Testing the Kerr black hole hypothesis with observations of Sgr A*

Éric Gourgoulhon

Laboratoire Univers et Théories (LUTH)
CNRS / Observatoire de Paris / Université Paris Diderot
Paris Sciences et Lettres Research University
92190 Meudon, France

http://luth.obspm.fr/~luthier/gourgoulhon/

based on a collaboration with
Philippe Grandclément, Marion Grould, Carlos Herdeiro, Zakaria Meliani, Jérôme Novak, Thibaut Paumard, Guy Perrin, Eugen Radu, Claire Somé, Odele Straub, Karim Van Aelst and Frédéric H. Vincent

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Outline

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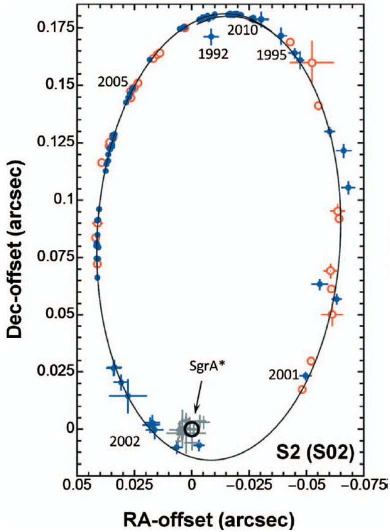
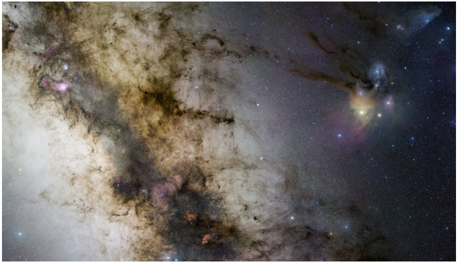
The black hole at the centre of our galaxy : Sgr A*





[ESO (2009)]

Mass of Sgr A* black hole deduced from stellar dynamics :

$$M_{\rm BH} = 4.3 \times 10^6 \, M_\odot$$

← Orbit of the star S2 around Sgr A*

$P = 16\,{\rm yr}$, $r_{\rm per} = 120\,{\rm UA} = 1400\,R_{\rm S}$, $V_{\rm per} = 0.02\,c$

[Genzel, Eisenhauer & Gillessen, RMP **82**, 3121 (2010)]

Next periastron passage : mid 2018

Can we see it from the Earth ?

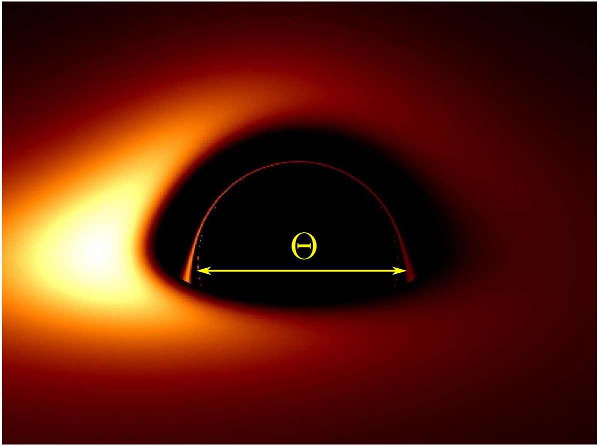


Image of a thin accretion disk around a Schwarzschild BH
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass M seen from a distance d :

$$\Theta = 6\sqrt{3}\,\frac{GM}{c^2 d} \simeq 2.60\,\frac{2R_S}{d}$$

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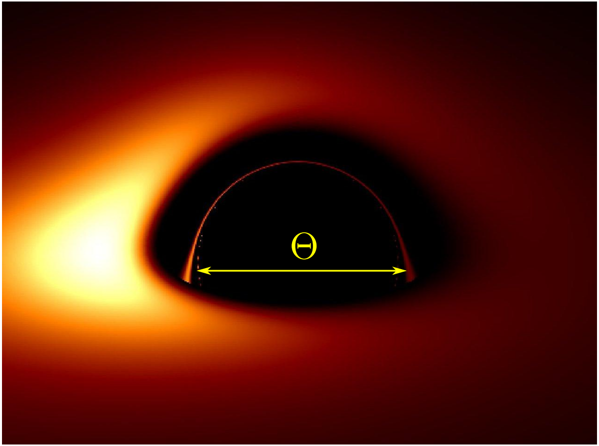

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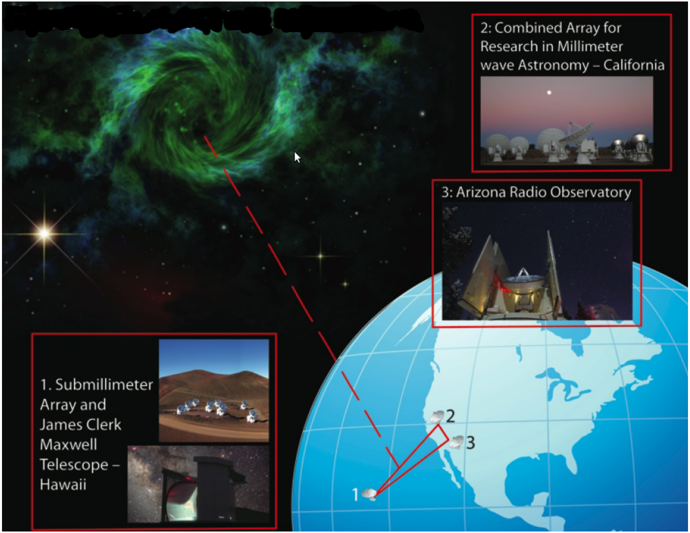
Largest black holes in the Earth's sky :

Sgr A* : $\Theta = 53\ \mu\mathrm{as}$
M87 : $\Theta = 21\ \mu\mathrm{as}$
M31 : $\Theta = 20\ \mu\mathrm{as}$

Remark : black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$

Reaching the μas resolution with VLBI



Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]

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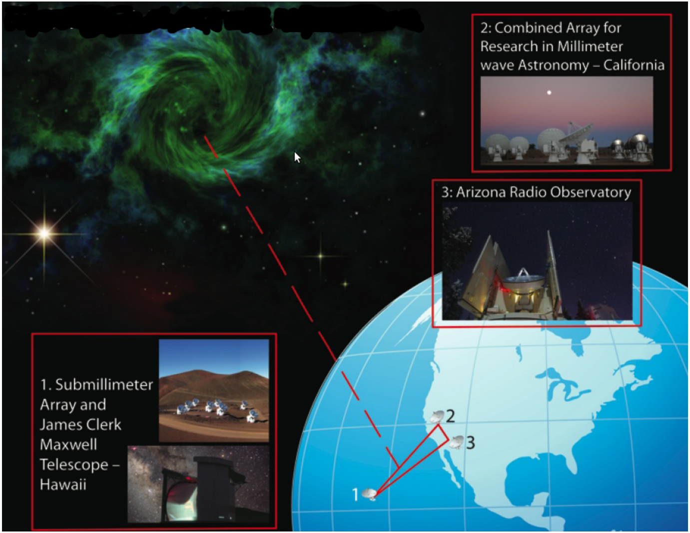

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Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far : VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only $37\ \mu$as [Doeleman et al., Nature **455**, 78 (2008)]

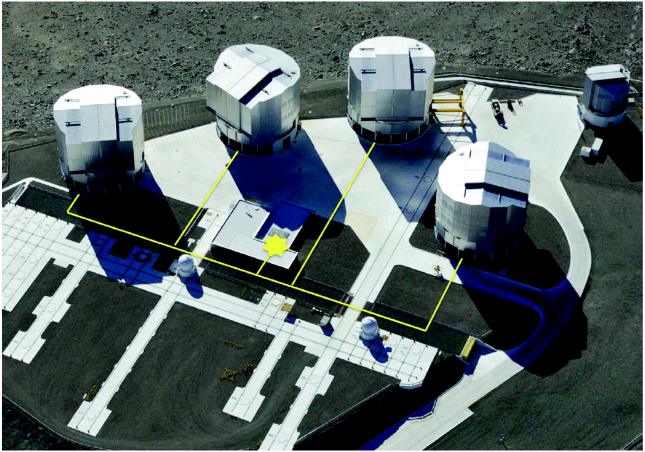
The near future : the Event Horizon Telescope

To go further :

- shorten the wavelength : $1.3\ \mathrm{mm} \to 0.8\ \mathrm{mm}$
- increase the number of stations ; in particular add ALMA

Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020
Spring 2017 : large observation campaign $\Longrightarrow$ first image ?

Near-infrared optical interferometry : GRAVITY



[Gillessen et al. 2010]

GRAVITY instrument at VLTI (2016)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits : $10\ \mu as$

Outline

The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a *stationary* black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 numbers :

- the total mass M
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

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Astrophysical black holes have to be electrically neutral :

- $Q = 0$: Kerr solution (1963)

Other special cases :

- $a = 0$: Reissner-Nordström solution (1916, 1918)
- $a = 0$ and $Q = 0$: Schwarzschild solution (1916)
- $a = 0$, $Q = 0$ and $M = 0$: Minkowski metric (1907)

The no-hair theorem : precise mathematical statement

Any spacetime $(\mathscr{M}, \boldsymbol{g})$ that

- is 4-dimensional
- is asymptotically flat
- is pseudo-stationary
- is a solution of the vacuum Einstein equation : $\mathrm{Ric}(\boldsymbol{g}) = 0$
- contains a black hole with a connected regular horizon
- has no closed timelike curve in the domain of outer communications
- is analytic

has a domain of outer communications that is isometric to the domain of outer communications of the Kerr spacetime.

domain of outer communications : black hole exterior

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Possible improvements : remove the hypotheses of analyticity and non-existence of closed timelike curves (analyticity removed recently but only for slowly rotating black holes [Alexakis, Ionescu & Klainerman, Duke Math. J. **163**, 2603 (2014)])

The Kerr solution

Roy Kerr (1963)

$$g_{\alpha\beta}\, dx^\alpha\, dx^\beta = -\left(1 - \frac{2GMr}{c^2\rho^2}\right) c^2 dt^2 - \frac{4GMar\sin^2\theta}{c^2\rho^2}\, c\, dt\, d\varphi + \frac{\rho^2}{\Delta}\, dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2GMa^2 r\sin^2\theta}{c^2\rho^2}\right)\sin^2\theta\, d\varphi^2$$

where

$\rho^2 := r^2 + a^2\cos^2\theta$, $\Delta := r^2 - \frac{2GM}{c^2} r + a^2$ and $\boxed{r \in (-\infty, \infty)}$

→ spacetime manifold : $\mathscr{M} = \mathbb{R}^2 \times \mathbb{S}^2 \setminus \{r = 0\ \&\ \theta = \pi/2\}$

→ 2 parameters : M : gravitational mass ; $a := \frac{J}{cM}$ reduced angular momentum

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→ Schwarzschild solution as the subcase $a = 0$:

$$g_{\alpha\beta}\,dx^\alpha\,dx^\beta = -\left(1-\frac{2GM}{c^2r}\right)c^2dt^2 + \left(1-\frac{2GM}{c^2r}\right)^{-1}dr^2 + r^2\left(d\theta^2+\sin^2\theta\,d\varphi^2\right)$$

Basic properties of Kerr metric

- Asymptotically flat ($r \to \pm\infty$)
- Pseudo-stationary : metric components independent from t, with $\partial/\partial t$ timelike at least asymptotically
- Axisymmetric : metric components independent from φ
- Not static when $a \neq 0$
- Contains a black hole $\iff$ $\boxed{0 \leq a \leq m}$, where $m := GM/c^2$

  event horizon : $r = r_+ := m + \sqrt{m^2 - a^2}$
- Contains a curvature singularity at $\rho = 0 \iff r = 0$ and $\theta = \pi/2$

The Kerr metric is specific to black holes

Spherically symmetric (non-rotating) bodies :

Birkhoff theorem

Within 4-dimensional general relativity, the spacetime outside any spherically symmetric body is described by Schwarzschild metric

$\Longrightarrow$ No possibility to distinguish a non-rotating black hole from a non-rotating dark star by monitoring orbital motion or fitting accretion disk spectra

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Rotating axisymmetric bodies :

No Birkhoff theorem

Moreover, no "reasonable" matter source has ever been found for the Kerr metric (the only known source consists of two counter-rotating thin disks of collisionless particles [Bicak & Ledvinka, PRL **71**, 1669 (1993)])

$\Longrightarrow$ The Kerr metric is specific to rotating black holes
(in 4-dimensional general relativity)

Lowest order no-hair theorem : quadrupole moment

Asymptotic expansion (large r) of the metric in terms of multipole moments $(\mathcal{M}_k, \mathcal{J}_k)_{k\in\mathbb{N}}$ [Geroch (1970), Hansen (1974)] :

- $\mathcal{M}_k$: mass 2^k-pole moment
- $\mathcal{J}_k$: angular momentum 2^k-pole moment

$\Longrightarrow$ For the Kerr metric, all the multipole moments are determined by (M, a) :

- $\mathcal{M}_0 = M$
- $\mathcal{J}_1 = aM = J/c$
- $\boxed{\mathcal{M}_2 = -a^2 M = -\frac{J^2}{c^2 M}}$ $(*)$ $\leftarrow$ mass quadrupole moment
- $\mathcal{J}_3 = -a^3 M$
- $\mathcal{M}_4 = a^4 M$
- $\cdots$

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Measuring the three quantities M, J, $\mathcal{M}_2$ provides a compatibility test w.r.t. the Kerr metric, by checking $(*)$

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Within general relativity

The compact object is not a black hole but

- boson stars
- gravastar
- dark stars
- ...

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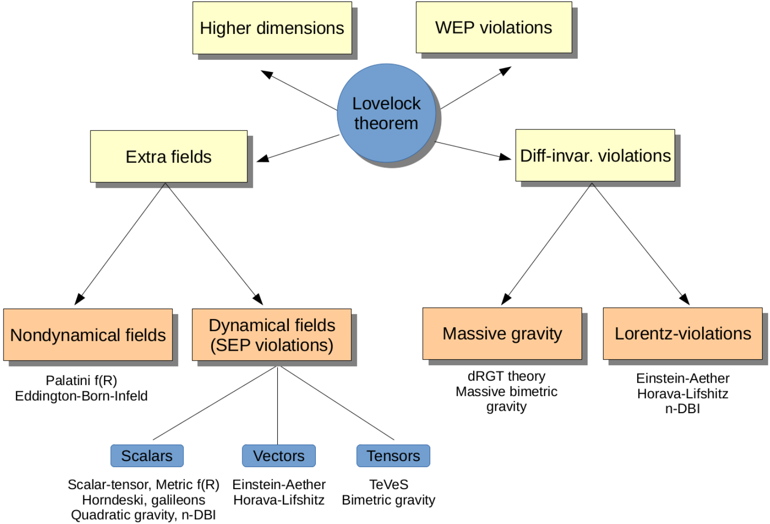
Beyond general relativity

The compact object is a black hole but in a theory that differs from 4-dimensional GR :

- Horndeski theories
- Chern-Simons gravity
- Hořava-Lifshitz gravity
- Higher-dimensional GR
- ...

Extensions of general relativity



[Berti et al., CGQ **32**, 243001 (2015)]

Viable scalar-tensor theories after GW170817

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [42] Brans-Dicke/$f(R)$ [43, 44] Kinetic Gravity Braiding [46]	quartic/quintic Galileons [13, 14] Fab Four [15, 16] de Sitter Horndeski [45] $G_{\mu\nu}\phi^{\mu}\phi^{\nu}$ [47], Gauss-Bonnet
beyond H.	Derivative Conformal (20) [18] Disformal Tuning (22) DHOST with $A_1 = 0$	quartic/quintic GLPV [19] DHOST [20, 48] with $A_1 \neq 0$
	Viable after GW170817	Non-viable after GW170817

[Ezquiaga & Zumalacárregui, arXiv:1710.05901]

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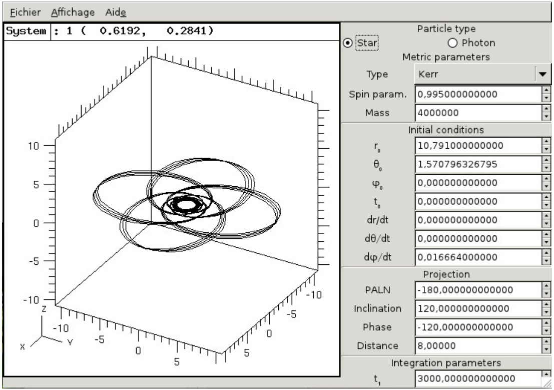
Need for a good and versatile **geodesic integrator**
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric

Gyoto code

Main developers : T. Paumard & F. Vincent



- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL) : http://gyoto.obspm.fr/

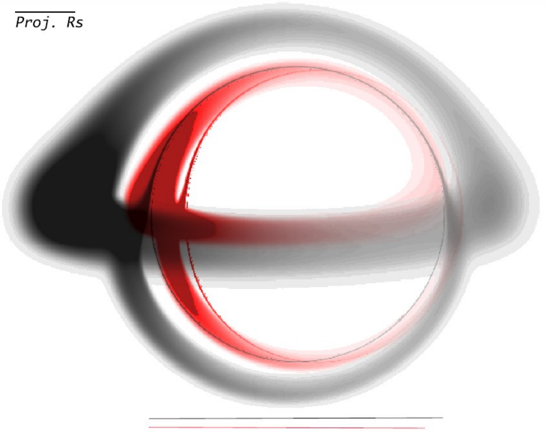
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

Measuring the spin from the black hole silhouette

Ray-tracing in the Kerr metric (spin parameter a)

Accretion structure around Sgr A* modelled as a ion torus, derived from the *polish doughnut* class [Abramowicz, Jaroszynski & Sikora (1978)]



Radiative processes included : thermal synchrotron, bremsstrahlung, inverse Compton

← Image of an ion torus computed with `Gyoto` for the inclination angle $i = 80°$:

- black : $a = 0.5M$
- red : $a = 0.9M$

[Straub, Vincent, Abramowicz, Gourgoulhon & Paumard, A&A **543**, A83 (2012)]

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Boson stars

Boson star = localized configurations of a self-gravitating complex scalar field Φ
$\equiv$ "Klein-Gordon geons" [Bonazzola & Pacini (1966), Kaup (1968), Ruffini & Bonazzola (1969)]

- Minimally coupled scalar field : $\mathcal{L} = \frac{1}{16\pi} R - \frac{1}{2} \left[\nabla_\mu \bar{\Phi} \nabla^\mu \Phi + V(|\Phi|^2) \right]$
- Scalar field equation : $\nabla_\mu \nabla^\mu \Phi = V'(|\Phi|^2)\, \Phi$
- Einstein equation : $R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = 8\pi T_{\alpha\beta}$

with $T_{\alpha\beta} = \nabla_{(\alpha} \bar{\Phi} \nabla_{\beta)} \Phi - \frac{1}{2} \left[\nabla_\mu \bar{\Phi} \nabla^\mu \Phi + V(|\Phi|^2) \right] g_{\alpha\beta}$

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Examples :

- free field : $V(|\Phi|^2) = \frac{m^2}{\hbar^2} |\Phi|^2$, $\quad m$: boson mass

  $\Longrightarrow$ field equation = Klein-Gordon equation : $\nabla_\mu \nabla^\mu \Phi = \frac{m^2}{\hbar^2} \Phi$
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Boson stars as black-hole mimickers

Boson stars can be very compact and are the less exotic alternative to black holes : they require only a scalar field and since 2012 we know that at least one fundamental scalar field exists in Nature : the Higgs boson !

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Maximum mass

- Free field : $M_{\rm max} = \alpha \frac{\hbar}{m} = \alpha \frac{m_{\rm P}^2}{m}$, with $\alpha \sim 1$
- Self-interacting field : $M_{\rm max} \sim \left(\frac{\lambda}{4\pi} \right)^{1/2} \frac{m_{\rm P}^2}{m} \times \frac{m_{\rm P}}{m}$

$m_{\rm P} = \sqrt{\hbar} = \sqrt{\hbar c / G} = 2.18 \; 10^{-8} \; {\rm kg}$: Planck mass

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m	$M_{\rm max}$ (free field)	$M_{\rm max}$ $(\lambda = 1)$
125 GeV (Higgs)	$2\ 10^{9}$ kg	$2\ 10^{26}$ kg
1 GeV	$3\ 10^{11}$ kg	$2\ M_\odot$
0.5 MeV	$3\ 10^{14}$ kg	$5\ 10^{6}\ M_\odot$

Framework

Hypotheses :

- stationarity $\Longrightarrow$ Killing vector ∂_t
- axisymmetry $\Longrightarrow$ Killing vector ∂_φ
- circularity : 2-surfaces of transitivity of the spacetime symmetry group $\mathbb{R} \times \mathrm{SO}(2)$ (surfaces of constant (r, θ)) orthogonal to surfaces of constant (t, φ)

$\Longrightarrow$ *quasi-isotropic coordinates* (t, r, θ, φ) (also called *Lewis-Papapetrou coordinates*) :

$$g_{\mu\nu}\, dx^\mu\, dx^\nu = -N^2 dt^2 + A^2 \left(dr^2 + r^2 d\theta^2\right) + B^2 r^2 \sin^2\theta \,(d\varphi + \beta^\varphi dt)^2$$

with

$$N = N(r,\theta), \quad \beta^\varphi = \beta^\varphi(r,\theta), \quad A = A(r,\theta), \quad B = B(r,\theta)$$

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with

$$N = N(r,\theta), \quad \beta^\varphi = \beta^\varphi(r,\theta), \quad A = A(r,\theta), \quad B = B(r,\theta)$$

Ansatz for the scalar field [Schunck & Mielke (1996)] :

$$\Phi(t, r, \theta, \varphi) = \Phi_0(r, \theta) e^{i(\omega t + k\varphi)}$$

with $\Phi_0(r, \theta)$ real function, $\omega \in \mathbb{R}$ and $k \in \mathbb{N}$ (regularity on the rotation axis)

Einstein equations

$$\Delta_3 \ln N = 4\pi A^2(E+S) + \frac{B^2 r^2 \sin^2\theta}{2N^2}\partial\beta^\varphi\partial\beta^\varphi - \partial\ln N\partial\ln(NB)$$

$$\tilde\Delta_3\left(\beta^\varphi r\sin\theta\right) = 16\pi\frac{NA^2}{B^2}\frac{P_\varphi}{r\sin\theta} + r\sin\theta\partial\beta^\varphi\partial(\ln N - 3\ln B)$$

$$\Delta_2\left[(NB-1)\,r\sin\theta\right] = 8\pi NA^2 Br\sin\theta\left(S^r{}_r + S^\theta{}_\theta\right)$$

$$\Delta_2\ln(AN) = 8\pi A^2 S^\varphi{}_\varphi + \frac{3B^2r^2\sin^2\theta}{4N^2}\partial\beta^\varphi\partial\beta^\varphi - \partial\ln N\partial\ln N$$

with

$$\Delta_3 := \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{1}{r^2\tan\theta}\frac{\partial}{\partial\theta}, \qquad \tilde\Delta_3 := \Delta_3 - \frac{1}{r^2\sin^2\theta}$$

$$\Delta_2 := \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}, \qquad \partial f\partial g := \frac{\partial f}{\partial r}\frac{\partial g}{\partial r} + \frac{1}{r^2}\frac{\partial f}{\partial\theta}\frac{\partial g}{\partial\theta}$$

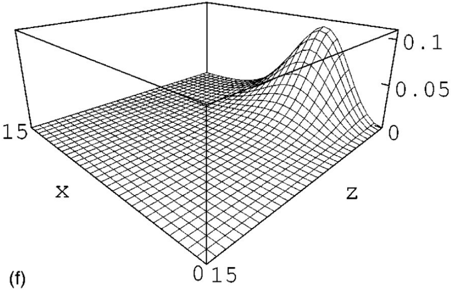
Scalar field equation

$$\Delta_3 \Phi_0 - \frac{k^2 \Phi_0}{r^2 \sin^2\theta} = A^2 \left[\frac{\mathrm{d}V}{\mathrm{d}\,|\Phi|^2} - \frac{1}{N^2}(\omega + k\beta^\varphi)^2 \right] \Phi_0 - \partial\Phi_0 \partial \ln(BN) + \left(\frac{A^2}{B^2} - 1 \right) \frac{k^2 \Phi_0}{r^2 \sin^2\theta}$$

$$\Longrightarrow \Phi(t, r, \theta, \varphi) = \Phi_0(r, \theta) e^{i(\omega t + k\varphi)}$$

Nonrotating and rotating boson stars

- $k = 0$: static and spherically symmetric boson stars
  $\Longrightarrow$ exterior spacetime $\simeq$ Schwarzschild (Φ decays fast)
- $k \geq 1$: stationary rotating "stars" with toroidal topology
  $\Longrightarrow$ exterior spacetime significantly different from Kerr



$\leftarrow$ Profile of $\Phi_0(r, \theta)$ for a free field with $k = 2$

z-axis = rotation axis :
$z = r\cos\theta$, $x = r\sin\theta\cos\varphi$

[Yoshida & Eriguchi, PRD **56**, 762 (1997)]

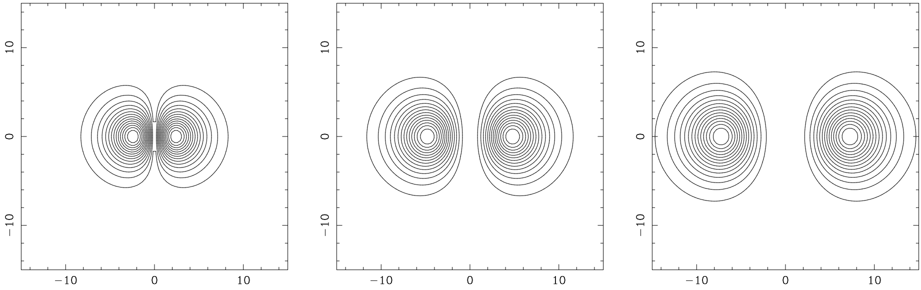
Rotating boson stars

Solutions computed by means of Kadath [Grandclément, JCP **229**, 3334 (2010)]

http://kadath.obspm.fr/

Isocontours of $\Phi_0(r, \theta)$ in the plane $\varphi = 0$ for $\omega = 0.8 \frac{m}{\hbar}$:

$k = 1$ $k = 2$ $k = 3$

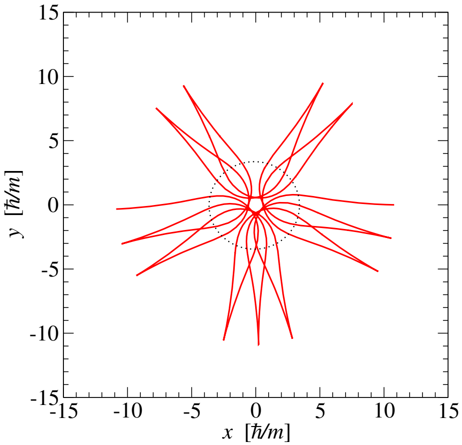


[Granclément, Somé & Gourgoulhon, PRD **90**, 024068 (2014)]

Initially-at-rest orbits around rotating boson stars

Orbit with a rest point around a rotating boson star based on the scalar field
$\Phi = \Phi_0(r,\theta)e^{i(\omega t + k\varphi)}$
with $k = 2$ and $\omega = 0.75\, m/\hbar$

Orbit = timelike geodesic computed by means of Gyoto



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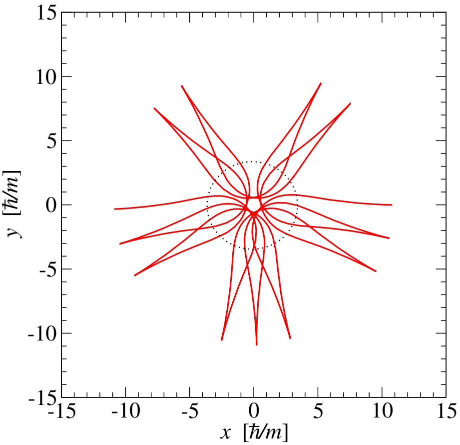
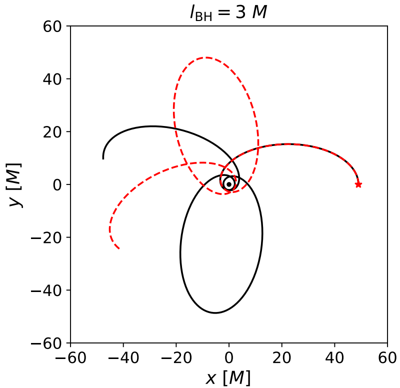


[Granclément, Somé & Gourgoulhon, PRD **90**, 024068 (2014)]

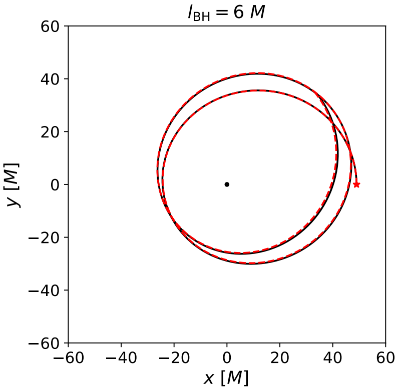
No equivalent in Kerr spacetime

Comparing orbits with a Kerr BH

Same reduced spin for the boson star and the Kerr BH : $a = 0.802\, M$
Boson star (BS) : $k = 1$ and $\omega = 0.8\, m/\hbar$
Same initial position : $r = 50\, M \Longrightarrow$ red dashed : BS, black solid : BH



Low orbital angular momentum

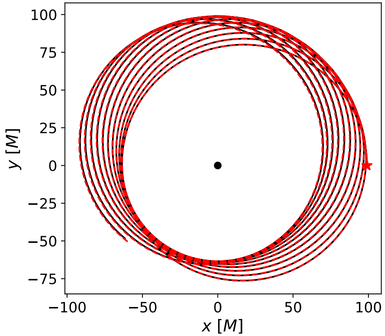


Higher orbital angular momentum

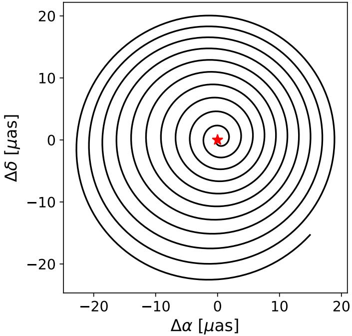
[Grould, Meliani, Vincent, Grandclément & Gourgoulhon, CQG **34**, 215007 (2017)]

Comparing orbits with a Kerr BH

Same reduced spin for the boson star and the Kerr BH : $a = 0.802\, M$
Boson star (BS) : $k = 1$ and $\omega = 0.8\, m/\hbar$
Orbit with pericenter of $60\, M$ and apocenter of $100\, M$



The two orbits

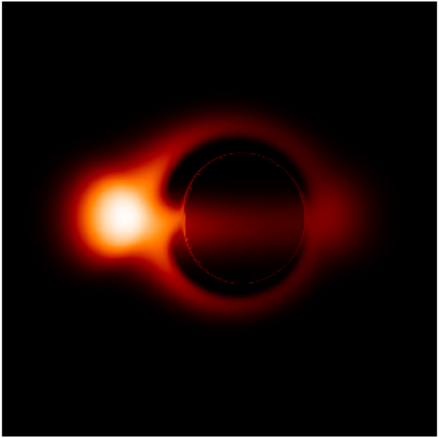


Difference between the BS orbit and the BH one for Sgr A*

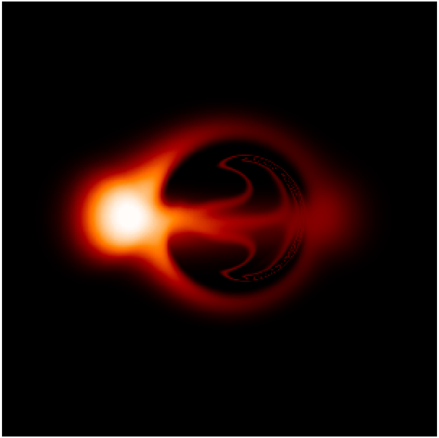
[Grould, Meliani, Vincent, Grandclément & Gourgoulhon, CQG **34**, 215007 (2017)]

Image of an accretion torus : comparing with a Kerr BH

Kerr BH $a/M = 0.9$

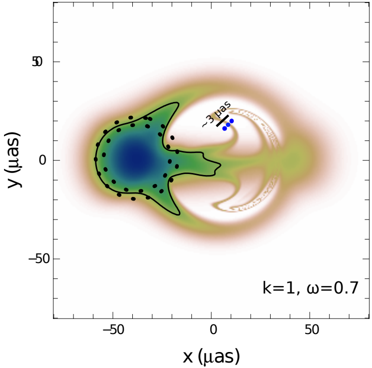


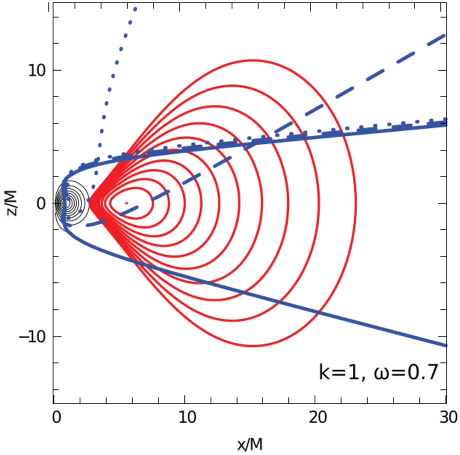
Boson star $k = 1$, $\omega = 0.70\, m/\hbar$



[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG **33**, 105015 (2016)]

Strong light bending in rotating boson star spacetimes





[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG **33**, 105015 (2016)]

Outline

Hairy black holes

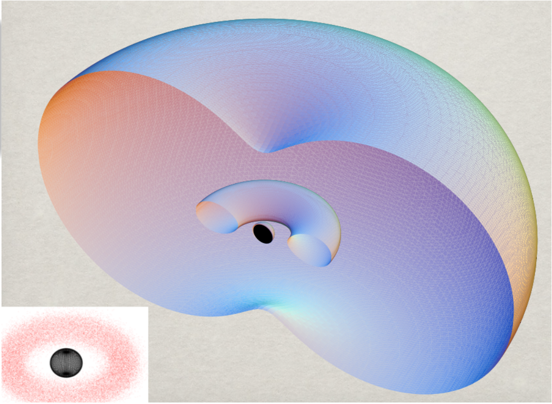
Herdeiro & Radu discovery (2014)

A black hole can have a complex scalar hair

Stationary axisymmetric configuration with a self-gravitating massive complex scalar field Φ and an event horizon

$\Phi(t, r, \theta, \varphi) = \Phi_0(r, \theta) e^{i(\omega t + k\varphi)}$

$\omega = k\Omega_{\rm H}$



[Herdeiro & Radu, PRL **112**, 221101 (2014)]

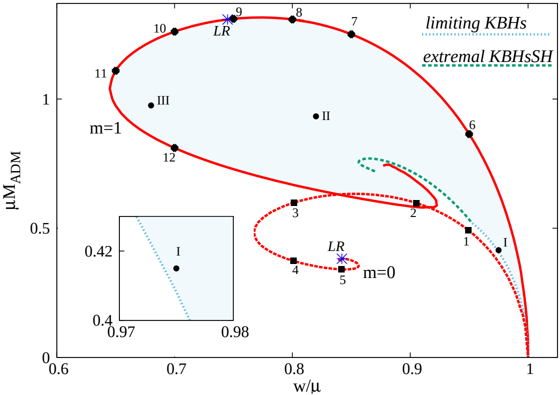
Herdeiro-Radu hairy black holes

- Configuration I : rather Kerr-like
- Configuration II : not so Kerr-like
- Configuration III : very non-Kerr-like

$\mu = \frac{m}{\hbar} = \frac{m}{m_{\rm Pl}^2} = \mathcal{M}^{-1}$

m=0 : non-rotating boson stars

m=1 : rotating boson stars with $k = 1$



[Cunha, Herdeiro, Radu Rúnarsson, PRL **115**, 211102 (2015)]

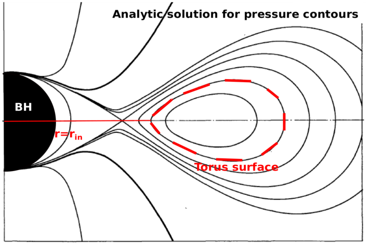
TABLE I. KBHsSH configurations considered in the present study. M is the ADM mass, M_H is the horizon's Komar mass, J is the total Komar angular momentum and J_H is the horizon's Komar angular momentum.

	M	M_H	J	J_H	$\frac{M_H}{M}$	$\frac{J_H}{J}$	$\frac{J}{M^2}$	$\frac{J_H}{M_H^2}$
Configuration I	$0.415\mathcal{M}$	$0.393\mathcal{M}$	$0.172\mathcal{M}^2$	$0.150\mathcal{M}^2$	95%	87%	0.999	0.971
Configuration II	$0.933\mathcal{M}$	$0.234\mathcal{M}$	$0.740\mathcal{M}^2$	$0.115\mathcal{M}^2$	25%	15%	0.850	2.10
Configuration III	$0.975\mathcal{M}$	$0.018\mathcal{M}$	$0.85\mathcal{M}^2$	$0.002\mathcal{M}^2$	1.8%	2.4%	0.894	6.20

Images of a magnetized accretion torus

Accretion torus model of [Vincent, Yan, Straub, Zdziarski & Abramowicz, A&A **574**, A48 (2015)]

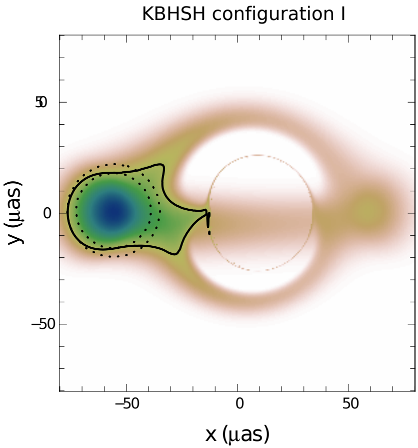
- non-self-gravitating perfect fluid
- polytropic EOS $\gamma = 5/3$
- constant specific angular momentum $\ell = u_\varphi/(-u_t) = 3.6\, M$ [Abramowicz, Jaroszynski & Sikora, A&A **63**, 221 (1978)]
- torus inner radius $r_{\rm in} \simeq 5.5\, M$
- max electron density : $n_{\rm e} = 6.3\ 10^{12}\ {\rm m}^{-3}$
- max electron temperature : $T_{\rm e} = 5.3\ 10^{10}\ {\rm K}$
- isotropized magnetic field $\Longrightarrow$ synchrotron radiation
- gas-to-magnetic pressure ration $\beta = 10$
- observer inclination angle : $\theta = 85°$

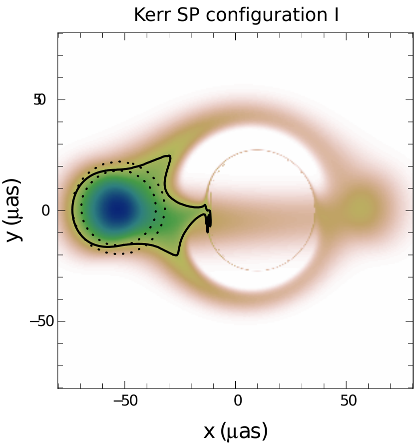


Configuration I

Gyoto-simulated images of Sgr A* at $f = 250$ GHz

hairy BH | Kerr BH with same (M, J)





[Vincent, Gourgoulhon, Herdeiro & Radu, PRD **94**, 084045 (2016)]

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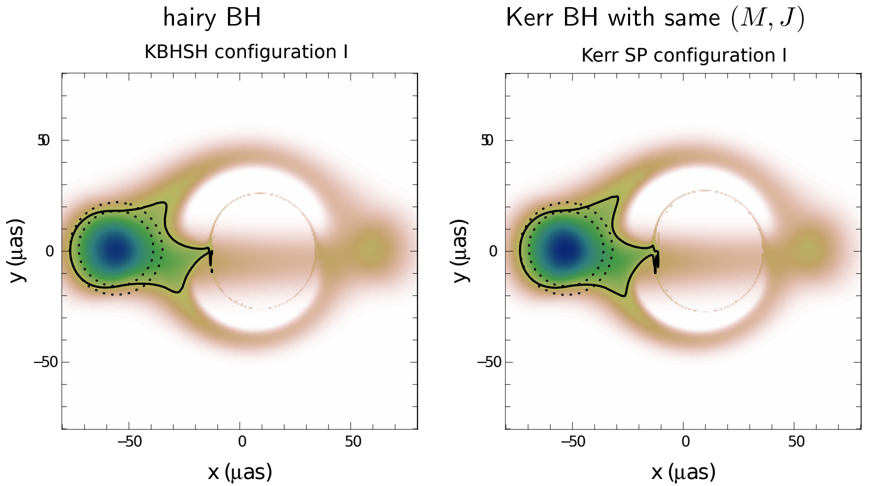


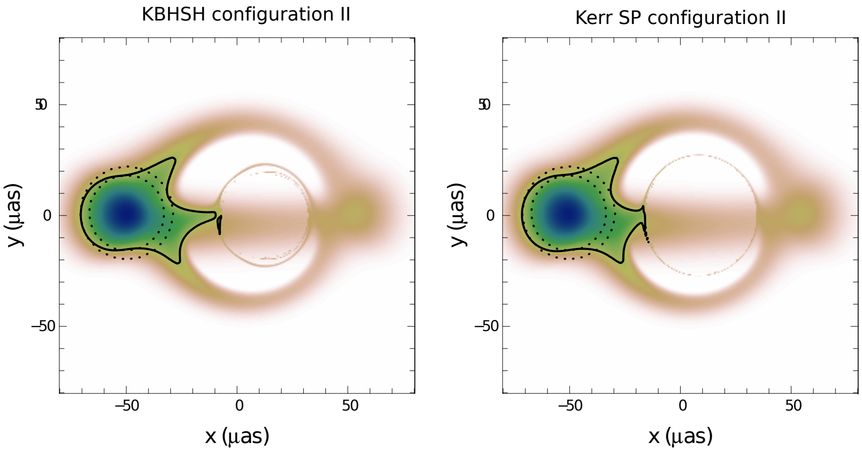
[Vincent, Gourgoulhon, Herdeiro & Radu, PRD **94**, 084045 (2016)]

5% difference in photon ring size $\Longrightarrow$ barely observable

Configuration II

Gyoto-simulated images of Sgr A* at $f = 250$ GHz

hairy BH | Kerr BH with same (M, J)

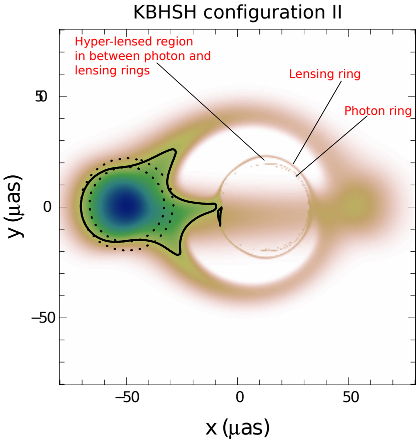


[Vincent, Gourgoulhon, Herdeiro & Radu, PRD **94**, 084045 (2016)]

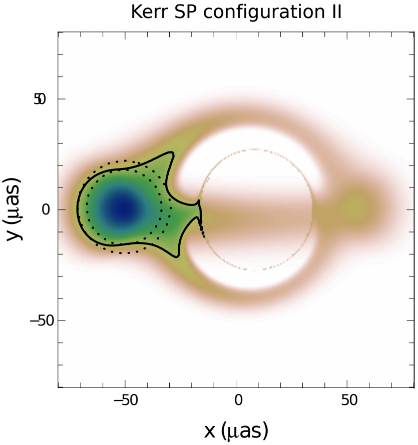
Configuration II

Gyoto-simulated images of Sgr A* at $f = 250$ GHz

hairy BH



Kerr BH with same (M, J)



[Vincent, Gourgoulhon, Herdeiro & Radu, PRD **94**, 084045 (2016)]

Configuration II

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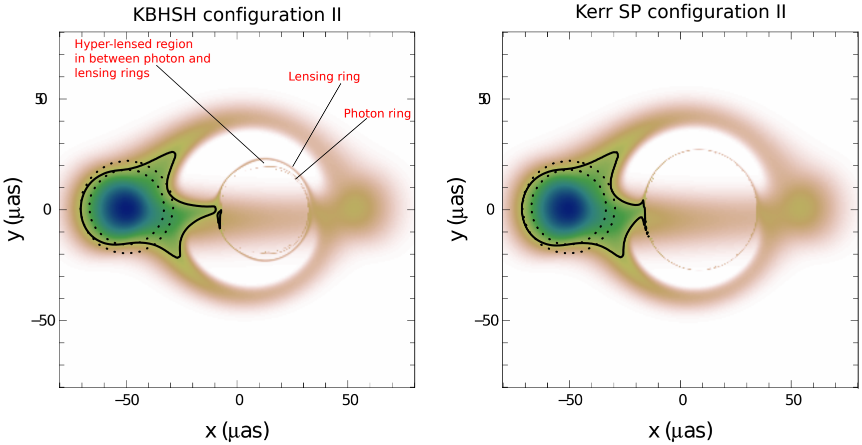
hairy BH | Kerr BH with same (M, J)



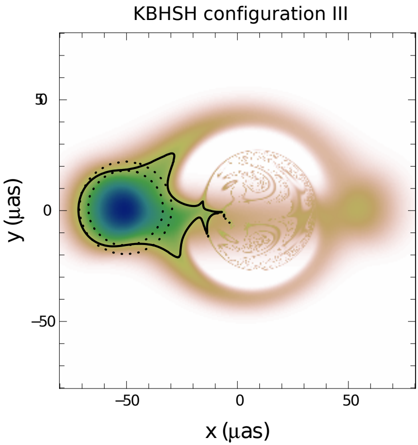
[Vincent, Gourgoulhon, Herdeiro & Radu, PRD **94**, 084045 (2016)]

20% difference between HBH-lensing and BH-photon rings $\Longrightarrow$ observable by EHT

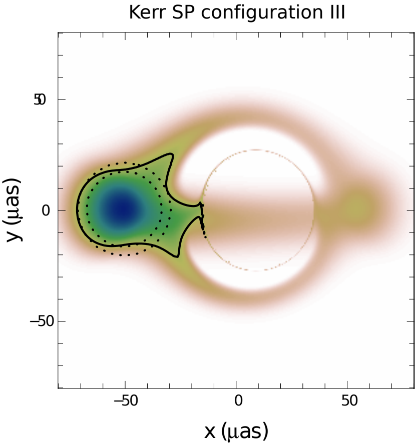
Configuration III

Gyoto-simulated images of Sgr A* at $f = 250\ \mathrm{GHz}$

hairy BH



Kerr BH with same (M, J)

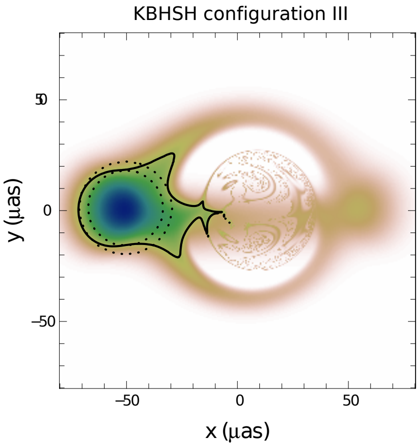


[Vincent, Gourgoulhon, Herdeiro & Radu, PRD **94**, 084045 (2016)]

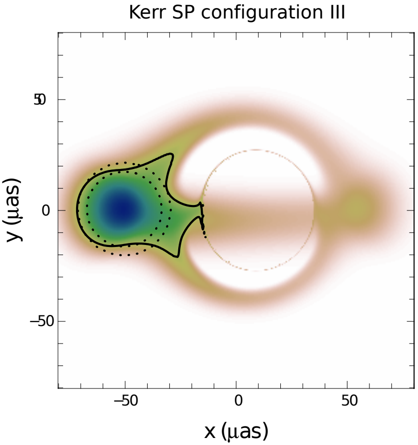
Configuration III

Gyoto-simulated images of Sgr A* at $f = 250$ GHz

hairy BH



Kerr BH with same (M, J)



[Vincent, Gourgoulhon, Herdeiro & Radu, PRD **94**, 084045 (2016)]

HBH : no sharp edge in the intensity distribution $\Longrightarrow$ detectable by EHT

Conclusions and perspective

After a century marked by the Golden Age (1965-1975), which culminated with the no-hair theorem, the first astronomical discoveries and the ubiquity of black holes in high-energy astrophysics, black hole physics is very much alive.

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It is entering a new observational era, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which provide unique opportunities to test general relativity in the strong field regime, notably by finding some voliation of the no-hair theorem.

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To conduct these tests, it is necessary to conduct studies of theoretical alternatives of the Kerr black hole.