Black hole physics
entering a new observational era

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Seminar at
Laboratoire de Mathématiques et Physique Théorique
Tours, 29 November 2012

Outline

1. Part 1
    - What is a black hole ?
    - Overview of the black hole theory
    - The current observational status of black holes
    - The near-future observations of black holes

2. Part 2
    - Tests of gravitation
    - The Gyoto tool
    - Ray-tracing in numerical spacetimes

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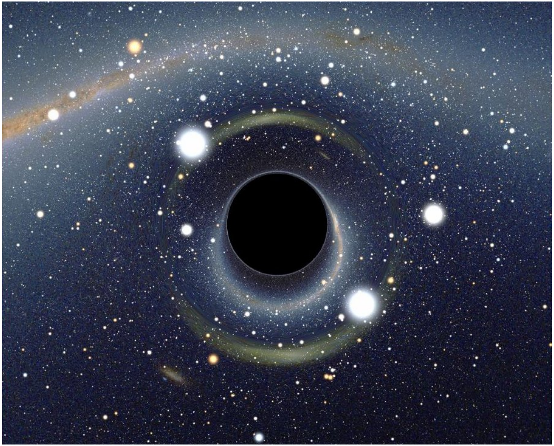

[Alain Riazuelo, 2007]

… for the layman:

A **black hole** is a region of spacetime from which nothing, not even light, can escape.

The (immaterial) boundary between the black hole interior and the rest of the Universe is called the **event horizon**.

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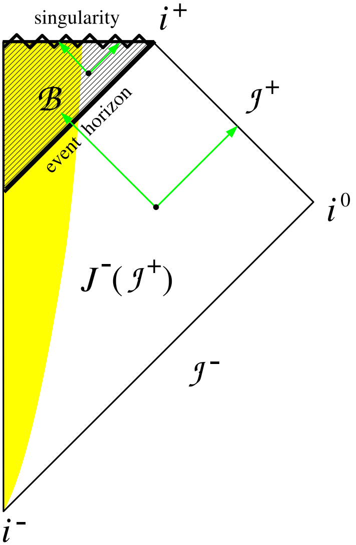

... for the mathematical physicist:

black hole: $\boxed{\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)}$

i.e. the region of spacetime where light rays cannot escape to infinity

- $(\mathscr{M}, \boldsymbol{g})$ = asymptotically flat manifold
- $\mathscr{I}^+$ = future null infinity
- $J^-(\mathscr{I}^+)$ = causal past of $\mathscr{I}^+$

event horizon: $\mathcal{H} := \partial J^-(\mathscr{I}^+)$
(boundary of $J^-(\mathscr{I}^+)$)

$\mathcal{H}$ smooth $\Longrightarrow$ $\mathcal{H}$ null hypersurface

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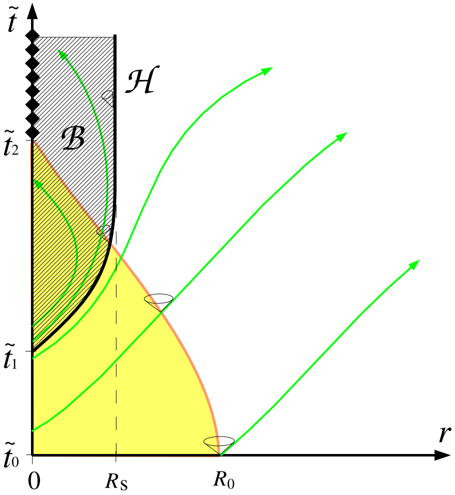

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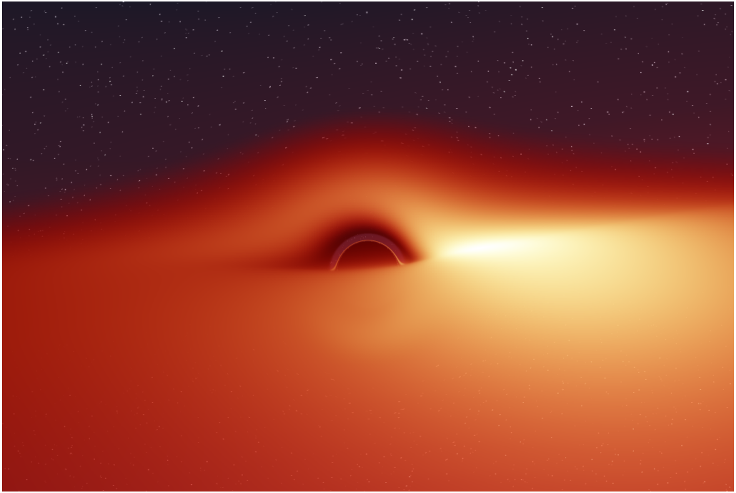
What is a black hole ?

... for the astrophysicist: a very deep gravitational potential well



[J.A. Marck, CQG **13**, 393 (1996)]

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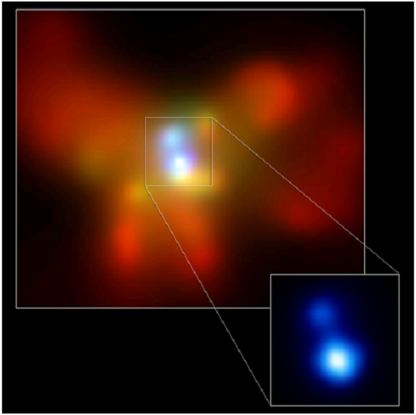
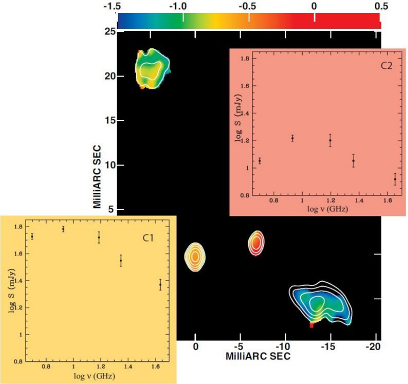


Binary BH in galaxy NGC 6240

$d = 1.4$ kpc

[Komossa et al., ApJ **582**, L15 (2003)]



Binary BH in radio galaxy 0402+379

$d = 7.3$ pc

[Rodriguez et al., ApJ **646**, 49 (2006)]

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- Georges Lemaître (1932) : the singularity at $r = R_{\rm s}$ is not physical *(coordinate singularity)*

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- John A. Wheeler (1967) : coined the name **black hole**

Main properties of black holes (1/3)

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- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation
- A black hole is not an infinitely dense object: on the contrary it is made of vacuum (except maybe at the singularity); black holes can form in spacetimes empty of any matter, by collapse of gravitational wave packets.

Main properties of black holes (2/3)

Uniqueness theorem

(Dorochkevitch, Novikov & Zeldovitch 1965, Israel 1967, Carter 1971, Hawking 1972) :
A black hole in equilibrium is necessarily a Kerr-Newmann black hole, which is a vacuum solution of Einstein described by only three parameters:

- the total mass M
- the total angular momentum J
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

- $Q = 0$ and $J = 0$: Schwarzschild solution (1916)
- $Q = 0$: Kerr solution (1963)

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- The mass M is not a measure of the “matter amount” inside the black hole, but rather a parameter characterizing the external gravitational field; it is measurable from the orbital period of a test particle in circular orbit around the black hole and far from it *(Kepler’s third law)*.

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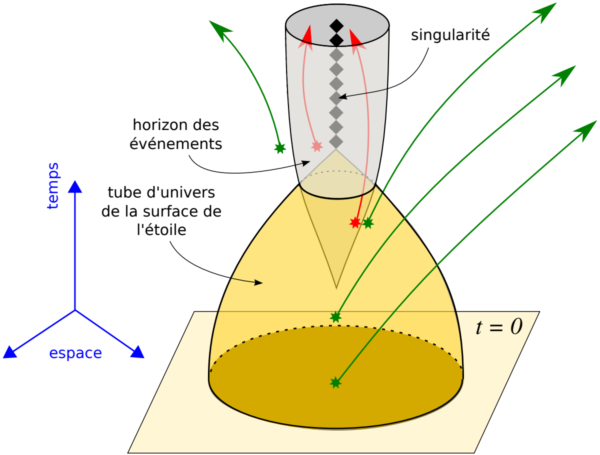
- The radius of a black hole is not a well defined concept: it *does not* correspond to some distance between the black hole "centre" (the singularity) and the event horizon. A well defined quantity is the area of the event horizon, A.
  The radius can be then defined from it: for a Schwarzschild black hole:

$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left(\frac{M}{M_\odot} \right) \ \mathrm{km}$$

Formation by gravitational collapse



Spacetime diagram of a gravitational collapse

Massive stars end their lives as **supernova**: the explosion is triggered by the gravitational collapse of the stellar iron core, via some bounce

Depending on the initial conditions, the collapse can be stopped by the *strong interaction* (the residual is then a *neutron star*) or be complete, leading to a black hole.

Other theoretical aspects

- The four laws of black hole dynamics
- Quantum properties (Bekenstein entropy, Hawking radiation)
- Black holes and gravitational waves
- Quasi-local approaches: trapping horizons, dynamical horizons, isolated horizons
- Black holes in higher dimensions

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Astrophysical black holes

There are three kinds of black holes in the Universe:

- **Stellar black holes**: supernova remnants:
  $M \sim 10 - 30\ M_\odot$ and $R \sim 30 - 90\ \mathrm{km}$

  example: Cyg X-1 : $M = 15\ M_\odot$ and $R = 45\ \mathrm{km}$

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- **Supermassive black holes**, in galactic nuclei:
  $M \sim 10^5 - 10^{10}\ M_\odot$ and $R \sim 3 \times 10^5\ \mathrm{km} - 200\ \mathrm{UA}$

  example: Sgr A* : $M = 4,3 \times 10^6\ M_\odot$ and
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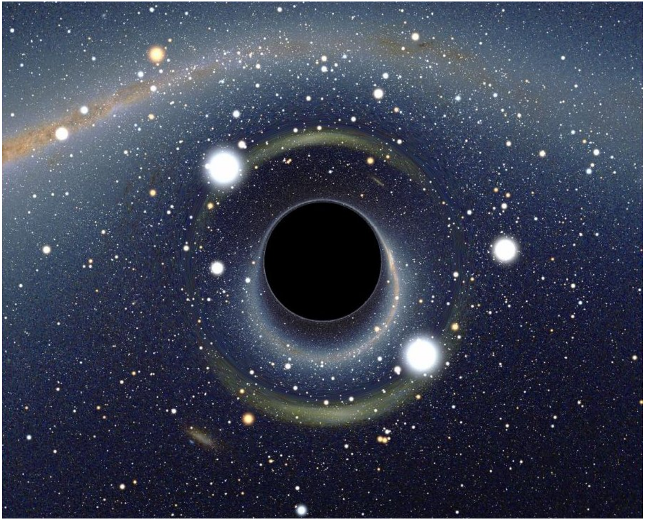
- **Intermediate mass black holes**, as ultra-luminous X-ray sources (?):
  $M \sim 10^2 - 10^4\ M_\odot$ and $R \sim 300\ \mathrm{km} - 3 \times 10^4\ \mathrm{km}$

  example: ESO 243-49 HLX-1 : $M > 500\ M_\odot$ and $R > 1500\ \mathrm{km}$

What we do not see yet...



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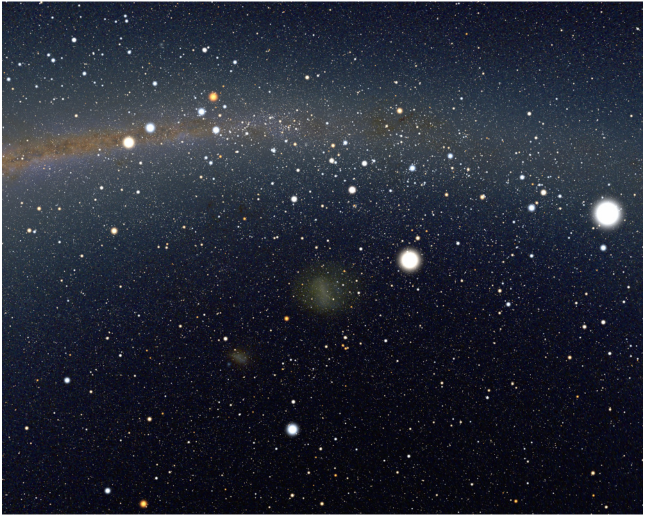

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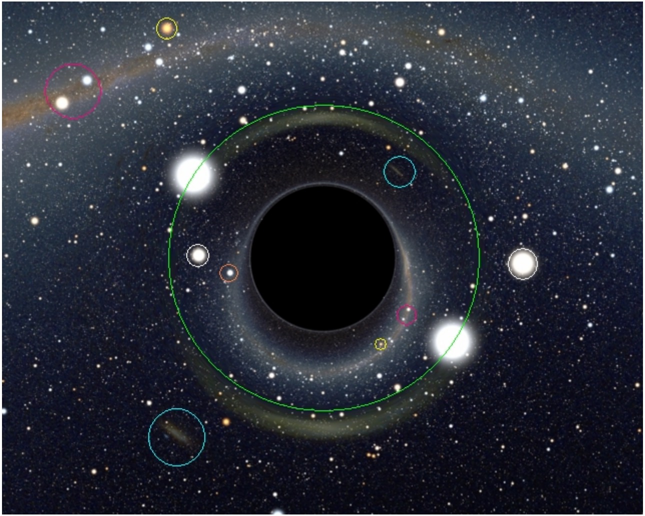

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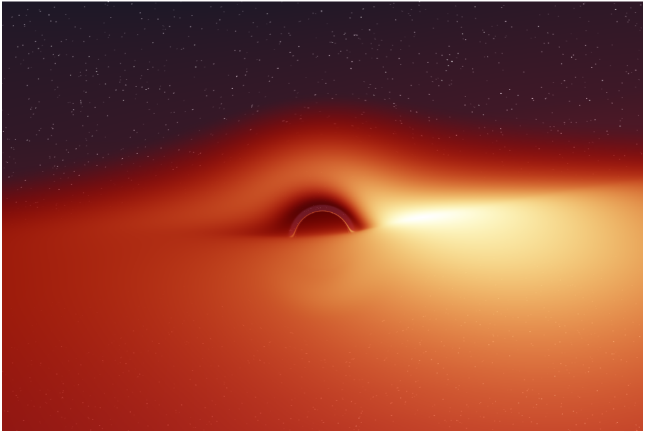
The black hole: a fantastic source of energy !

Release of potential gravitational energy by **accretion** on a black hole: up to 42% of the mass-energy mc^2 of accreted matter !

NB: thermonuclear reaction release less than 1% mc^2

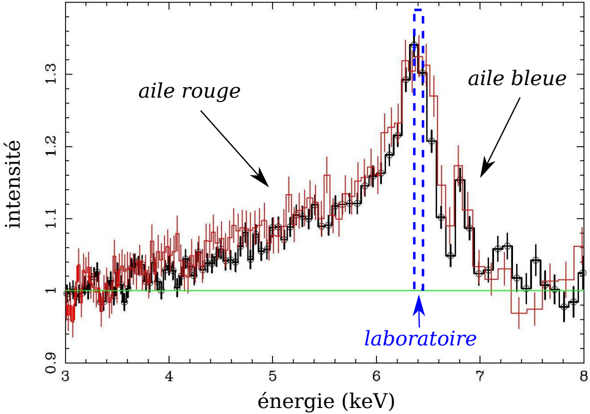


Matter falling in a black hole forms an **accretion disk** [Donald Lynden-Bell (1969), Nicolaï Shakura & Rachid Sunayev (1973)]

[J.-A. Marck (1996)]

The accretion disk as a spacetime probe

X-ray spectrum of the accretion disk around the supermassive black hole in the nucleus of the galaxy MCG-6-30-15 :

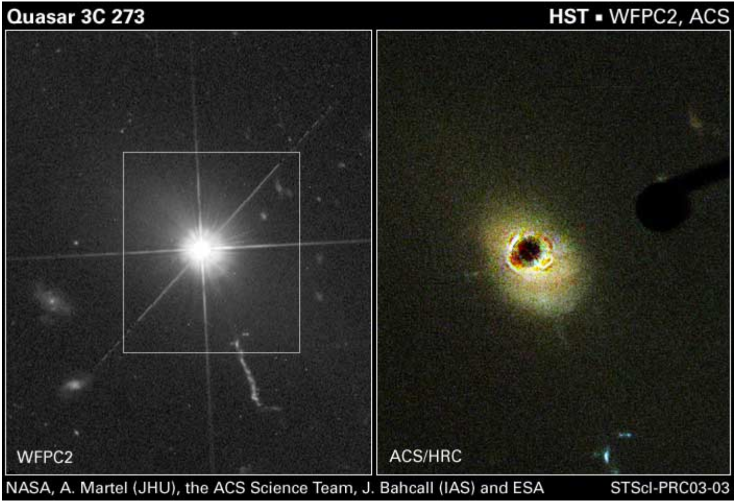


Kα line: X fluorescence line of Fe atoms in the accretion disk (the Fe atoms are excited by the X-ray emitted from the plasma corona surrounding the disk).

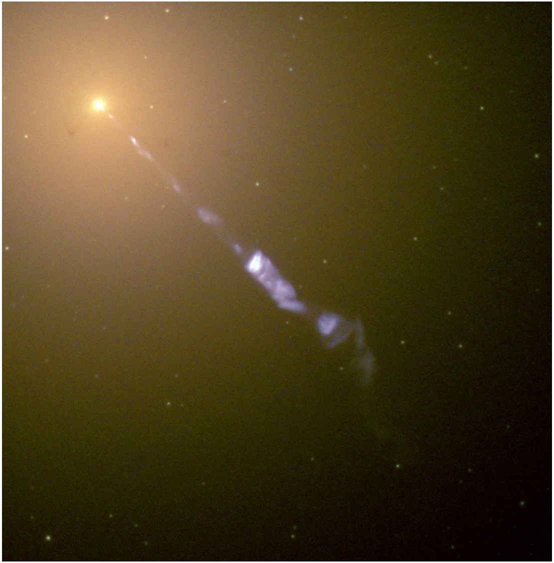
Redshift ⇒ time dilatation

Kα line observed by the satellites XMM-Newton (red) and Suzaku (black) (adapted from [Miller (2007)])

Black holes in the core of quasars



Black holes in active galactic nuclei (AGN)

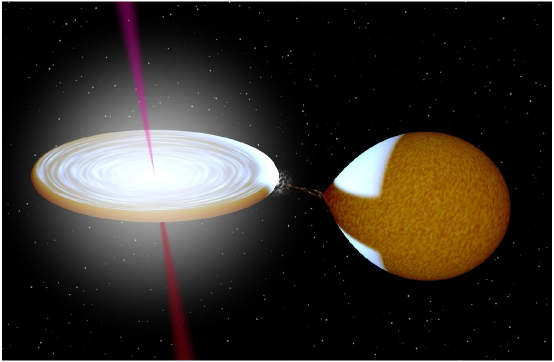


Jet emitted by the nucleus of the giant elliptic galaxy M87, at the centre of Virgo cluster [HST]

$M_{\rm BH} = 3 \times 10^9 \, M_\odot$

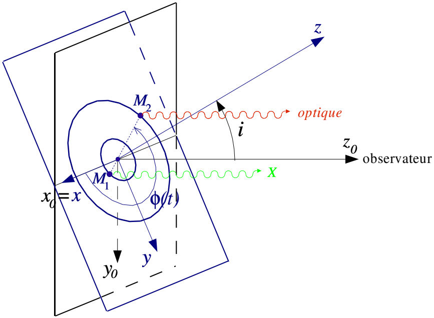
$V_{\rm jet} \simeq 0.99 \, c$

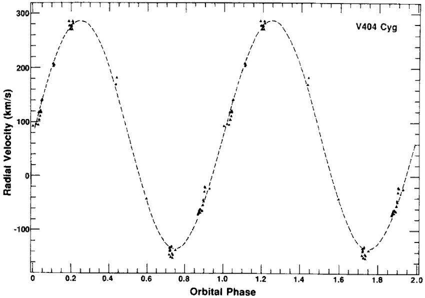
Black holes in X-ray binaries



$\sim$ 20 identified stellar black holes in our galaxy

Detection of a black hole in a X-ray binary





$$V_{\rm rad}(t) = K_2 \cos(2\pi t/P) + V_0 \Rightarrow K_2,\ P$$

Kepler's third law: $f := \dfrac{M_1^3 \sin^3 i}{(M_1 + M_2)^2} = \dfrac{K_2^3 P}{2\pi G}$

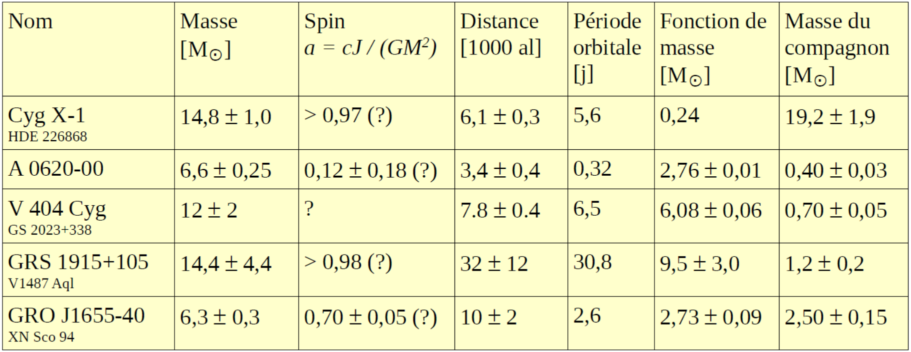
f is a lower bound on M_1 : $M_1 > f$

Mass criterion: $M_1 > M_{\rm max}(\text{neutron star}) \simeq 3\, M_\odot$

Black holes in X-ray binaries

The first black hole identified via the mass criterion was *Cygnus X-1* in 1972. Since then, around 20 black holes have been identified in this way.

Selection of 5 black holes in X-ray binaries:



Nom	Masse $[M_\odot]$	Spin $a = cJ/(GM^2)$	Distance [1000 al]	Période orbitale [j]	Fonction de masse $[M_\odot]$	Masse du compagnon $[M_\odot]$
Cyg X-1 HDE 226868	14,8 ± 1,0	> 0,97 (?)	6,1 ± 0,3	5,6	0,24	19,2 ± 1,9
A 0620-00	6,6 ± 0,25	0,12 ± 0,18 (?)	3,4 ± 0,4	0,32	2,76 ± 0,01	0,40 ± 0,03
V 404 Cyg GS 2023+338	12 ± 2	?	7.8 ± 0.4	6,5	6,08 ± 0,06	0,70 ± 0,05
GRS 1915+105 V1487 Aql	14,4 ± 4,4	> 0,98 (?)	32 ± 12	30,8	9,5 ± 3,0	1,2 ± 0,2
GRO J1655-40 XN Sco 94	6,3 ± 0,3	0,70 ± 0,05 (?)	10 ± 2	2,6	2,73 ± 0,09	2,50 ± 0,15

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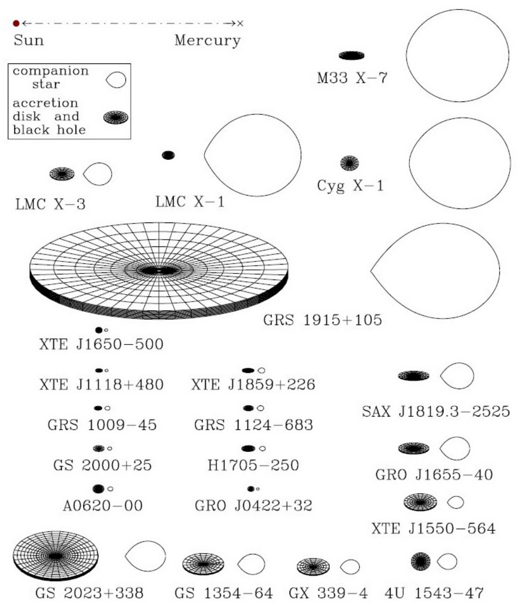
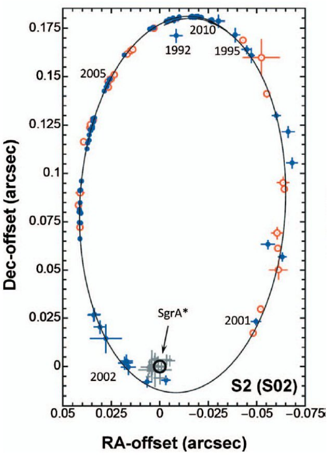


[McClintock et al. (2011)]

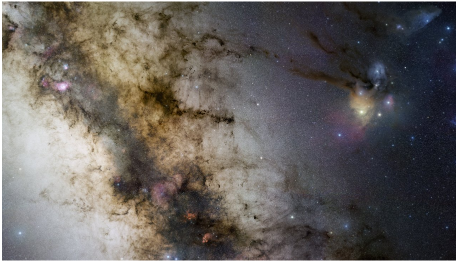
The black hole at the centre of our galaxy



Orbit of the star S2 around the black hole Sgr A*

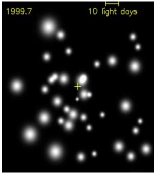
[Genzel et al. (2010)]

$$M_{\rm BH} = 4.3 \times 10^6 \, M_\odot$$



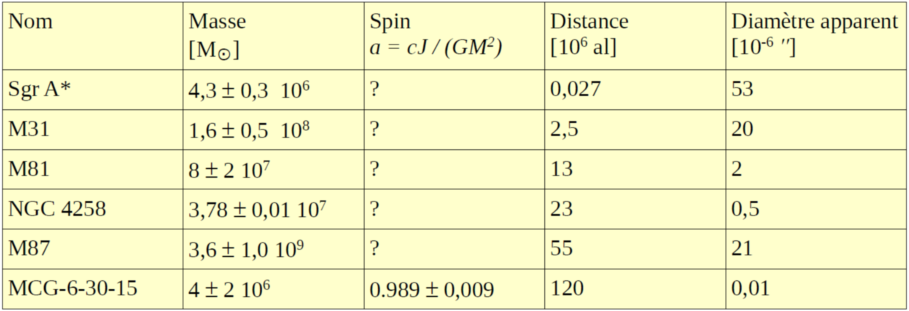
[ESO (2009)]

Detection via the stellar dynamics



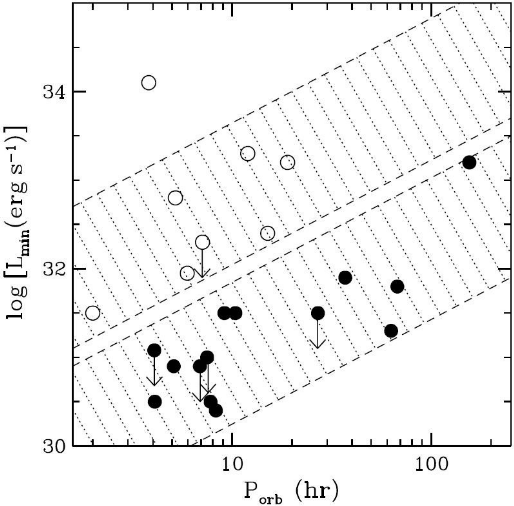
Supermassive black holes

Selection of 6 supermassive black holes:



Nom	Masse [$M_\odot$]	Spin $a = cJ/(GM^2)$	Distance [10^6 al]	Diamètre apparent [10^{-6} ″]
Sgr A*	$4{,}3 \pm 0{,}3 \; 10^6$	?	0,027	53
M31	$1{,}6 \pm 0{,}5 \; 10^8$	?	2,5	20
M81	$8 \pm 2 \; 10^7$	?	13	2
NGC 4258	$3{,}78 \pm 0{,}01 \; 10^7$	?	23	0,5
M87	$3{,}6 \pm 1{,}0 \; 10^9$	?	55	21
MCG-6-30-15	$4 \pm 2 \; 10^6$	$0.989 \pm 0{,}009$	120	0,01

Better than the mass criterion: clues for a horizon !



Luminosity of X-ray binaries during quiescent stages:

the systems with a black hole (•) are ~ 100 times less luminous than those with a neutron star (∘)

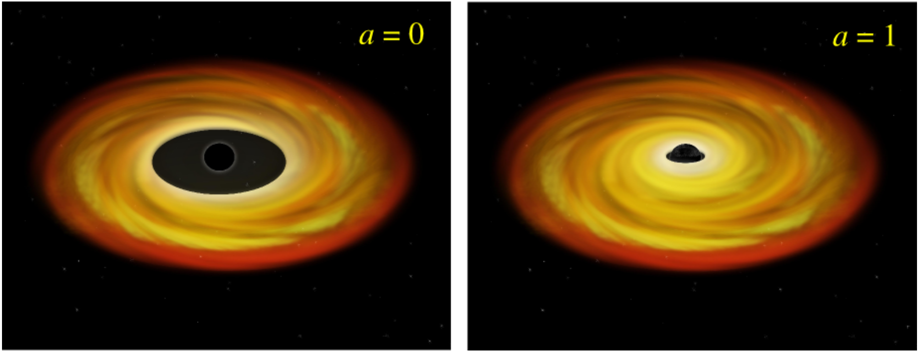
[Narayan & McClintock, New Astron. Rev. **51**, 733 (2008)]

Beyond the mass: measuring the spin

Innermost Stable Circular Orbit (ISCO):

$$R_{\rm ISCO}(a=0) = \frac{6GM}{c^2} \quad \text{and} \quad R_{\rm ISCO}(a=1) = \frac{GM}{c^2}$$

The internal edge of the accretion disk is located at the ISCO

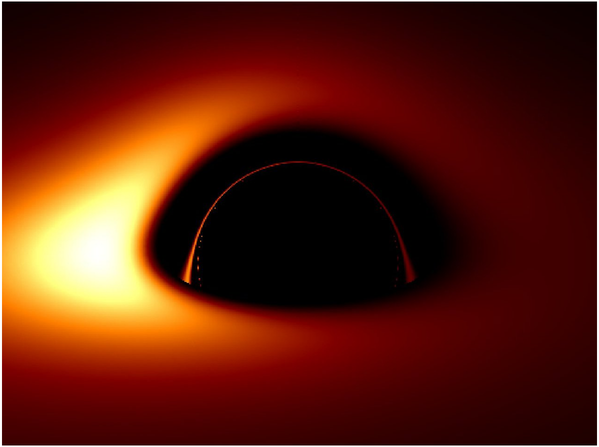


[NASA/CXC/M. Weiss]

Comparison of the X-ray spectrum to an emission model $\Longrightarrow$ estimation of a

Outline

Seeing the black hole shadow



Thin accretion disk

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Largest black-hole apparent sizes in the Earth's sky:

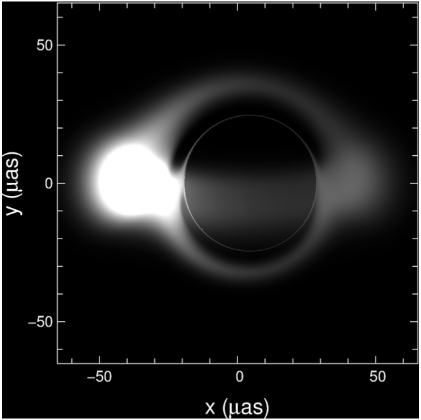
Sgr A* : $D = 53\ \mu\mathrm{as}$
M87 : $D = 21\ \mu\mathrm{as}$
M31 : $D = 20\ \mu\mathrm{as}$

Rem. 1: black holes in X-ray binaries are $\sim 10^5$ times smaller, for $D \propto M/d$

Rem. 2: HST angular resolution:

$$D_{\min} \sim 10^5\ \mu\mathrm{as}\ !$$

Seeing the black hole shadow



Ion torus

[Straub, Vincent, Abramowicz, Gourgoulhon & Paumard, A&A **543**, A83 (2012)]

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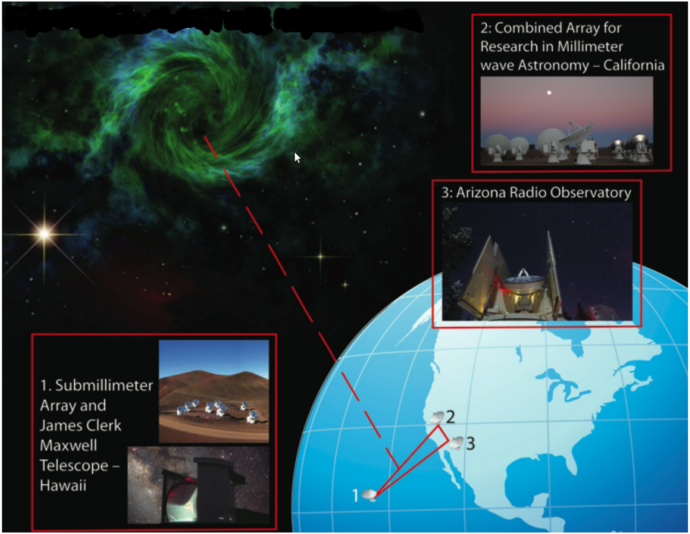
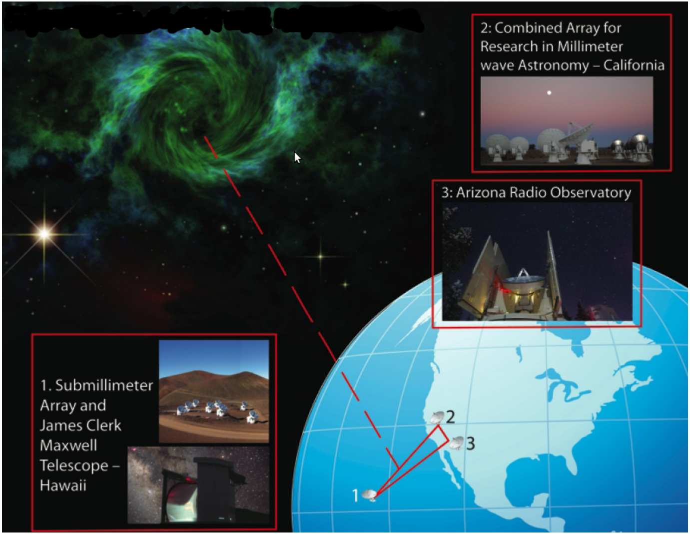
The solution to reach the μas regime: interferometry !



Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]

The solution to reach the μas regime: interferometry !


[redacted]

Existing American VLBI network [Doeleman et al. 2011]

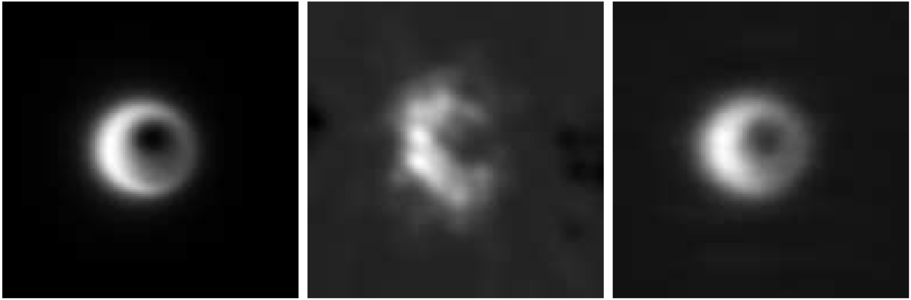
Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far: VLBI observations at 1.3 mm have shown that the size of the emitting region in Sgr A* is only $37\ \mu\mathrm{as}$.

The near future: the Event Horizon Telescope

Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020

The near future: the Event Horizon Telescope



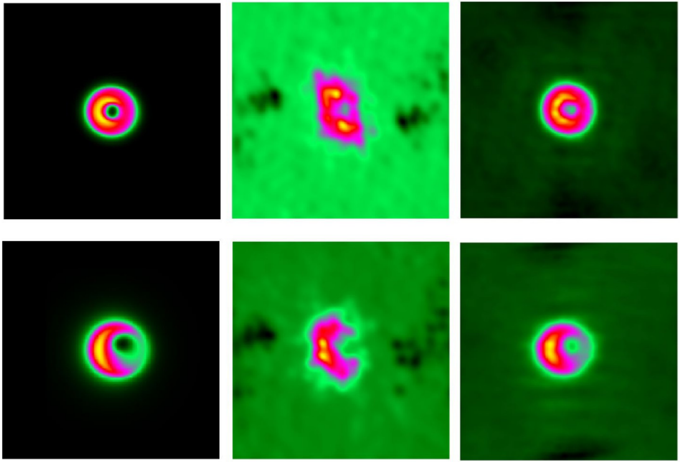
Simulations of VLBI observations of Sgr A* at $\lambda = 0.8\ \mathrm{mm}$

left: perfect image, *centre:* 7 stations (~ 2015), *right:* 13 stations (~ 2020)

$a = 0,\ i = 30^\circ$

[Fish & Doeleman, arXiv:0906.4040 (2009)]

The near future: the Event Horizon Telescope



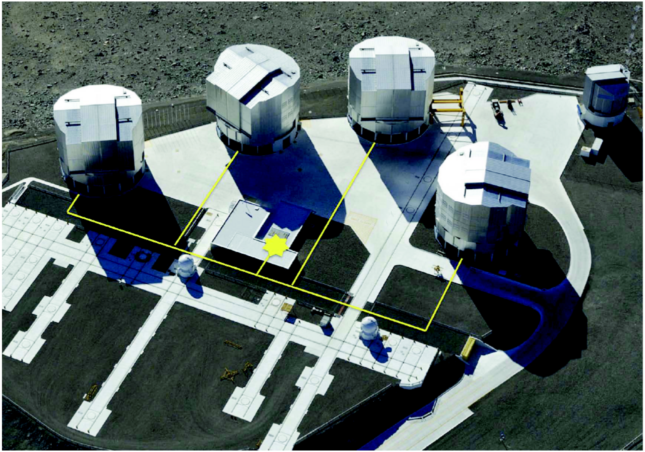
Simulations of VLBI observations of Sgr A* at $\lambda = 0.8$ mm

left: perfect image, *centre:* 7 stations (~ 2015), *right:* 13 stations (~ 2020)

top: $a = 0.5$, $i = 85°$; *bottom:* $a = 0$, $i = 60°$

[Doeleman et al. (2009)]

Near-infrared optical interferometry

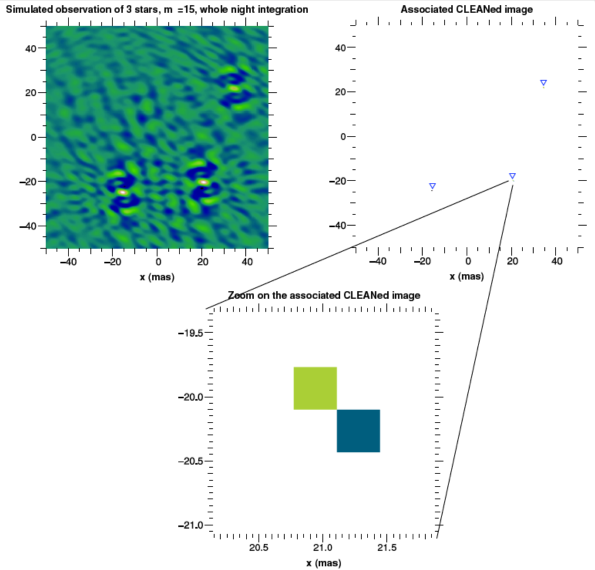


[Gillessen et al. 2010]

GRAVITY instrument at VLT (2014)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes) $\Longrightarrow$ astrometric precision of $10\ \mu\mathrm{as}$

Simulations of GRAVITY observations



Observation of 3 stars of magnitude 15 during a whole night.

[Vincent et al., MNRAS **412**, 2653 (2011)]

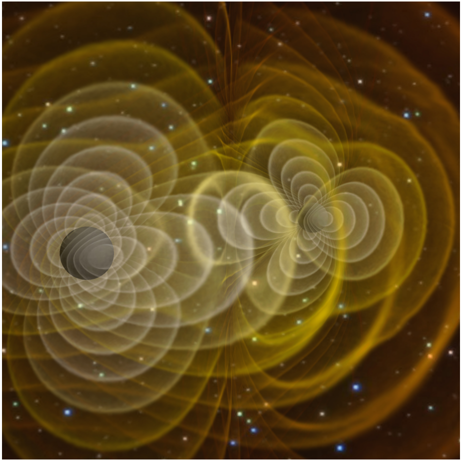
Testing the no-hair theorem

GRAVITY is expected to observe stars on relativistic orbites (closer than S2)
Measure of relativistic effects:

- periastron advance
- Lense-Thirring precession

$\Longrightarrow$ constraints on the spacetime metric in the vicinity of the central object
$\Longrightarrow$ is it really the Kerr metric (a, M) ?

Another future observational mean: gravitational waves



[Baker et al., 2006]

gravitational waves = perturbations in the spacetime curvature

- reveal the spacetime **dynamics**
- generated by matter or black hole acceleration
- far from sources, are propagating at the speed of light
- NB: **electromagnetic waves** are perturbation of the electromagnetic field propagating *within* spacetime, whereas **gravitational waves** are waves of spacetime *itself*

Detection of gravitational waves

Interferometric detector VIRGO at Cascina, near Pisa [CNRS/INFN]

Outline

1. Part 1
    - What is a black hole ?
    - Overview of the black hole theory
    - The current observational status of black holes
    - The near-future observations of black holes
2. Part 2
    - Tests of gravitation
    - The Gyoto tool
    - Ray-tracing in numerical spacetimes

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Theoretical alternatives to the Kerr black hole

Within general relativity

- boson stars
- gravastar
- Q-star
- dark stars
- ...

Beyond general relativity

"hairy" black holes

- in Einstein-Yang-Mills
- in Einstein-Gauss-Bonnet with dilaton
- in Chern-Simons gravity
- ...

How to test the alternatives ?

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)
- accretion disk spectra different from those arising in Kerr metric (X-ray observatories)
- images of the black hole shadow different from that of a Kerr black hole (EHT)

How to test the alternatives ?

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Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric

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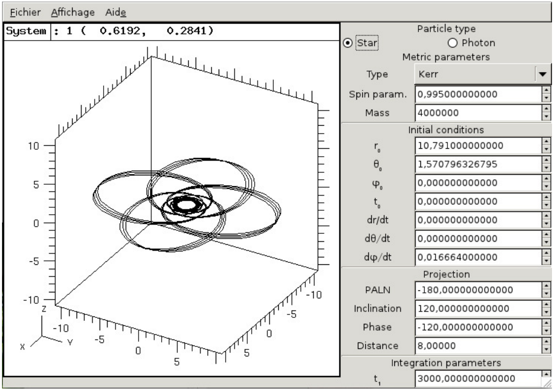
Gyoto code



System : 1 (0.6192, 0.2841)

Particle type

● Star ○ Photon

Metric parameters

Type: Kerr

Spin param.: 0,995000000000

Mass: 4000000

Initial conditions

r_0: 10,791000000000

θ_0: 1,570796326795

φ_0: 0,000000000000

t_0: 0,000000000000

dr/dt: 0,000000000000

dθ/dt: 0,000000000000

dφ/dt: 0,016664000000

Projection

PALN: -180,000000000000

Inclination: 120,000000000000

Phase: -120,000000000000

Distance: 8,00000

Integration parameters

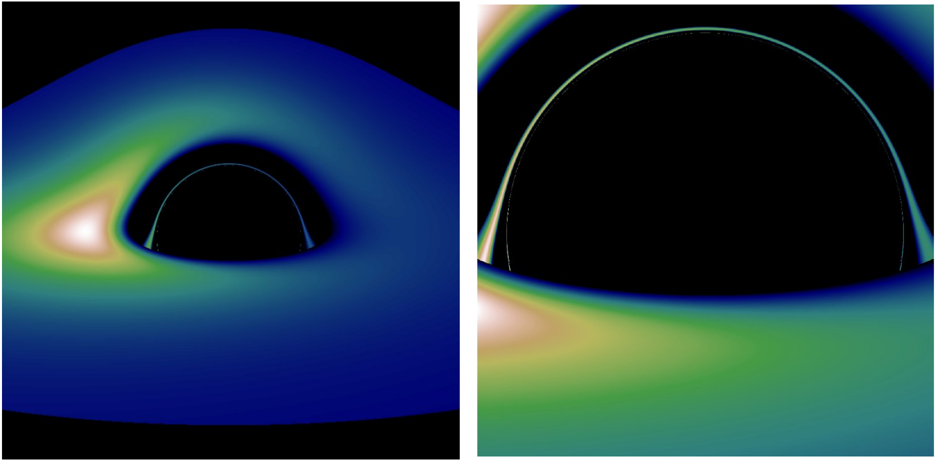
t_1: 3000,000000000000

- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick interface
- Free software (GPL) : `http://gyoto.obspm.fr/`

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

Gyoto code

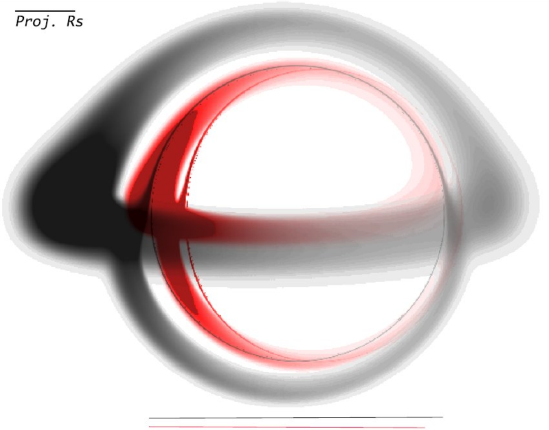


Computed images of a thin accretion disk around a Schwarzschild black hole

Measuring the spin from the black hole silhouette

Spin parameter of a Kerr black hole : $a = \dfrac{J}{M}$

Accretion structure around Sgr A* modeled as a ion torus, derived from the *polish doughnut* class [Abramowicz, Jaroszynski & Sikora (1978)]



Radiative transfer included (thermal synchrotron, bremsstrahlung, inverse Compton)

← Image of an ion torus computed with Gyoto for the inclination angle $i = 80°$:

- black: $a = 0.5M$
- red: $a = 0.9M$

[Straub, Vincent, Abramowicz, Gourgoulhon & Paumard, A&A **543**, A83 (2012)]

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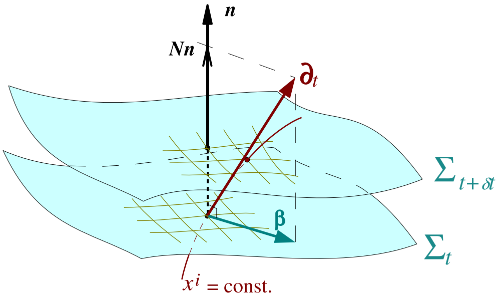
3+1 formalism for general relativity

Numerical spacetimes are generally computed within the 3+1 formalism



4-dimensional spacetime $(\mathcal{M}, \boldsymbol{g})$ foliated by spacelike hypersurfaces $(\Sigma_t)_{t\in\mathbb{R}}$

Unit timelike normal: $\underline{\boldsymbol{n}} = -N\boldsymbol{\nabla}t$

Induced metric: $\boldsymbol{\gamma} = \boldsymbol{g} + \underline{\boldsymbol{n}} \otimes \underline{\boldsymbol{n}}$

Shift vector of adapted coordinates (t, x^i): vector $\boldsymbol{\beta}$ tangent to Σ_t such that $\partial/\partial t = N\boldsymbol{n} + \boldsymbol{\beta}$

$$\boxed{g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + \gamma_{ij}(\mathrm{d}x^i + \beta^i\mathrm{d}t)(\mathrm{d}x^j + \beta^j\mathrm{d}t)}$$

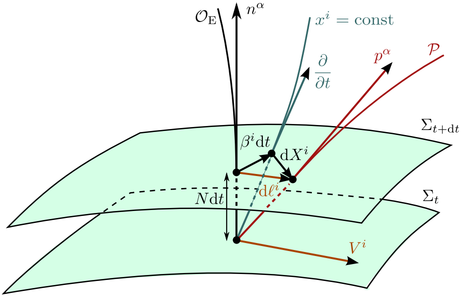
3+1 decomposition of the geodesic equation (1/2)

A particle $\mathcal{P}$ of 4-momentum vector $\boldsymbol{p}$ follows a geodesic iff $\boxed{\boldsymbol{\nabla}_{\boldsymbol{p}}\,\boldsymbol{p} = 0}$

3+1 decomposition of $\boldsymbol{p}$: $\boxed{\boldsymbol{p} = E(\boldsymbol{n} + \boldsymbol{V})}$, with

- E : particle's energy with respect to the Eulerian observer (4-velocity $\boldsymbol{n}$)
- $\boldsymbol{V}$: vector tangent to Σ_t, representing the particle's 3-velocity with respect to the Eulerian observer

3+1 decomposition of the geodesic equation (2/2)



Equation of $\mathcal{P}$'s worldline in terms of the 3+1 coordinates : $x^i = X^i(t)$

The physical 3-velocity $\boldsymbol{V}$ is related to the coordinate velocity $\dot{X}^i := {\rm d}x^i/{\rm d}t$ by

$$V^i = \frac{1}{N}\left(\dot{X}^i + \beta^i\right)$$

Orth. projection of $\boldsymbol{\nabla}_{\boldsymbol{p}}\,\boldsymbol{p} = 0$ along $\boldsymbol{n}$:

$$\boxed{\frac{{\rm d}E}{{\rm d}t} = E\left(N K_{jk} V^j V^k - V^j \partial_j N\right)}$$

Orth. projection of $\boldsymbol{\nabla}_{\boldsymbol{p}}\,\boldsymbol{p} = 0$ onto Σ_t:

$$\boxed{\left\{\begin{array}{ll} \dfrac{{\rm d}X^i}{{\rm d}t} & = N V^i - \beta^i \\ \dfrac{{\rm d}V^i}{{\rm d}t} & = N V^j \left[V^i\left(\partial_j \ln N - K_{jk} V^k\right) + 2K^i_{\ \, j} - {}^3\Gamma^i_{\ jk} V^k\right] - \gamma^{ij}\partial_j N - V^j \partial_j \beta^i \end{array}\right.}$$

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

Numerical procedure

Metric in 3+1 form is obtained from

- analytic solution (*e.g.* Kerr) $\Longrightarrow$ tests
- rotating neutron star model code `LORENE/nrotstar`
- simulation of a neutron star collapsing to a black hole with `CoCoNuT`

Gravitational fields are computed using spectral methods and represented by a set of coefficients $\{c_{i\ell m}\}_{(i,\ell,m)\in[0\ldots N]}$:

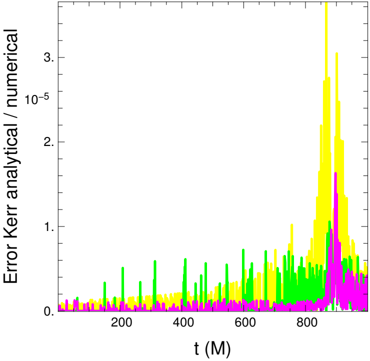
$$f(r,\theta,\varphi) = \sum_{i,\ell,m} c_{i\ell m}\, T_i(r)\, Y_\ell^m(\theta,\varphi)$$

$\Longrightarrow$ metric fields can be evaluated at any spatial point with this triple sum.

Integration of geodesics is done backward: from observer to the object, using a RK4, with adaptive step.

Test on Kerr spacetime

Integration of a null geodesic in the Kerr metric, using “numerical” (`LORENE`-prepared) metric fields in Boyer-Lindquist coordinates and 3+1 approach.



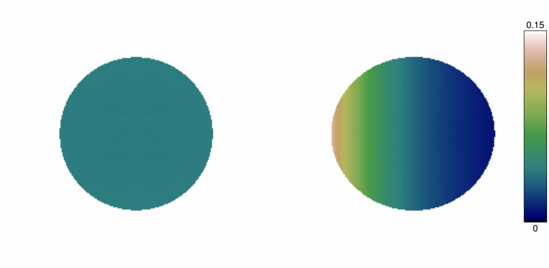
Comparison with integration using analytical expressions for the metric, with $a = 0.5M$.
Accuracy on $(r(t), \theta(t), \varphi(t))$ for:

- $t = 1000M, r = 100M \to t = 0, r = 865M$
- the smallest distance $r = 4.3M$ @ $t \sim 900M$.

Stationary neutron star

Rapidly rotating neutron star generated by `LORENE/nrotstar`

- EOS of Akmal, Pandharipande & Ravenhall
- $1.4\,M_\odot$ gravitational mass
- static or rotating with $f = 716$ Hz
- optically thick, emitting as a blackbody at 10^6 K



Map of specific intensity in $\mathrm{W\ m^{-2}\ ster^{-1}\ Hz^{-1}}$

$\Longrightarrow$ check of conservation of $p_t\ (10^{-6}), p_\varphi\ (10^{-4})$ and $p_\mu p^\mu\ (10^{-5})$ along the geodesics

Ray-tracing in dynamical spacetimes: collapse to a black hole

Spacetime generated by the `CoCoNuT` code

Initial data:

- spherically symmetric neutron star on the unstable branch
- polytropic EoS, $\gamma = 2$, $M_{\rm grav} = 1.62 M_\odot$, $M_{\rm bar} = 1.77 M_\odot$
- initial perturbation $\rho \to \rho \left[1 + 0.01 \sin\left(\frac{\pi r}{10\ {\rm km}}\right)\right]$

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sent to `CoCoNuT`, run with 500 radial cells.

- at $t = 0.438$ ms, appearance of the apparent horizon
- at $t = 0.495$ ms, 99.99% of matter is inside the AH
- run is stopped when too strong gradients appear on metric (maximal slicing)

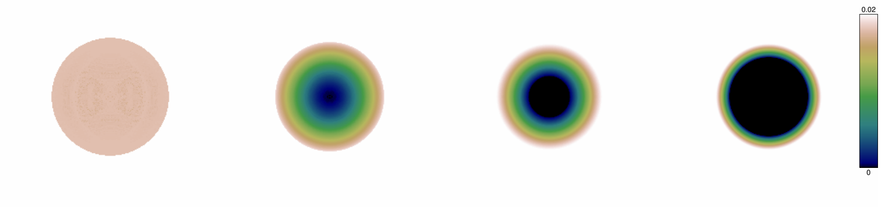
$\Longrightarrow$ 3+1 metric $\left(N, \beta^i, \gamma_{ij}\right)$, K_{ij}, fluid velocity u^μ, radius of the star and/or AH exported at every time-step to `GYOTO`

$\Longrightarrow$ $3^{\rm rd}$-order interpolation in time to integrate geodesic equations

Ray-tracing in dynamical spacetimes: collapse to a black hole

Integration backward until reaching the star's surface or the apparent horizon

Surface of the star: blackbody at 10^6 K. Intensity given in logarithmic scale



- coordinate radius of the star 7 km (left) → 2.9 km (right)
- relativistic bending of light rays $\Longrightarrow$ apparent radius larger
- event horizon first appear at the centre, closer to the observer