 0.09 \text{ UA} = \frac{1}{4} \times \text{radius of Mercury's orbit}$$

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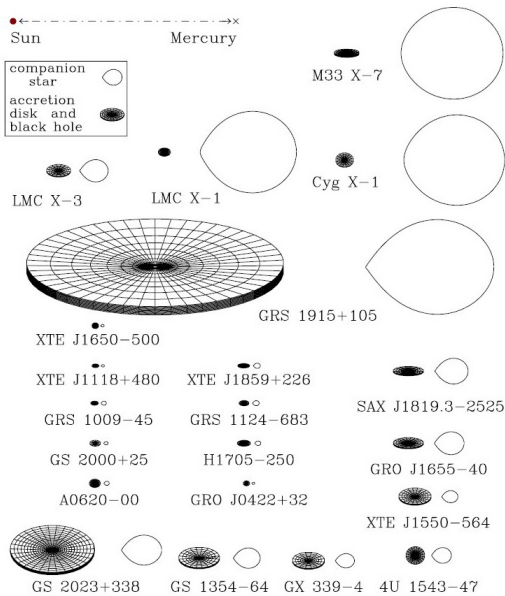
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 $R = 13 \times 10^6 \text{ km} = 18 R_{\odot} = 0.09 \text{ UA} = \frac{1}{4} \times \text{radius of Mercury's orbit}$

- **Intermediate mass black holes**, as ultra-luminous X-ray sources :

$$M \sim 10^2 - 10^4 M_{\odot} \text{ and } R \sim 300 \text{ km} - 3 \times 10^4 \text{ km}$$

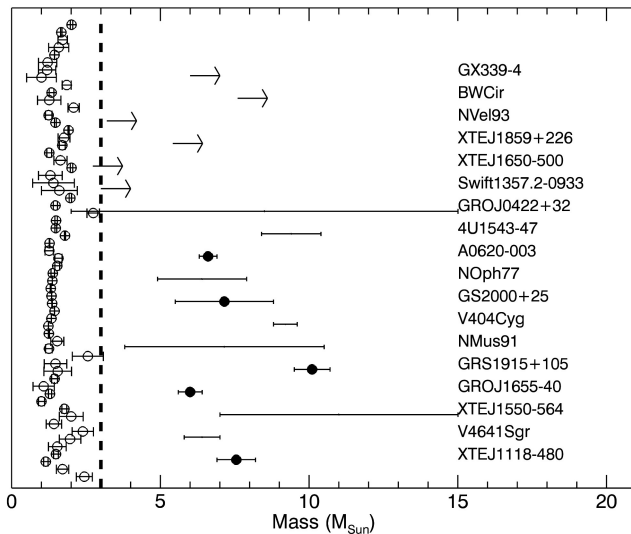
example : ESO 243-49 HLX-1 : $M > 500 M_{\odot}$ and $R > 1500 \text{ km}$

Stellar black holes in X-ray binaries



[McClintock et al. (2011)]

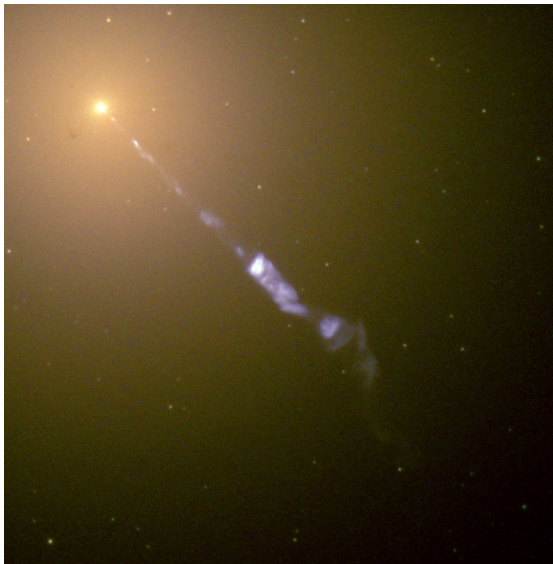
Stellar black holes in X-ray binaries



Dynamically measured masses of black holes in transient low-mass X-ray binaries (right), compared with measured masses of neutron stars (left)

[Corral-Santana et al., *A&A* 587, A61 (2016)]

Supermassive black holes in active galactic nuclei (AGN)

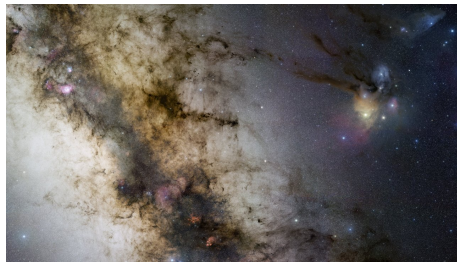
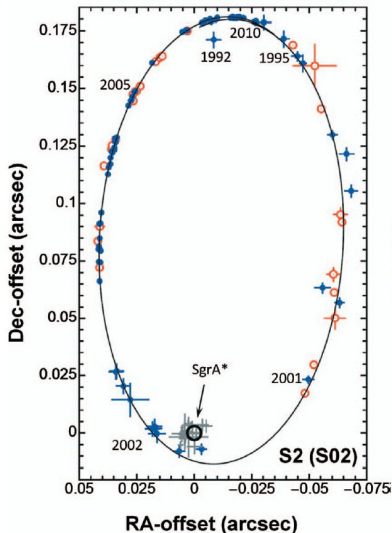


Jet emitted by the nucleus of the giant elliptical galaxy M87, at the centre of Virgo cluster [HST]

$$M_{\text{BH}} = 3 \times 10^9 M_{\odot}$$

$$V_{\text{jet}} \simeq 0.99 c$$

The black hole at the centre of our galaxy : Sgr A*



[ESO (2009)]

Measure of the mass of Sgr A* black hole by stellar dynamics :

$$M_{\text{BH}} = 4.3 \times 10^6 M_{\odot}$$

← Orbit of the star S2 around Sgr A*

$$P = 16 \text{ yr}, \quad r_{\text{per}} = 120 \text{ UA} = 1400 R_{\text{S}},$$

$$V_{\text{per}} = 0.02 c$$

[Genzel, Eisenhauer & Gillessen, RMP 82, 3121 (2010)]

Can we see a black hole from the Earth ?

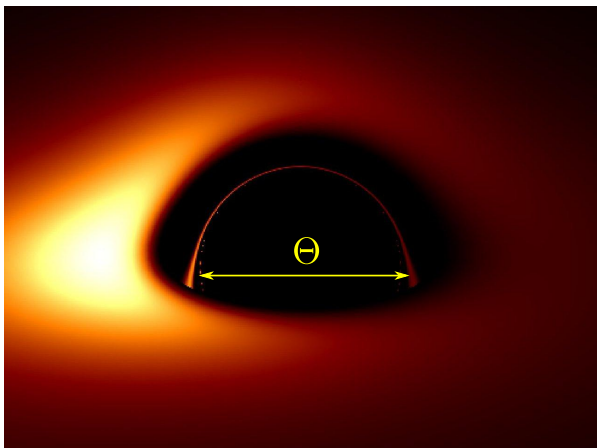


Image of a thin accretion disk around a Schwarzschild BH

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass M seen from a distance d :

$$\Theta = 6\sqrt{3} \frac{GM}{c^2 d} \simeq 2.60 \frac{2R_S}{d}$$

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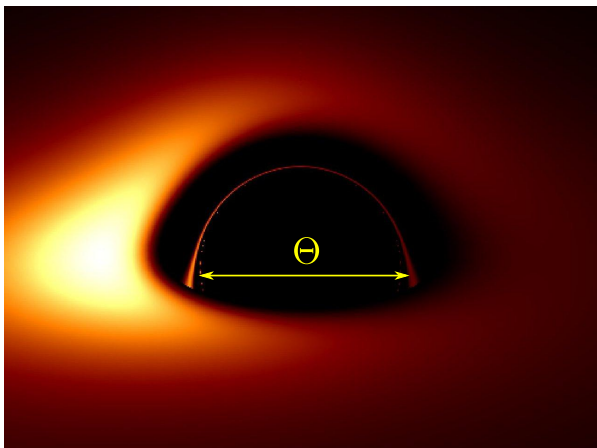


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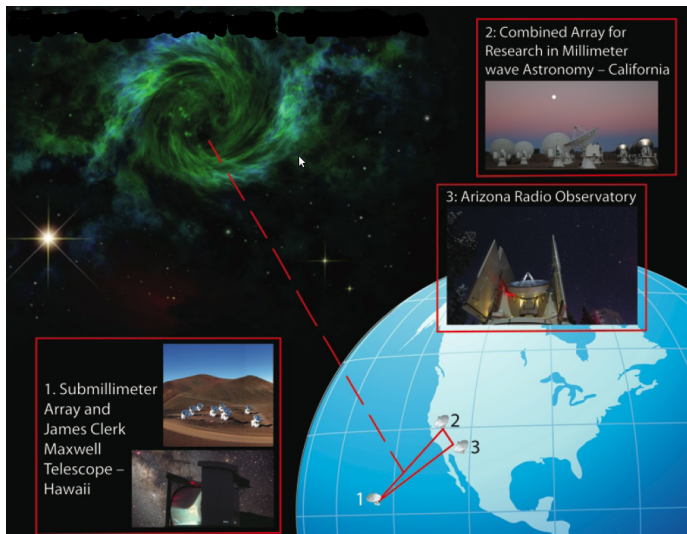
Largest black holes in the Earth's sky :

Sgr A* : $\Theta = 53 \mu\text{as}$

M87 : $\Theta = 21 \mu\text{as}$

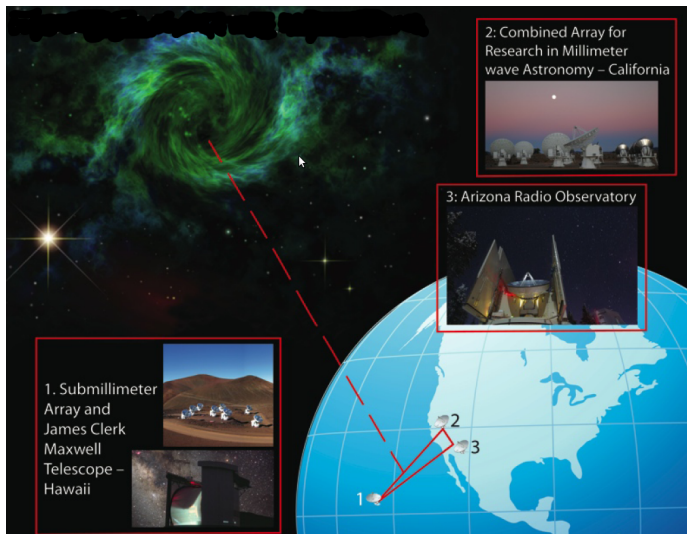
M31 : $\Theta = 20 \mu\text{as}$

Remark : black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$

Reaching the μas resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]

Reaching the μas resolution with VLBI

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far : VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only $37 \mu\text{as}$

[Doeleman et al., Nature 455, 78 (2008)]

Existing American VLBI network [Doeleman et al. 2011]

The near future : the Event Horizon Telescope

To go further :

- shorten the wavelength : **1.3 mm** → **0.8 mm**
- increase the number of stations; in particular add ALMA

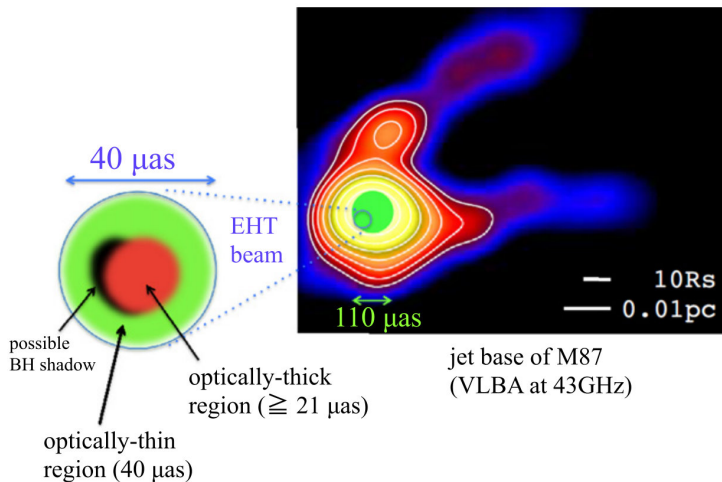


Atacama Large Millimeter Array (ALMA)

part of the **Event Horizon Telescope (EHT)** to be completed by 2020

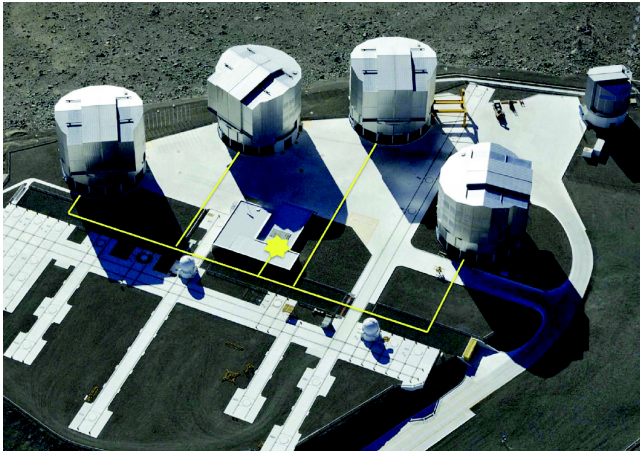
August 2015 : VLBI observations involving ALMA and VLBA

VLBA and EHT observations of M87



[Kino et al., ApJ 803, 30 (2015)]

Near-infrared optical interferometry : GRAVITY



[Gillessen et al. 2010]

GRAVITY instrument at VLTI (2016)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits : $10 \mu\text{as}$

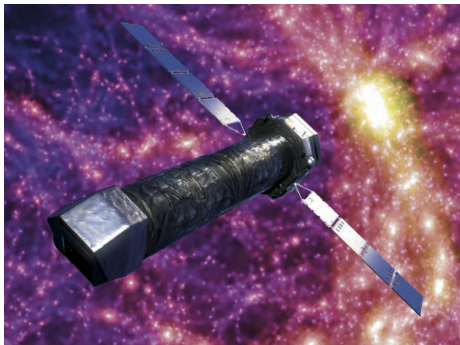
Near-infrared optical interferometry : GRAVITY



[MPE/GRAVITY team]

July 2015 : GRAVITY
shipped to Chile and
successfully assembled
at the Paranal
Observatory
Fall 2016 : observations
have started !

Observing black holes at high energy : the Athena mission



[Athena team]

X-ray observatory Athena : the future L2 mission of ESA (launch \sim 2028)

Among the scientific objectives :

- Determine the formation and early growth of supermassive black holes, via the observation of a large sample of AGN at $z \sim 6 - 8$
- Measure the spins of supermassive black holes
- Measure the spins of stellar black holes

Outline

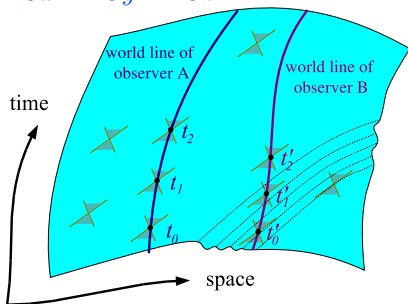
- 1 Black holes : a century-old history
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Gravitational waves

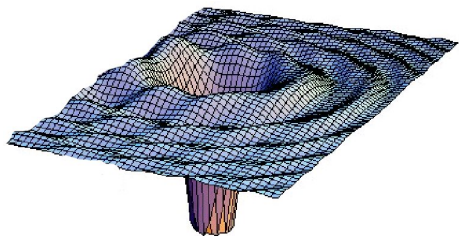
Linearization of Einstein equation in weak field : $g = \eta + h$,
 η = Minkowski metric²

$$\Rightarrow \text{wave equation : } \square \bar{h} = -\frac{16\pi G}{c^4} T \quad (\text{Lorenz gauge})$$

$$\text{with } \square = -\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \quad \bar{h} = h - \frac{1}{2} h \eta \quad \text{and} \quad h = \text{Trace}(h).$$



Black holes and gravitational waves



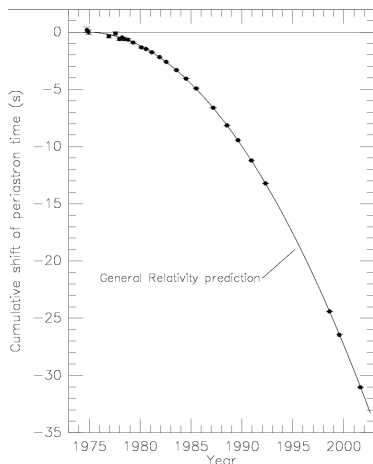
Link between black holes and gravitational waves :

Both are **spacetime distortions** :

- extreme distortions (black holes)
- small distortions (gravitational waves)

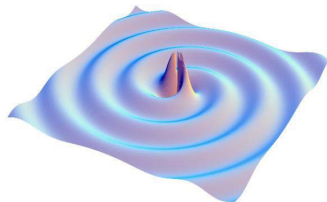
In particular, black holes and gravitational waves are both **vacuum solutions** of Einstein equation

Observational evidence for gravitational waves



[Weisber & Taylor (2002)]

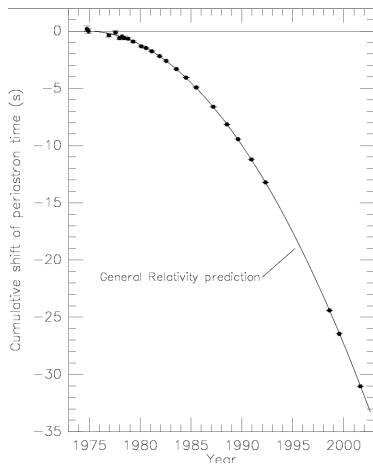
Emission of gravitational waves by the neutron star binary system PSR B1509-58 (*binary pulsar*)



← Observed decay of the orbital period

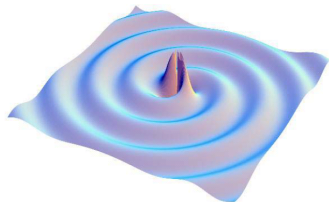
$P = 7$ h 45 min of the binary pulsar PSR B1509-58 produced by the *reaction to gravitational radiation*
 \implies coalescence in 140 millions year.

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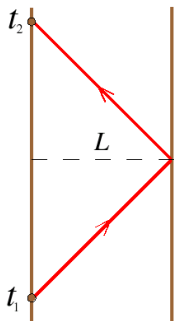
Nobel Prize in Physics to R. Hulse & J. Taylor (1993)

Measurable effects of a gravitational wave passage



Measure of the distance L between two free masses by a “radar” method :

$$L = \frac{1}{2} c(t_2 - t_1)$$



Variation of length L when a gravitational wave passes by :

$$\delta L \simeq h L$$

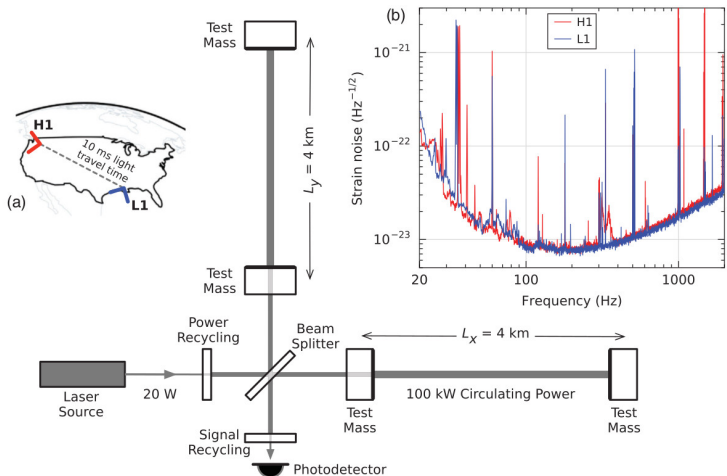
h = amplitude of the gravitational wave

In practice, h is so small that our senses are not sensitive to it :

for the most important **astrophysical sources** :

$$h \sim 10^{-21} !!!$$

Advanced LIGO detectors



[Abbott et al., PRL 116, 061102 (2016)]

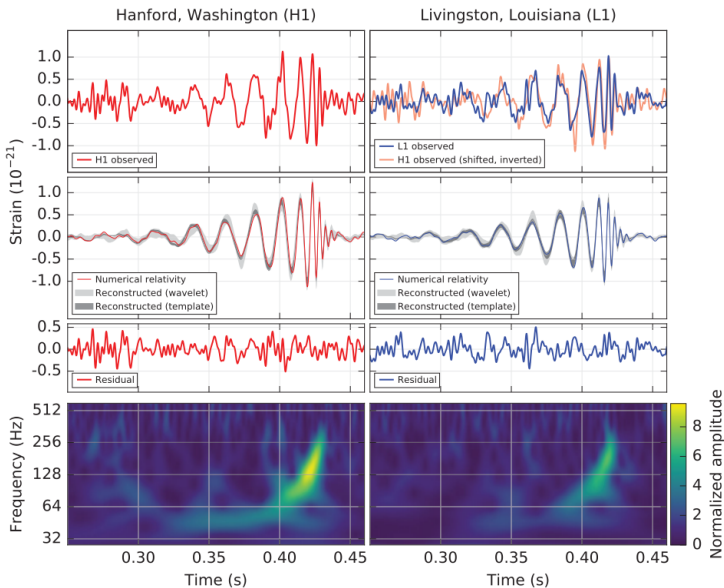
Advanced ground-based GW detectors



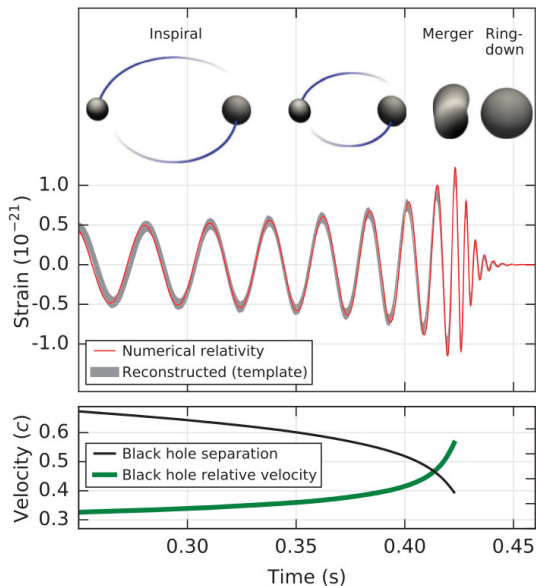
- **Adv. LIGO** : started Sept. 2015
- **Adv. Virgo** : will start in spring 2017
- **KAGRA** (Japan) : 2018

Gravitational wave detector VIRGO in Cascina, near Pisa (Italy) [CNRS/INFN]

September 14, 2015, 09:50:45 UTC



GW150914 event



Signal :

$$\Delta t = 0.2 \text{ s}$$

$$f : 35 \rightarrow 250 \text{ Hz}$$

$$h_{\text{max}} = 1.0 \cdot 10^{-21}$$

Matched filter :

$$S/N = 24$$

$$F_{\text{false}} = 1/203,000 \text{ yr}$$

$$M_1 = 36 \pm 5 M_{\odot}$$

$$M_2 = 29 \pm 4 M_{\odot}$$

$$d = 410 \pm 180 \text{ Mpc}$$

$$z = 0.09 \pm 0.04$$

$$M_{\text{final}} = 62 \pm 4 M_{\odot}$$

$$\Rightarrow E_{\text{rad}}^{\text{GW}} = 3.0 \pm 0.5 M_{\odot} c^2$$

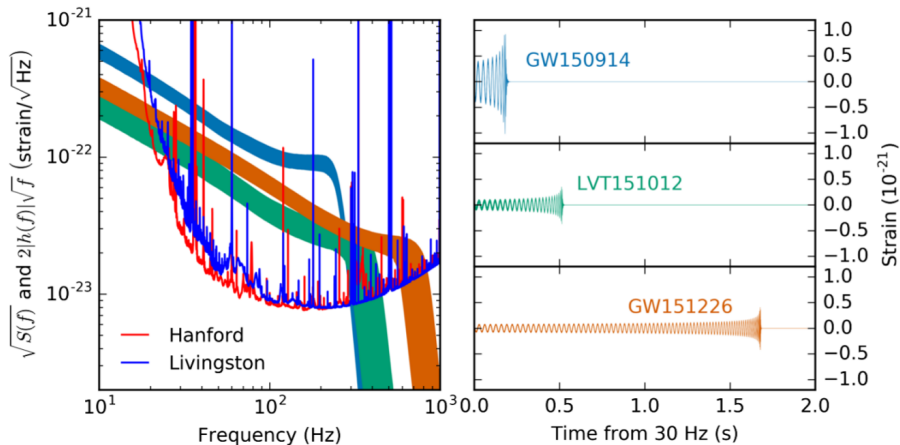
$$a_1 < 0.7, a_2 < 0.9$$

$$a_{\text{final}} = 0.67 \pm 0.07$$

[Abbott et al., PRL 116, 061102

(2016)]

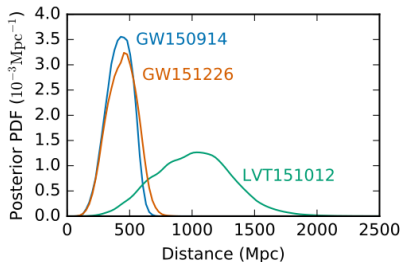
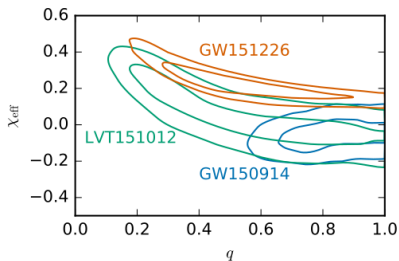
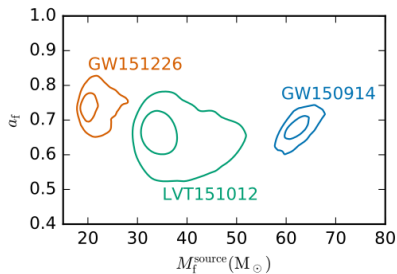
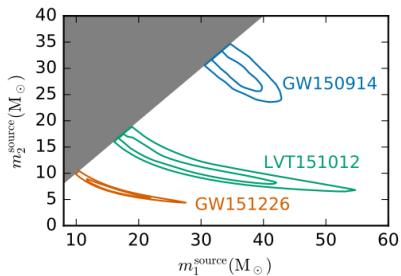
LIGO events in run O1 (12 Sept. 2015 - 19 Jan. 2016)



[Abbott et al., PRX 6, 041015 (2016)]

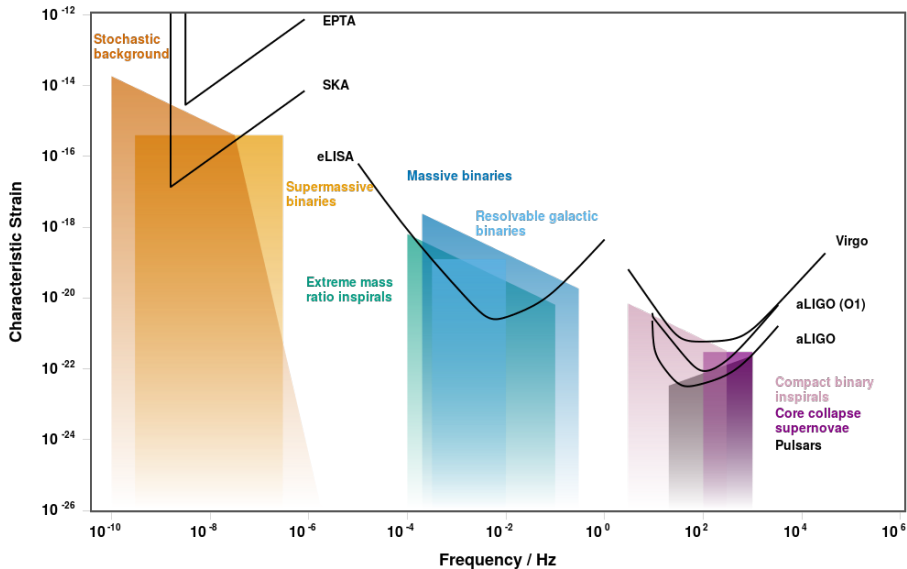
NB : LVT = LIGO-Virgo Trigger (not significant enough to be a detection)

LIGO events in run O1 (12 Sept. 2015 - 19 Jan. 2016)



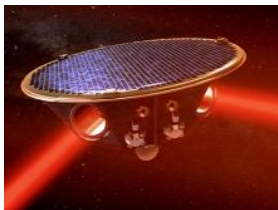
[Abbott et al., PRX 6, 041015 (2016)]

GW detectors in different bandwidths



Space detector LISA (ESA)

Interferometric gravitational wave detector in solar orbit

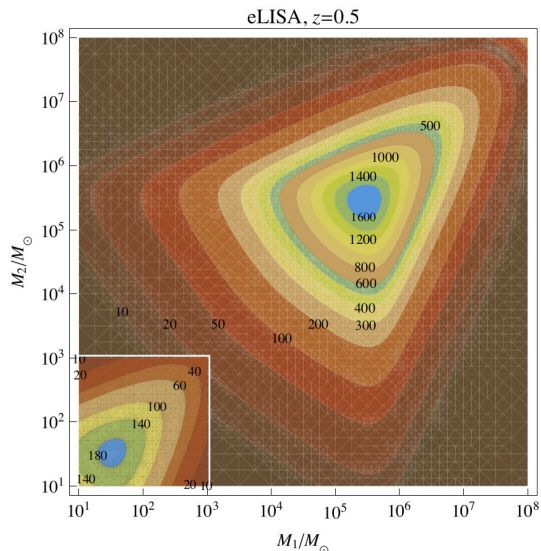


[eLISA / NGO]

- theme selected by ESA in 2013 for the L3 mission
- launch ~ 2034
- technology demonstrator **LISA Pathfinder** launched on 3 December 2015 ; successful results announced in June 2016 !



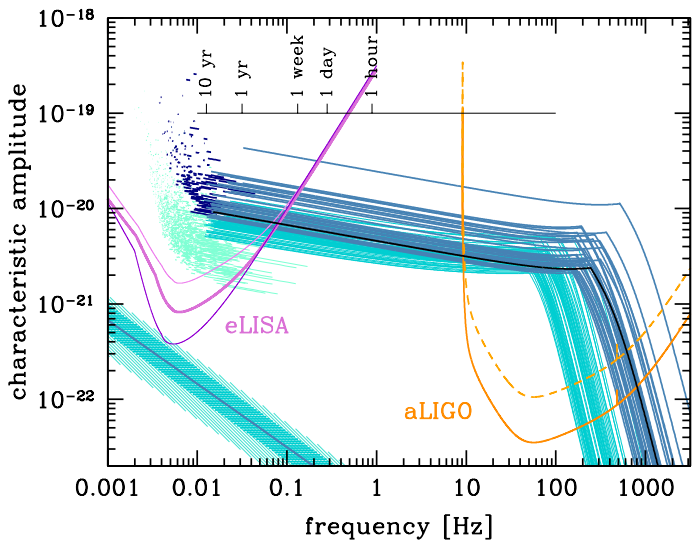
LISA observations of massive binary BH mergers



Signal-to-noise ratio for
gravitational waves from the
inspiral of a BH binary at
 $z = 0.5$

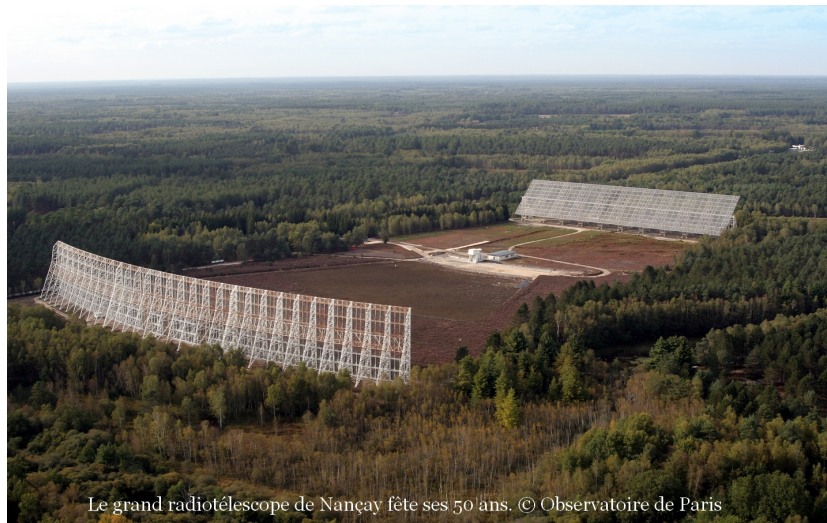
[Barusse et al., J. Phys. Conf. Ser. **610**, 012001 (2015)]

The promise of multi-band gravitational wave astronomy



[Sesana, PRL 116, 231102 (2016)]

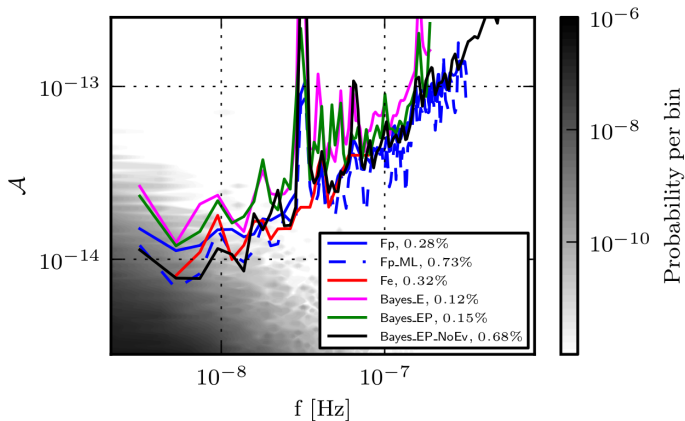
Detecting gravitational waves by pulsar timing



Le grand radiotélescope de Nançay fête ses 50 ans. © Observatoire de Paris

EPTA results on supermassive BH binaries

EPTA : European Pulsar Timing Array



[Babak et al., MNRAS 455, 1665 (2016)]

Outline

- 1 Black holes : a century-old history
- 2 Some recent developments
- 3 Black holes in the sky
- 4 Observing black holes via gravitational waves : a dream come true
- 5 Testing general relativity with black holes

Is general relativity unique?

Yes if we assume

- a 4-dimensional spacetime
- gravitation only described by a metric tensor g
- field equation involving only derivatives of g up to second order
- diffeomorphism invariance
- $\nabla \cdot T = 0$ (\implies weak equivalence principle)

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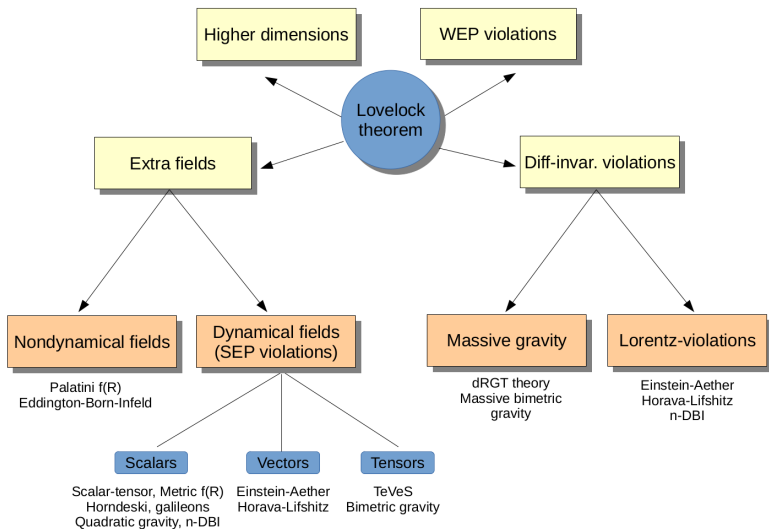
However, GR is certainly not the ultimate theory of gravitation :

- it is not a quantum theory
- cosmological constant / dark energy problem

GR is generally considered as a low-energy limit of a more fundamental theory :

- string theory
- loop quantum gravity
- ...

Extensions of general relativity



[Berti et al., CGQ 32, 243001 (2015)]

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Observational tests

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- **stellar orbits** deviating from Kerr timelike geodesics (**GRAVITY**)
- **accretion disk spectra** different from those arising in Kerr metric (**X-ray observatories, e.g. Athena**)
- **images of the black hole silhouette** different from that of a Kerr BH (**EHT**)

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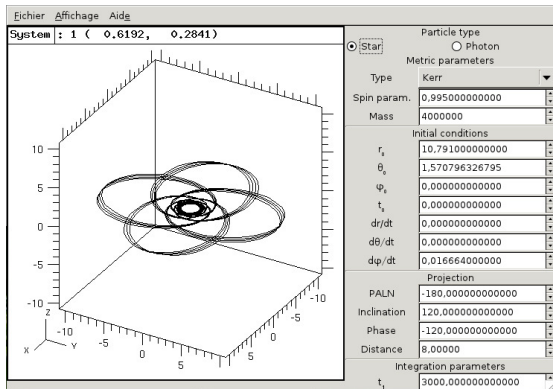
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Need for a good and versatile geodesic integrator

to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric

Gyoto code

Main developers : T. Paumard & F. Vincent



- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL) : <http://gyoto.obspm.fr/>

[Vincent, Paumard, Gourgoulhon & Perrin, CQG 28, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG 29, 245005 (2012)]

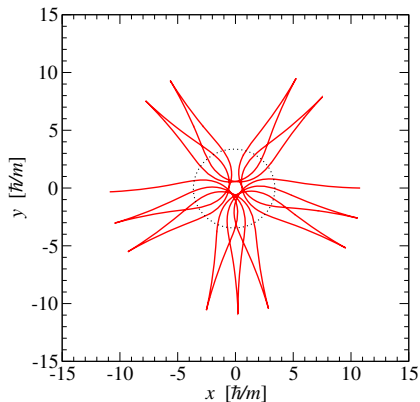
An example : rotating boson stars

Boson star = localized configurations of a self-gravitating massive complex scalar field $\Phi \equiv$ “Klein-Gordon geons”

[Bonazzola & Pacini (1966), Kaup (1968)]

Boson stars may behave as black-hole mimickers

- Solutions of the *Einstein-Klein-Gordon* system computed by means of **Kadath** [Grandclément, JCP 229, 3334 (2010)]
- Timelike geodesics computed by means of **Gyoto**

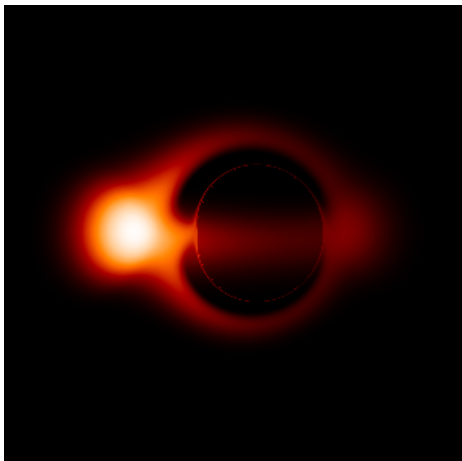
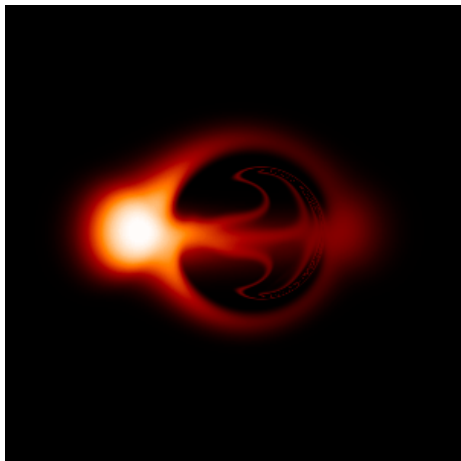


Pointy petal orbit around a rotating boson star for a free scalar field

$$\Phi = \phi(r, \theta) e^{i(\omega t + 2\varphi)}, \quad \omega = 0.75 m/\hbar$$

[Grandclément, Somé & Gourgoulhon, PRD 90, 024068 (2014)]

Image of an accretion torus

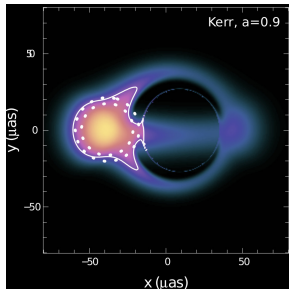
Kerr BH $a/M = 0.9$ Boson star $k = 1, \omega = 0.70 m/\hbar$ 

[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG 33, 105015 (2016)]

Black holes with scalar hair

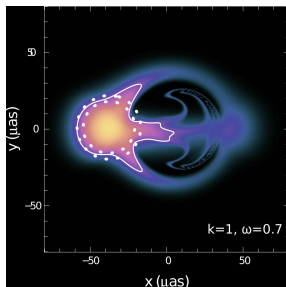
Kerr black hole

$$a/M = 0.9$$



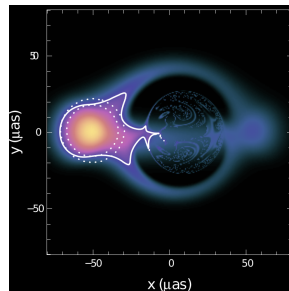
boson star [1]

$$k=1, \omega=0.7 m/h$$



hairy black hole [2]

$$a/M = 0.9$$



Kadath → metric

HR code → metric

(via Lorene)

Gyoto → ray-tracing

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[1. Vincent, Meliani, Grandclément, Gourgoulhon & Straub, *Class. Quantum Grav.* **33**, 105015 (2016)][2. Vincent, Gourgoulhon, Herdeiro & Radu, *Phys. Rev. D* **94**, 084045 (2016)]

Conclusions

After a century marked by the Golden Age (1965-1975), the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, **black hole physics** is very much alive.

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The **GW150914 event** was both the first direct detection of gravitational waves and the first observation of the merger of two black holes — the most dynamical event in relativistic gravity. The waveform was found consistent with general relativity.