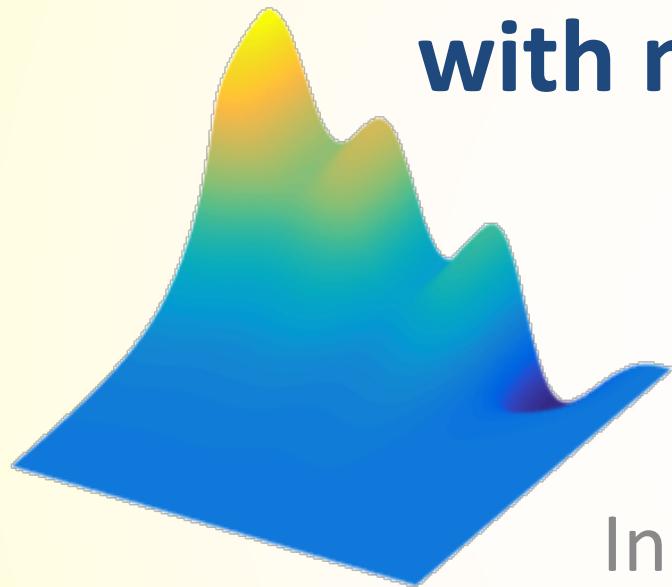


# Spherical collapse in quintessential cosmologies with numerical relativity



André Füzfa

In collaboration with

Isabel Cordero-Carrion (Valencia, Spain)

Jeremy Rekier (Brussels, Belgium)

Based on :

- [arXiv:1409.3476](https://arxiv.org/abs/1409.3476), PRD 91, 024025 (2015)
- [arXiv:1509.08354](https://arxiv.org/abs/1509.08354), submitted to PRD

# Motivations

## ★ Cosmic acceleration:

- Supported by several cosmological observations
- Well accounted for by cosmological constant  $\Lambda$
- Fine-tuning & coincidence problems, status of  $\Lambda$   
⇒ new physical mechanism dubbed « Dark Energy » (DE)

## ★ DE vs $\Lambda$ :

- Space-time variation : clustering of DE
- DE fluid: anisotropic stresses and momentum transfer
- Collapse of DE is model dependent
- Large-scale structure formation in presence of DE  
⇒ Window on the nature of DE beyond  $\Lambda$

# Quintessence Cosmology

★ Raychaudhuri equation:  $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho + 3p)$

$$\rho = \frac{\dot{\phi}^2}{2} + V$$

$$p = \frac{\dot{\phi}^2}{2} - V$$

$$\ddot{a} > 0 \Leftrightarrow \frac{\dot{\phi}^2}{2} \ll V$$

– Add a real scalar field with only self and gravitational interactions

**(minimal coupling)**

– Klein-Gordon equation:

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{dV}{d\phi} = 0$$

– Examples :

◆ Thawing models:  $\Lambda$  early, « matter » lately

◆ Freezing models:  $\Lambda$  lately, « chameleon » early

# Quintessence clustering

## How is quintessence distributed in space?

### 1) Perturbative approach:

- Evolutions of scalar field perturbations in matter-dominated era:

Hwang & Noh, PRD 64, 103509

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} + \left(k^2/a^2 + V_{,\phi\phi}\right)\delta\phi = \dot{\phi}\dot{\delta_m}$$

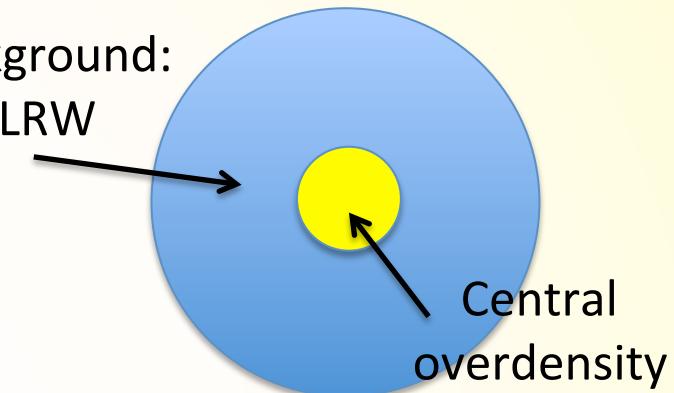
- Quintessence perturbations grow up exponentially for comoving length scales above the Compton length of the quintessence field
- Quintessence field very light: Compton length  $\sim$  Horizon
- scalar field prevented from collapsing? => **pressures**
- Non-linear regime? Toward the collapse?

# Quintessence clustering

## 2) Phenomenological method:

### Two-regions approach:

Background:  
FLRW



#### Background

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\bar{\rho} \quad \frac{d}{dt}\bar{\rho} + 3\frac{\dot{a}}{a}(\bar{\rho}(1+w)) = 0$$

#### Local

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3}\bar{\rho}(1+\delta) - \frac{k}{R^2}$$

$$\frac{d}{dt}(\bar{\rho}(1+\delta)) + 3\frac{\dot{R}}{R}[\bar{\rho}(1+\delta)(1+w)] = \Gamma$$

Local Hubble Rate →  $\left(\frac{\dot{R}}{R}\right)^2$   
 Local curvature →  $\frac{k}{R^2}$   
 Overdensity →  $\bar{\rho}(1+\delta)$   
 Pheno Energy transfer →  $\Gamma$

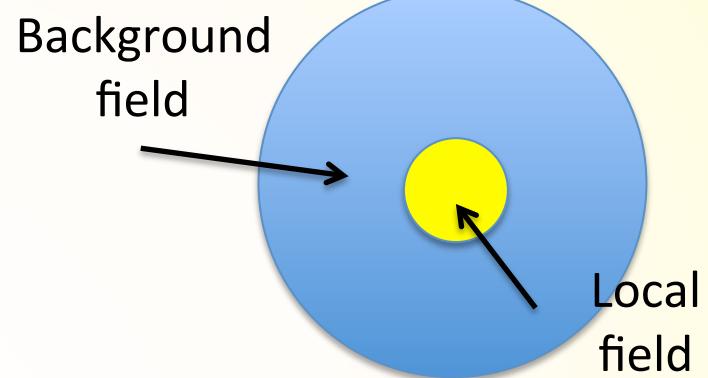
Padmanabhan 1993 ;Weinberg 2003

Wang & Steinhardt 1998

# Quintessence clustering

## 2) Phenomenological method:

**Two-regions approach for quintessence:**



$$\ddot{\phi}_{\text{loc}} + 3 \left[ \frac{\dot{R}}{R} \right] \dot{\phi}_{\text{loc}} + \frac{dV}{d\phi} \Big|_{\phi_{\text{loc}}} = 0$$

**Fully local:**  
 Inefficient energy transfer

$$\ddot{\phi}_{\text{loc}} + 3 \left[ \frac{\dot{a}}{a} \right] \dot{\phi}_{\text{loc}} + \frac{dV}{d\phi} \Big|_{\phi_{\text{loc}}} = 0$$

**Semi local:**  
 Fully efficient energy transfer

# Quintessence clustering

## 3) Non-perturbative methods

- Pheno approaches allows glimpsing at non-linear regime
- 2-regions: no access to space-time inhomogeneities!
- How to choose the energy transfer? (scale dependent)
- Solution: study spherical collapse in asymptotically Friedmann space-times with full GR!
- Generalization of Lemaître-Tolman-Bondi (LTB) models :

Source:

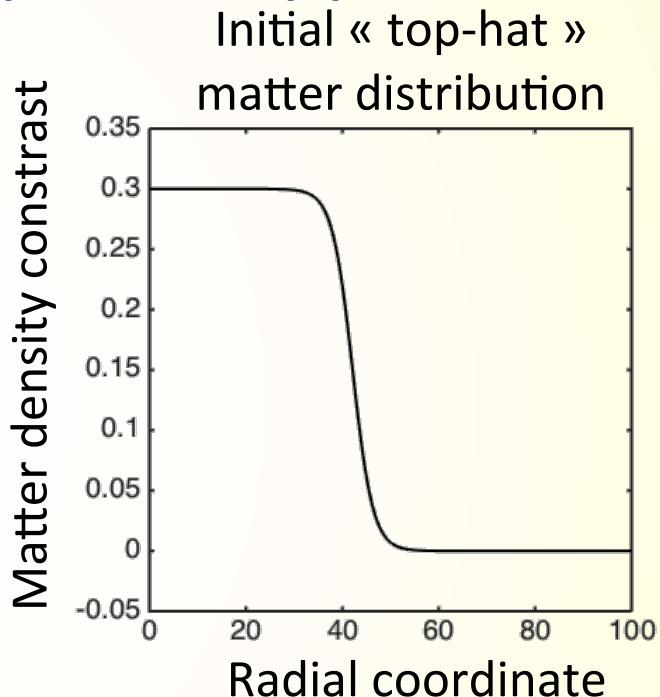
$$T_{\mu}^{\nu} = \begin{pmatrix} -E & j_r & 0 & 0 \\ j_r & p_r & 0 & 0 \\ 0 & 0 & p_t & 0 \\ 0 & 0 & 0 & p_t \end{pmatrix}$$

E : energy density  
 $j_r$ : radial momentum transfer  
 $p_r$ : radial pressure  
 $p_t$ : tangential pressure

# A Numerical Relativity Approach

## ★ Initial state for comparison with pheno approaches:

- ★ Only matter density fluctuation
- ★ Homogeneous expansion rate
- ★ Homogeneous scalar field and field velocity



- ★ Baumgarte-Shapiro-Shibata-Nakamura **BSSN** formalism
- ★ Geodesic slicing (for comparison with phenos and LTB)
- ★ Generalised Sommerfeld boundary conditions  
(expanding background)

# A Numerical Relativity Approach

## ★ Free-evolution scheme for the BSSN system

- Initial value problem:
  - find BSSN initial conformal factor
  - Boundary value problem from combined Hamiltonian and Momentum constraints (Lané-Emden-like equation)
- Propagation of initial conditions:
  - Partially-Implicit Runge-Kutta method (PIRK) of second order
  - 4th order finite difference scheme for spatial derivatives
  - 4th order Kreiss-Oliger numerical dissipation
  - Parity conditions around the origin of coordinates

## ★ Code validation:

- ★ Pure gauge dynamics and scalar field pulse on expanding backgrounds
- ★ Comparison with  $\Lambda$ -LTB analytical solutions
- ★ Consistency checks with constraints ( $> 2^{\text{nd}}$  order convergence)

## ★ Code publicly available at

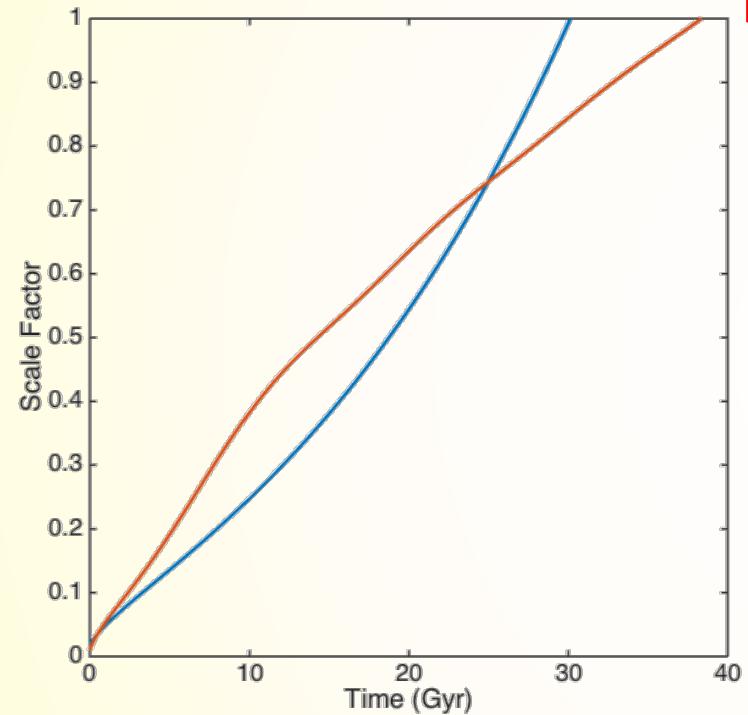
- ★ <https://github.com/jrekier/FORTCosmoSS>

- [arXiv:1409.3476](https://arxiv.org/abs/1409.3476)
- [arXiv:1509.08354](https://arxiv.org/abs/1509.08354)

# Background space-times

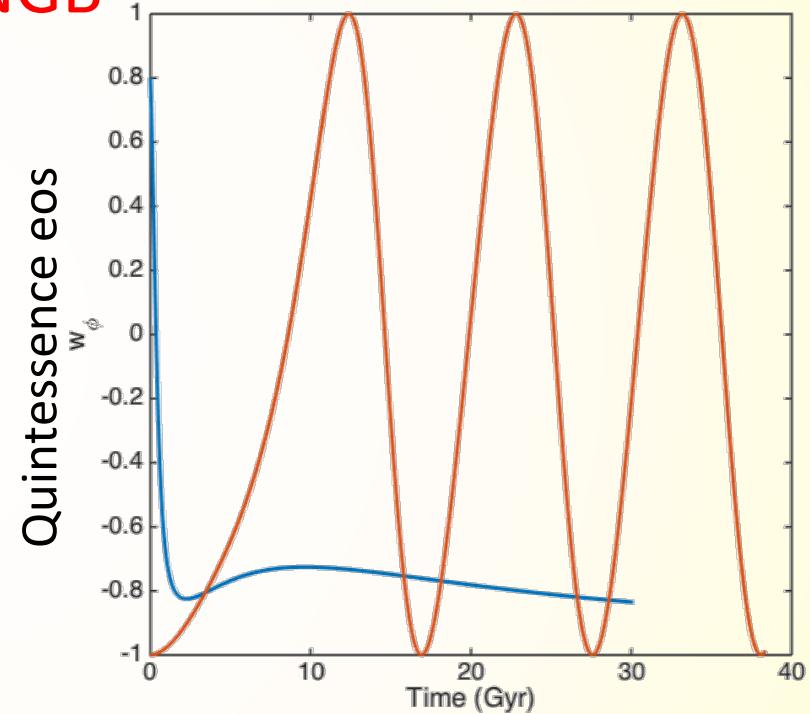
Freezing: Ratra-Peebles

Thawing: PNGB



Ratra-Peebles potential:

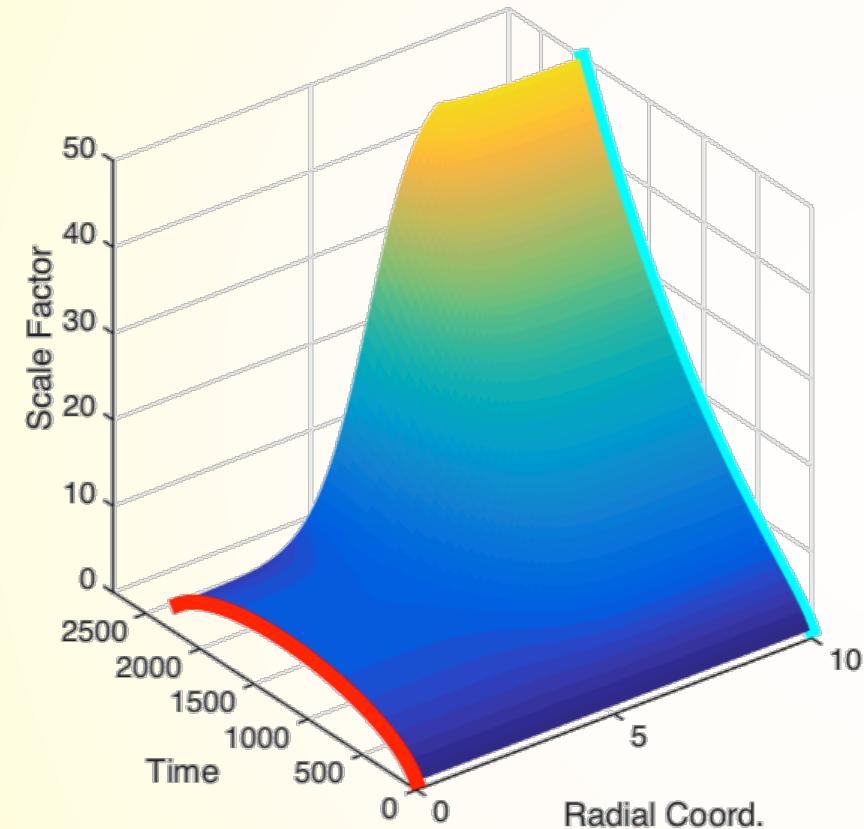
$$V(\phi) = \frac{M^{4+n}}{\phi^n}$$



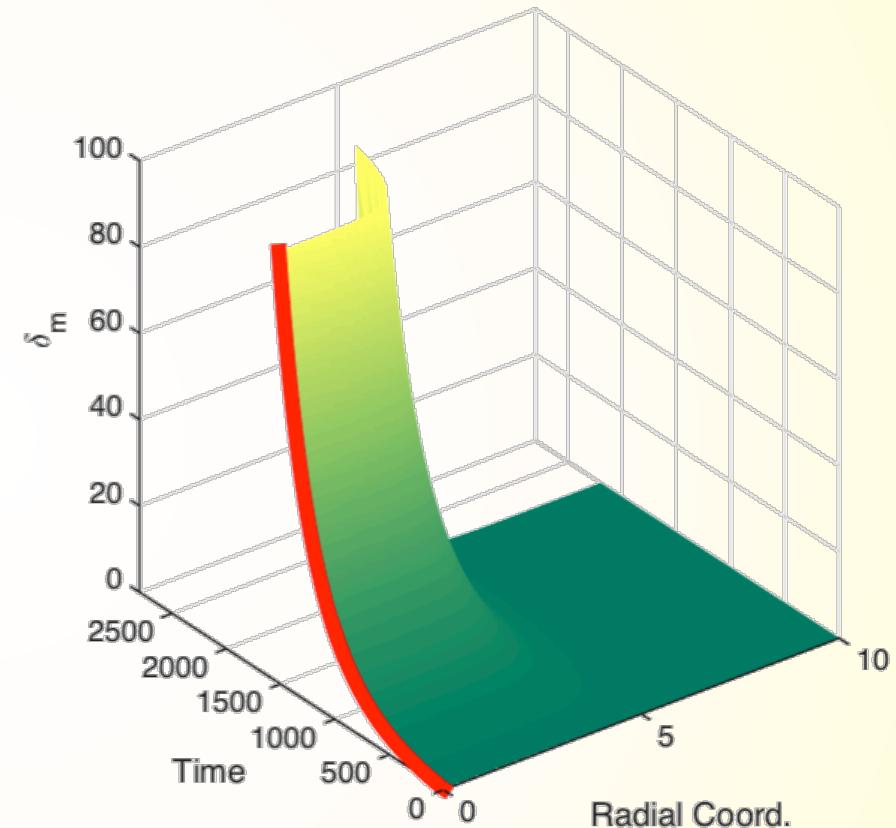
Pseudo-Nambu-Goldstone Boson potential:

$$V(\phi) = \mu^4 \left( 1 + \cos \left( \frac{\phi}{f} \right) \right)$$

# Inhomogeneous space-time

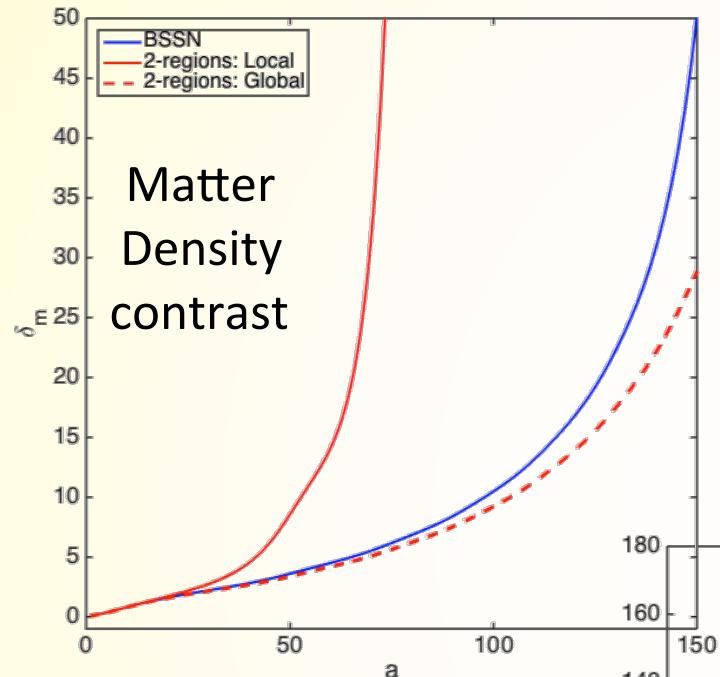


Scale Factor



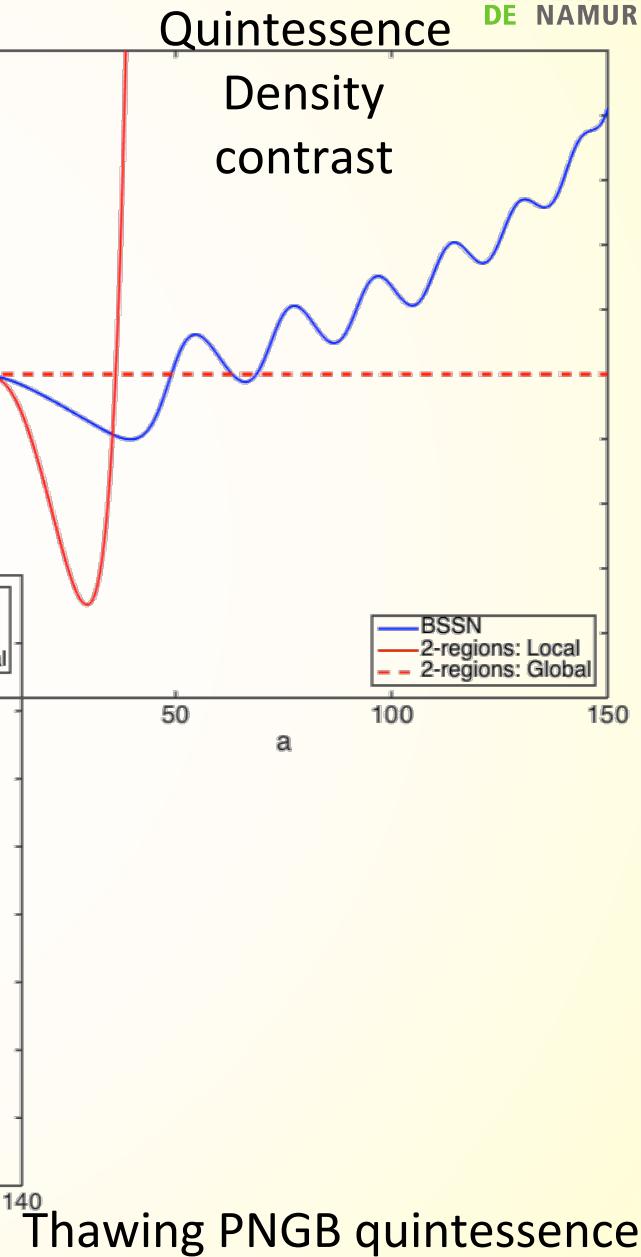
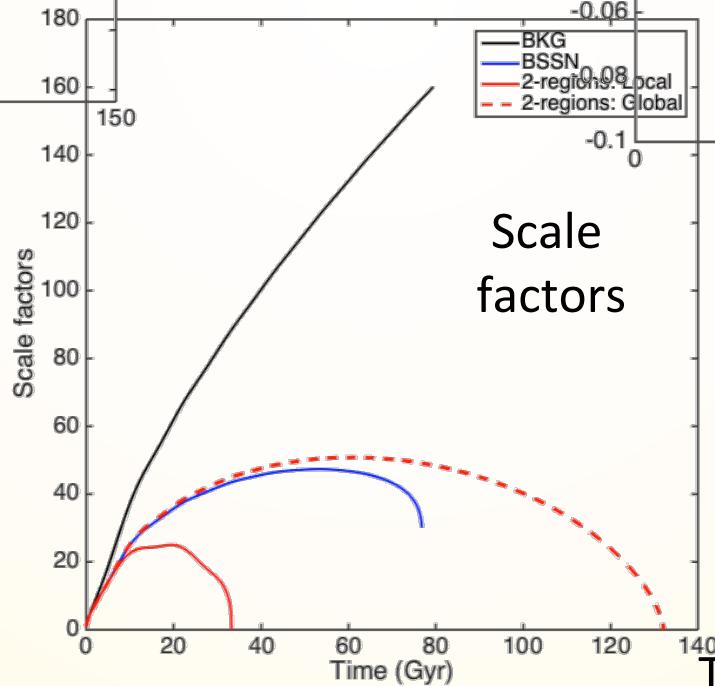
Matter Density Contrast

# BSSN versus phenos



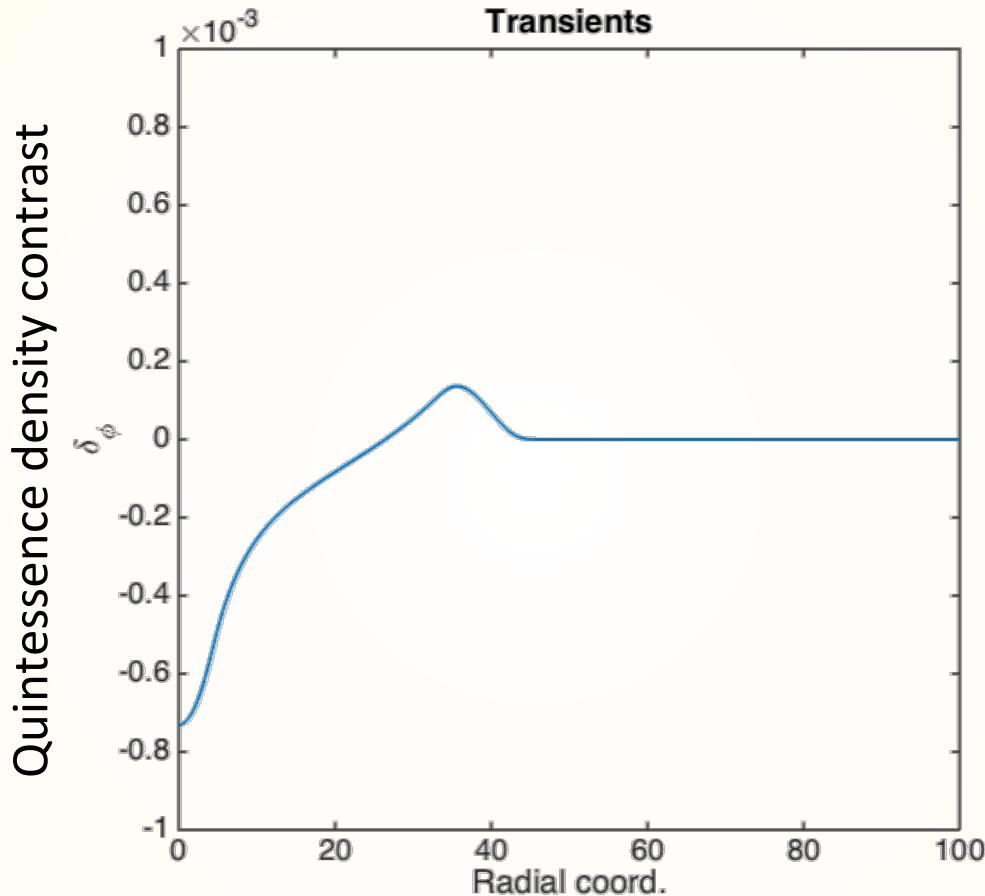
Efficiency  
Of the  
Energy transfer  
Depends  
On fluctuation  
characteristics

Quantities  
At the centre



Thawing PNGB quintessence

# Quintessence clusters!

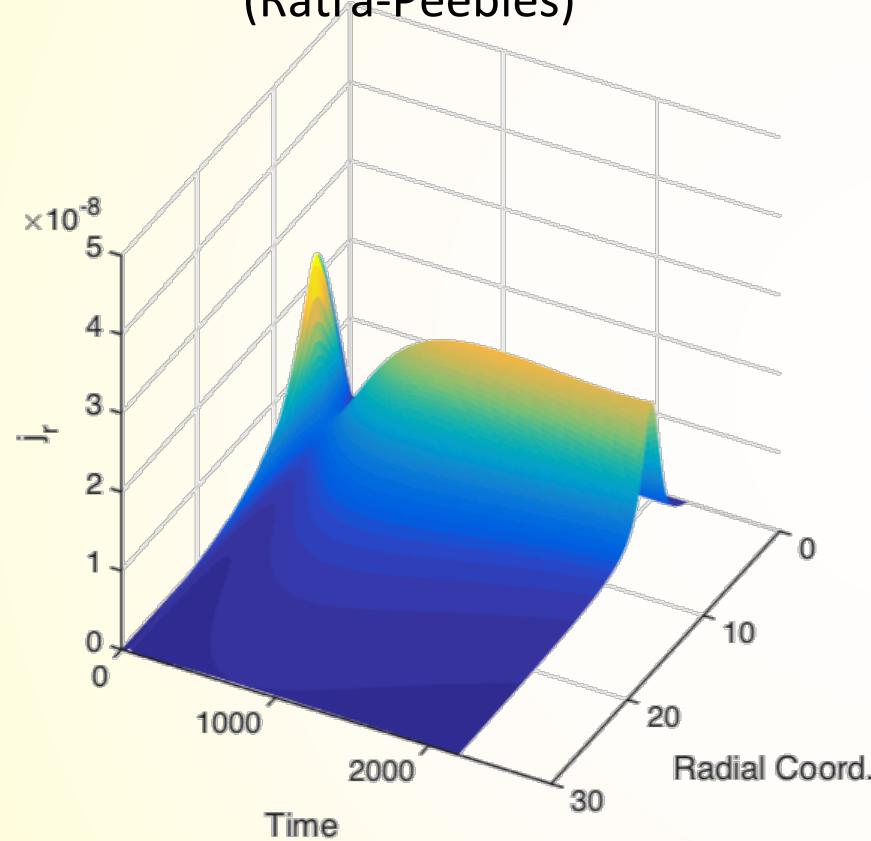


Video:  
pulse propagation

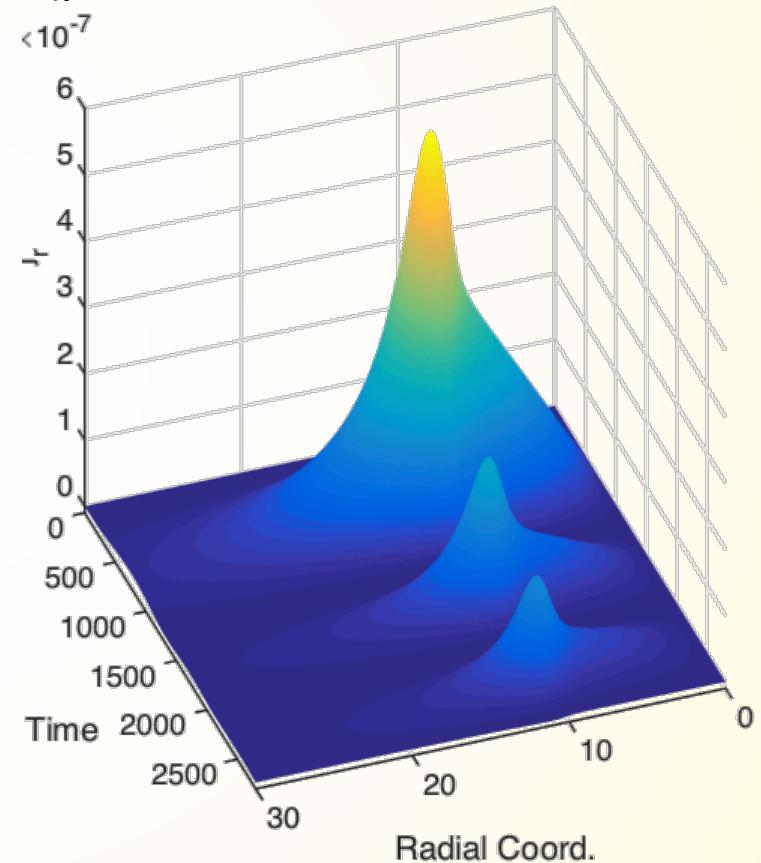
Initially homogeneous quintessence  
quickly get inhomogeneous

# Momentum transfer

Freezing quintessence  
(Ratra-Peebles)



Thawing quintessence  
(pseudo-Nambu Goldstone boson)



Development of Momentum transfer slows down quintessence collapse  
And depends on quintessence potential

# Some Conclusions & Perspectives

## ★ Application of numerical relativity to cosmology:

- Quintessence clusters on small scales !

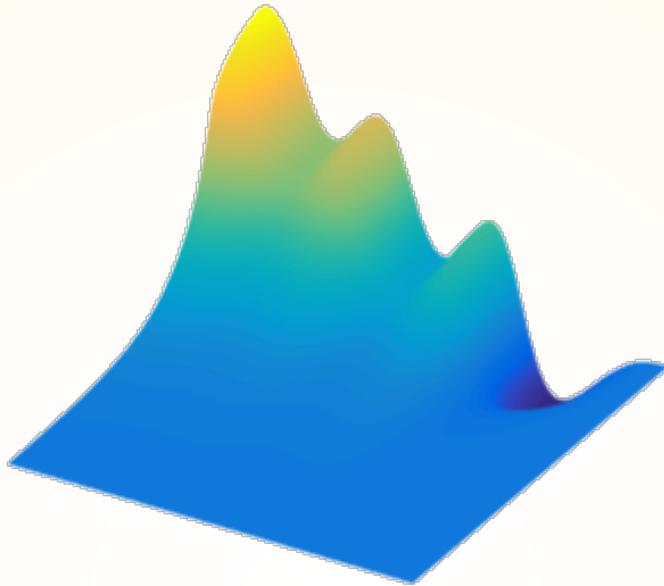
## ★ Impact on matter fluctuations growth

- Real collapse of quintessence intermediate between full clustering and no clustering
- Momentum transfer and pressure anisotropies prevent quintessence from collapsing too quickly
- the steeper the structure the higher the momentum transfer
- Quintessence/matter collapse model & scale dependent

## ★ Full computations of Space-time inhomogeneities

- Backreactions on measured expansion
- Gravitational lensing in dynamical space-times

## ★ Non-minimal coupling: direct impact on matter!



Quintessence density contrast evolution for PBNG

**THANK YOU FOR YOUR ATTENTION**