Exploring black hole spacetimes with SageManifolds

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The mission

Create a viable free open source alternative to Magma, Maple, Mathematica and Matlab.

Some advantages of SageMath

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Freedom means

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- Python is a very powerful *object oriented language*, with a neat syntax

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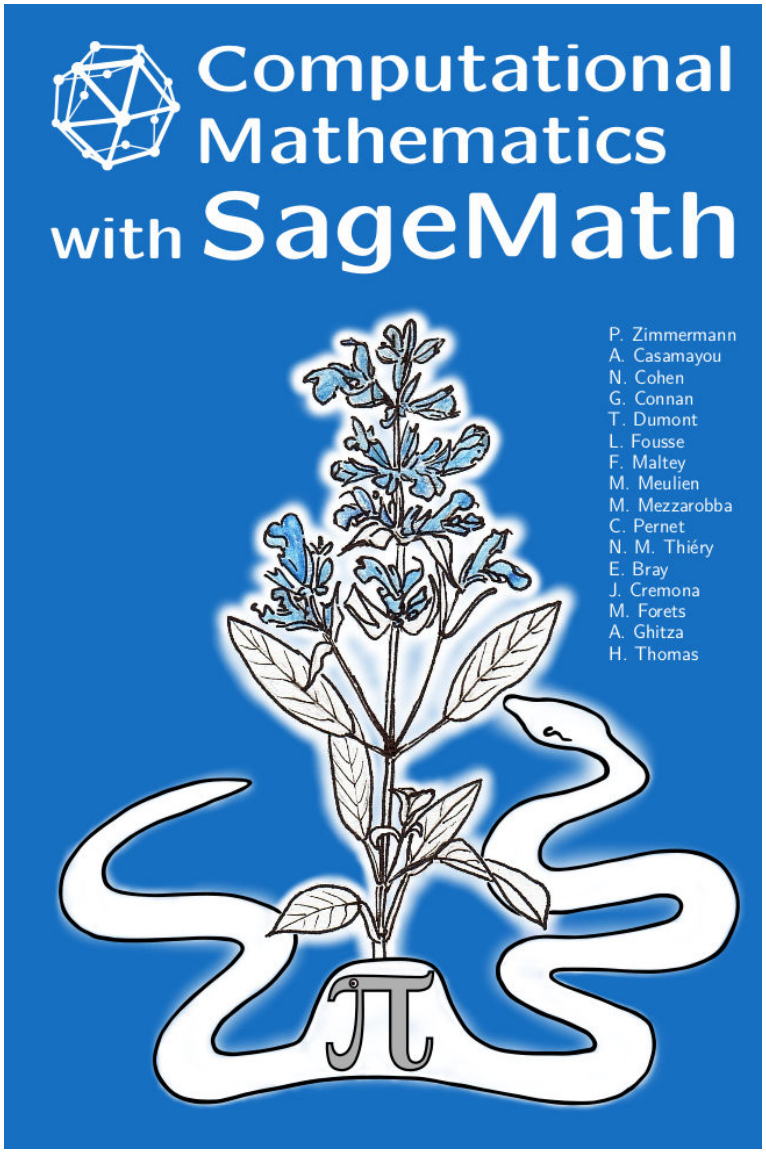
SageMath is developing and spreading fast

...sustained by an enthusiastic community of developers

The Sage book



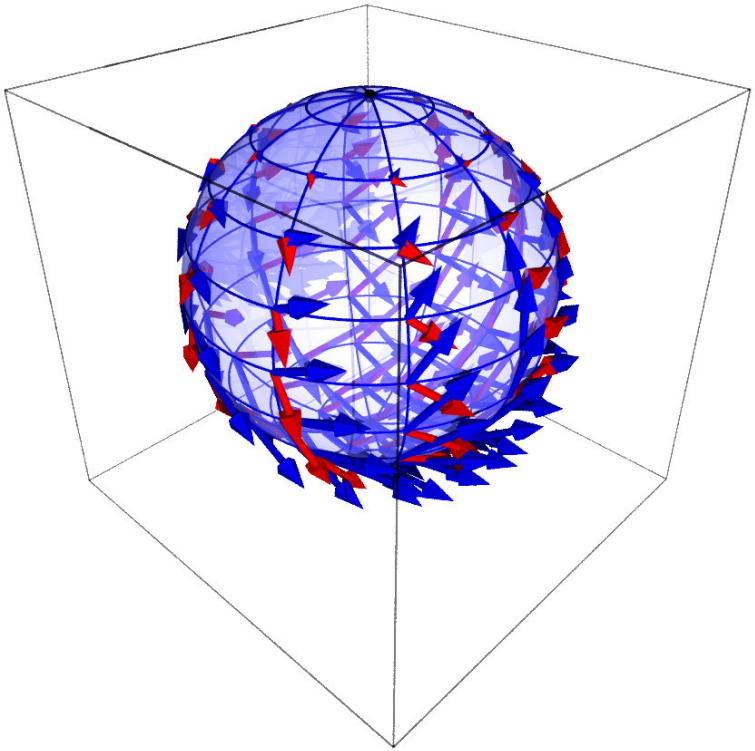
by P. Zimmermann, A. Casamayou, N. Cohen, G. Connan, T. Dumont, L. Fousse, F. Maltey, M. Meulien, M. Mezzarobba, C. Pernet, N.M. Thiéry, E. Bray, J. Cremona, M. Forets, A. Ghitza & H. Thomas (2018)

Released under *Creative Commons* license

Freely downloadable from
http://sagebook.gforge.inria.fr/english.html

Tensor calculus with SageMath

SageManifolds project: extends SageMath towards **differential geometry** and **tensor calculus**



Stereographic-coordinates frame on $\mathbb{S}^2$

- http://sagemanifolds.obspm.fr
- fully included in SageMath
- a dozen of contributors (developers and reviewers)
  cf. http://sagemanifolds.obspm.fr/authors.html
- want to stay tuned: subscribe to the mailing list
- help: https://ask.sagemath.org

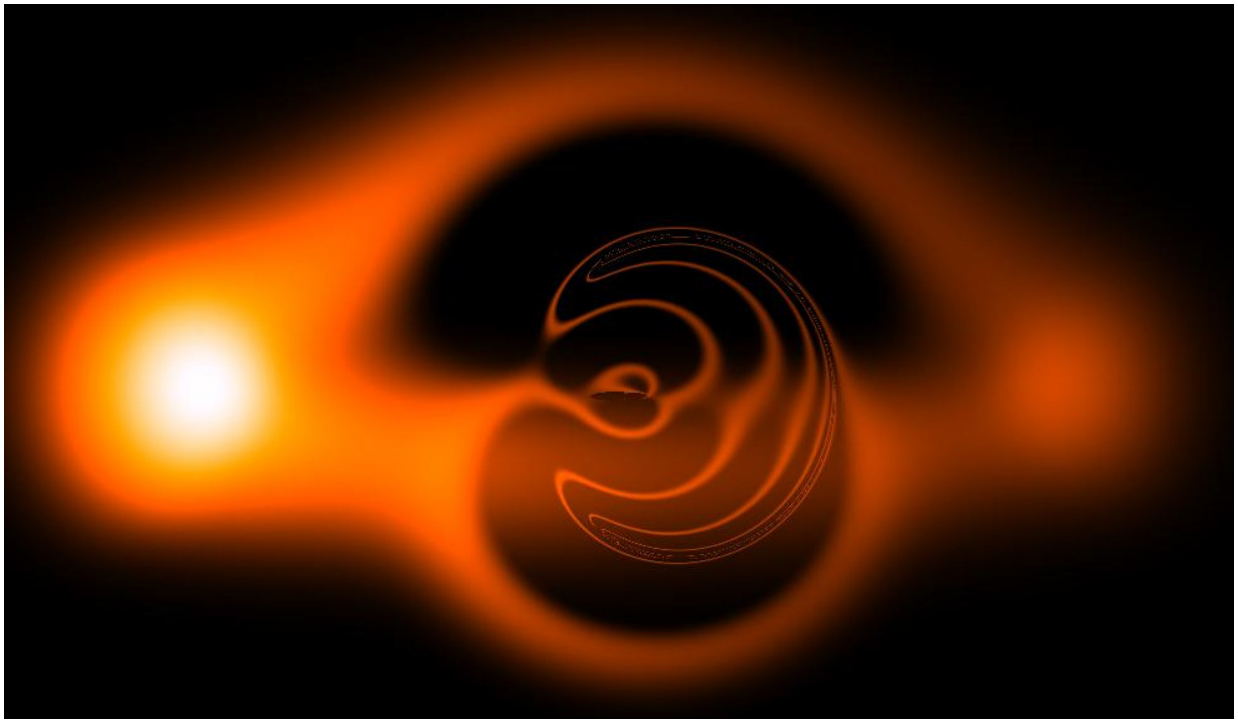
Everybody is very welcome to contribute:
visit https://sagemanifolds.obspm.fr/contrib.html

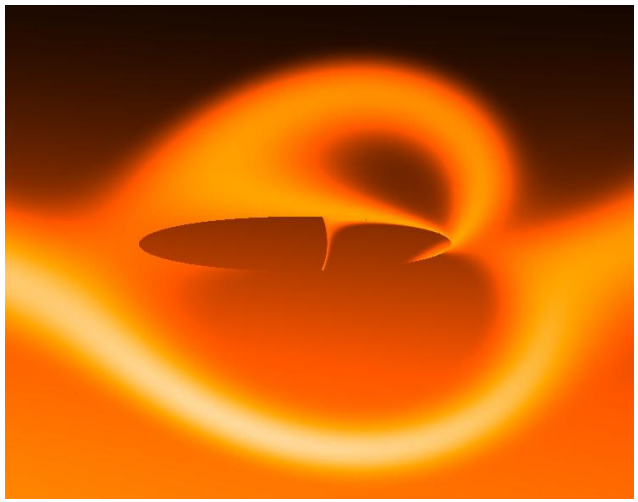
A short demo

- Schwarzschild spacetime:
  http://nbviewer.jupyter.org/github/sagemanifolds/SageManifolds/blob/master/Worksheets/v1.3/SM_basic_Schwarzschild.ipynb
- More examples at
  https://luth.obspm.fr/~luthier/gourgoulhon/leshouches18/sage.html
  and https://sagemanifolds.obspm.fr/examples.html

An example of application: naked rotating wormhole

Regular (singularity-free) spacetime with wormhole topology ($\mathbb{R}^2 \times \mathbb{S}^2$), sustained by exotic matter, asymptotically close a to Kerr spacetime with a naked singularity ($a > M$) and surrounded by an accretion torus



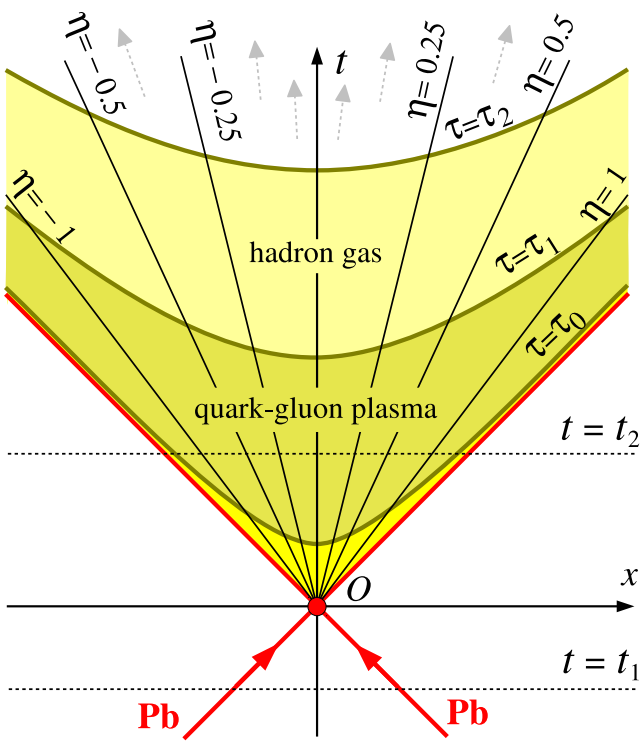


zoom on the central region

[Lamy, Gourgoulhon, Paumard & Vincent, CQG **35**, 115009 (2018)]

- Derivation of the geodesic equation: `SageMath`
- Integration of the geodesic equation: `Gyoto`

Another application: quark-gluon plasma in the gauge/gravity duality



Spacetime diagram of a heavy-ion collision (LHC)

$\tau_0 \simeq 0.2$ fm$/c = 6\ 10^{-25}$ s

$\tau_1 \sim 10\tau_0$

Gauge/gravity duality ("holographic principle")

4D strongly-coupled gauge theory $\equiv$ 5D gravitation

Example: AdS/CFT correspondence

Quark-gluon plasma (QGP) in heavy-ion collisions: low-viscosity fluid with *anisotropic* pressure ($p_x < p_y$)

Thermalization of QGP $\equiv$ 5D black hole formation

Gauge theory: QCD

Gravity: 5D Lifshitz-like spacetime (*anisotropic* generalization of AdS_5) with formation of a black brane (Vaidya-type collapse); new exact solutions with the help of `SageManifolds`

Results: faster thermalization in the transversal direction; evolution of the entanglement entropy

[Aref'eva, Golubtsova & Gourgoulhon, J. High Ener. Phys. **09(2016)**, 142 (2016)]