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Gravitational-wave emission in shift-symmetric Horndeski theories

Meudon, Mini-workshop on
"Gravitation and scalar fields"
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based on EB & K. Yagi arXiv:1509.04539

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Outline

- Modifications of GR and the equivalence principle
- Binary pulsars as tests of the "strong" equivalence principle (i.e. the universality of free fall for objects with strong internal gravity)
- How to calculate the deviations from the strong equivalence principle in modified gravity theories
- The case of shift-symmetric Horndeski gravity

Beyond GR: how?

Lovelock's theorem

In a 4-dimensional spacetime, the only divergence-free symmetric rank-2 tensor constructed only from the metric $g_{\mu\nu}$ and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term, i.e. $G_{\mu\nu} + \Lambda\, g_{\mu\nu}$

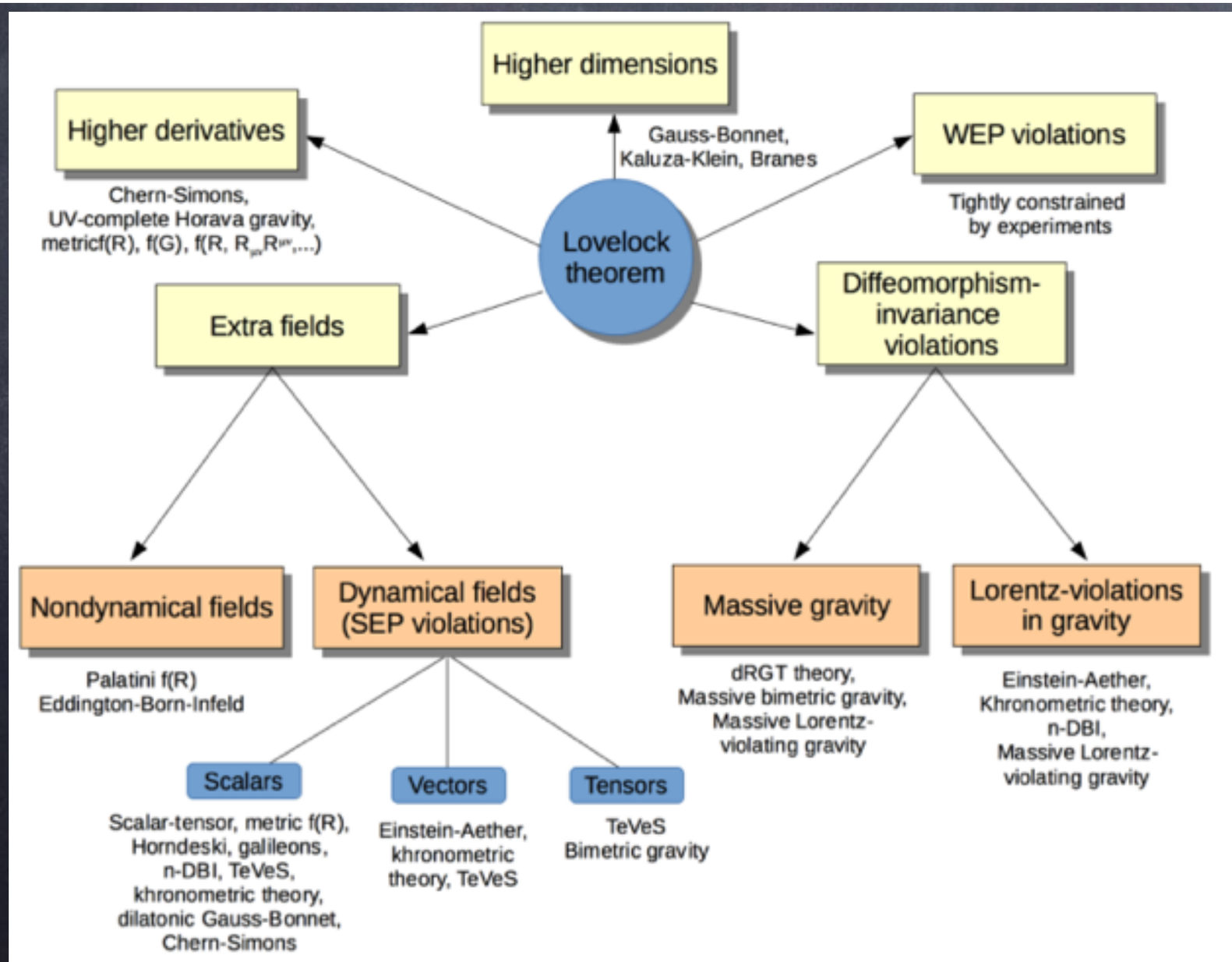


Figure adapted from Berti et al 2015

Generic way to modify GR is to add extra fields!

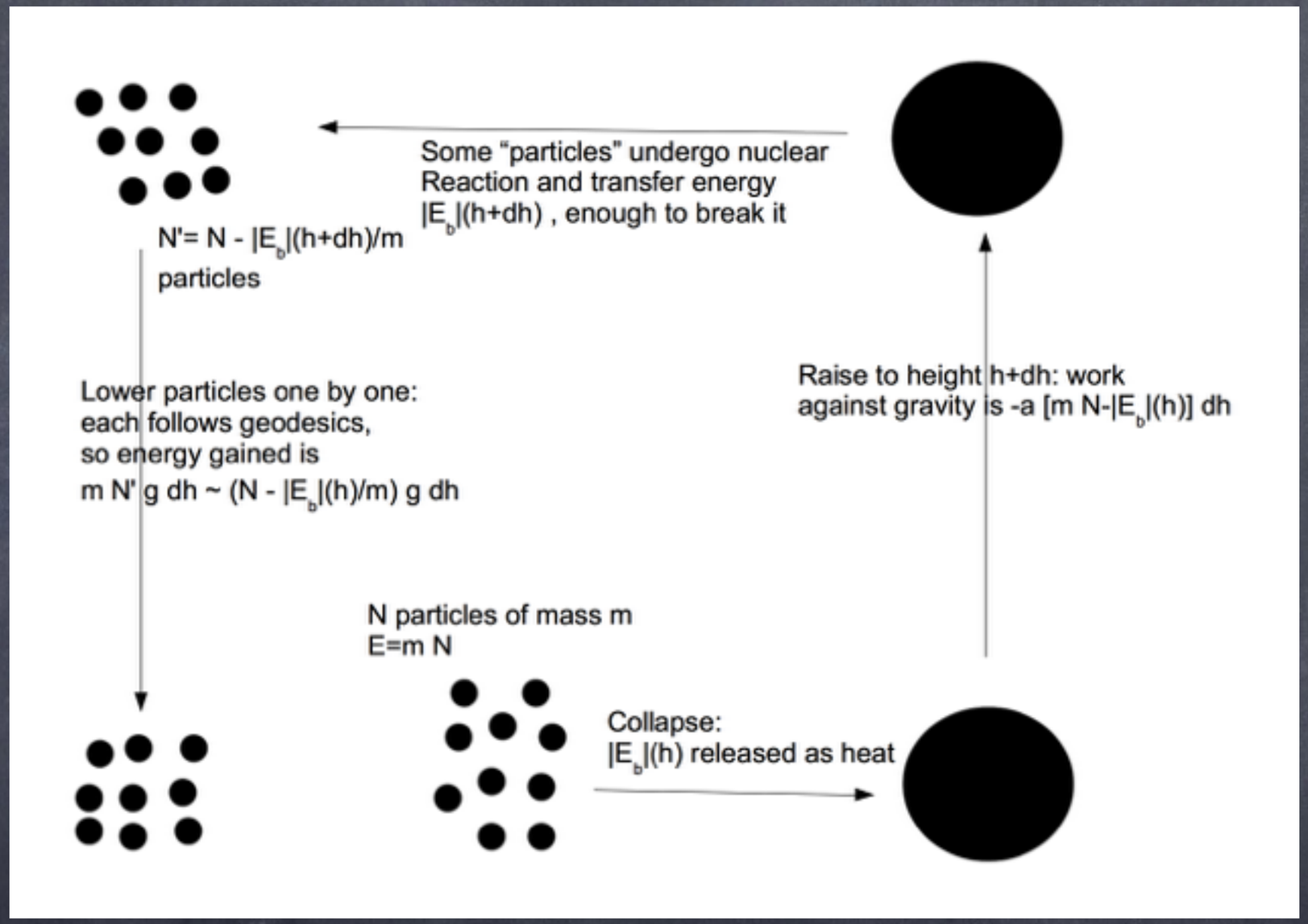
How to couple extra fields?

- Satisfy weak equivalence principle (i.e. universality of free fall for bodies with weak self-gravity) by avoiding coupling extra fields to matter

$$S_m(\psi, g_{\mu\nu})$$

- But extra fields usually couple non-minimally to metric, so gravity mediates effective interaction between matter and new field in strong gravity regimes (Nordvedt effect)
- Equivalence principle violated for strongly gravitating bodies
- For strongly gravitating bodies, gravitational binding energy gives large contribution to total mass, but binding energy depends on extra fields!
  $m_{inertial}/m_{gravitational}$ depends on local field value and may be $\neq 1$

Strong-equivalence principle violations by thought experiments (Dicke 1969)



Energy balance gives $(-g+a)(Nn-E_b) = -\frac{dE_b}{dh} = -\frac{dE_b}{dU}\frac{dU}{dh} = -\frac{dE_b}{dU}g$

$$m_{in}a = m_{grav}g \qquad m_{in} = Nm - E_b, \quad m_{grav} = m_{in} - \frac{dE_b}{dU}$$

A few examples

- Brans-Dicke, scalar-tensor theories: $G_{eff} \propto G_N/\varphi$, but φ in which star is immersed depends on cosmology, presence of other star → $m_{inertial}/m_{gravitational}$ changes with time

- Lorentz-violating gravity (Einstein-aether, Horava): preferred frame exists for gravitational physics → gravitational mass of strongly gravitating bodies depends on velocity wrt preferred frame → $m_{inertial} \neq m_{gravitational}$ for binary pulsars because v changes with time

- If gravitational mass depends on fields, deviations from GR motion already at geodesics level

$$S = \sum_n \int m_n(\psi) ds \qquad u_n^\mu \nabla_\mu (m_n u^\nu) \sim \mathcal{O}(s_n) \qquad s_n \equiv \frac{\partial m_n}{\partial \psi}$$

sensitivities or charges,
i.e. response to change in field
boundary conditions

Strong-equivalence principle violations in the dissipative sector

- Whenever strong-equivalence principle is violated, monopolar and dipolar radiation may be produced

- In electromagnetism, no monopolar radiation because electric charge conservation is implied by Maxwell eqs

- In GR, no monopolar or dipolar radiation because energy and linear momentum conservation is implied by Einstein eqs

e.g. $M_1 \sim \int \rho x^i d^3x \qquad h \sim \frac{G}{c^3}\dot{M}_1 \sim \frac{G}{c^3}\frac{P}{r}$ not a wave!

- In alternative theories, effective coupling matter-extra fields in strong gravity regimes ⟹ energy and momentum transfer between bodies and extra field

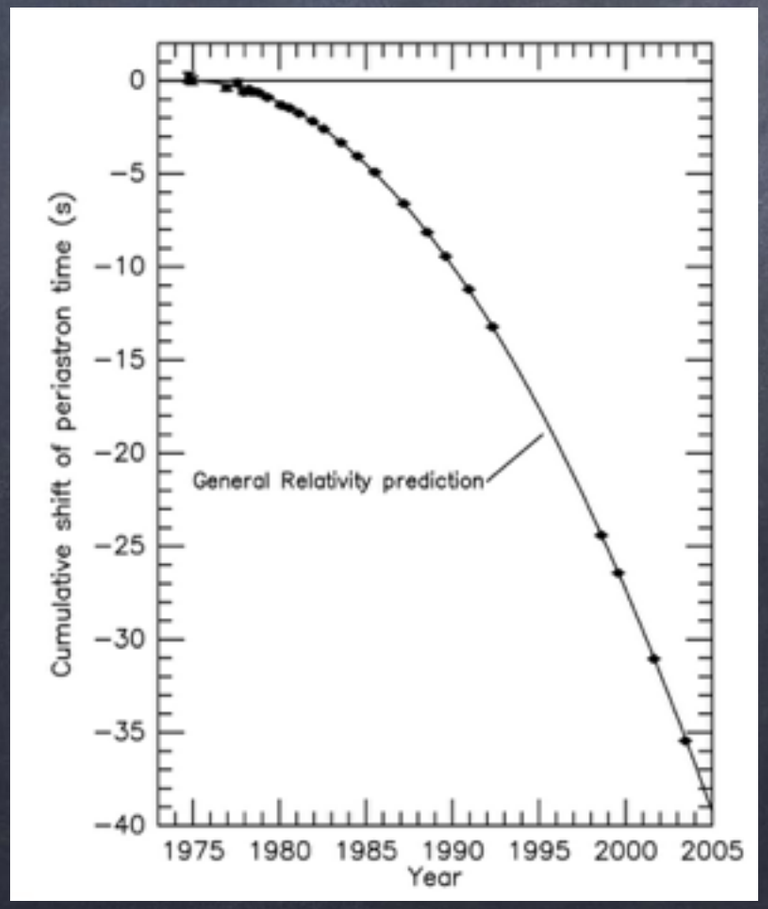
$$h \sim \frac{G}{c^3}\dot{M}_1 \sim \frac{G}{c^3}\frac{d}{dt}\left(m_1(\psi)x_1 + m_2(\psi)x_2\right) \sim \frac{G}{c^3}\mathcal{O}(s_1 - s_2)$$

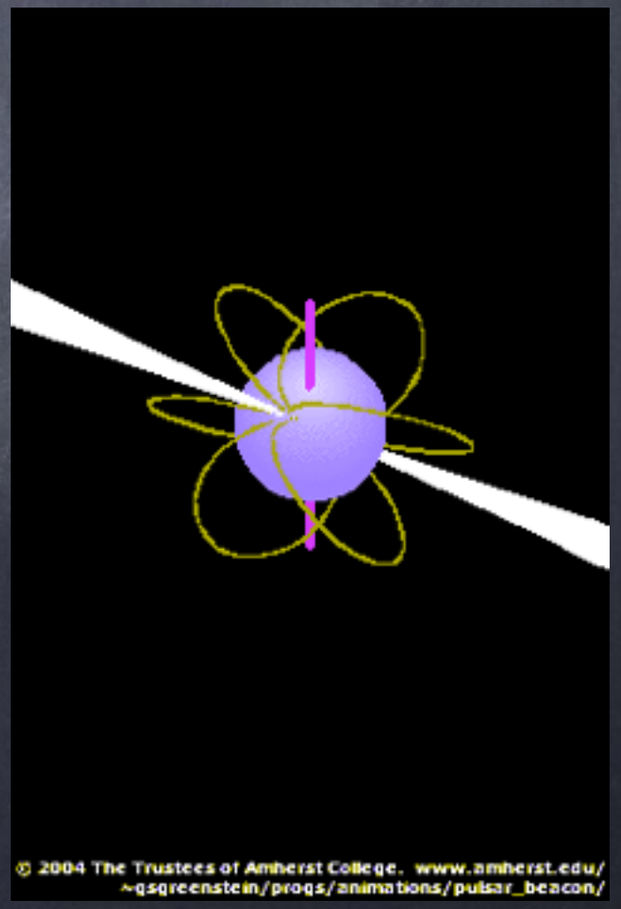
1.5 PN effect vs 2.5 PN in GR! Testable with binary pulsars!

Binary pulsars

- Binary system of stars on circular orbits has time changing mass quadrupole ⟹ GW emission
- GWs carry energy and angular momentum away from system, binding energy gets more and more negative and binary shrinks
- Indirect detection by binary pulsar systems (e.g. Hulse-Taylor pulsar)
- Violations of strong equivalence principle and dipolar fluxes regulated by "sensitivities"

$$s_{Q_A} = \frac{1}{M}\frac{\partial M}{\partial Q_A}\bigg|_{N,\Sigma}$$





How to calculate sensitivites

$$S = \int \mathcal{L}(Q_A, \partial_\mu Q_A) d^4x \quad \Longrightarrow \quad \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu Q_A)} \right) - \frac{\partial \mathcal{L}}{\partial Q_A} = 0$$

Use invariance under time shifts to write canonical mass for solution with $\pi_A \partial_t Q_A = 0$, with $\pi_A = \partial\mathcal{L}/\partial(\partial_t Q_A)$ (e.g. stationary solution)

$$M = \int d^3x \left(\pi_A \partial_t Q_A - \mathcal{L} \right) = - \int d^3x \mathcal{L}$$

Compute mass difference between two neighbouring stellar solutions with same baryonic mass and entropy, but different local values of the field

$$\delta M = - \int d^3x \partial_t \left(\pi_A \delta Q_A \right) - \int d^2 S_i \frac{\partial \mathcal{L}}{\partial(\partial_i Q_A)} \delta Q_A$$

[Use field eqs to get rid of bulk terms]

(Essentially same technique used by Damour & Esposito Farese for ST theories)

How to calculate sensitivites

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m(\psi_B, \partial_\mu \psi_B, g_{\mu\nu}) + \mathcal{L}_\phi(\phi_A, \partial_\mu \phi_A, g_{\mu\nu}, \partial_\alpha g_{\mu\nu})$$

Lagrangian for extra dof's

$$\mathcal{L}_g = \sqrt{-g} g^{\mu\nu} (\Gamma^\alpha_{\mu\lambda}\Gamma^\lambda_{\nu\alpha} - \Gamma^\lambda_{\mu\nu}\Gamma^\alpha_{\lambda\alpha})/(16\pi G)$$

Einstein Lagrangian

$$\mathcal{L}_m = \mathcal{L}_{\text{fluid}} = -\sqrt{-g}\rho(n,\sigma) - \varphi \partial_\mu J^\mu - \theta \partial_\mu(\sigma J^\mu) - \alpha_A \partial_\mu(\beta_A J^\mu)$$

$$J^\mu = \sqrt{-g} n U^\mu$$

perfect-fluid Lagrangian
(Dirac, Schutz, Brown...)

with Einstein Lagrangian canonical mass = ADM mass

How to calculate sensitivites

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m(\psi_B, \partial_\mu \psi_B, g_{\mu\nu}) + \mathcal{L}_\phi(\phi_A, \partial_\mu \phi_A, g_{\mu\nu}, \partial_\alpha g_{\mu\nu})$$

$$\delta M = -\int d^2 S_i \frac{\partial \mathcal{L}_g}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu} - \int d^3 x \partial_t \left(\pi_{\psi_B} \delta \psi_B\right) - \int d^2 S_i \frac{\partial \mathcal{L}_m}{\partial(\partial_i \psi_B)} \delta \psi_B$$
$$- \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \phi_A)} \delta \phi_A - \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu}$$

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$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m(\psi_B, \partial_\mu \psi_B, g_{\mu\nu}) + \mathcal{L}_\phi(\phi_A, \partial_\mu \phi_A, g_{\mu\nu}, \partial_\alpha g_{\mu\nu})$$

$$\delta M = -\cancel{\int d^2 S_i \frac{\partial \mathcal{L}_g}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu}} - \int d^3x \partial_t \left(\pi_{\psi_B} \delta\psi_B\right) - \int d^2 S_i \frac{\partial \mathcal{L}_m}{\partial(\partial_i \psi_B)} \delta\psi_B$$

$$- \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \phi_A)} \delta\phi_A - \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu}$$

$$- \int d^2 S_i \frac{\partial \mathcal{L}_g}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu} = 0 \quad \text{if} \quad g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)\,,\ \delta g_{\mu\nu} = \mathcal{O}(1/r)$$

How to calculate sensitivites

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m(\psi_B, \partial_\mu \psi_B, g_{\mu\nu}) + \mathcal{L}_\phi(\phi_A, \partial_\mu \phi_A, g_{\mu\nu}, \partial_\alpha g_{\mu\nu})$$

$$\delta M = -\cancel{\int d^2 S_i \frac{\partial \mathcal{L}_g}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu}} - \int d^3 x \partial_t \left(\pi_{\psi_B} \delta \psi_B\right) - \cancel{\int d^2 S_i \frac{\partial \mathcal{L}_m}{\partial(\partial_i \psi_B)} \delta \psi_B}$$

$$- \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \phi_A)} \delta \phi_A - \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu}$$

$$- \int d^2 S_i \frac{\partial \mathcal{L}_g}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu} = 0 \quad \text{if} \quad g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)\,,\ \delta g_{\mu\nu} = \mathcal{O}(1/r)$$

$$- \int d^2 S_i \frac{\partial \mathcal{L}_m}{\partial(\partial_i \psi_B)} \delta \psi_B = 0$$

by adopting coordinate comoving with the fluid

How to calculate sensitivites

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m(\psi_B, \partial_\mu \psi_B, g_{\mu\nu}) + \mathcal{L}_\phi(\phi_A, \partial_\mu \phi_A, g_{\mu\nu}, \partial_\alpha g_{\mu\nu})$$

$$\delta M = -\int d^2 S_i \frac{\partial \mathcal{L}_g}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu} - \int d^3 x \partial_t \left(\pi_{\psi_B} \delta \psi_B\right) - \int d^2 S_i \frac{\partial \mathcal{L}_m}{\partial(\partial_i \psi_B)} \delta \psi_B$$

$$- \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \phi_A)} \delta \phi_A - \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu}$$

$$-\int d^2 S_i \frac{\partial \mathcal{L}_g}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu} = 0$$ if $g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r)\,,\ \delta g_{\mu\nu} = \mathcal{O}(1/r)$

$$-\int d^2 S_i \frac{\partial \mathcal{L}_m}{\partial(\partial_i \psi_B)} \delta \psi_B = 0$$

by adopting coordinate comoving with the fluid

$$-\int d^3 x \partial_t \left(\pi_{\psi_B} \delta \psi_B\right) = -(h - \sigma T) U_t \delta N - T U_t \delta \Sigma = 0$$

if baryon number N and entropy Σ are the same for the two neighboring solutions

How to calculate sensitivites

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_m(\psi_B, \partial_\mu \psi_B, g_{\mu\nu}) + \mathcal{L}_\phi(\phi_A, \partial_\mu \phi_A, g_{\mu\nu}, \partial_\alpha g_{\mu\nu})$$

$$\delta M = -\int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \phi_A)} \delta\phi_A - \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu}$$

Expression applied to FJBD and DEF scalar tensor theories (Damour & Esposito Farese) and to Lorentz-violating gravity (Yagi, Blas, EB & Yunes, 2014)

Can we generalize to Lagrangian $\mathcal{L}_\phi(\phi_A, \partial_\mu \phi_A, \partial_\mu \partial_\nu \phi, g_{\mu\nu}, \partial_\alpha g_{\mu\nu})$?

Lagrangian order reduction

$$S = \int \mathcal{L}(Q_A, \partial_\mu Q_A, \partial_\nu \partial_\mu Q_A) d^4x$$

Define $X_{A\mu} \equiv \partial_\mu Q_A$ and enforce definition by Lagrange multipliers

$$S = \int \left[\mathcal{L}(Q_A, X_{A\mu}, \partial_\nu X_{A\mu}) + \lambda^{A\mu}(X_{A\mu} - \partial_\mu Q_A) \right] d^4x$$

Two actions are equivalent, and same procedure as before gives

$$\delta M = -\int d^2S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \phi_A)} \delta\phi_A - \int d^2S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})} \delta g_{\mu\nu} - \int d^2S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \partial_j \phi_A)} \partial_j \delta\phi_A$$
$$- \int d^2S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \partial_j g_{\mu\nu})} \partial_j \delta g_{\mu\nu} + \int d^2S_i \partial_j \left(\frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \partial_j \phi_A)} \right) \delta\phi_A + \int d^2S_i \partial_j \left(\frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \partial_j g_{\mu\nu})} \right) \delta g_{\mu\nu}$$

Horndeski theories (aka generalized galileons)

- Most generic scalar-tensor theories with 2nd-order field eqs

$$\mathcal{L}_\phi = \frac{\sqrt{-g}}{16\pi G}\Big\{K(\phi,X) - G_3(\phi,X)\Box\phi + G_4(\phi,X)R + \partial_X G_4(\phi,X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$+G_5(\phi,X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}\partial_X G_5(\phi,X)\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right]\Big\}$$

$$X \equiv -\nabla_\mu\phi\nabla^\mu\phi/2 \quad (\nabla_\mu\nabla_\nu\phi)^2 \equiv \nabla_\mu\nabla^\nu\phi\nabla_\nu\nabla^\mu\phi \quad (\nabla_\mu\nabla_\nu\phi)^3 \equiv \nabla_\mu\nabla^\rho\phi\nabla_\rho\nabla^\nu\phi\nabla_\nu\nabla^\mu\phi$$

- Galileon interactions also arise in massive gravity

- Very non-linear field eqs allow Vainshtein mechanism

$$\Box\phi + \partial_X G_3[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2 - R_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi] + ... = ...$$

$$\frac{d\phi}{dr} \propto \frac{r^3}{r_V^3}\left[\sqrt{1+\frac{r_V^3}{r^3}} - 1\right]\frac{GM(r)}{r^2}$$

Scalar effects only arise for r>> r_V (Vainhstein radius)

Shift symmetric Horndeski theories

$$\mathcal{L}_\phi = \frac{\sqrt{-g}}{16\pi G}\Big\{K(X) - G_3(X)\Box\phi + G_4(X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
$$+G_5(X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{G_{5X}}{6}\left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\right] + \chi\phi\mathcal{G}\Big\}$$

- Invariant under shift symmetry $\phi \to \phi + \mathrm{const}$
- Assume analytic K, G_3, G_4, G_5 [i.e. $K(X)=X+O(X)^2$, $G_i=O(X)$ (i=1,2,3)]...
- ...but include Gauss-Bonnet term $\chi\phi\mathcal{G}$ (which comes from log |X| divergence in G5)

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  $K(X)=X+O(X)^2$, $G_i=O(X)$ $(i=1,2,3)$

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  $K(X)=X+O(X)^2$, $G_i=O(X)$ $(i=1,2,3)$

- Evaluate $$\delta M = -\int d^2S_i \frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\phi_A)}\delta\phi_A - \int d^2S_i \frac{\partial\mathcal{L}_\phi}{\partial(\partial_i g_{\mu\nu})}\delta g_{\mu\nu} - \int d^2S_i \frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j\phi_A)}\partial_j\delta\phi_A$$
$$-\int d^2S_i \frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j g_{\mu\nu})}\partial_j\delta g_{\mu\nu} + \int d^2S_i\partial_j\left(\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j\phi_A)}\right)\delta\phi_A + \int d^2S_i\partial_j\left(\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j g_{\mu\nu})}\right)\delta g_{\mu\nu}$$

  using $\phi = \mathcal{O}(1/r)\,,\quad g_{\mu\nu} = \eta_{\mu\nu} + \mathcal{O}(1/r) \Longrightarrow \delta g_{\mu\nu} \sim \delta\phi \sim \mathcal{O}(1/r)$

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- E.g. contribution of K to first term:
$$\delta M \sim \int d^2S_i \frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\phi)}\delta\phi \sim r^2|\nabla\phi|\frac{1}{r} \sim \frac{1}{r} \to 0 \quad \text{as} \quad r\to\infty$$

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  $K(X)=X+O(X)^2$, $G_i=O(X)$ (i=1,2,3)

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$$-\int d^2S_i \frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j g_{\mu\nu})}\partial_j\delta g_{\mu\nu} + \int d^2S_i\partial_j\left(\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j\phi_A)}\right)\delta\phi_A + \int d^2S_i\partial_j\left(\frac{\partial\mathcal{L}_\phi}{\partial(\partial_i\partial_j g_{\mu\nu})}\right)\delta g_{\mu\nu}$$

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- Similar reasoning shows that all terms vanish, i.e. $\delta M=0$

Shift symmetric Horndeski theories

- $$\mathcal{L}_\phi = \frac{\sqrt{-g}}{16\pi G}\Big\{K(X) - G_3(X)\Box\phi + G_4(X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$
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  $K(X)=X+O(X)^2$, $G_i=O(X)$ (i=1,2,3)

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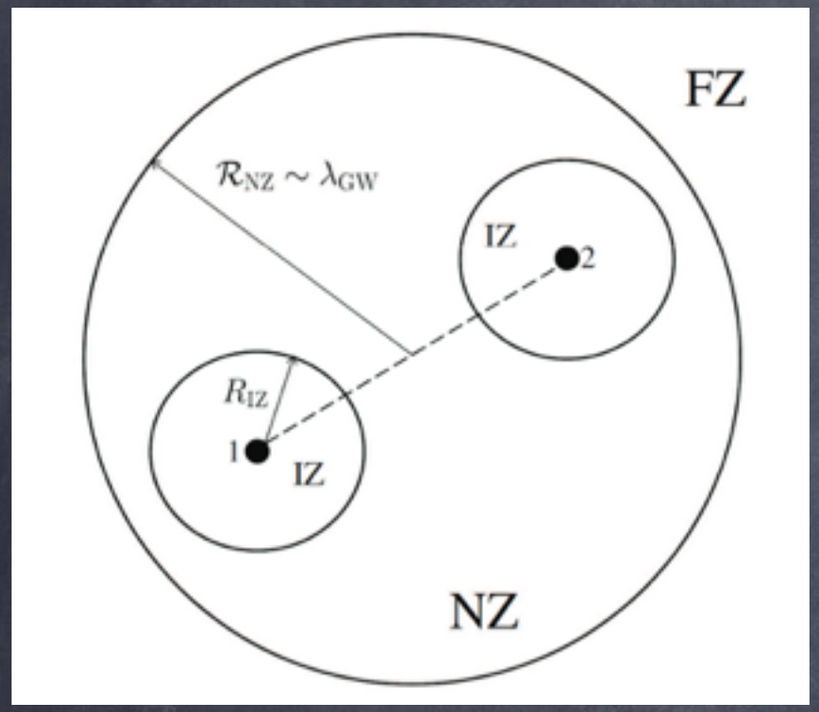
Sensitivities of stars vanish!

PN expansion for shift-symmetric Horndeski theories



- Define inner, near and far zones
- Inner zone must be >> star and << λ_{GW}
- In near and far zone, fields decay as 1/r and stars described as point particles with mass

$$m(\phi) = m + \frac{\partial m}{\partial \phi}\delta\phi + \ldots = m + s\delta\phi + \ldots$$

- Expand dynamics in orders of v/c

$$\bar{h}^{\mu\nu} = \eta^{\mu\nu} - \sqrt{-g}g^{\mu\nu}$$

$$\Box_\eta \bar{h}^{\mu\nu} = -16\pi G[(1 - \bar{h}^\alpha{}_\alpha)T^{\mu\nu} + \tau^{\mu\nu}] + \ldots$$

$$\Box_\eta \delta\phi = \mathcal{O}(s_\phi^{(1)}, s_\phi^{(2)}) - \chi\delta\mathcal{G} + \ldots.$$

$$\partial_\mu \bar{h}^{\mu\nu} = 0 \qquad \tau^{\mu\nu} = \mathcal{O}(\bar{h}^2)$$

- No monopolar or dipolar emission, same quadrupole as GR
- Deviations from GR at 3PN (2PN) order in the dissipative (conservative) sector

What assumptions are we making?

- Shift symmetry for the total action (i.e. no conformal coupling of the scalar to matter): crucial!

  If no shift symmetry, $\phi = \phi_\infty + \frac{\alpha}{r} + ... \Rightarrow \delta\phi = \delta\phi_\infty + O(1/r)$

  $$\delta M \sim \int d^2 S_i \frac{\partial \mathcal{L}_\phi}{\partial(\partial_i \phi)} \delta\phi \sim r^2 |\nabla\phi| \delta\phi_\infty \propto \alpha \delta\phi_\infty \Rightarrow s_\phi \propto \alpha$$

  NB: Conformal coupling is present in FJBD, DEF and massive gravity: in those cases sensitivities can NOT be neglected

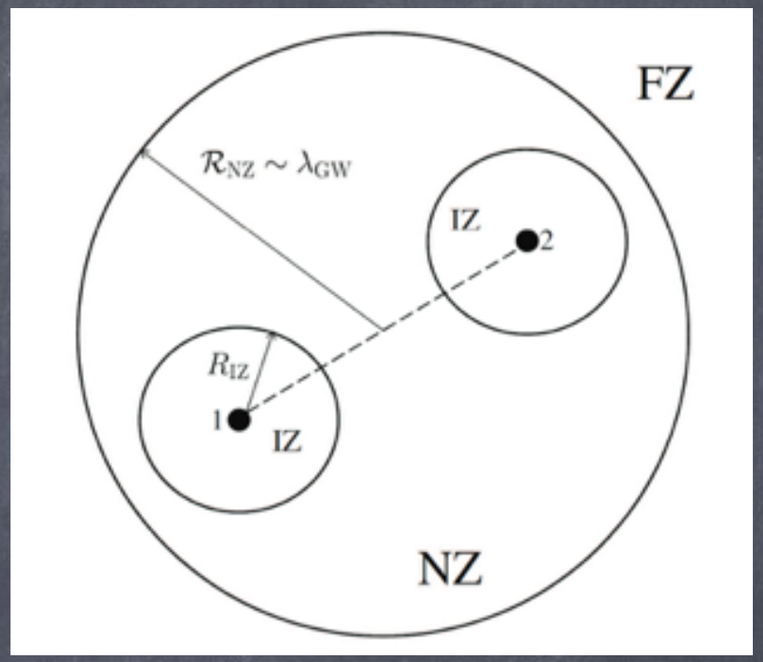
- No direct scalar-matter coupling: non-crucial as long as coupling is shift-symmetric, e.g. matter coupled to disformal metric $g_{\mu\nu} + \kappa\, \partial_\mu \phi \partial_\nu \phi$

What assumptions are we making?

- If Vainshtein screening present, Vainshtein radius $r_V \ll \lambda_{GW}$: crucial!

  Needed to use PN formalism!

  r_V is "effective" star radius, i.e. if $r_V \ll \lambda_{GW}$ finite size effects at high PN orders



  $\lambda_{GW} \sim 10^9$ km for binary pulsars, and 10^3 km for LIGO sources

- If $r_V \gtrsim \lambda_{GW}$, PN formalism is not applicable, i.e. dynamics is non-perturbative (or "strongly coupled"). Problematic because binary pulsars' dynamics is perturbative

  Unclear how to even do calculation. Possible approach: WKB approximation (de Rham, Tolley and Wesley 2013; Chu & Trodden 2013), but conclusion that all multipoles radiate with comparable strength hard to reconcile with binary pulsars

What assumptions are we making?

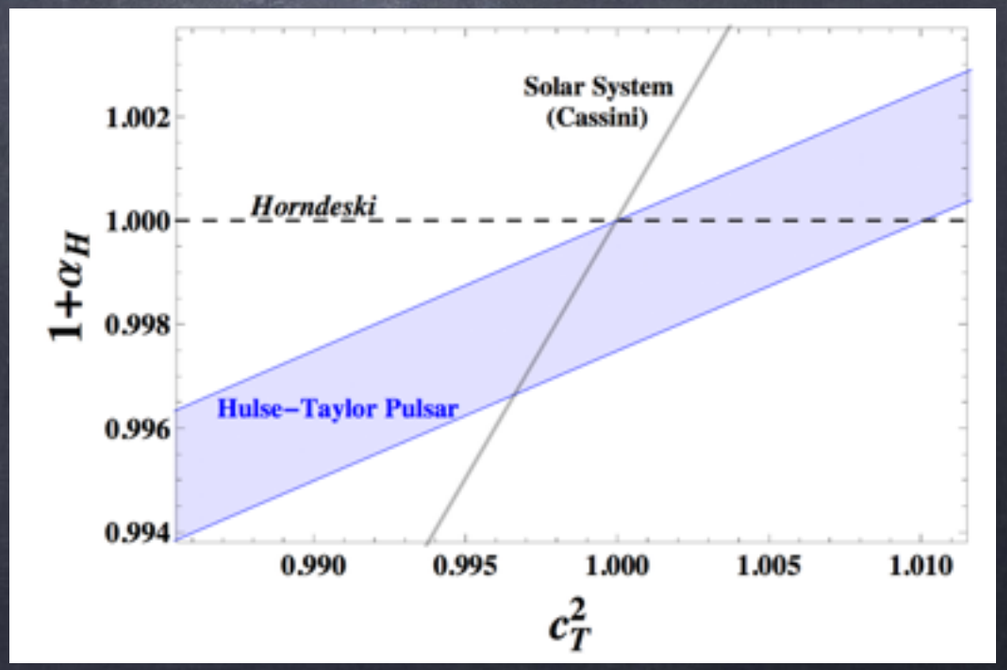
Neglect cosmological-expansion effects: potentially important

- In all scalar-tensor theories, where sensitivities for stars and BHs affected by scalar field's cosmological expansion (Jacobson, Charmousis, Babichev, Esposito Farese, etc)

- A more subtle effect in Horndenski/beyond Horndenski theories:
  - Screening works for spatial but not time derivatives
  - Time derivatives change GW-matter coupling in the quadrupole formula (G_{gw}), and GW propagation speed c_T

$$\gamma_{PPN} = \frac{1+\alpha_H}{c_T^2} \qquad G_{gw} = \left(\frac{1+\alpha_H}{c_T}\right)^2 G_N$$

α_H and c_T are theory's parameters
($\alpha_H = 0$ in Horndenski)



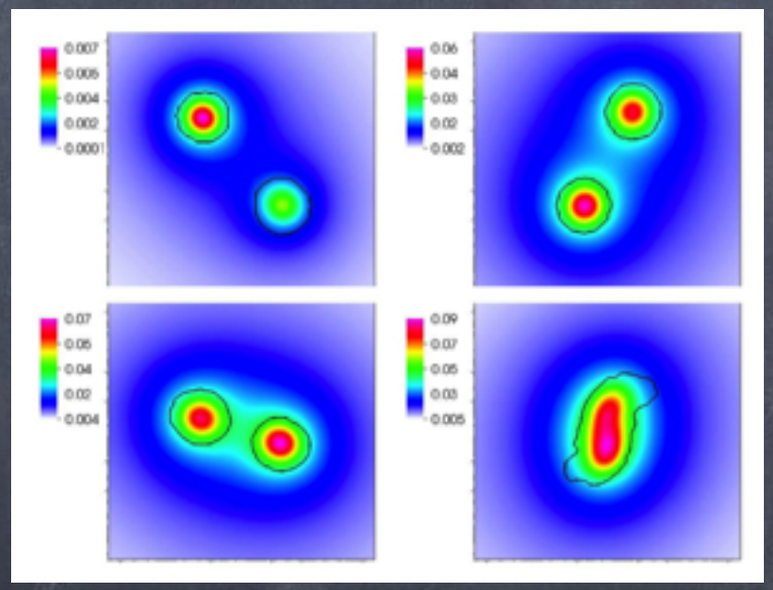
Jimenez, Piazza, Velten 2015

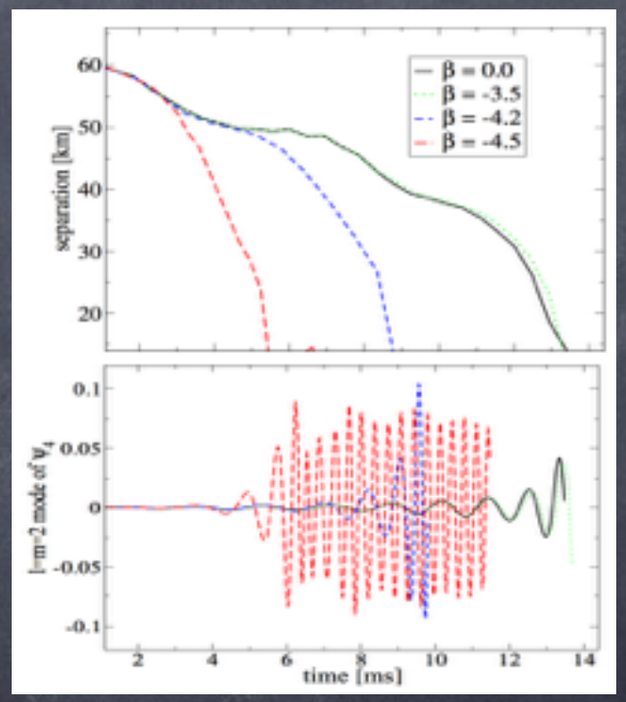
Possible smoking-gun scalar effects?

- Like pornography, "When you see it, you know it"! (Supreme Court Justice Potter Stewart, 1964)

- Abrupt waveform termination/earlier plunge than in GR for LIGO NS-NS sources, in DEF scalar tensor theories

EB, Palenzuela, Ponce & Lehner 2014





- Caused by induced scalarization of one (spontaneously scalarized) star on the other, by dynamical scalarization of an initially non-scalarized binary

Spontaneous/dynamical scalarization as "phase transitions"

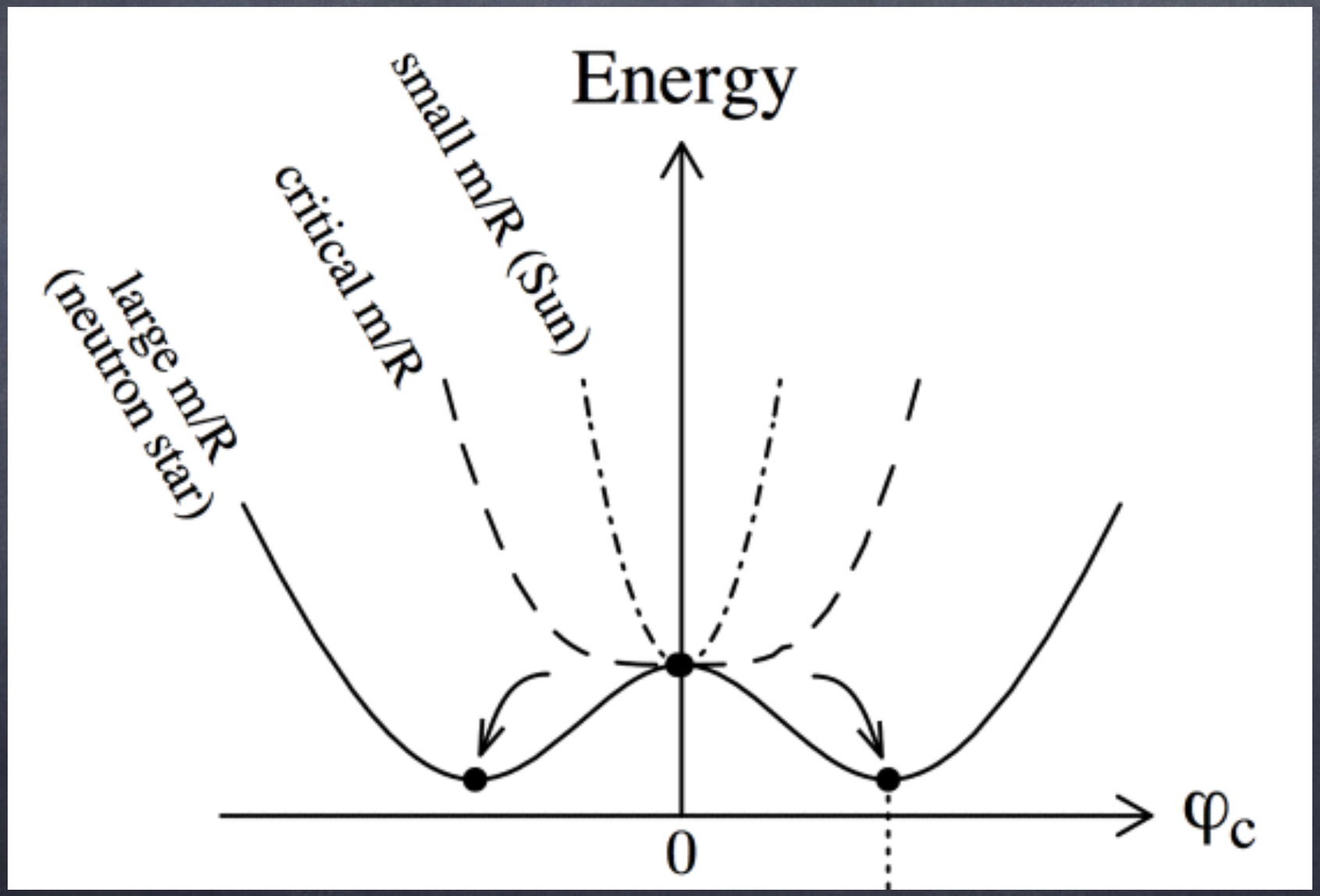


Figure from Esposito-Farese, gr-qc/0402007

Conclusions

- Modifications of GR generally introduce violations of the strong equivalence principle via the sensitivities, i.e. free fall of bodies with strong internal gravity is not universal
- Binary pulsars test strong equivalence principle and are the most dreaded theory killer
- Sensitivities can be calculated from asymptotic behavior of solutions for isolated stars
- In Horndeski theories, sensitivities are NOT zero in general, but vanish if shift-symmetry imposed on scalar and on matter-scalar coupling, provided that the dynamics is perturbative and cosmological-expansion effects can be neglected
- If Vainshtein screening present, counter-intuitive result that deviations from GR are screened if Vainshtein radius is small