# Black holes: new theoretical approaches and applications to numerical relativity

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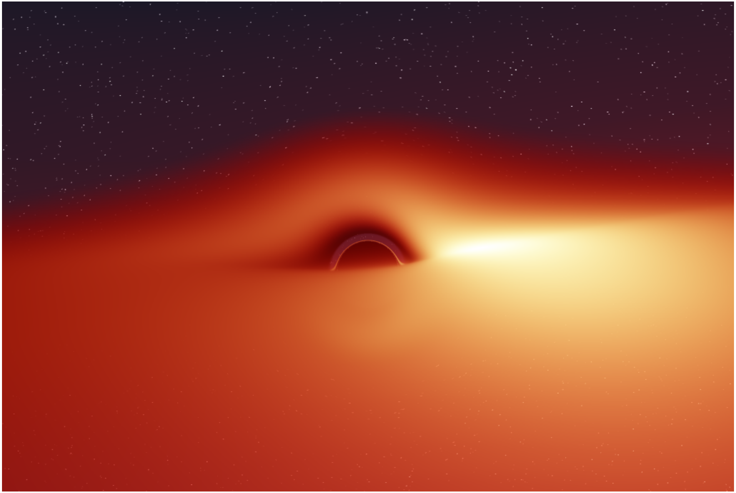
# Plan

1. New horizons
2. A Navier-Stokes-like equation
3. Applications to numerical relativity

# Outline

# What is a black hole ?

... for the astrophysicist: a very deep gravitational potential well



[J.A. Marck, CQG **13**, 393 (1996)]

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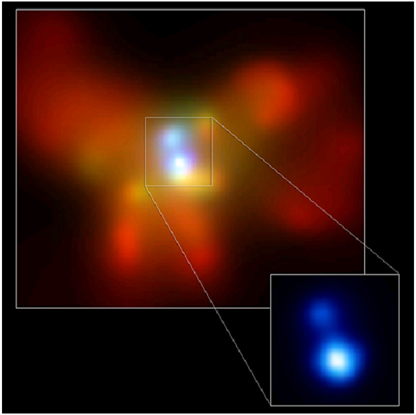
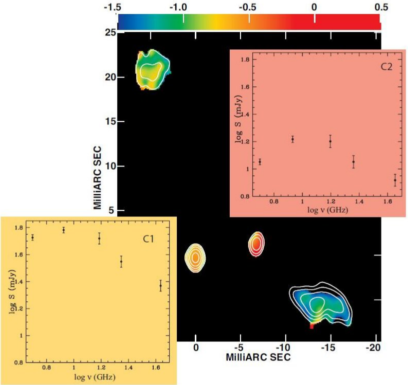


Binary BH in galaxy NGC 6240

$d = 1.4$ kpc

[Komossa et al., ApJ **582**, L15 (2003)]




Binary BH in radio galaxy 0402+379

$d = 7.3$ pc

[Rodriguez et al., ApJ in press, astro-ph/0604042]

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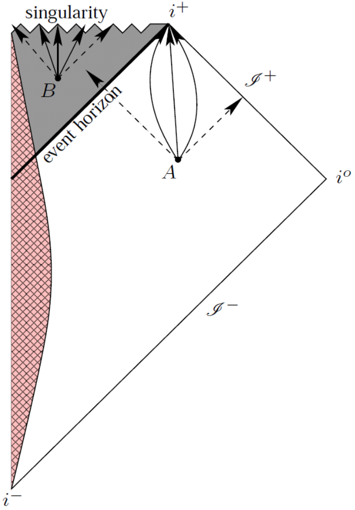


... for the mathematical physicist:

$$\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)$$

i.e. the region of spacetime where light rays cannot escape to infinity

- $\mathscr{M}$ = asymptotically flat manifold
- $\mathscr{I}^+$ = future null infinity
- $J^-(\mathscr{I}^+)$ = causal past of $\mathscr{I}^+$

*event horizon:* $\mathcal{H} := \dot{J}^-(\mathscr{I}^+)$
(boundary of $J^-(\mathscr{I}^+)$)

$\mathcal{H}$ smooth $\Longrightarrow$ $\mathcal{H}$ null hypersurface

[Booth, Can. J. Phys. **83**, 1073 (2005)]

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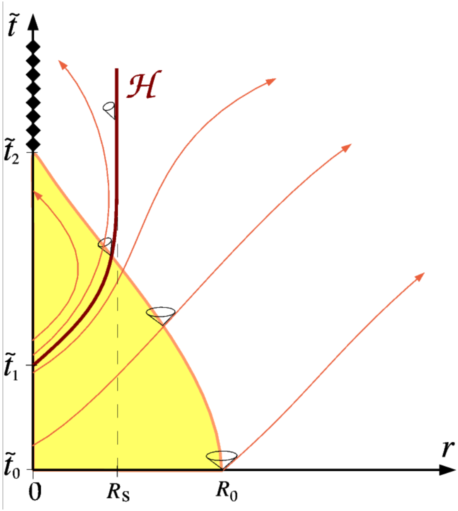


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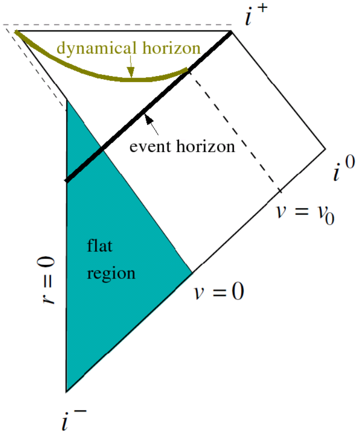
The determination of the boundary of $J^-(\mathscr{I}^+)$ requires the knowledge of the entire future null infinity. Moreover this is not locally linked with the notion of strong gravitational field:




Example of event horizon in a **flat** region of spacetime:

Vaidya metric, describing incoming radiation from infinity:

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dv\, dr + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

with

| | | |
|---|---|---|
| with | $m(v) = 0$ | for $v < 0$ |
| | $dm/dv > 0$ | for $0 \leq v \leq v_0$ |
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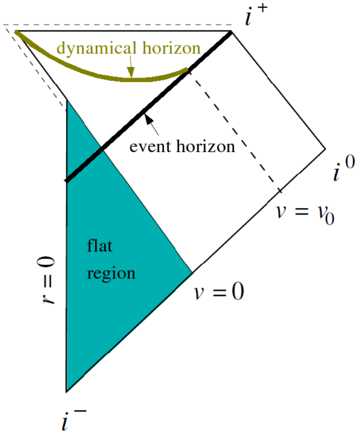


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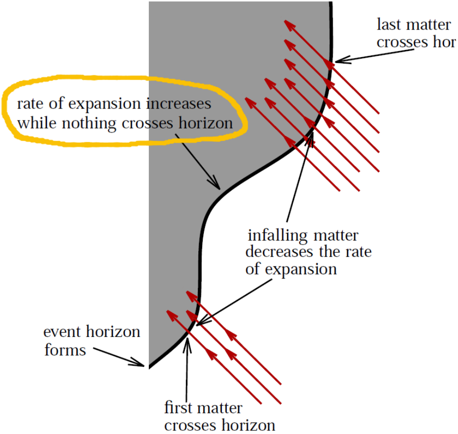
$\Rightarrow$ no local physical experiment whatsoever can locate the event horizon

# Another non-local feature: teleological nature of event horizons

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The classical black hole boundary, i.e. the event horizon, responds in advance to what will happen in the future.

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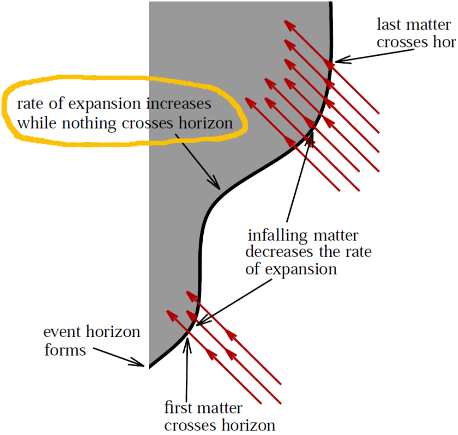


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The classical black hole boundary, i.e. the event horizon, responds in advance to what will happen in the future.

To deal with black holes as physical objects, a local definition would be desirable

# Local characterizations of black holes

Recently a **new paradigm** appeared in the theoretical approach of black holes: instead of event horizon, black holes are described by

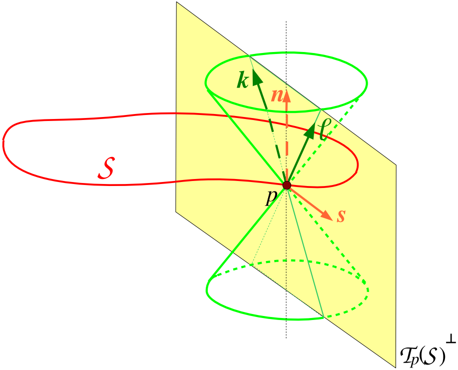
- trapping horizons (Hayward 1994)
- isolated horizons (Ashtekar et al. 1999)
- dynamical horizons (Ashtekar and Krishnan 2002)

All these concepts are **local** and are based on the notion of trapped surfaces

*Motivations:* quantum gravity, numerical relativity

# Trapped surfaces

$\mathcal{S}$ : closed (i.e. compact without boundary) spacelike 2-dimensional surface embedded in spacetime $(\mathscr{M}, \boldsymbol{g})$




∃ two future-directed null directions (light rays) orthogonal to $\mathcal{S}$:
$\boldsymbol{\ell}$ = outgoing, expansion $\theta^{(\boldsymbol{\ell})}$
$\boldsymbol{k}$ = ingoing, expansion $\theta^{(\boldsymbol{k})}$

In flat space, $\theta^{(\boldsymbol{k})} < 0$ and $\theta^{(\boldsymbol{\ell})} > 0$

- $\mathcal{S}$ is trapped $\iff$ $\theta^{(\boldsymbol{k})} \leq 0$ and $\theta^{(\boldsymbol{\ell})} \leq 0$
- $\mathcal{S}$ is marginally trapped $\iff$ $\theta^{(\boldsymbol{k})} \leq 0$ and $\theta^{(\boldsymbol{\ell})} = 0$

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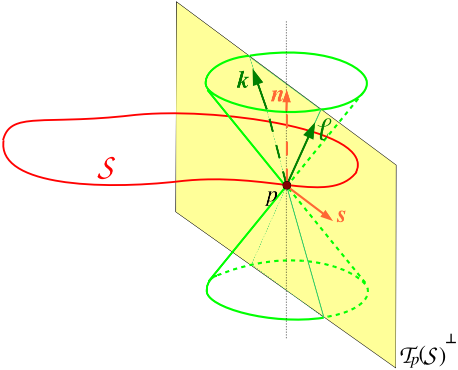


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*trapped surface* = **local** concept characterizing very strong gravitational fields

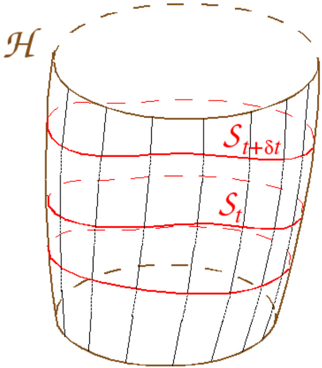
# Connection with singularities and black holes

*Proposition* [Penrose (1965)]: provided that the weak energy condition holds, $\exists$ a trapped surface $\mathcal{S} \Longrightarrow \exists$ a singularity in $(\mathscr{M}, \boldsymbol{g})$ (in the form of a future inextendible null geodesic)

*Proposition* [Hawking & Ellis (1973)]: provided that the cosmic censorship conjecture holds, $\exists$ a trapped surface $\mathcal{S} \Longrightarrow \exists$ a black hole $\mathcal{B}$ and $\mathcal{S} \subset \mathcal{B}$

# Local definitions of "black holes"

A hypersurface $\mathcal{H}$ of $(\mathscr{M}, \boldsymbol{g})$ is said to be




- a **future outer trapping horizon (FOTH)** iff
  (i) $\mathcal{H}$ foliated by marginally trapped 2-surfaces $(\theta^{(\boldsymbol{k})} < 0$ and $\theta^{(\boldsymbol{\ell})} = 0)$
  (ii) $\mathcal{L}_{\boldsymbol{k}} \, \theta^{(\boldsymbol{\ell})} < 0$
  [Hayward, PRD **49**, 6467 (1994)]

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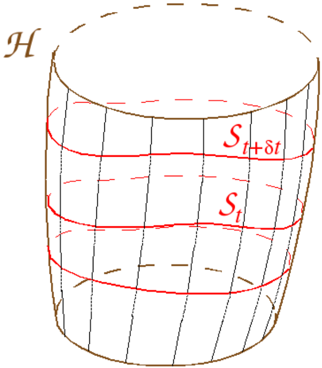


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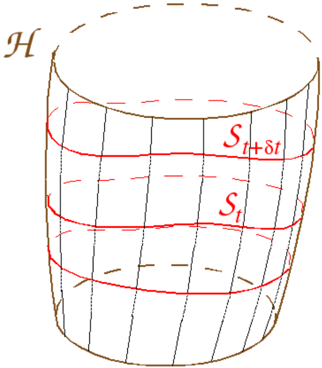


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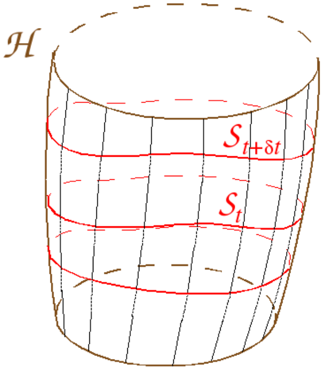


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- an **isolated horizon** iff
  (i) $\mathcal{H}$ is a non-expanding horizon
  (ii) $\mathcal{H}$'s full geometry is not evolving along the null generators: $[\boldsymbol{\mathcal{L}}_{\boldsymbol{\ell}}\, , \hat{\boldsymbol{\nabla}}] = 0$
  [Ashtekar, Beetle & Fairhurst, CQG **16**, L1 (1999)]

# Dynamics of these new horizons

The *dynamical horizons* and *trapping horizons* have their **own dynamics**, ruled by the Einstein equation.
In particular, one can establish for them

- first and second laws of black hole mechanics
  [Ashtekar & Krishnan, PRD **68**, 104030 (2003)], [Hayward, PRD **70**, 104027 (2004)]
- a Navier-Stokes like equation ⇒ viscous membrane behavior as for the event horizon ("membrane paradigm")
  [Gourgoulhon, PRD **72**, 104007 (2005)]

# Outline

# Concept of black hole viscosity

- Hartle and Hawking (1972, 1973): introduced the concept of **black hole viscosity** when studying the response of the *event horizon* to external perturbations
- Damour (1979): 2-dimensional **Navier-Stokes** like equation for the event horizon $\Longrightarrow$ *shear viscosity* and *bulk viscosity*
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Shall we restrict the analysis to the event horizon ?

Can we extend the concept of viscosity to the local characterizations of black hole recently introduced, i.e. future outer trapping horizons and dynamical horizons ?

NB: *event horizon* = null hypersurface
*future outer trapping horizon* = null or spacelike hypersurface
*dynamical horizon* = spacelike hypersurface

# Navier-Stokes equation in Newtonian fluid dynamics

$$\rho \left( \frac{\partial v^i}{\partial t} + v^j \nabla_j v^i \right) = -\nabla^i P + \mu \Delta v^i + \left( \zeta + \frac{\mu}{3} \right) \nabla^i (\nabla_j v^j) + f^i$$

or, in terms of fluid momentum density $\pi_i := \rho v_i$,

$$\boxed{\frac{\partial \pi_i}{\partial t} + v^j \nabla_j \pi_i + \theta \pi_i = -\nabla_i P + 2\mu \nabla^j \sigma_{ij} + \zeta \nabla_i \theta + f_i}$$

where $\theta$ is the fluid expansion:

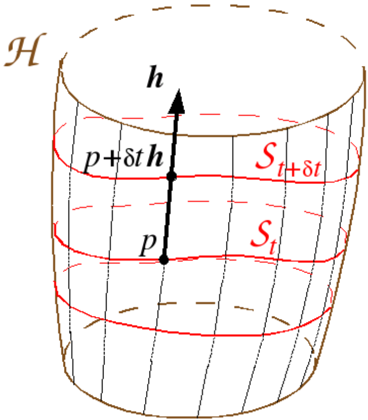
$$\theta := \nabla_j v^j$$

and $\sigma_{ij}$ the velocity shear tensor:

$$\sigma_{ij} := \frac{1}{2} (\nabla_i v_j + \nabla_j v_i) - \frac{1}{3} \theta \, \delta_{ij}$$

$P$ is the pressure, $\mu$ the shear viscosity, $\zeta$ the bulk viscosity and $f_i$ the density of external forces

# Evolution vector on the horizon




Vector field $\boldsymbol{h}$ on $\mathcal{H}$ defined by

- (i) $\boldsymbol{h}$ is tangent to $\mathcal{H}$
- (ii) $\boldsymbol{h}$ is orthogonal to $\mathcal{S}_t$
- (iii) $\boldsymbol{\mathcal{L}}_{\boldsymbol{h}}\, t = h^\mu \partial_\mu t = \langle \mathbf{d}t, \boldsymbol{h} \rangle = 1$

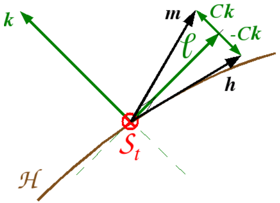
NB: (iii) $\Longrightarrow$ the 2-surfaces $\mathcal{S}_t$ are Lie-dragged by $\boldsymbol{h}$

# Generalized Damour-Navier-Stokes equation

For a future trapping horizon, one can derive from Einstein equation the following relation [Gourgoulhon, PRD **72**, 104007 (2005)]

$$ {}^{\mathcal{S}}\mathcal{L}_{\boldsymbol{h}}\, \boldsymbol{\Omega}^{(\boldsymbol{\ell})} + \theta^{(\boldsymbol{h})}\, \boldsymbol{\Omega}^{(\boldsymbol{\ell})} = \boldsymbol{\mathcal{D}}\kappa - \boldsymbol{\mathcal{D}}\cdot \vec{\boldsymbol{\sigma}}^{(\boldsymbol{m})} - \frac{1}{2}\boldsymbol{\mathcal{D}}\theta^{(\boldsymbol{h})} - \theta^{(\boldsymbol{k})}\boldsymbol{\mathcal{D}}C + 8\pi \vec{\boldsymbol{q}}^{*}\boldsymbol{T}\cdot\boldsymbol{m} $$

- $\boldsymbol{\Omega}^{(\boldsymbol{\ell})}$ : normal fundamental form of $\mathcal{S}_t$ associated with null normal $\boldsymbol{\ell}$
- $\theta^{(\boldsymbol{h})}$, $\theta^{(\boldsymbol{m})}$ and $\theta^{(\boldsymbol{k})}$: expansion scalars of $\mathcal{S}_t$ along the vectors $\boldsymbol{h}$, $\boldsymbol{m}$ and $\boldsymbol{k}$
- $\boldsymbol{\mathcal{D}}$ : covariant derivative within $(\mathcal{S}_t, \boldsymbol{q})$
- $\kappa = -\boldsymbol{k}\cdot\boldsymbol{\nabla}_{\boldsymbol{h}}\boldsymbol{h}$ : "surface-gravity"
- $\boldsymbol{\sigma}^{(\boldsymbol{m})}$ : shear tensor of $\mathcal{S}_t$ along the vector $\boldsymbol{m}$
- $C$ : half the scalar square of $\boldsymbol{h}$



# Equivalent form

$$\boxed{{}^{\mathcal{S}}\!\mathcal{L}_{\boldsymbol{h}}\,\boldsymbol{\pi} + \theta^{(h)}\boldsymbol{\pi} = -\boldsymbol{\mathcal{D}}P + 2\mu\boldsymbol{\mathcal{D}}\cdot\vec{\boldsymbol{\sigma}}^{(m)} + \zeta\boldsymbol{\mathcal{D}}\theta^{(h)} - \theta^{(k)}\boldsymbol{\mathcal{D}}C + \boldsymbol{f}}$$

with $\boldsymbol{\pi} := -\frac{1}{8\pi}\boldsymbol{\Omega}^{(\ell)}$ momentum surface density

$P := \frac{\kappa}{8\pi}$ pressure

$\mu := \frac{1}{16\pi}$ shear viscosity

$\zeta := \frac{1}{16\pi}$ bulk viscosity

$\boldsymbol{f} := -\vec{\boldsymbol{q}}^{*}\boldsymbol{T}\cdot\boldsymbol{m}$ external force surface density ($\boldsymbol{T}$ = stress-energy tensor)

compare

Similar to the Navier-Stokes-like equation obtained by Damour (1978) for an event horizon, except for the positive bulk viscosity.

# Generalized angular momentum

Definition [Booth & Fairhurst, CQG **22**, 4545 (2005)]: Let $\varphi$ be a vector field on $\mathcal{H}$ which

- is tangent to $\mathcal{S}_t$
- has closed orbits
- has vanishing divergence with respect to the induced metric: $\mathcal{D} \cdot \varphi = 0$

The *generalized angular momentum associated with* $\varphi$ is then defined by

$$J(\varphi) := -\frac{1}{8\pi} \oint_{\mathcal{S}_t} \langle \mathbf{\Omega}^{(\ell)}, \varphi \rangle \, {}^{\mathcal{S}}\epsilon,$$

*Remark 1:* does not depend upon the choice of null vector $\ell$, thanks to the divergence-free property of $\varphi$

*Remark 2:*

- coincides with Ashtekar & Krishnan's definition for a dynamical horizon
- coincides with Brown-York angular momentum if $\mathcal{H}$ is timelike and $\varphi$ a Killing vector

# Angular momentum flux law

Under the supplementary hypothesis that $\varphi$ is transported along the evolution vector $\boldsymbol{h}$ : $\boldsymbol{\mathcal{L}}_{\boldsymbol{h}}\,\boldsymbol{\varphi} = 0$, the generalized Damour-Navier-Stokes equation leads to

$$\frac{d}{dt}J(\varphi) = -\oint_{\mathcal{S}_t} \boldsymbol{T}(\boldsymbol{m},\boldsymbol{\varphi})\,{}^{\mathcal{S}}\boldsymbol{\epsilon} - \frac{1}{16\pi}\oint_{\mathcal{S}_t}\left[\vec{\boldsymbol{\sigma}}^{(\boldsymbol{m})} : \boldsymbol{\mathcal{L}}_{\boldsymbol{\varphi}}\,\boldsymbol{q} - 2\theta^{(\boldsymbol{k})}\boldsymbol{\varphi}\cdot\boldsymbol{\mathcal{D}}C\right]{}^{\mathcal{S}}\boldsymbol{\epsilon}$$

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Two interesting limiting cases:

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- $\mathcal{H}$ = null hypersurface : $C = 0$ and $\boldsymbol{m} = \boldsymbol{\ell}$ :

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- $\mathcal{H}$ = future trapping horizon :

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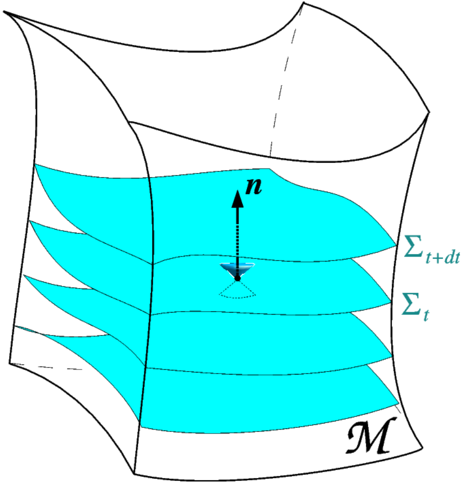
# 3+1 numerical relativity




**3+1 formalism:** slicing of the spacetime manifold $\mathscr{M}$ by a family of spacelike hypersurfaces $(\Sigma_t)_{t\in\mathbb{R}}$

$t$ = coordinate time
$\Sigma_t$ = "the 3-dimensional space" at instant $t$

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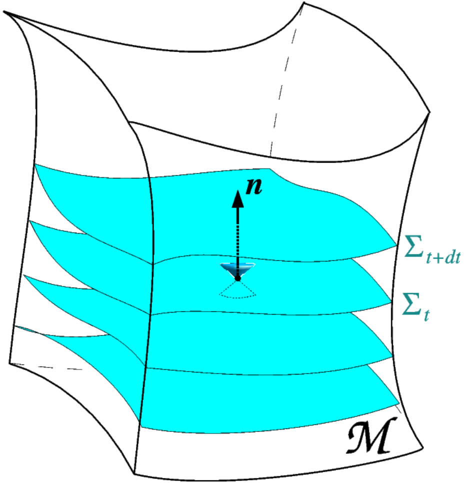


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$\Rightarrow$ **resolution of Einstein equation = Cauchy problem**
i.e. time evolution from initial data given on some hypersurface $\Sigma_0$

# 3+1 numerical relativity and black holes

**black hole** $\Rightarrow \exists$ singularity in spacetime

$\Rightarrow$ divergent quantities in the 3+1 formalism

However, there is no need to numerically evolve the region around the singularity since it is hidden behind the event horizon and causally disconnected from the exterior.

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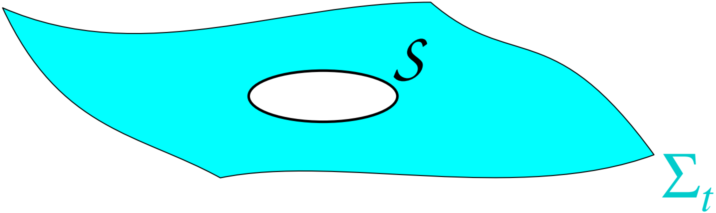
However, there is no need to numerically evolve the region around the singularity since it is hidden behind the event horizon and causally disconnected from the exterior.

Idea: excise from the numerical domain a region containing the singularity



# 3+1 numerical relativity and black holes

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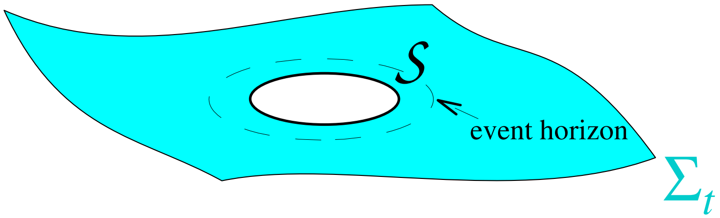
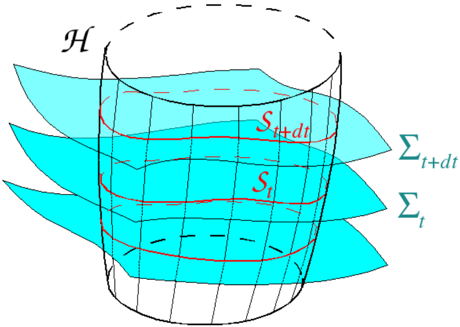



Provided that the excised region is located within the even horizon, the choice of it does not affect the exterior spacetime

# Our project

Choose the excision boundary $\mathcal{S}_t$ to be a **marginally trapped surface** for each time $t$




The tube $\mathcal{H} = \bigcup_{t\in\mathbb{R}} \mathcal{S}_t$

is then a **trapping horizon**

- geometrically well defined excision boundary
- ensures $\mathcal{S}_t$ is located inside the event horizon reminder
- easy to implement with spherical coordinates and spectral methods

# Status

- Equilibrium conditions (isolated horizon) expressed in terms of the quantities of the 3+1 formalism
  - [Jaramillo, Gourgoulhon & Mena Marugán, PRD **70**, 124036 (2004)]
  - [Gourgoulhon & Jaramillo, Phys. Rep. **423**, 159 (2006)]
- Analytical study of the dynamical case completed
- Numerical implementation has started in the framework of the constrained scheme for 3+1 Einstein equations (Dirac gauge)
  - [Bonazzola, Gourgoulhon, Grandclément & Novak, PRD **70**, 104007 (2004)]