Spacetimes of rotating stars and black holes

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Meudon, France

NS Synergy 2012
Opava, Czech Republic
1-6 October 2012

An illustrative rotating neutron star solution

Star of gravitational mass $M = 1.4\,M_\odot$, rigidly rotating at $\Omega/(2\pi) = 716\ \mathrm{Hz}$ (the highest rotation frequency among observed neutron stars, achieved by the pulsar PSR J1748-2446ad)

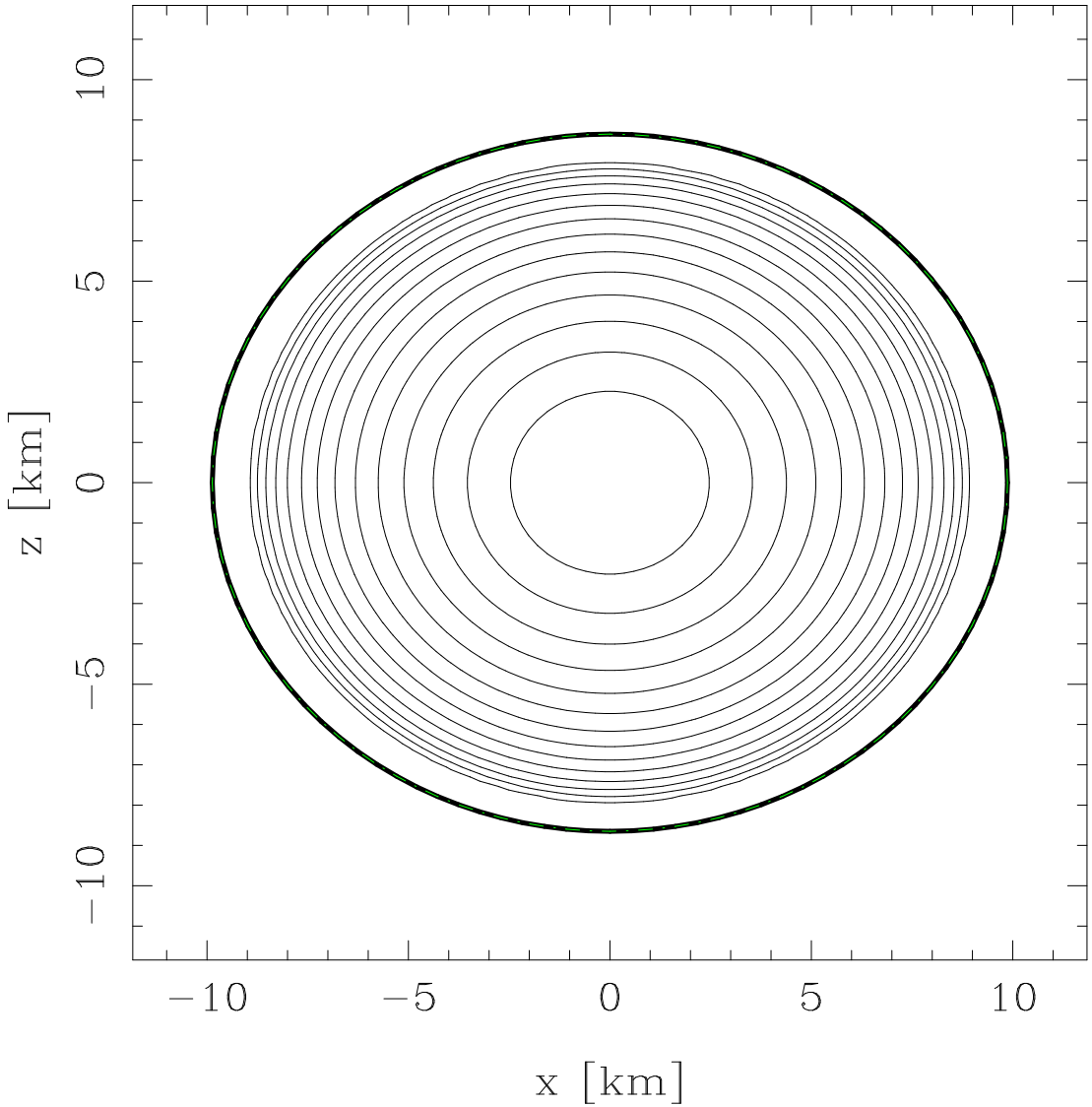
Equation of state: *(cf. Micaela Oertel's lecture)*

- *core:* model A18+δv+UIX* of Akmal, Pandharipande & Ravenhall (1998), describing a matter of neutrons, protons, electrons and muons via a Hamiltonian including two-body and three-body interactions, as well as relativistic corrections
- *inner crust:* SLy4 model of Douchin & Haensel (2001)
- *outer crust:* Haensel & Pichon (1994), which is based on the experimental masses of neutron rich nuclei.

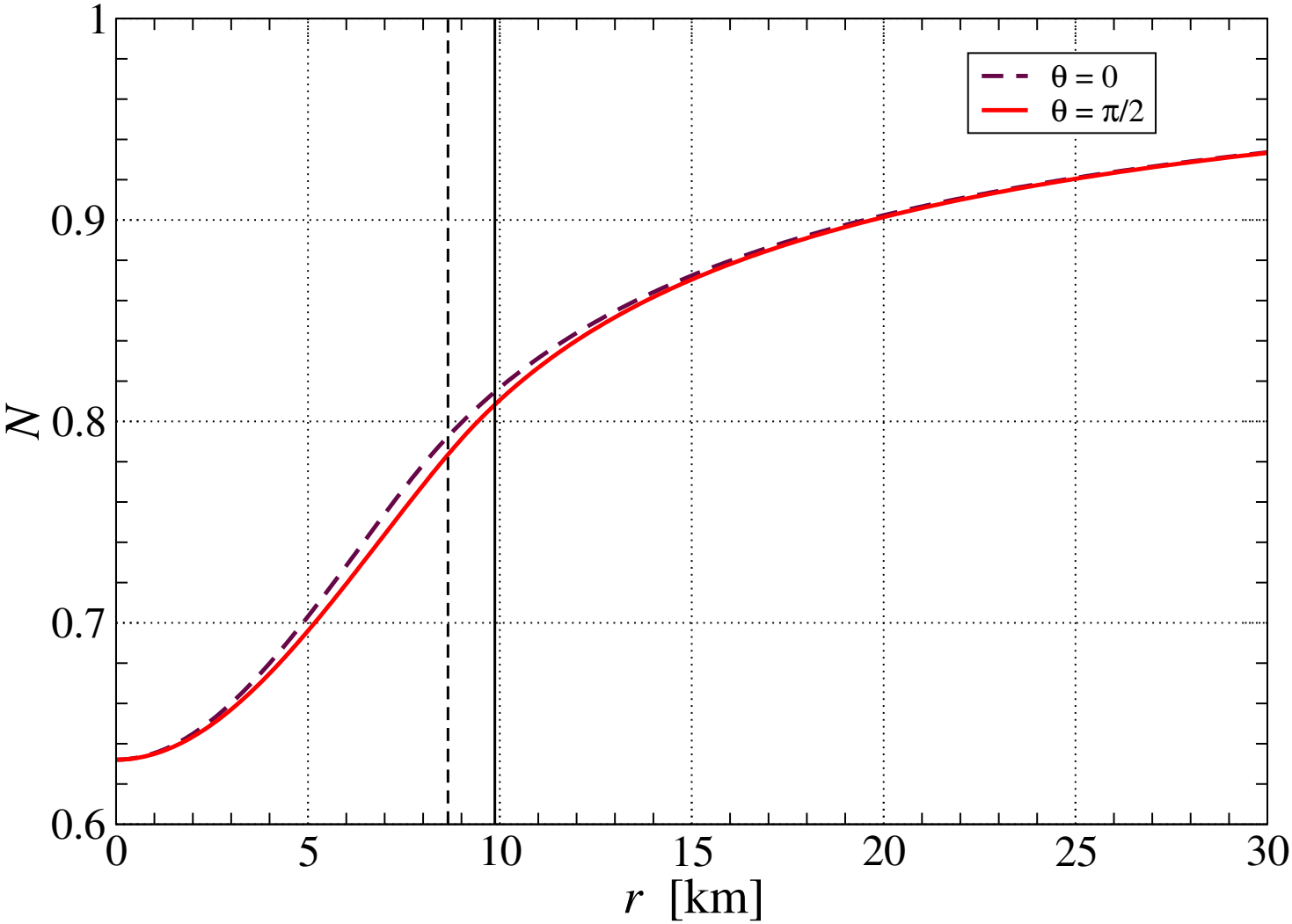
Computation: code `Lorene/nrotstar` (`http://www.lorene.obspm.fr`)

An illustrative rotating neutron star solution

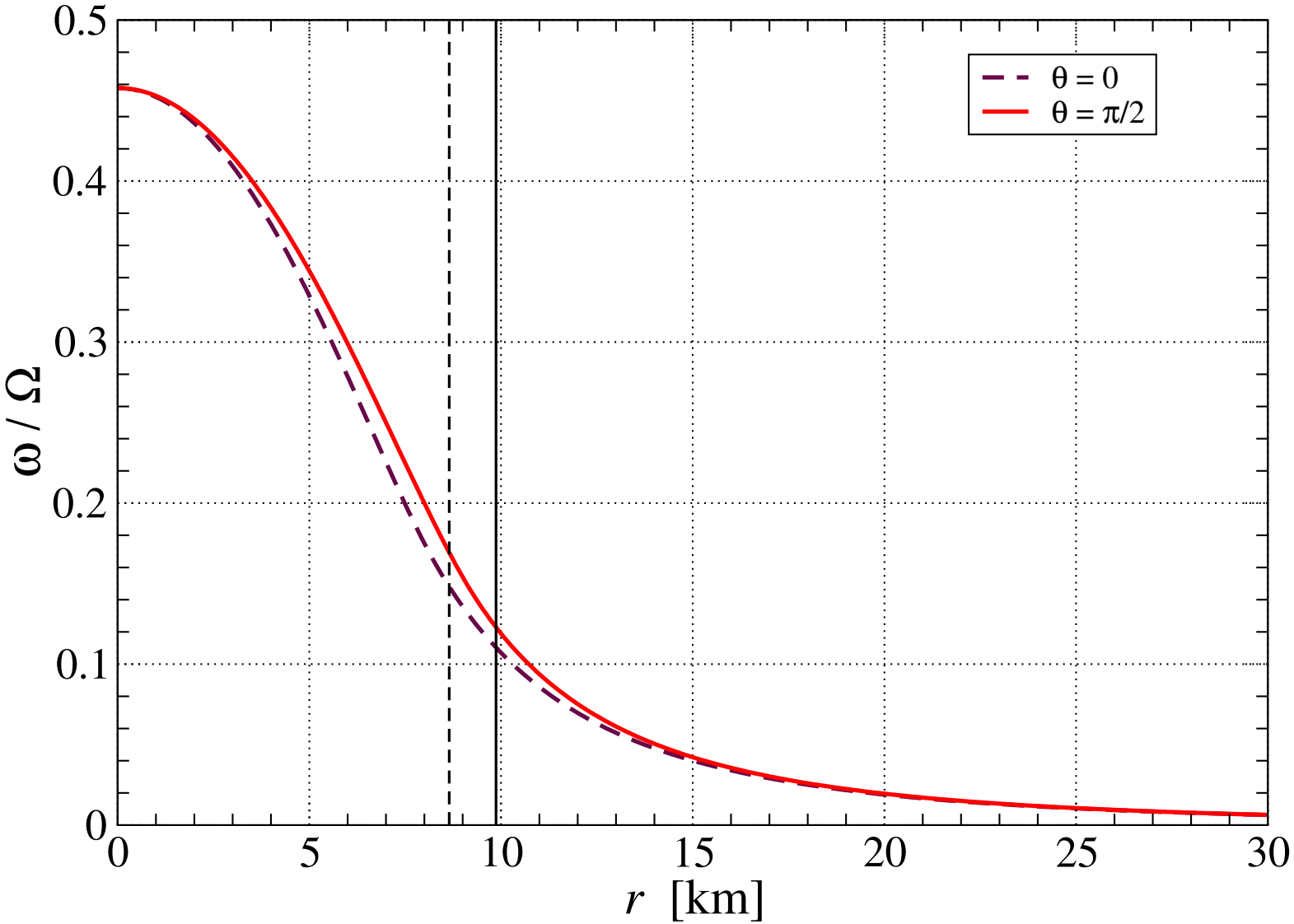
Gravitational mass M	$1.400\,M_\odot$
Baryon mass M_b	$1.542\,M_\odot$
Rotation frequency $\Omega/(2\pi)$	716 Hz
Central log-enthalpy H_c	$0.2262\,c^2$
Central proper baryon density $n_\mathrm{b,c}$	0.5301 fm^{-3}
Central proper energy density ε_c	$5.7838\,\rho_\mathrm{nuc}c^2$
Central pressure p_c	$0.8628\,\rho_\mathrm{nuc}c^2$
Coordinate equatorial radius r_eq	9.867 km
Coordinate polar radius r_p	8.649 km
Axis ratio $r_\mathrm{p}/r_\mathrm{eq}$	0.8763
Circumferential equat. radius R_circ	12.08 km
Compactness $GM/(c^2R_\mathrm{circ})$	0.1711
Angular momentum J	$0.7238\,GM_\odot^2/c$
Kerr parameter $cJ/(GM^2)$	0.3693
Moment of inertia I	1.417×10^{38} kg m^2
Kinetic energy ratio T/W	0.0348
Velocity at the equator U_eq	$0.1967\,c$
Redshift from equator, backward z^b_eq	0.5529
Redshift from pole z_p	0.2618



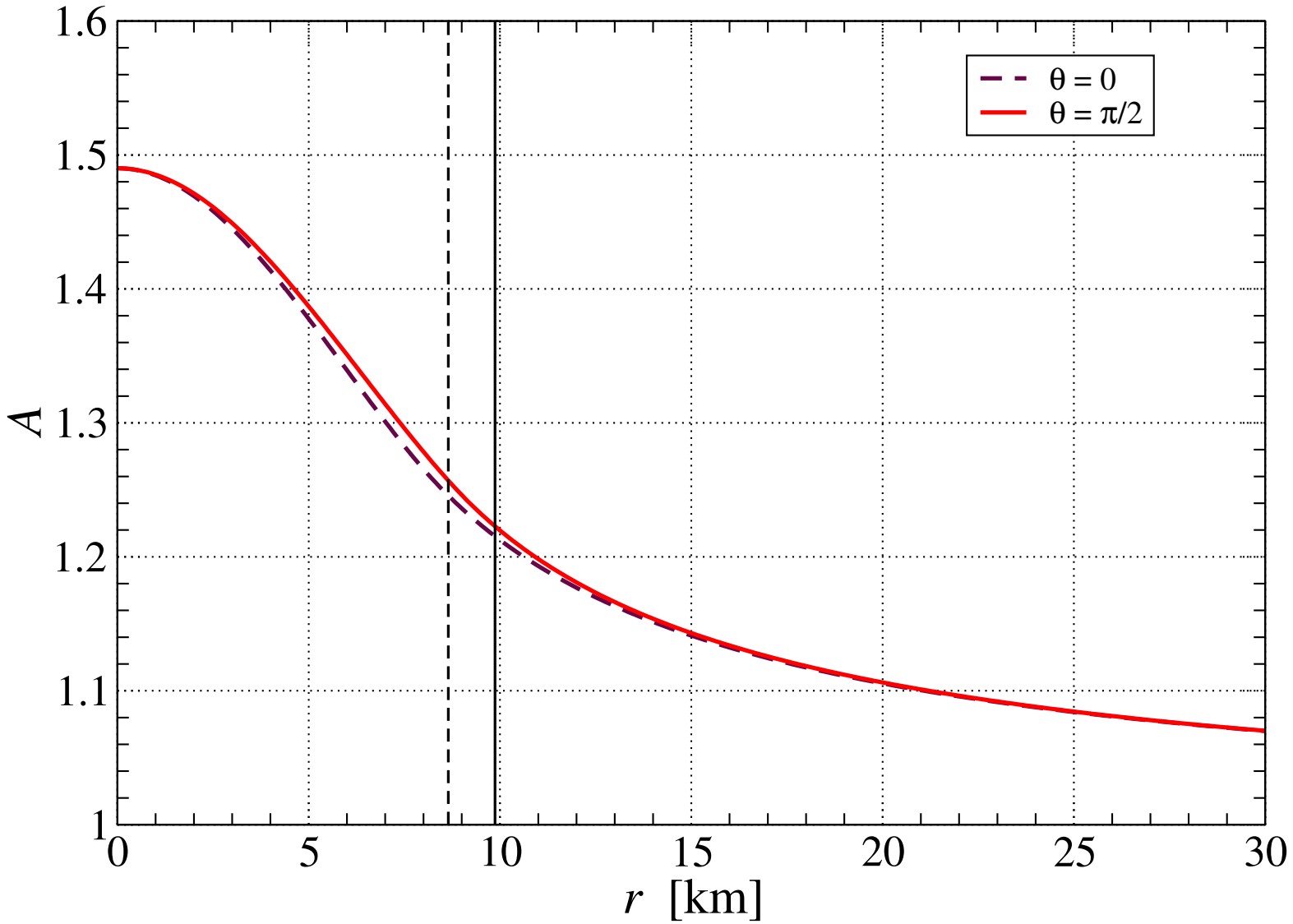
Isocontours of the proper energy density ε in the meridional plane $\varphi = 0$.
The coordinates (x, z) are defined by $x := r \sin\theta$ and $z := r \cos\theta$.
The thick solid line marks the stellar surface.



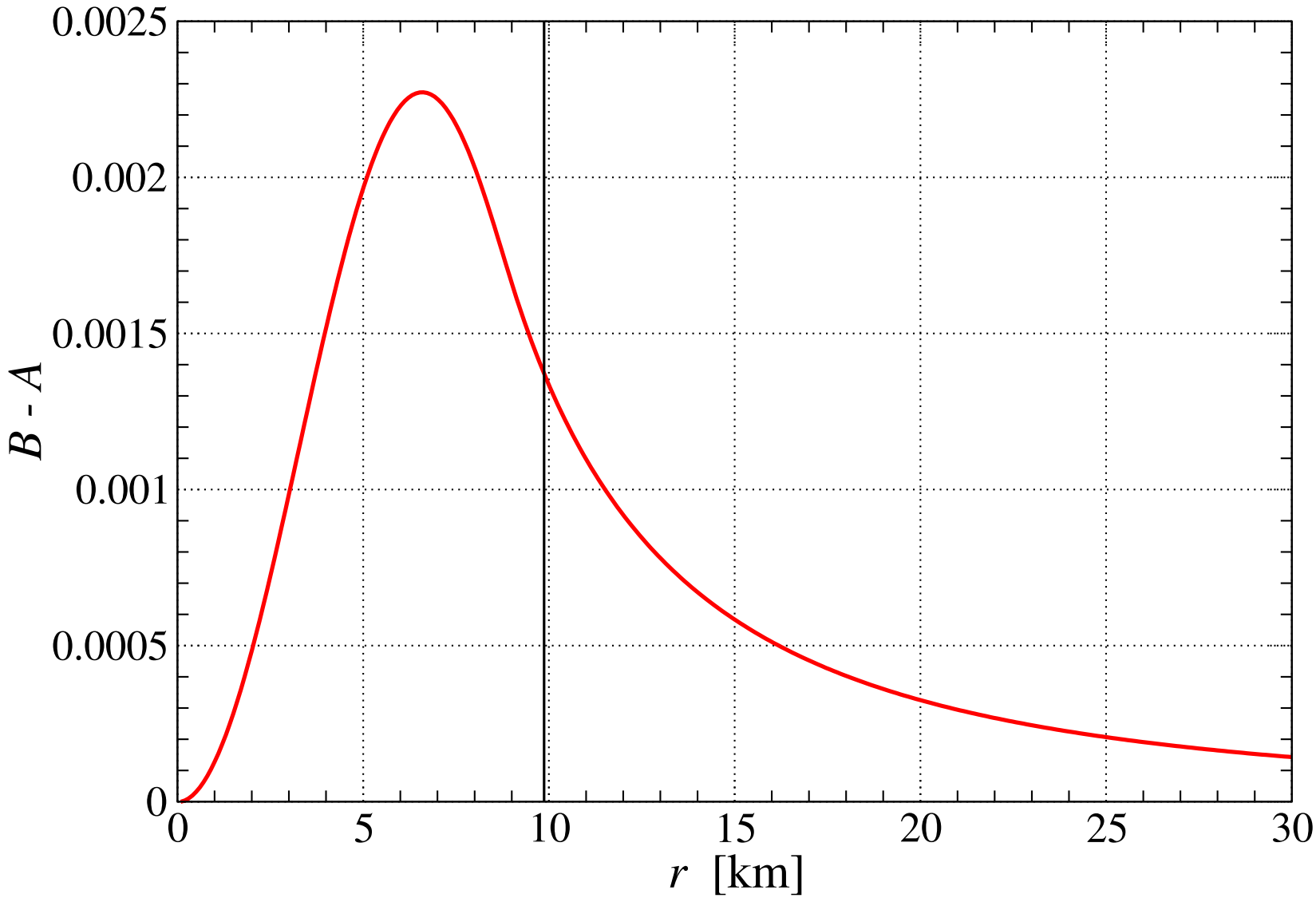
Profile of the lapse function N in two different directions: $\theta = 0$ (rotation axis) and $\theta = \pi/2$ (equatorial plane).
The vertical solid line (resp. dashed line) marks the location of the stellar surface in the equatorial plane (resp. along the rotation axis).



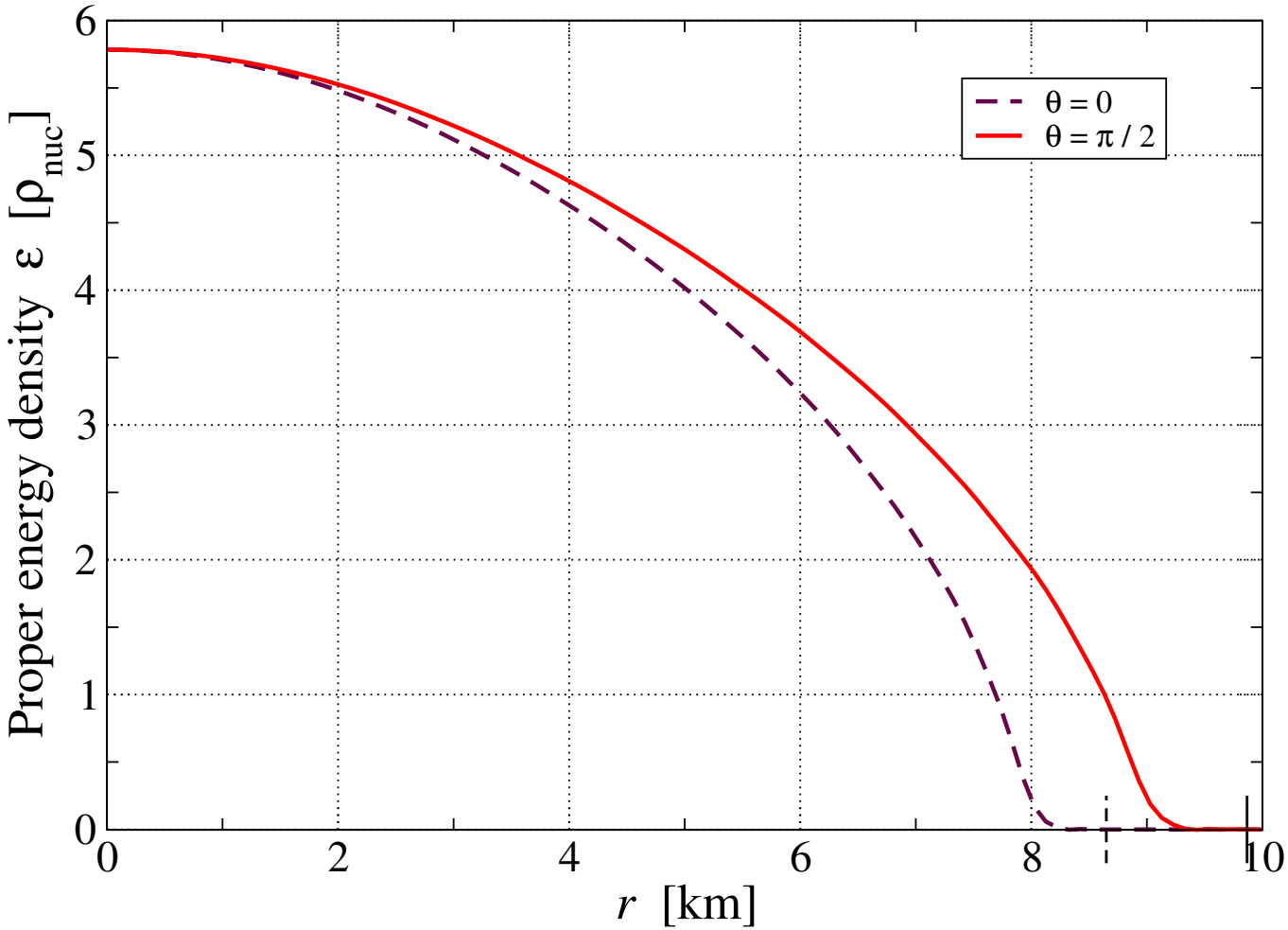
Profile of the shift vector component $\omega = -\beta^{\varphi}$



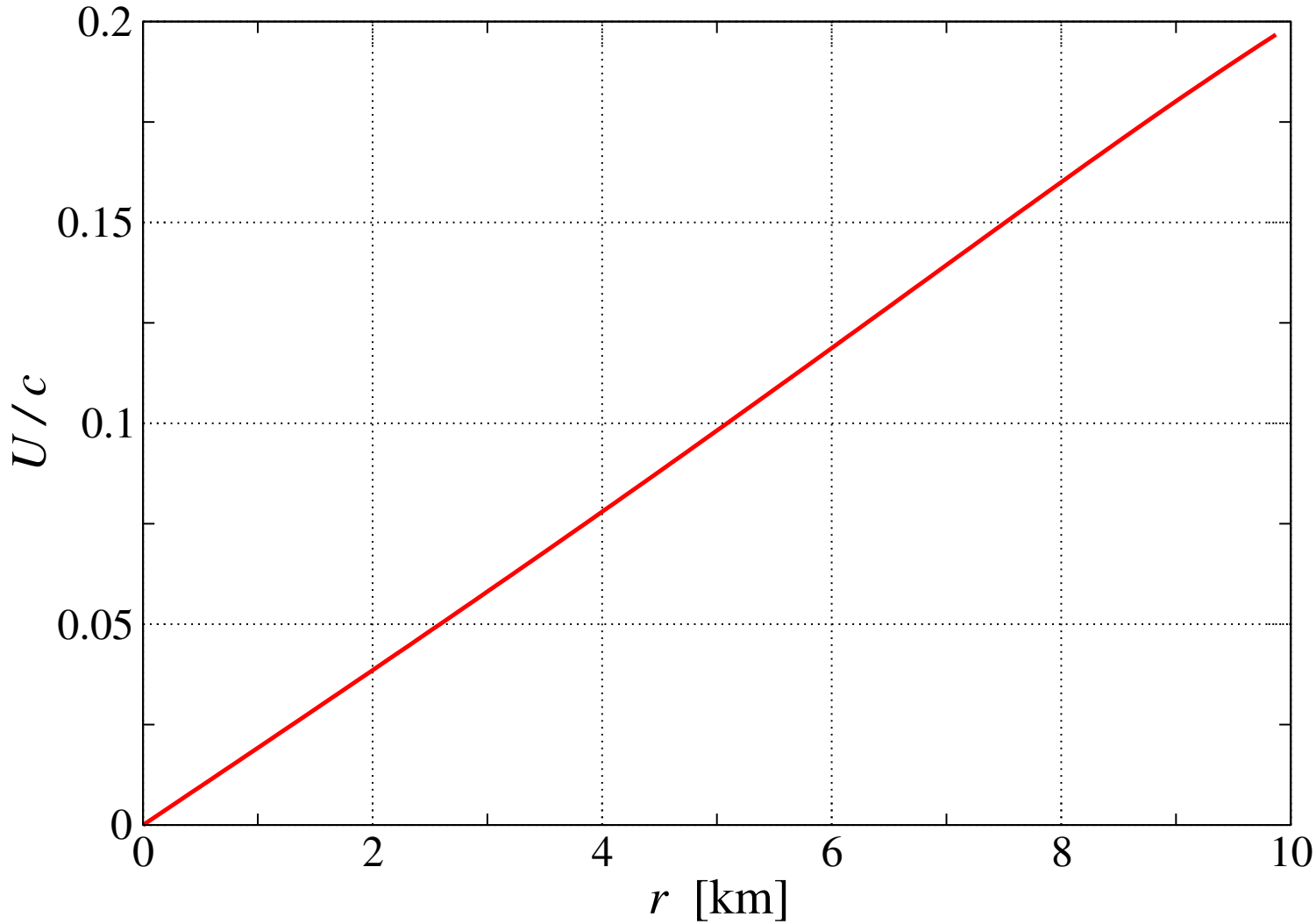
Profile of the metric coefficient A



Profile of the difference between the metric coefficients B and A in the equatorial plane. The vertical solid line marks the location of the stellar surface in that plane.



Profile of the fluid proper energy density ε in two different directions: $\theta = 0$ (rotation axis) and $\theta = \pi/2$ (equatorial plane).
The small vertical solid line (resp. dashed line) marks the location of the stellar surface in the equatorial plane (resp. along the rotation axis).
$\rho_{\rm nuc} := 1.66 \times 10^{17}\ {\rm kg\,m^{-3}}$.



Profile of the norm U of the fluid velocity with respect to the ZAMO, in the equatorial plane.

Bibliography

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