Lasts orbits of inspiralling binary black holes

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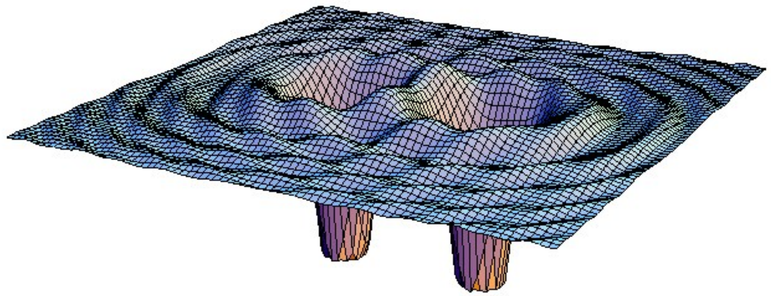
Plan

1. Binary black holes inspiral and coalescence

2. Last stable orbit

3. Treatment within the helical Killing vector approach

Binary black holes inspiral and coalescence



From the GW detection point of view:

- the most promising source [Lipunov, Postnov & Prokhorov, New Astron. **2**, 43 (1997)]

From the theoretical point of view:

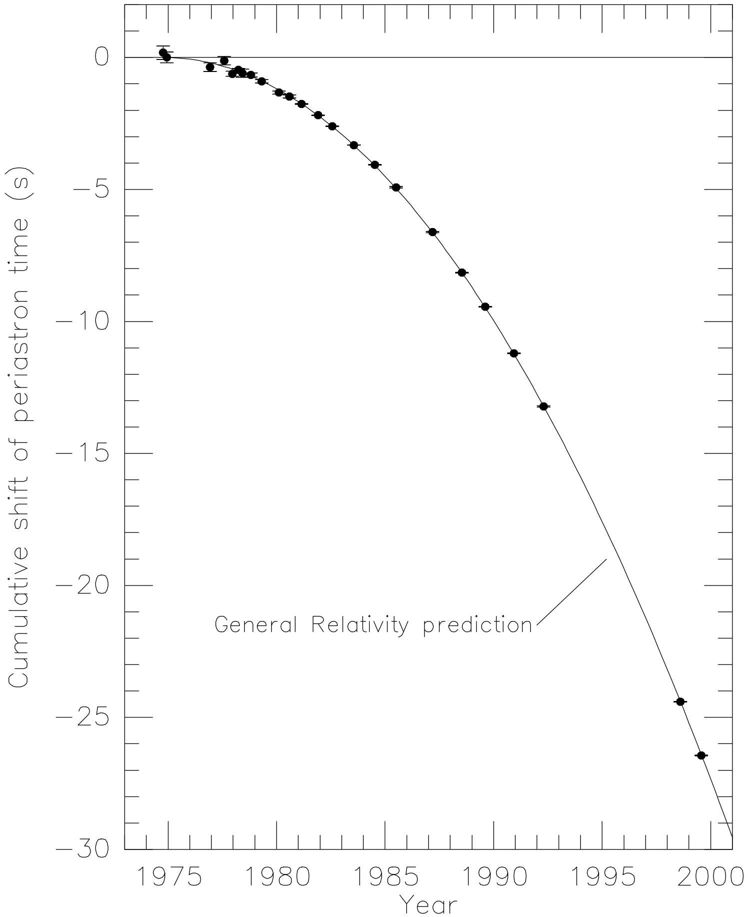
- Binary BH = the two body problem in General Relativity
- Extreme GR $\Longrightarrow$ probes GR in the strong field regime

From the astrophysical point of view:

- Rate of binary black hole coalescences $\Longrightarrow$ massive star evolution
- Inspiral GW signal $\Longrightarrow$ precise measure of Hubble constant H_0
- GW observations of supermassive BH at high z $\Longrightarrow$ formation of large structures

Evolution of binary black holes

Contrary to Newtonian 2-body problem, no stationary solution for 2 bodies in GR :
Energy and angular momentum loss due to gravitational radiation $\Longrightarrow$ shrink of the orbits

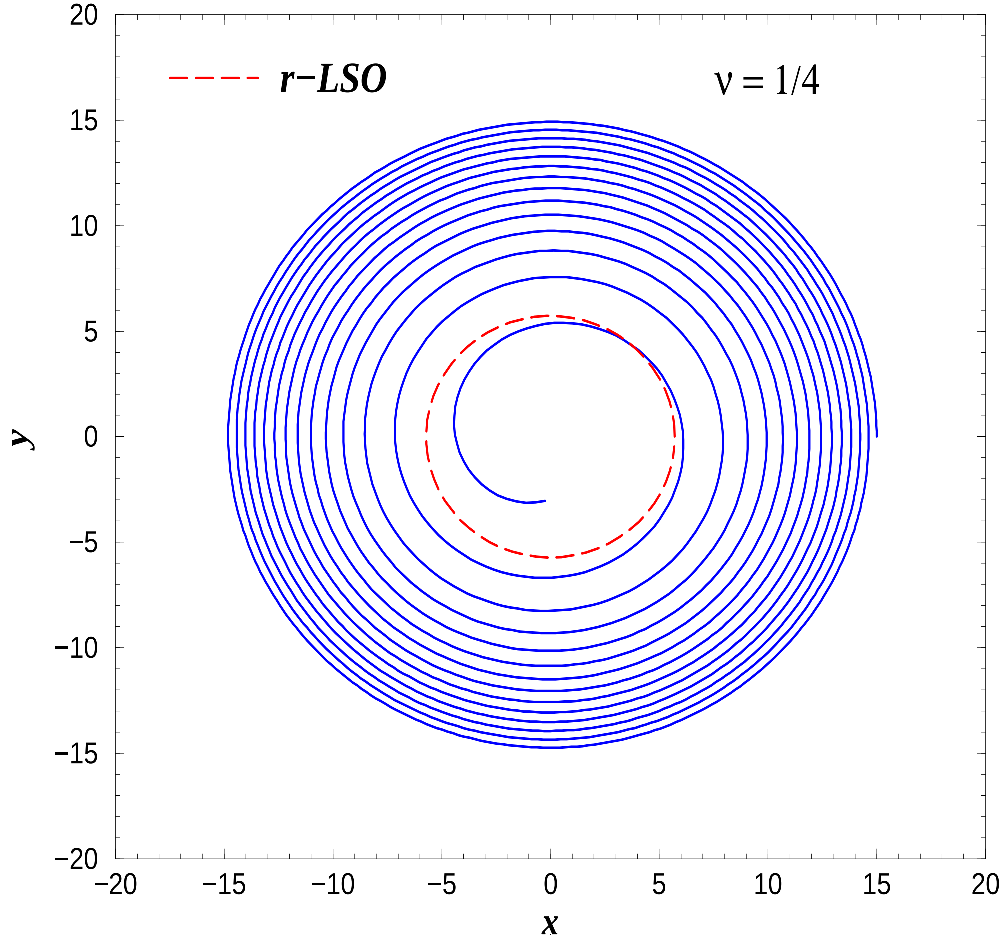


[from Lorimer (2001)]

$\longleftarrow$ Observed decay of the orbital period ($P = 7\,\mathrm{h}\,45\,\mathrm{min}$) of the binary pulsar PSR B1913+16 due to gravitational radiation reaction $\Longrightarrow$ merger in 140 Myr.

Another effect of gravitational wave emission:
circularization of the orbits: $e \rightarrow 0$

Inspiralling motion



2-PN Effective One Body computation

[Buonanno & Damour, PRD **62**, 064015 (2000)]]

Two types of binary BH coalescence

(1) Coalescence of stellar BH: from evolution of massive stars

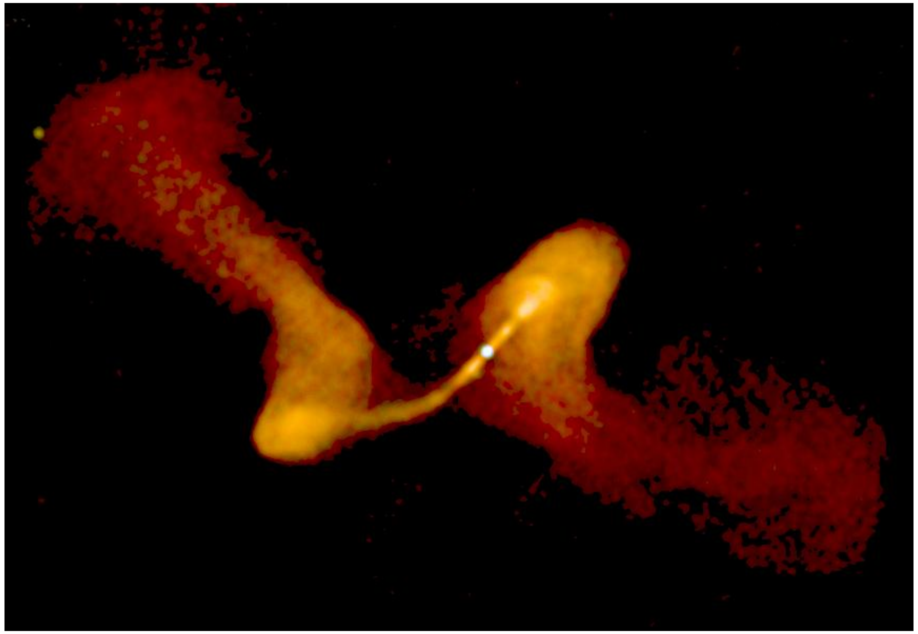
event rate: • up to $\sim 20/\mathrm{Myr}$ per galaxy [Belczynski, Kalogera, Bulik, ApJ **572**, 407 (2002)]

• $1.6 \times 10^{-7}\ \mathrm{yr}^{-1}\mathrm{Mpc}^{-3}$ from binary BH formation in globular clusters [Portegies Zwart & McMillan, ApJ **528**, L17 (2000)]

(2) Coalescence of supermassive BH:

from galaxy encounters

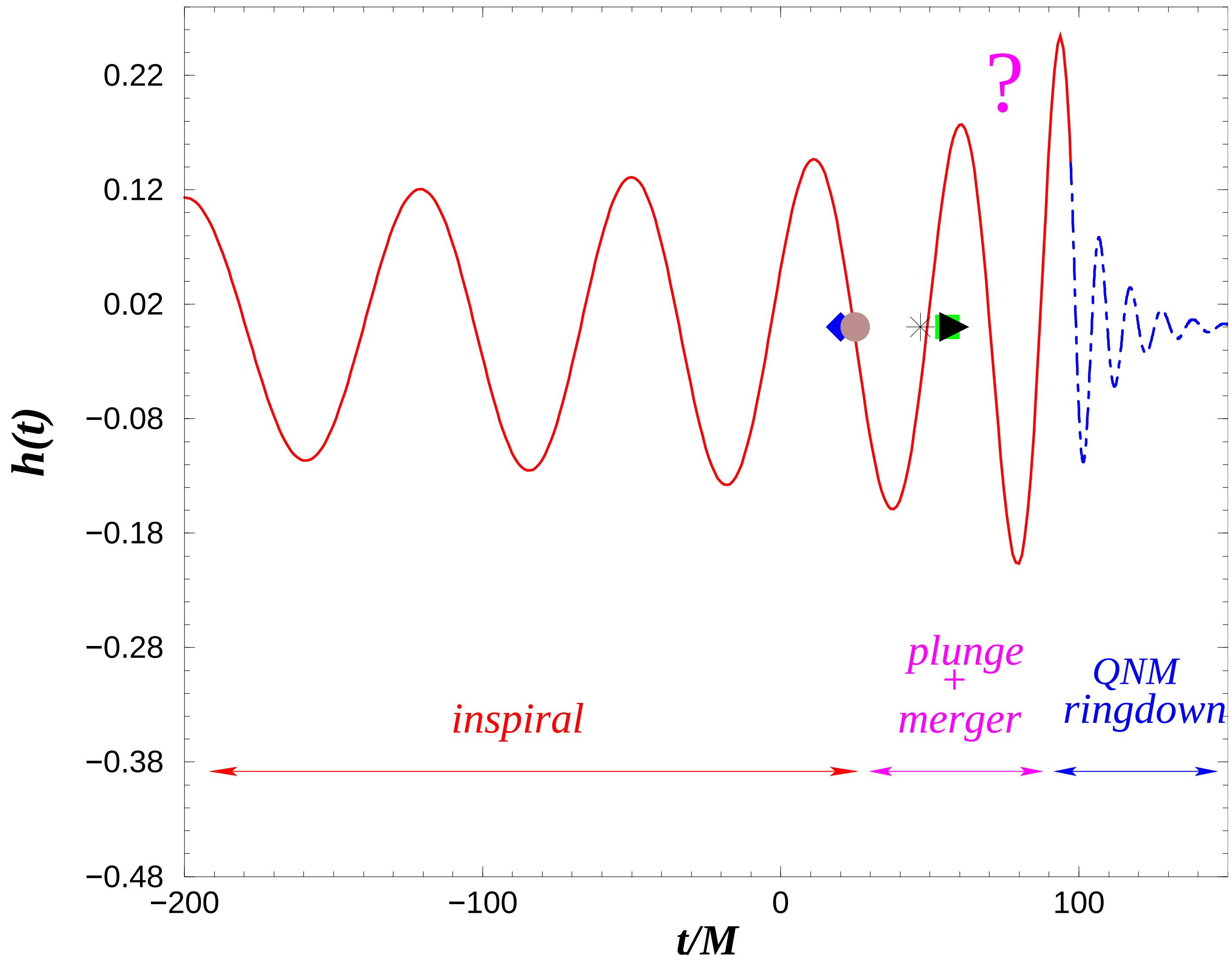
event rate : possibly large



NGC 326 [Merrit & Eckers, Science **297**, 1310 (2002)]

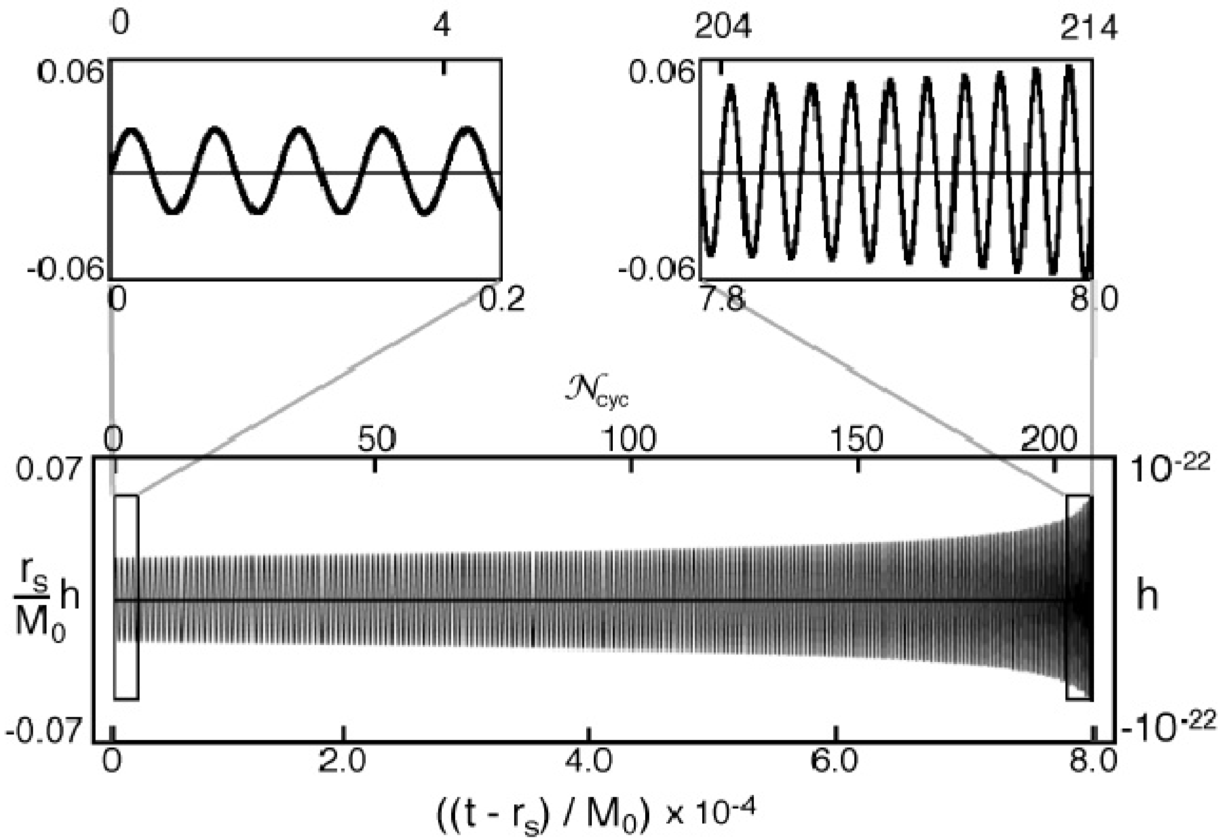
NB: Same physics (scaling with M)

Gravitational waveform



[from Buonanno & Damour, PRD **62**, 064015 (2000)]

Inspiral waveform: the most understood phase



[from Duez, Baumgarte & Shapiro, PRD **63**, 084030 (2001)]

Chirp signal:

Lowest order:

$$h_+ \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \cos(2\pi f t)$$

$$h_\times \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \sin(2\pi f t)$$

$$f = K_0\, \mathcal{M}^{-5/8} (t_{\rm coal} - t)^{-3/8}$$

with the constant:

$$K_0 = \frac{5^{3/8}}{8\pi} \left(\frac{c^3}{G}\right)^{5/8}$$

and the "chirp mass":

$$\mathcal{M} = (M_1 M_2)^{3/5} (M_1 + M_2)^{-1/5}$$

More precise formulae:

- More harmonics in $h_+(t)$ and $h_\times(t)$ (up to 6 at the 2.5PN level)
- Orbital phase ($\Longrightarrow$ number of cycles) at the 3.5PN level:

$$
\begin{aligned}
\phi(t) = &-\frac{1}{\nu}\Bigg\{\tau^{5/8} + \left(\frac{3715}{8064} + \frac{55}{96}\nu\right)\tau^{3/8} - \frac{3}{4}\pi\tau^{1/4} \\
+ &\left(\frac{9275495}{14450688} + \frac{284875}{258048}\nu + \frac{1855}{2048}\nu^2\right)\tau^{1/8} + \left(-\frac{38645}{172032} - \frac{15}{2048}\nu\right)\pi\ln\left(\frac{\tau}{\tau_0}\right) \\
&+ \left(\frac{831032450749357}{57682522275840} - \frac{53}{40}\pi^2 - \frac{107}{56}C + \frac{107}{448}\ln\left(\frac{\tau}{256}\right)\right. \\
&+ \left[-\frac{123292747421}{4161798144} + \frac{2255}{2048}\pi^2 + \frac{385}{48}\lambda - \frac{55}{16}\theta\right]\nu + \frac{154565}{1835008}\nu^2 \\
&\left.-\frac{1179625}{1769472}\nu^3\right)\tau^{-1/8} + \left(\frac{188516689}{173408256} + \frac{140495}{114688}\nu - \frac{122659}{516096}\nu^2\right)\pi\tau^{-1/4}\Bigg\}
\end{aligned}
$$

[Blanchet, Faye, Iyer & Joguet, PRD **65**, 061501(R) (2002)]

Chirp time

Characteristic evolution time at the frequency f:

$$\tau := \frac{f}{\dot{f}} = \frac{8}{3}(t_{\rm coal} - t) = \frac{5}{96\pi^{8/3}} \frac{c^5}{G^{5/3}} \mathcal{M}^{-5/3} f^{-8/3}$$

- for stellar black holes ($M_1 = M_2 = 10\, M_\odot \Rightarrow \mathcal{M} = 8.7\, M_\odot$):

$$\tau = 100 \text{ s} \left(\frac{10\,\rm Hz}{f}\right)^{8/3} \left(\frac{8.7\, M_\odot}{\mathcal{M}}\right)^{5/3}$$

- for supermassive black holes ($M_1 = M_2 = 10^6\, M_\odot \Rightarrow \mathcal{M} = 8.7 \times 10^5\, M_\odot$):

$$\tau = 116 \text{ d} \left(\frac{10^{-4}\,\rm Hz}{f}\right)^{8/3} \left(\frac{8.7 \times 10^5 M_\odot}{\mathcal{M}}\right)^{5/3}$$

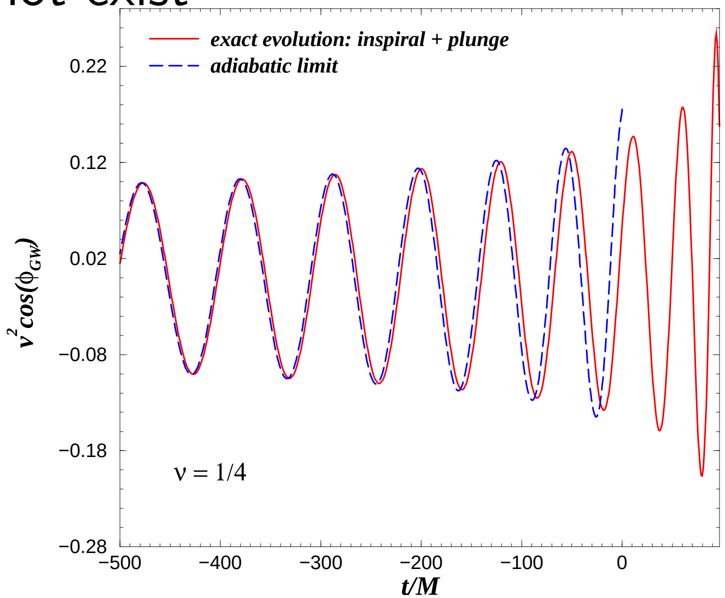
NB: $h\tau f^2 = \frac{K}{r}$ with K independent of $\mathcal{M} \Longrightarrow$ standard candle

End of inspiral: the last stable orbit

Very small mass ratio (Schwarzschild spacetime) : there exists an *innermost stable circular orbit (ISCO)* :

$$R^{\rm Schw}_{\rm ISCO} = 6M \qquad \Omega^{\rm Schw}_{\rm ISCO} = 6^{-3/2} M^{-1} \simeq 0.068\, M^{-1}$$

Equal mass ratio : gravitational radiation dissipation $\Longrightarrow$ strictly circular orbits do not exist



The ISCO is then defined in terms of the conservative part in the equation of motions, which give rise to circular orbits (adiabatic approximation). Consider a sequence of circular orbits of smaller and smaller radius, mimicking the inspiral. The ISCO is defined as the *turning point* in the *binding energy* of this sequence.

$\longleftarrow$ [Buonanno & Damour, PRD **62**, 064015 (2000)]

Binary BH ISCO computations

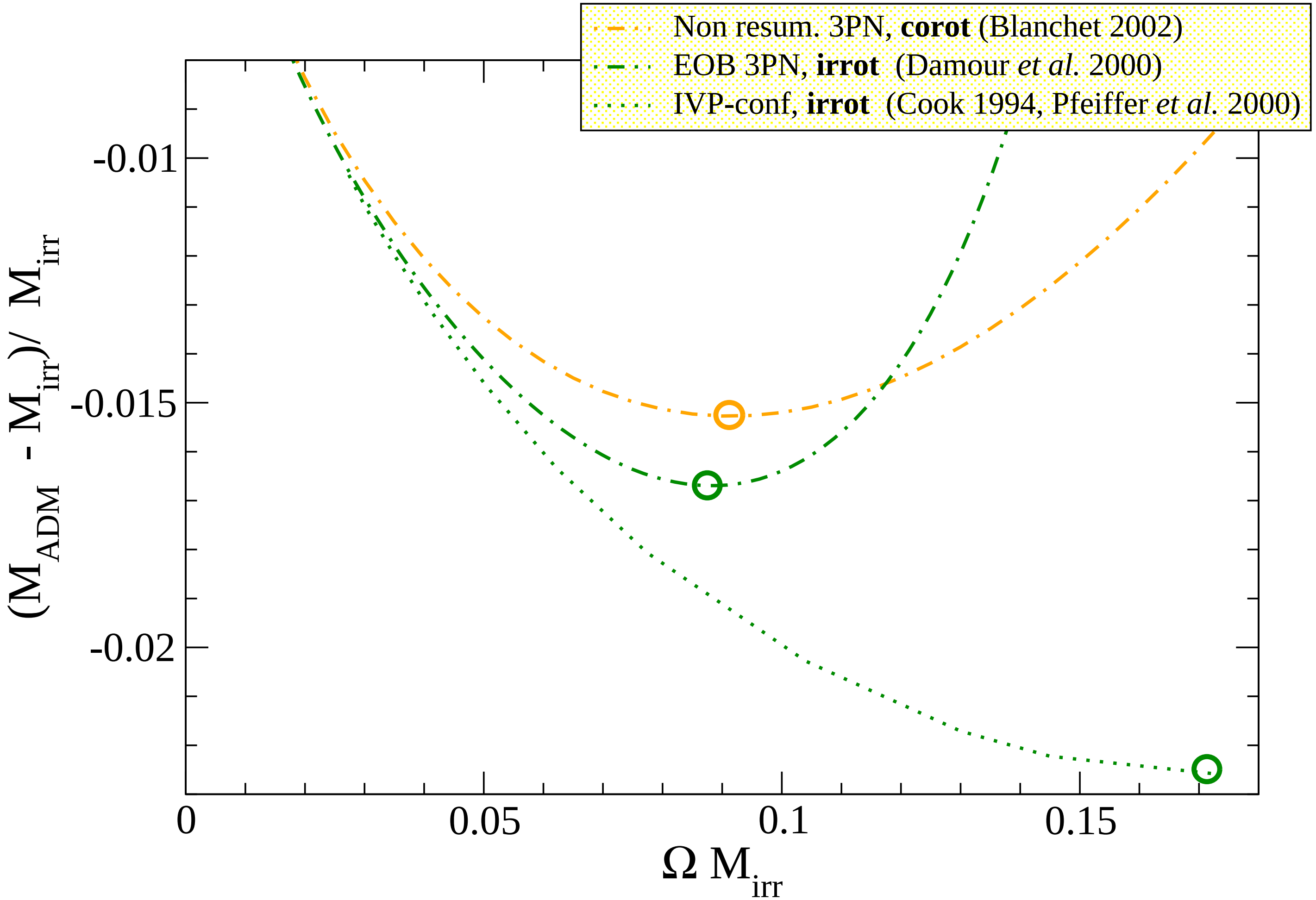
- **Post-Newtonian computations :** at the 3-PN level:
  - Effective One Body approach (EOB) : Damour, Jaranowski & Schäfer 2000 [PRD **62**, 084011 (2000)]
  - Non-resummed Taylor expansion : Blanchet 2002 [PRD **65**, 124009 (2002)]

- **Numerical computations** (before 2001): based on the initial value problem (IVP) :
  - Cook 1994 [PRD **50**, 5025 (1994)]
  - Pfeiffer, Teukolsky & Cook 2000 [PRD **62**, 104018 (2000)]
  - Baumgarte 2000 [PRD **62**, 024018 (2000)]

$\longrightarrow$ Big discrepancy between the two types of computations

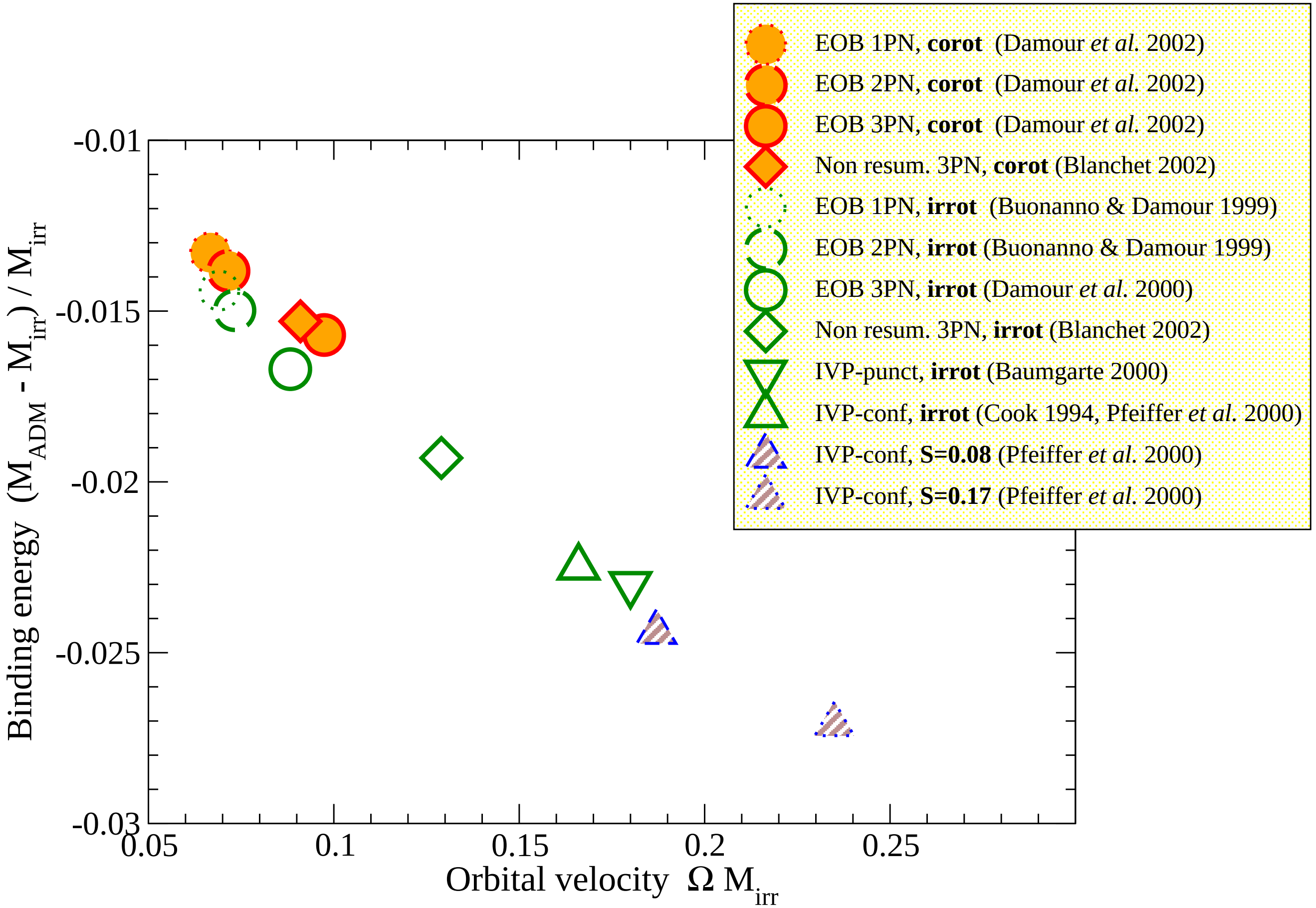
Discrepancy between analytical and numerical methods

Binding energy along an evolutionary sequence of equal-mass binary black holes



Discrepancy between analytical and numerical methods

Location of the ISCO



Gravitational wave frequency:

$$f = 320\ \frac{\Omega M_{\rm ir}}{0.1}\ \frac{20\ M_\odot}{M_{\rm ir}}\ {\rm Hz}$$

A new numerical approach

[Gourgoulhon, Grandclément & Bonazzola, PRD **65**, 044020 (2002)]

[Grandclément, Gourgoulhon & Bonazzola, PRD **65**, 044021 (2002)]

Problem treated:
Binary black holes in the pre-coalescence stage
⇒ the notion of **orbit** has still some meaning

Basic idea:
Construct an approximate, but full spacetime (i.e. **4-dimensional**) representing 2 orbiting black holes
Previous numerical treatments (IVP) : 3-dimensional (initial value problem on a spacelike 3-surface)
4-dimensional approach ⇒ rigorous definition of orbital angular velocity

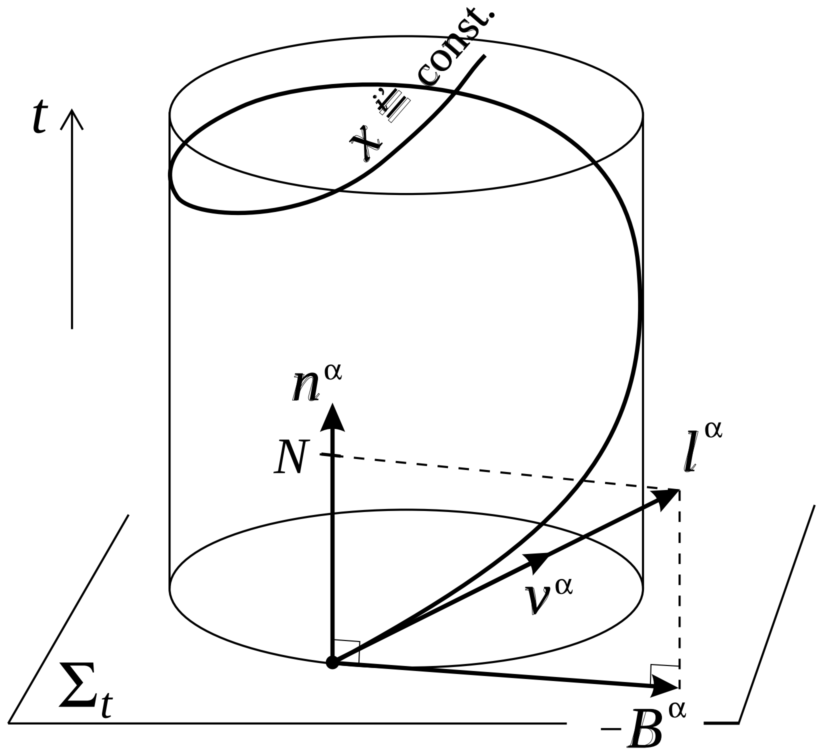
Helical symmetry

Physical assumption: when the two holes are sufficiently far apart, the radiation reaction can be neglected ⇒ closed orbits
Gravitational radiation reaction circularizes the orbits ⇒ circular orbits

Geometrical translation: there exists a Killing vector field $\boldsymbol{\ell}$ such that:

far from the system (asymptotically inertial coordinates $(t_0, r_0, \theta_0, \varphi_0)$),

$$\boldsymbol{\ell} \to \frac{\partial}{\partial t_0} + \Omega \frac{\partial}{\partial \varphi_0}$$



Helical symmetry: discussion

Helical symmetry is exact

- in **Newtonian gravity** and in **2nd order Post-Newtonian gravity**
- in general relativity for a non-axisymmetric system (binary) only with **standing gravitational waves**

But a spacetime with a helical Killing vector and standing gravitational waves cannot be asymptotically flat in full GR [Gibbons & Stewart 1983].

We have used a truncated version of GR (the **Isenberg-Wilson-Mathews** approximation, which will be described below) which (i) admits the helical Killing vector and (ii) is asymptotically flat.

Rotation state of each black hole

Choice: rotation synchronized with the orbital motion (corotating system)

Justifications:

- the only rotation state fully compatible with the helical symmetry
  [Friedman, Uryu & Shibata, PRD **65**, 064035 (2002)]
- for close systems, black hole "effective viscosity" might be very efficient in synchronizing the spins with the orbital motion
  [e.g. Price & Whelan, PRL **87**, 231101 (2001)]

Geometrical translation: the two horizons are Killing horizons associated with $\boldsymbol{\ell}$:

$$\boldsymbol{\ell}\cdot\boldsymbol{\ell}|_{\mathcal{H}_1} = 0 \qquad \text{and} \qquad \boldsymbol{\ell}\cdot\boldsymbol{\ell}|_{\mathcal{H}_2} = 0 \ .$$

[cf. the rigidity theorem for a Kerr black hole]

Consequence on the shift vector: $\boldsymbol{\ell}\cdot\boldsymbol{\ell} = -N^2 + \boldsymbol{\beta}\cdot\boldsymbol{\beta}$
$\Rightarrow$ boundary conditions on the horizons: $\boldsymbol{\beta}|_{\mathcal{H}_1} = 0 \quad$ and $\quad \boldsymbol{\beta}|_{\mathcal{H}_2} = 0$.

Einstein equations

Framework: 3+1 formalism with maximal slicing: $K = 0$

Isenberg-Wilson-Mathews approximation: conformally flat spatial metric: $\boldsymbol{\gamma} = \Psi^4 \boldsymbol{f}$

$\Rightarrow$ spacetime metric : $ds^2 = -N^2 dt^2 + \Psi^4 f_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$

Amounts to solve 5 of the 10 Einstein equations (one more than IVP !) :

$$\Delta\Psi = -\frac{\Psi^5}{8}\hat{A}_{ij}\hat{A}^{ij} \quad \text{(Lichnerowicz equation)} \quad \textit{(Hamiltonian constraint)}$$

$$\Delta\beta^i + \frac{1}{3}\bar{D}^i\bar{D}_j\beta^j = 2\hat{A}^{ij}\left(\bar{D}_j N - 6N\bar{D}_j \ln\Psi\right) \quad \textit{(momentum constraint)}$$

$$\Delta N = N\Psi^4\hat{A}_{ij}\hat{A}^{ij} - 2\bar{D}_j \ln\Psi\, \bar{D}^j N \quad \textit{(trace of } \frac{\partial K_{ij}}{\partial t} = \cdots\textit{)}$$

with $\hat{A}_{ij} := \Psi^{-4}K_{ij}$ and $\hat{A}^{ij} := \Psi^4 K^{ij}$

Kinematical relation between γ and $\boldsymbol{K}$:

$$\hat{A}^{ij} = \frac{1}{2N}(L\beta)^{ij} \text{ with } (L\beta)^{ij} := \bar{D}^i\beta^j + \bar{D}^j\beta^i - \frac{2}{3}\bar{D}_k\beta^k f^{ij} \quad \textit{(traceless part)}$$

$$\bar{D}_i\beta^i = -6\beta^i\bar{D}_i \ln\Psi \quad \textit{(trace part)}$$

Determination of Ω

Virial assumption: $O(r^{-1})$ part of the metric ($r \to \infty$) same as Schwarzschild

[The only quantity "felt" at the $O(r^{-1})$ level by a distant observer is the total mass of the system.]

A priori

$$\Psi \sim 1 + \frac{M_{\rm ADM}}{2r} \qquad \text{and} \qquad N \sim 1 - \frac{M_{\rm K}}{r}$$

Hence

$$\text{(virial assumption)} \iff M_{\rm ADM} = M_{\rm K}$$

Note

$$\text{(virial assumption)} \iff \Psi^2 N \sim 1 + \frac{\alpha}{r^2}$$

Defining an evolutionary sequence

An evolutionary sequence is defined by:

$$\left.\frac{dM_{\rm ADM}}{dJ}\right|_{\rm sequence} = \Omega$$

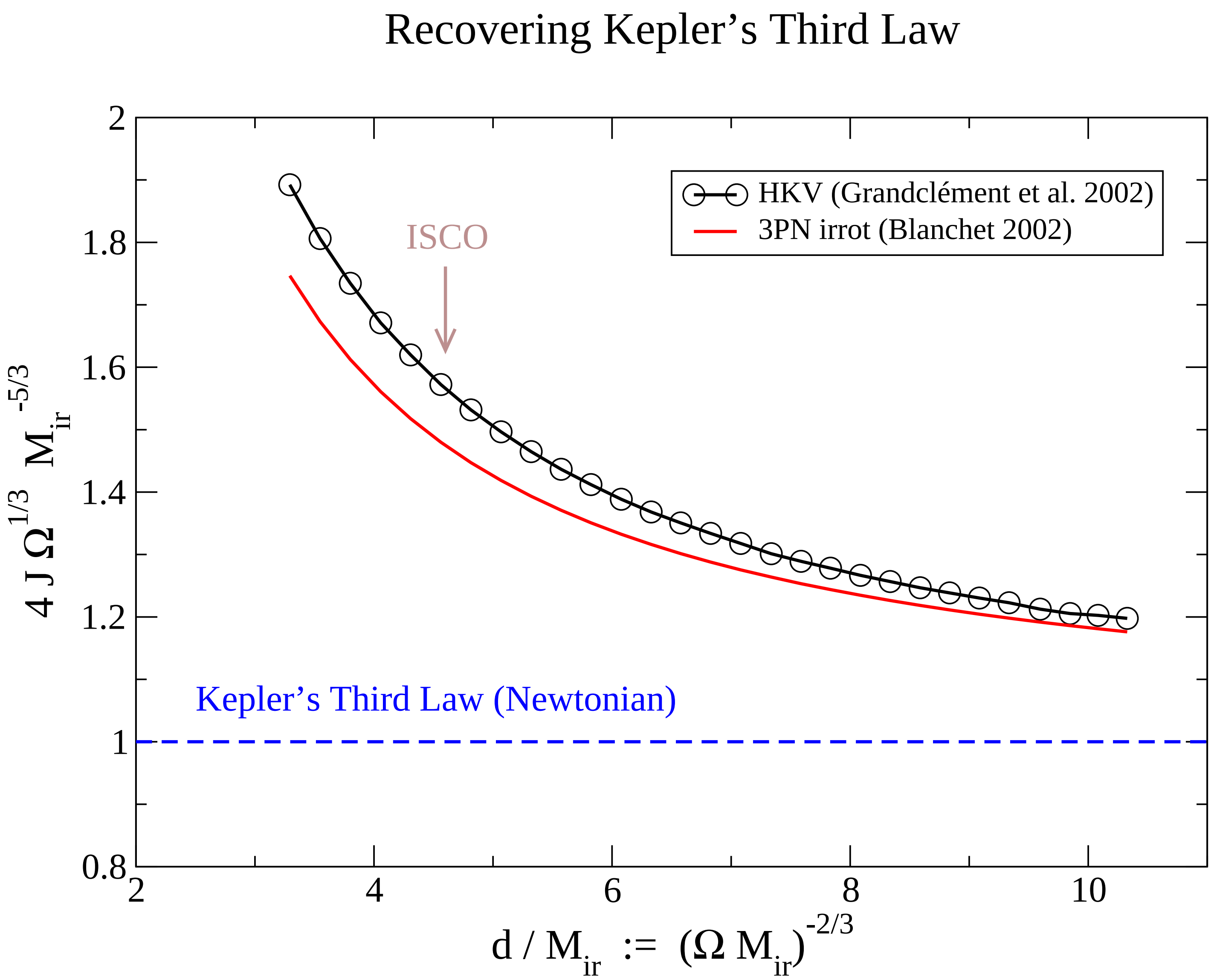
This is equivalent to requiring the constancy of the horizon area of each black hole, by virtue of the First law of thermodynamics for binary black holes :

$$dM_{\rm ADM} = \Omega\, dJ + \frac{1}{8\pi}\left(\kappa_1\, dA_1 + \kappa_2\, dA_2\right)$$

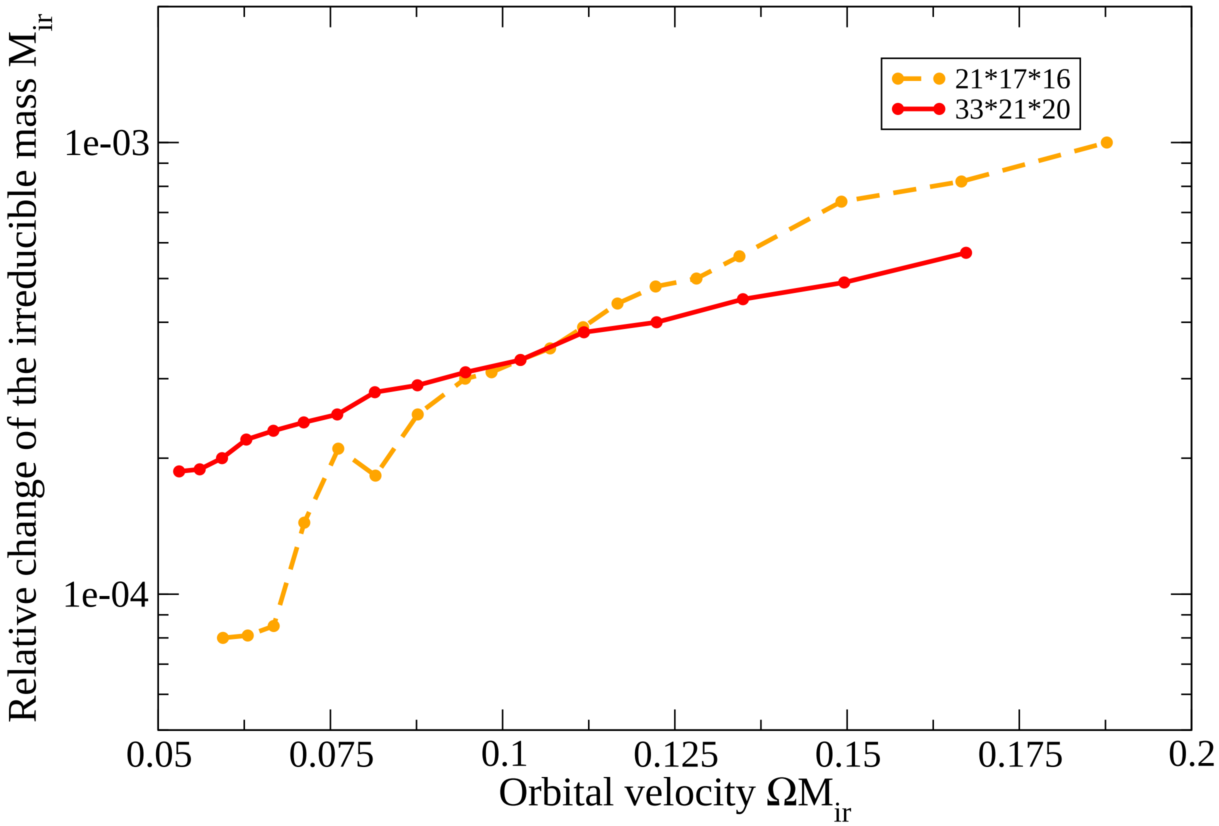
recently established by Friedman, Uryu & Shibata [PRD **65**, 064035 (2002)].

Note: Within the helical symmetry framework, a minimum in $M_{\rm ADM}$ along a sequence at fixed horizon area locates a change of orbital stability (ISCO) [Friedman, Uryu & Shibata, PRD **65**, 064035 (2002)].

Test : getting Kepler's third law at large separation



Test: conservation of the horizon area along a sequence

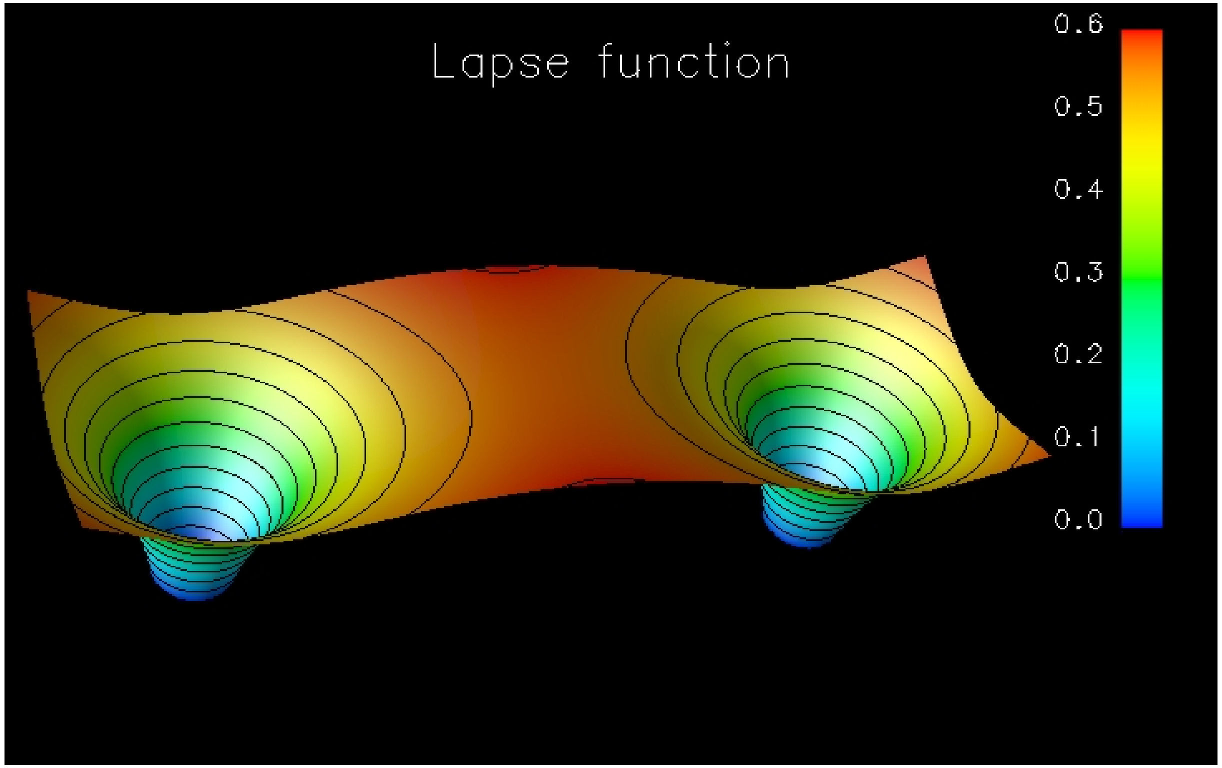


Relative change of the horizon area along an evolutionary sequence

Quasi-equilibrium sequences of binary black hole on circular orbits

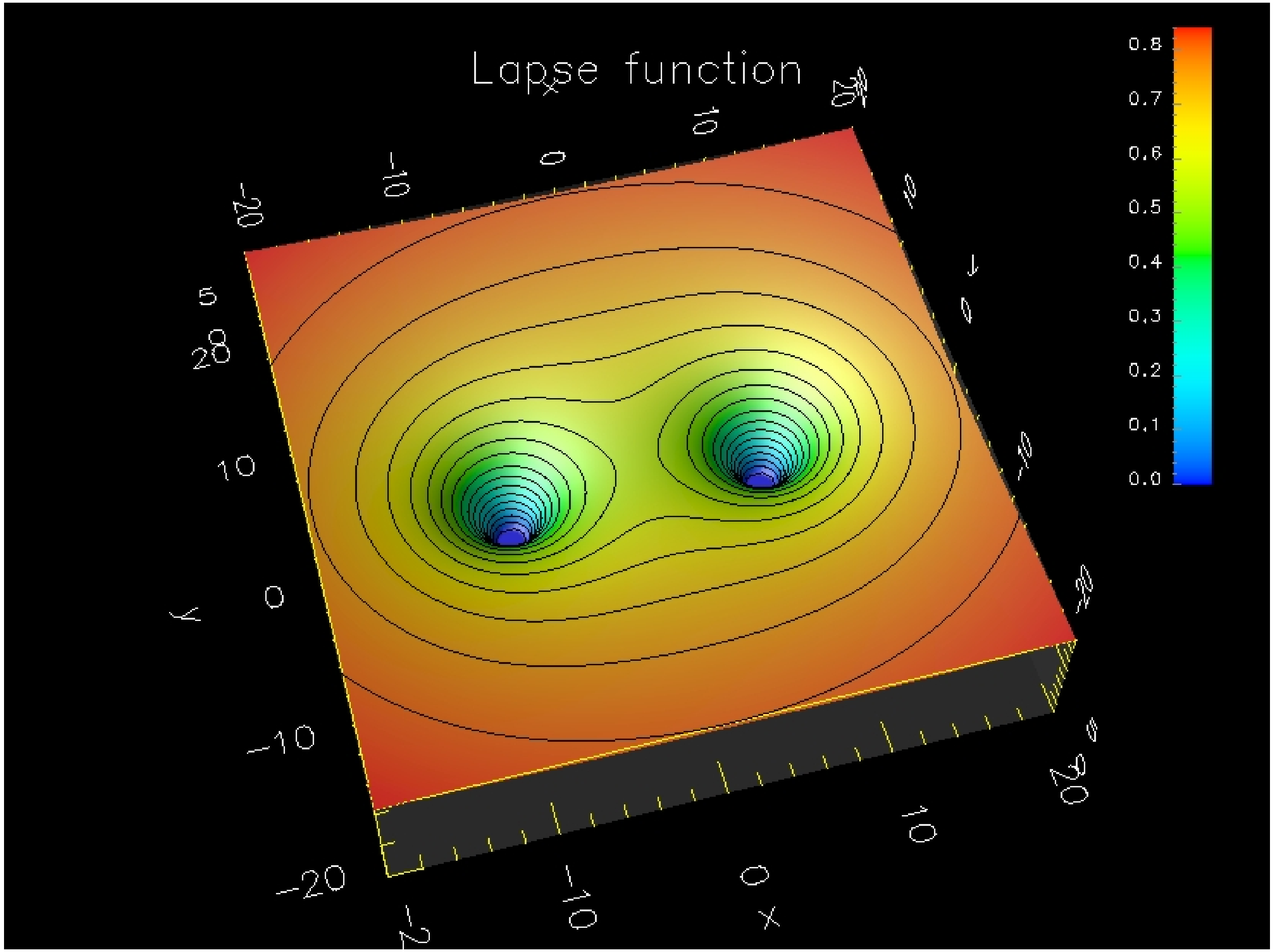
Computations performed by means of multi-domain spectral methods (LORENE C++ based library)

ISCO configuration



[Grandclément, Gourgoulhon, Bonazzola, PRD **65**, 044021 (2002)]

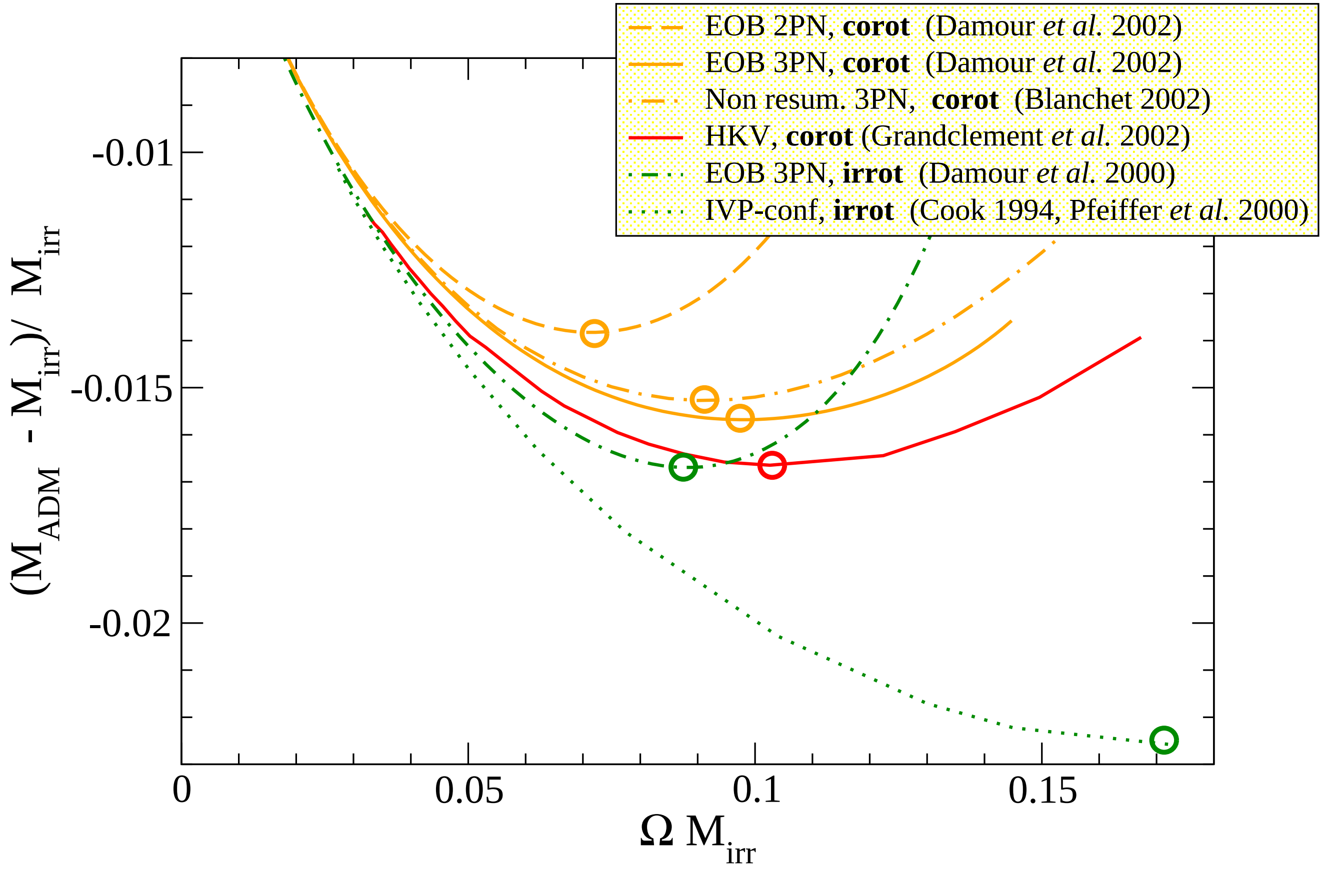
ISCO configuration



[Grandclément, Gourgoulhon, Bonazzola, PRD **65**, 044021 (2002)]

Comparison with Post-Newtonian computations

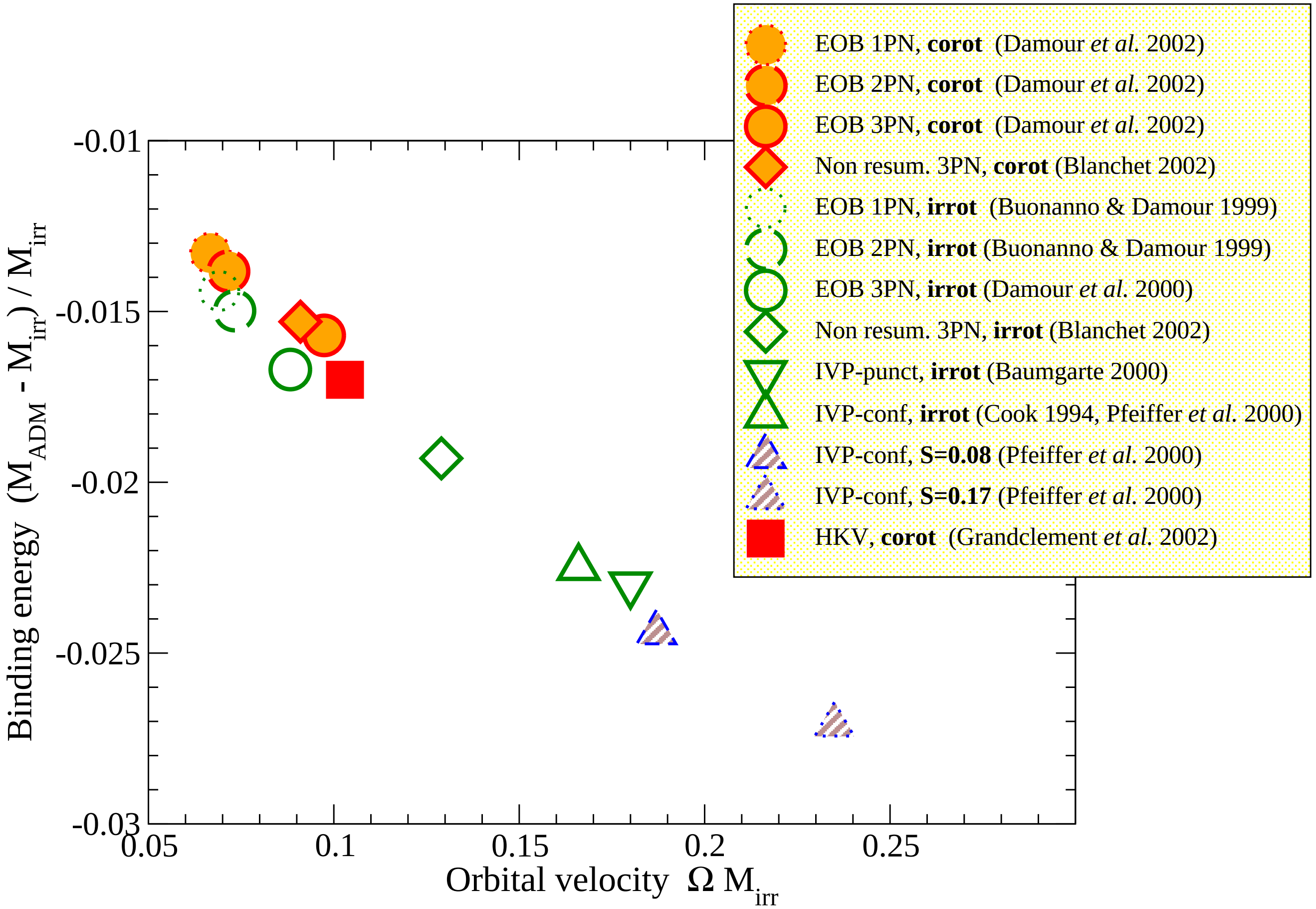
Binding energy along an evolutionary sequence of equal-mass binary black holes



[from Damour, Gourgoulhon, Grandclément, PRD **66**, 024007 (2002)]

Location of the ISCO

Comparison with Post-Newtonian computations



Gravitational wave frequency:

$$f = 320 \, \frac{\Omega M_{\rm ir}}{0.1} \, \frac{20 \, M_\odot}{M_{\rm ir}} \ {\rm Hz}$$

[from Damour, Gourgoulhon, Grandclément, PRD **66**, 024007 (2002)]

Energy emitted by gravitational radiation

Absolute upper bounds:

Hawking (1971) : $\frac{E_{\rm rad}}{M} < 0.5$ for merger of maximaly rotating Kerr BH, such that the final BH does not rotate

$\frac{E_{\rm rad}}{M} < 0.29$ for merger of non-rotating BH

Inspiral stage: $\frac{E_{\rm rad}}{M} \simeq 0.017$

Plunge + merger phase: $\frac{E_{\rm rad}}{M} \sim 0.1$? [Flanagan & Hughes, PRD **57**, 4535 (1998)]

$\frac{E_{\rm rad}}{M} \sim 0.007$? [Damour, Iyer & Sathyaprakash, PRD **63**, 044023 (2001)]

Ringdown phase: $\frac{E_{\rm rad}}{M} \sim 0.03$? [Brandt & Seidel, PRD **52**, 870 (1995)]

[Flanagan & Hughes, PRD **57**, 4535 (1998)]

$\frac{E_{\rm rad}}{M} \sim 0.007$? [Damour, Iyer & Sathyaprakash, PRD **63**, 044023 (2001)]

Range of detection and expected event rate

Stellar BH ($2 \times 10\, M_\odot$):
Detection range:

- first generation (VIRGO, LIGO-I): $d_{\rm max} \simeq 100$ Mpc
- second generation: $d_{\rm max} \simeq 1$ Gpc

Expected event rate:

- first generation (VIRGO, LIGO-I): ~ 1 per year
- second generation: daily

Supermassive BH ($2 \times 10^6\, M_\odot$):
$d_{\rm max} >$ Hubble radius for LISA $\Longrightarrow$ expected rate: a few per year up to 10^3 per year

Conclusion and perspectives

- There is now a good agreement between numerical computations (3+1 formalism with boundary conditions on two horizons), and analytical ones (high order post-Newtonian expansions for point-mass particles) about the location of the last stable orbit (ISCO): the GW frequency at the ISCO is

$$f \sim 300 \, \frac{20 \, M_\odot}{M_1 + M_2} \; \mathrm{Hz} \quad \text{(VIRGO)} \qquad f \sim 3 \times 10^{-3} \, \frac{2 \times 10^6 \, M_\odot}{M_1 + M_2} \; \mathrm{Hz} \quad \text{(LISA)}$$

- Around the ISCO, radiation reaction becomes important and must be taken into account $\Rightarrow$ start of plunge.

- We have begun to compute the dynamical evolution of the binary system, which will lead to the merger. Other groups (AEI, Cornell, Austin) are also trying to compute it, using other numerical methods.

- Contrary to binary neutron stars, most of the energy is likely to be emitted in the merger phase and not in the inspiral phase.