Geons in Asymptotically Anti-de Sitter spacetimes

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AdS/CFT correspondance

(Anti-de Sitter/Conformal Field Theory)

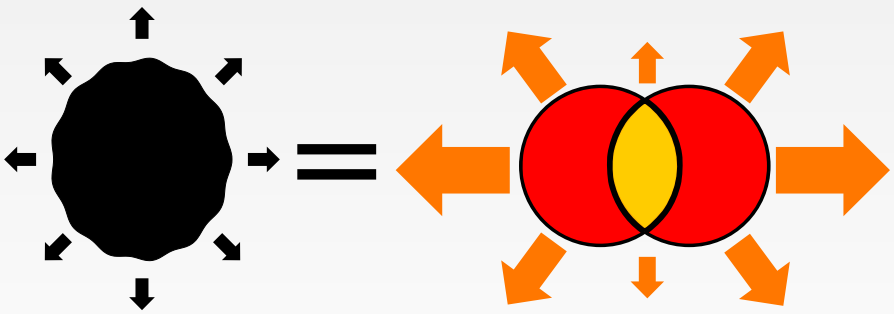
AdS/CFT

Seminal paper

The Large N Limit of Superconformal Field Theories and Supergravity
J. M. Maldacena, *Adv. Theor. Math. Phys.* **2** (1998) 231

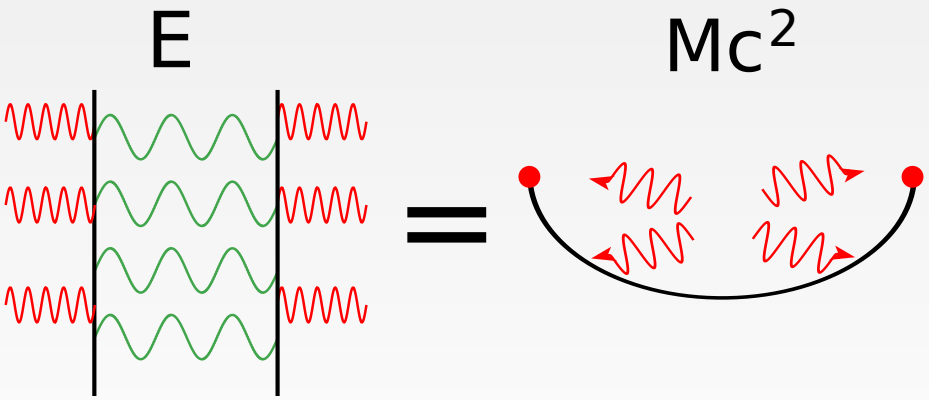
More than 11 000 citations !

AdS to CFT



Schwarzschild Quark-gluon plasma

CFT to AdS



Casimir AdS_5

Holographic principle

Strongly coupled 4D gauge theory = Grav. theory in 5D AAdS

- QCD, QED at strong coupling is hard
- Super gravity in AAdS much easier
- AdS_5 dual to $\mathcal{N} = 4$ Super Yang-Mills

Black hole thermodynamics

AAdS without Black Hole : $T = 0$
AAdS with Black Hole : $T \neq 0$

Anti-de Sitter (AdS) spacetime

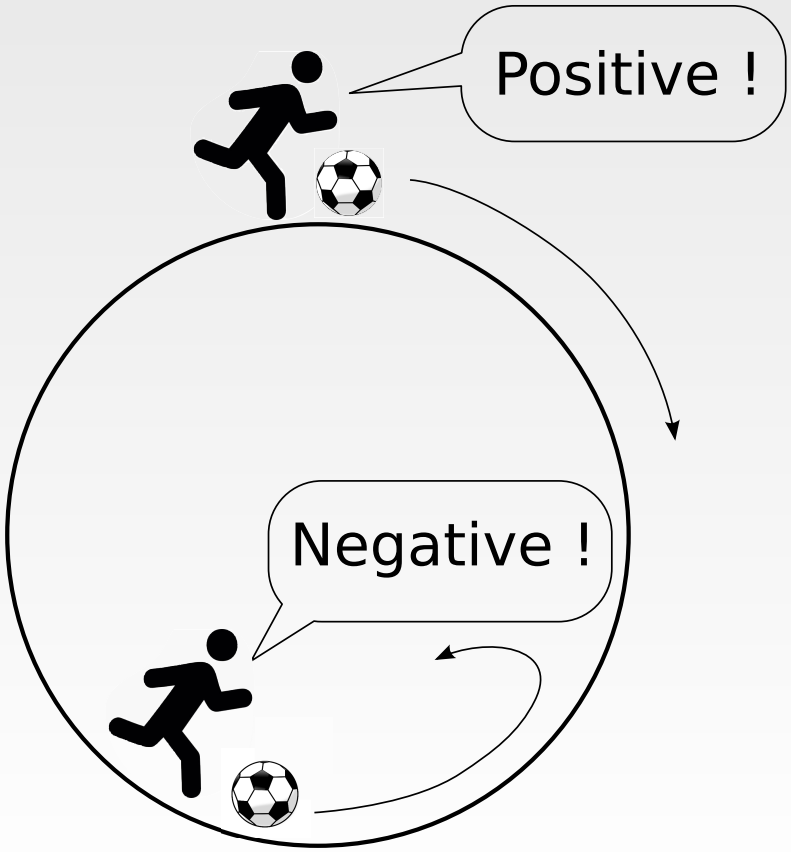
Negative cosmological constant

AdS = unique maximally symmetric solution of Einstein with $\Lambda < 0$

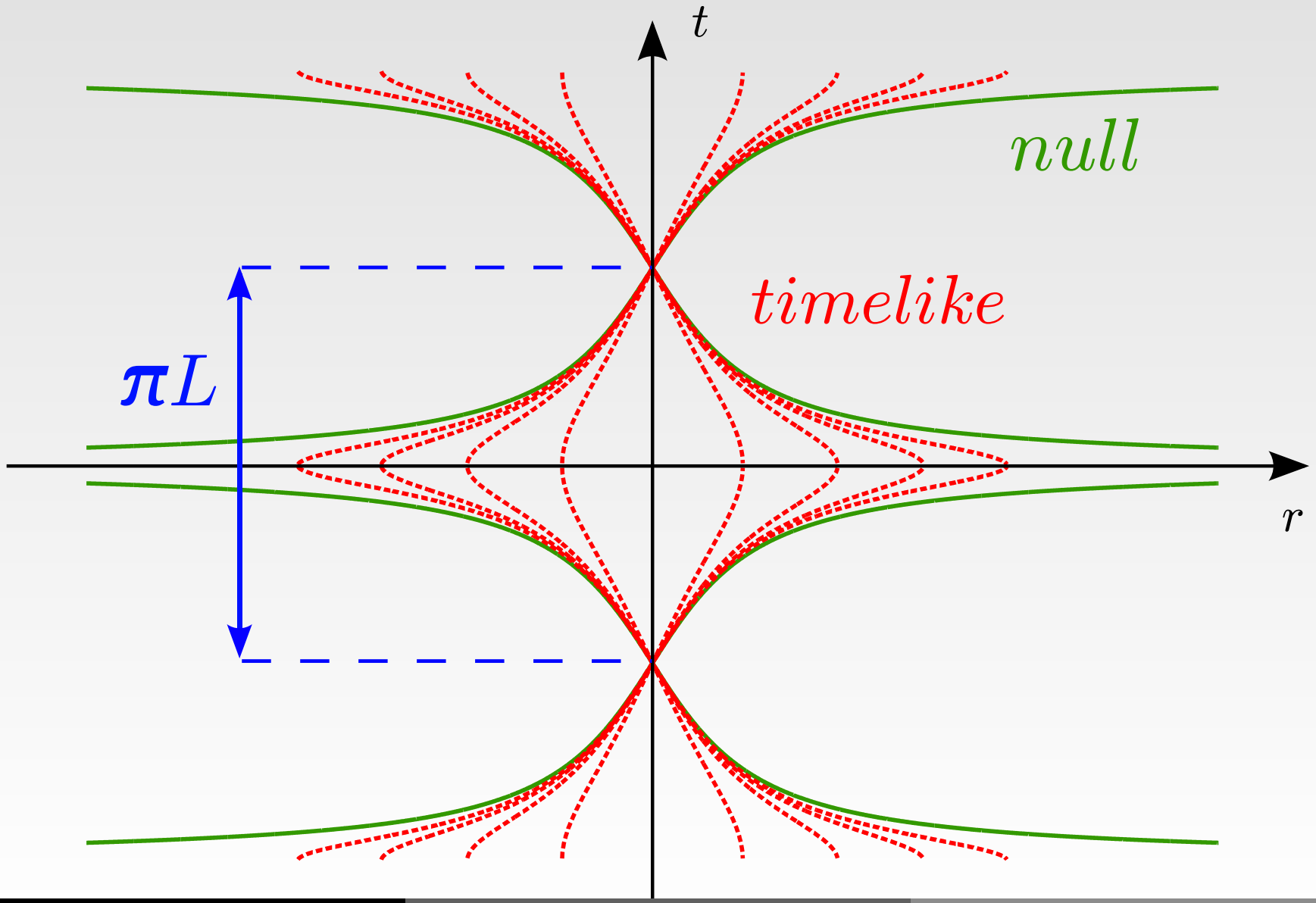
$$R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

AdS

- 10 Killing vectors
- AdS length L : $\boxed{\Lambda = -\frac{3}{L^2}}$
- Constant curvature : $\boxed{R = -\frac{12}{L^2}}$



Radial geodesics

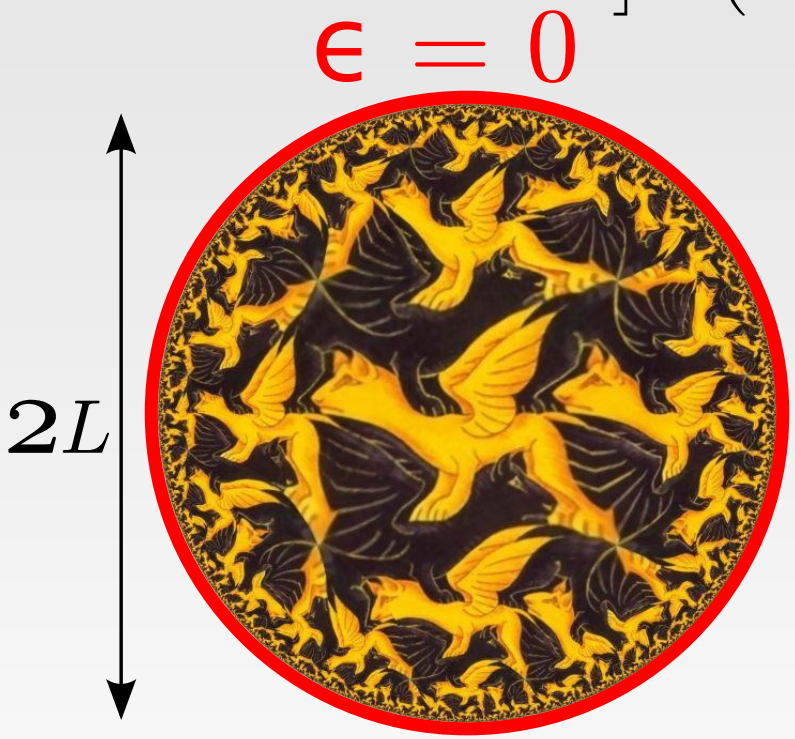


Conformal representation

$$ds^2 = -\left(\frac{1+\rho^2}{1-\rho^2}\right)^2 dt^2 + \frac{4}{(1-\rho^2)^2}\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \quad \left(\rho = \frac{r}{L}\right)$$

Properties

- compactified $r \in [0, L]$
- boundary at $r = L$
- $\boxed{\varepsilon = \frac{1-\rho^2}{1+\rho^2}}$
- $g_{\alpha\beta} = O\left(\frac{1}{\varepsilon^2}\right)$
- $\hat{g}_{\alpha\beta} = \varepsilon^2 g_{\alpha\beta} = O(1)$

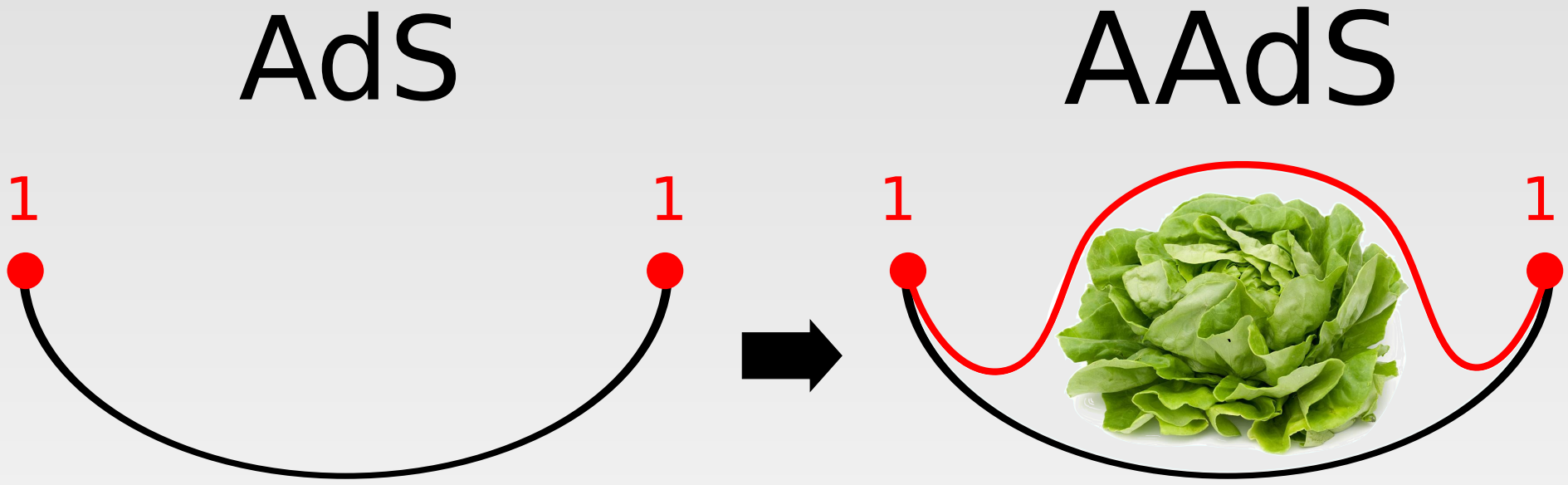


Conformal metric :

$$\boxed{d\hat{s}^2 = -dt^2 + \frac{4}{(1+\rho^2)^2}\left[dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]}$$

Asymptotically AdS (AAdS) spacetime

AdS + something



Problem : $g_{\alpha\beta}$ diverges like $O\left(\frac{1}{\varepsilon^2}\right)$ at the boundary !

Cure : conformal structure at the boundary

AAdS boundary conditions (Ashtekar and Magnon 1984)

$$\varepsilon^2 g_{\alpha\beta} \underset{r=L}{=} \mathrm{diag}(-1,1,1,1)$$

Global quantities

Weyl tensor

Definition :

$$C_{\alpha\beta\mu\nu} = R_{\alpha\beta\mu\nu} - \text{ [trace part]}$$

$$C^{\alpha}_{\beta\alpha\nu} = C^{\alpha}_{\alpha\mu\nu} = C^{\alpha}_{\beta\mu\alpha} = 0$$

Conformal invariance :

$$\boxed{\hat{g}_{\alpha\beta} = \varepsilon^2 g_{\alpha\beta} \Rightarrow \hat{C}_{\alpha\beta\mu\nu} = C_{\alpha\beta\mu\nu}}$$

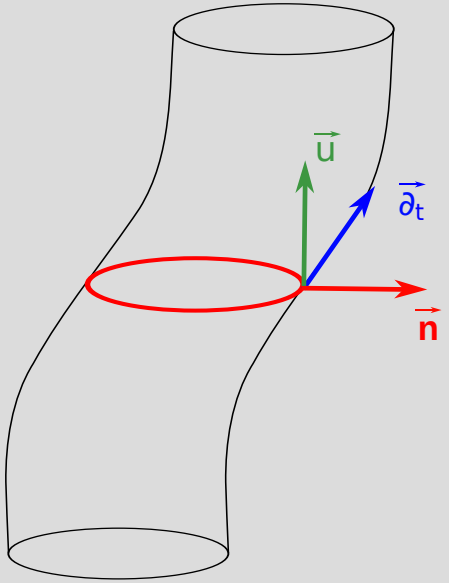
AdS boundary : $\boxed{\Rightarrow C_{\alpha\beta\mu\nu} = O(\varepsilon)}$

Global quantities (Ashtekar and Das 2000)

Define : $\hat{K}_{\alpha\beta\mu\nu} = \dfrac{\hat{C}_{\alpha\beta\mu\nu}}{\varepsilon} = O(1)$, $n_\alpha = \nabla_\alpha \varepsilon$ and $n^\alpha = \hat{g}^{\alpha\beta}\nabla_\beta \varepsilon$

$$M = \frac{L^3}{8\pi} \oint_{\mathcal{S}^\infty} \hat{K}_{\alpha\beta\mu\nu} n^\beta n^\nu (\partial_t)^\alpha u^\mu \sqrt{\hat{\sigma}} d^2 y$$

$$J = \frac{L^3}{8\pi} \oint_{\mathcal{S}^\infty} \hat{K}_{\alpha\beta\mu\nu} n^\beta n^\nu (\partial_\varphi)^\alpha u^\mu \sqrt{\hat{\sigma}} d^2 y$$



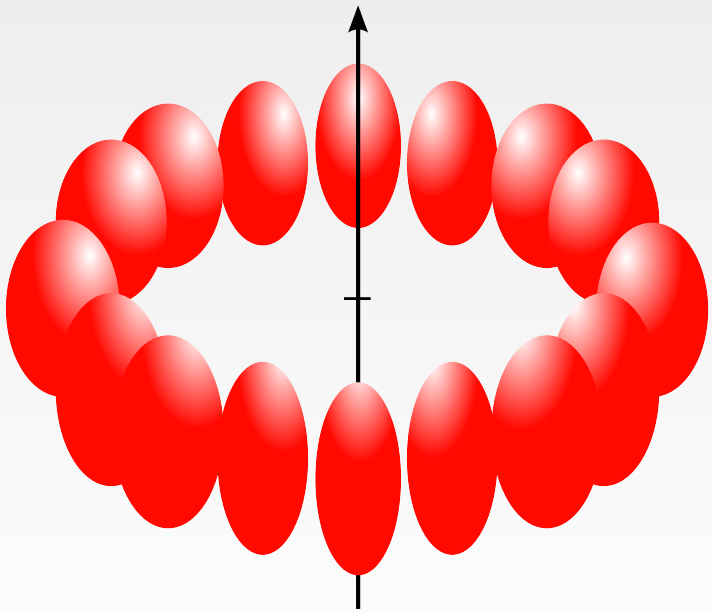
Geons in AAdS spacetime

What's a geon ?

GEON = Electro-Gravitational Entity

Seminal papers : 50's and 60's

Wheeler, Power, Brill, Ernst, Melvin, Hartle, Thorne, Kaup



At the beginning :

- asymptotically flat
- cylindrical, toroidal, spherical
- EM/ν/GW/ϕ wave packet

Geon properties

- need rotation to avoid collapse
- photon self attraction
- Black body radiation

Linear geon in AAdS

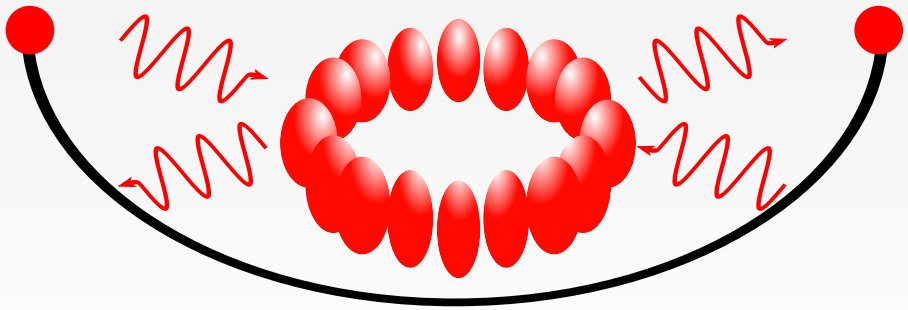
Seminal papers (AAdS) : 2010's

Bizon, Rostworowski, Maliborski, Dias, Horowitz, Santos, Kodama, Ishibashi, Seto, Wald

Properties

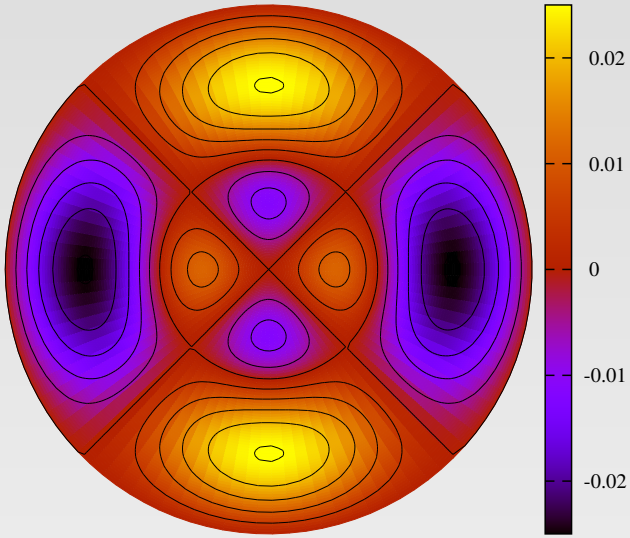
- vaccum solution
- need rotation to avoid collapse
- stationnary in corotating frame

$\boxed{\Rightarrow \text{Helical Killing vector}}$

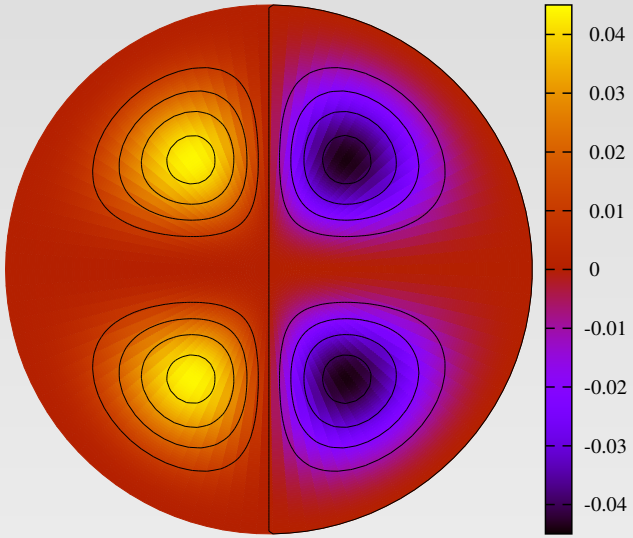


Mathematics

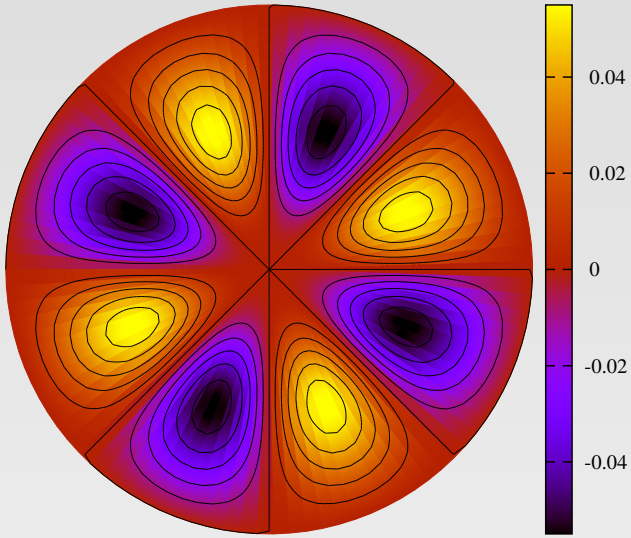
- $g_{\alpha\beta} = \bar{g}_{\alpha\beta} + \mathcal{A} h_{ij}$ with $\mathcal{A} \ll 1$
- $R_{\mu\nu} - \frac{R}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = O(\mathcal{A}^2)$
- Tensor spherical harmonics



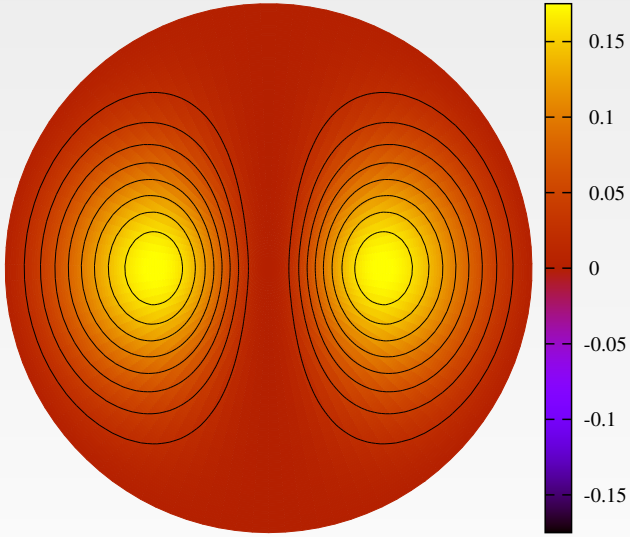
$\hat{n}$ in z = 0 plane



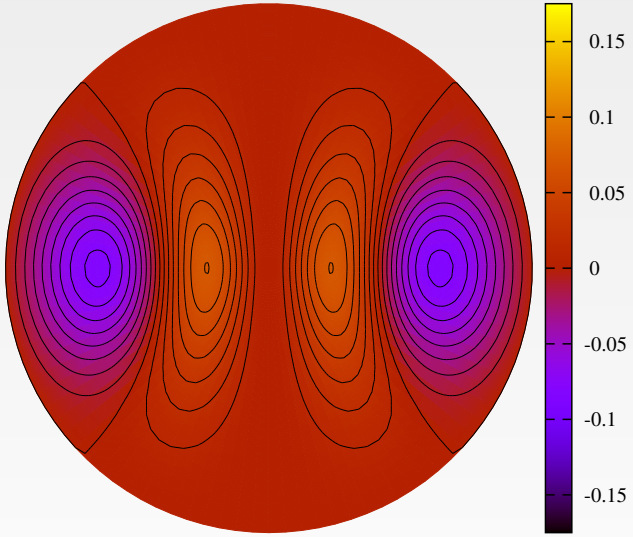
B^y in z = 0 plane



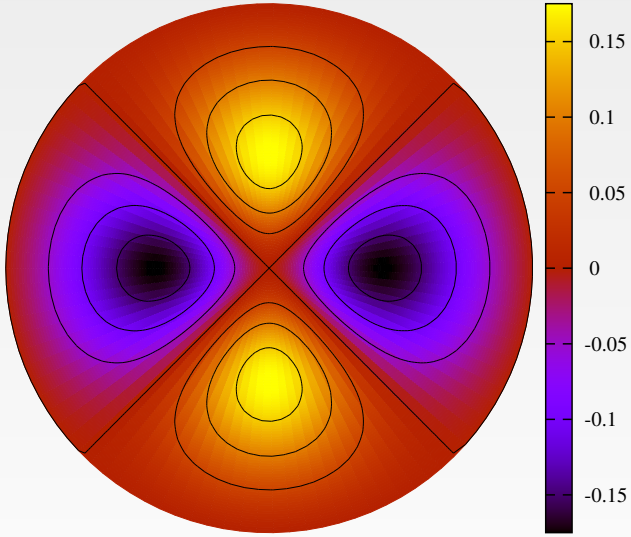
$\hat{h}_{xy}$ in z = 0 plane



$\hat{h}_{xx}$ in x = 0 plane



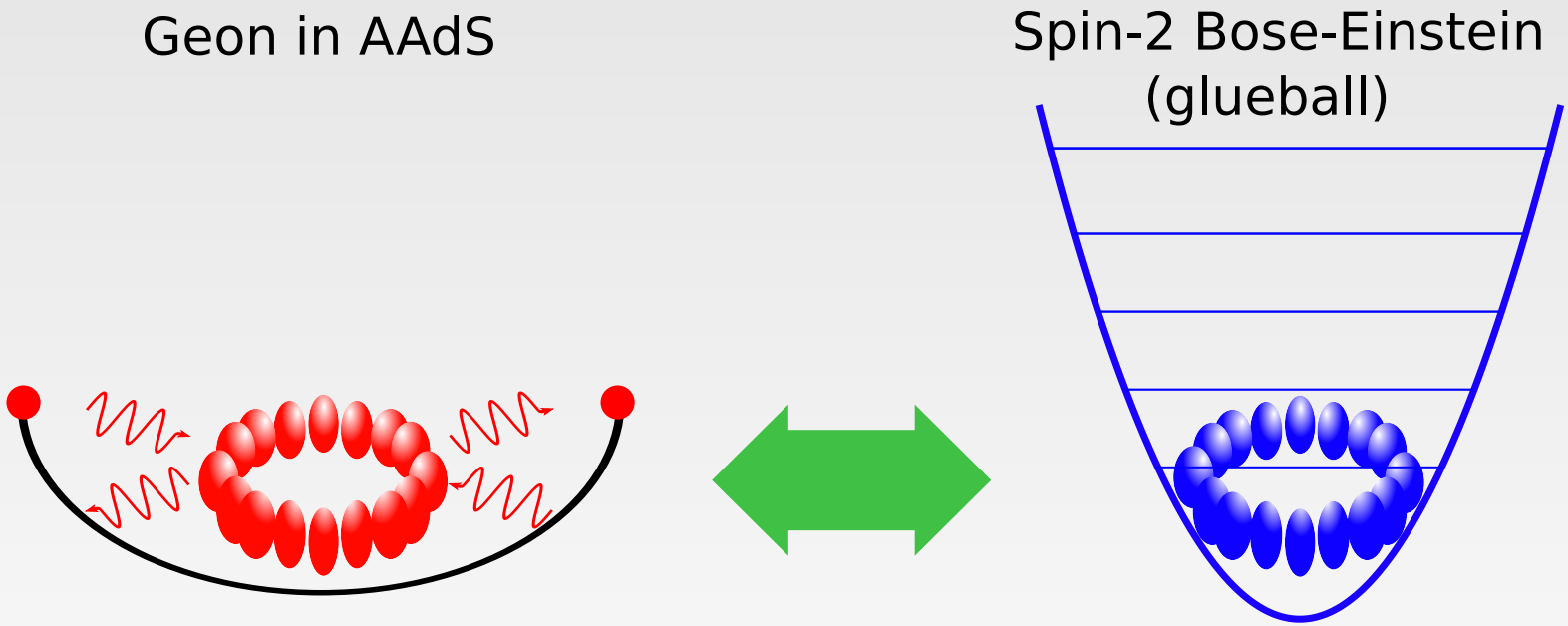
$\hat{h}_{yy}$ in x = 0 plane



$\hat{h}_{zz}$ in z = 0 plane

What is a geon useful for ?

- AdS/CFT dual representation : No black hole $\Rightarrow \boxed{T = 0}$



- Explore a non-linear stability island of AdS spacetime [1, 2]

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1. P. Bizoń and A. Rostworowski. *Physical Review Letters*, 107, July 2011
2. P. Bizoń, M. Maliborski, and A. Rostworowski. *ArXiv*, June 2015, 1506.03519

Non-linear geon

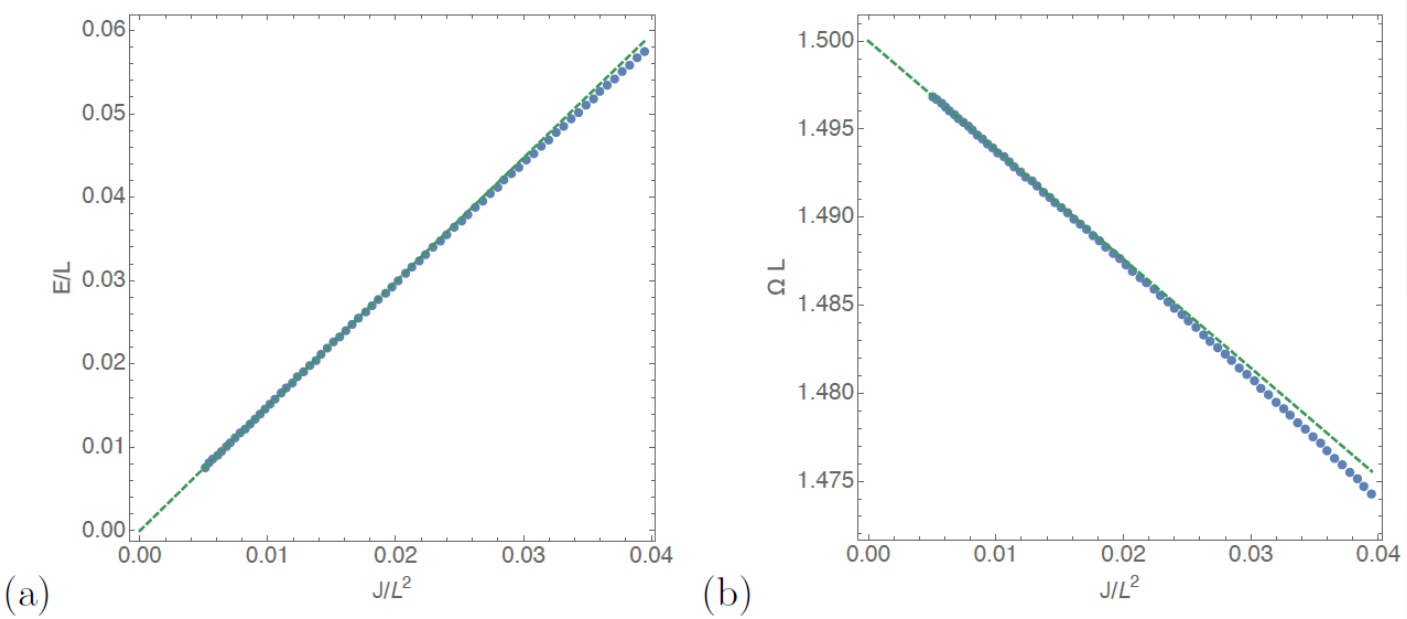
Step by step

1. Find linear geon
2. AAdS-AdS formulation
3. Choose a gauge
4. Invert Einstein system

What you need :

- Kodama-Ishibashi formalism
- Spectral method
- Numerical library : KADATH
- Newton-Raphson in dimension 10^4

Horowitz and Santos 2014



Conclusion

Conclusion

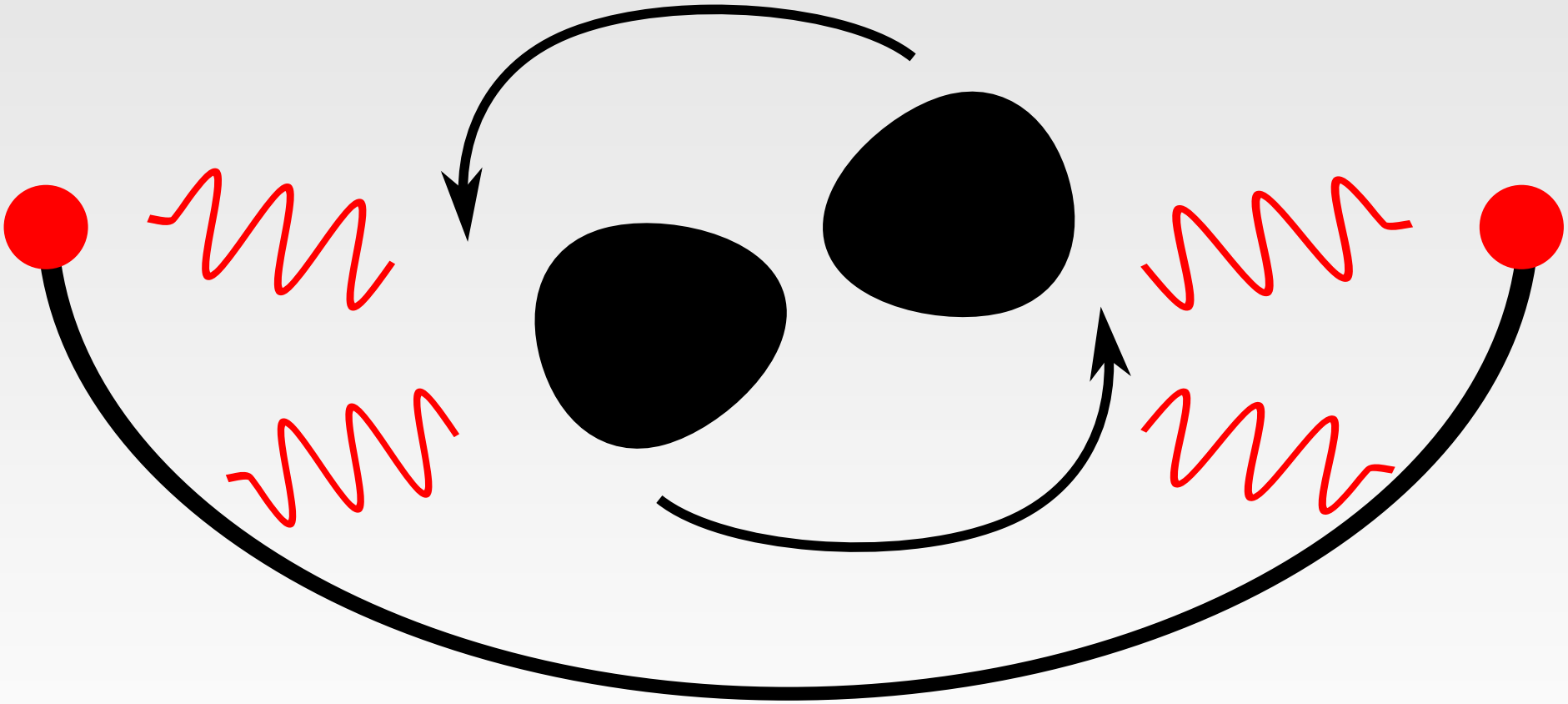
State of the art :

- small sequences of geons computed with central amplitude $\sim 10\%$ of AdS at several resolutions

TO-DO list :

- compute larger geon sequences
- maximum mass and maximum angular momentum of a geon ?
- check stability with evolution code
- AdS/CFT interpretation
- add ingredients (black holes, boson stars. . .)

Holy grail of AdS gravitational systems :
Exact non-coalescing binaries with exact helical symmetry



Thank you for your attention

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