Kerr black holes with self-interacting scalar hair:

hairier but not heavier

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Outline

Boson stars: maximal masses

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- For spherically symmetric BSs, i.e. $m = 0$, $\alpha_{\mathrm{BS}} = 0.633$ [S. L. Liebling and C. Palenzuela (2012)]
- while for rotating ones, e.g. $m = 1; 2$, $\alpha_{\mathrm{BS}} = 1.315; 2.216$ [S. Yoshida and Y. Eriguchi (1997)];[P. Grandclement, C. Somé and E. Gourgoulhon (2014)]

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- Therefore, if one considers a scalar field mass of $\mu \sim 1$ GeV and reasonably low m, this maximal mass is very low.
- Due to this, these boson stars are known as mini boson stars.

Self-interacting spherically symmetric solutions

- This maximal mass can be considerably increased if one considers quartic self-interactions of the form $V(|\Psi|) = \lambda|\Psi|^4$. [M. Colpi, S. L. Shapiro and I. Wasserman (1986)]

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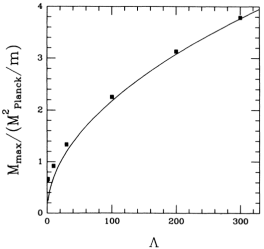

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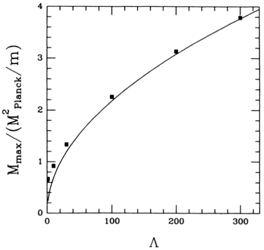

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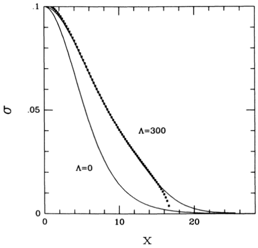

- Eqn. (2) found from numerics and analytical study of the high coupling limit.
- More extended than mini-BSs.

Self-interacting rotating solutions

- Quartic BSs have been generalized to the rotating case, usually for a single value of the coupling.
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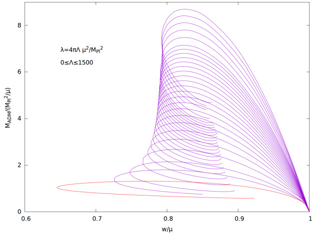

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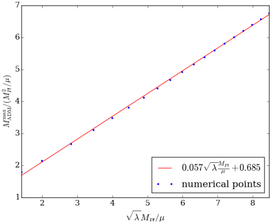

How do quartic self-interactions affect KBHsSH?

- Our interest lies in studying these quartic BSs with a black hole at their centre.
- So far, we have considered two possible cases, quartic and hexic self interactions [arXiv:1509.02923]
  1. $V(\phi) = \mu^2\phi^2 + \lambda\phi^4$,
  2. $V(\phi) = \mu^2\phi^2 - \beta\phi^4 + \gamma\phi^6$.

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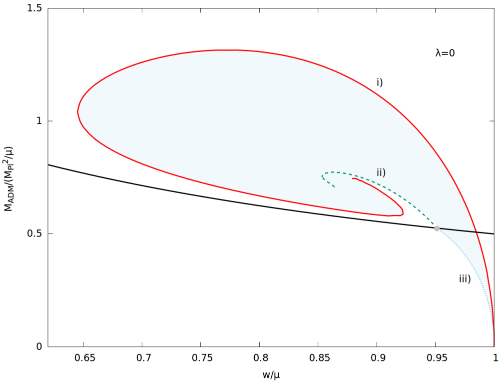

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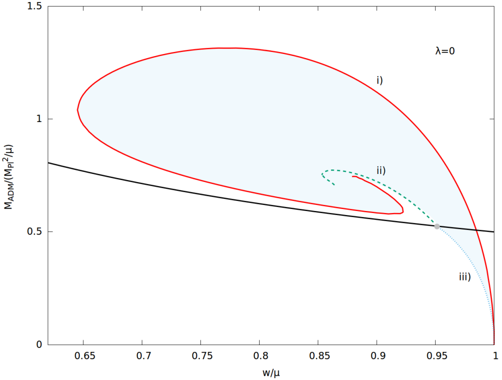

Envelope made up of 3 parts:

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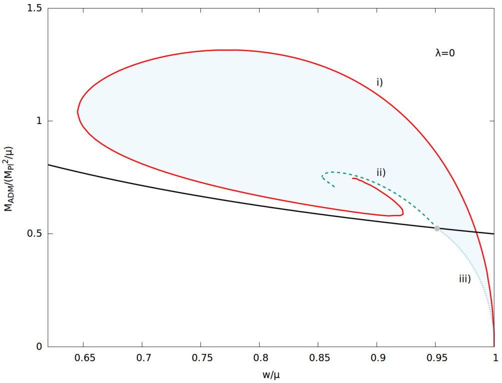

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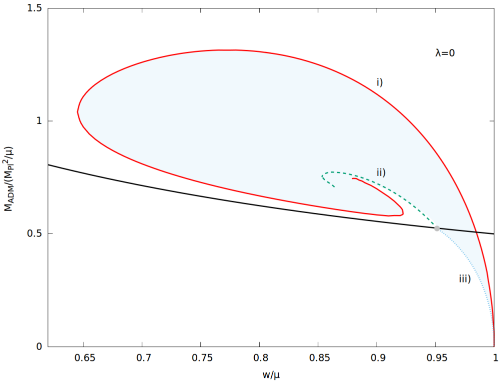

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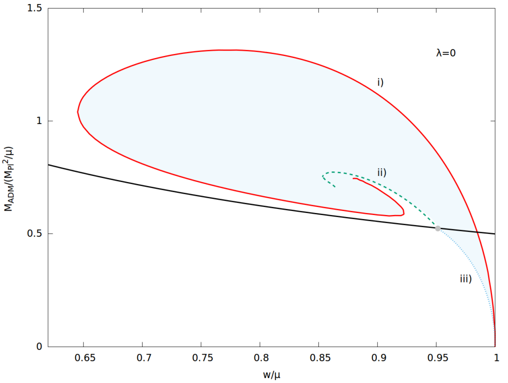

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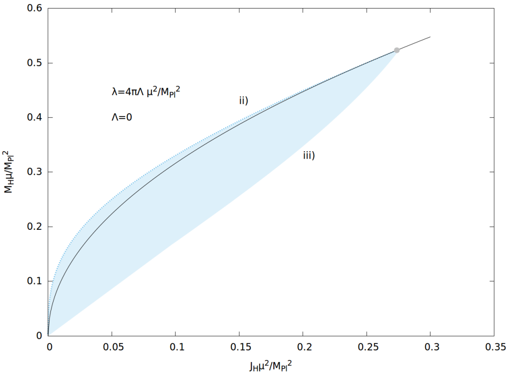

Envelope made up of 2 parts:

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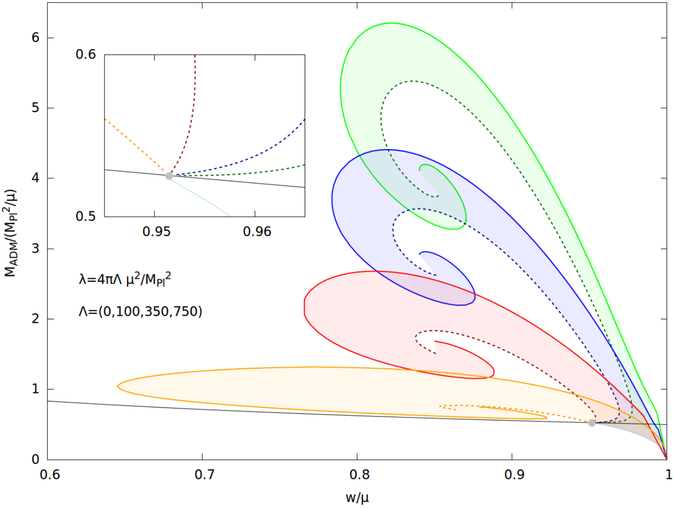
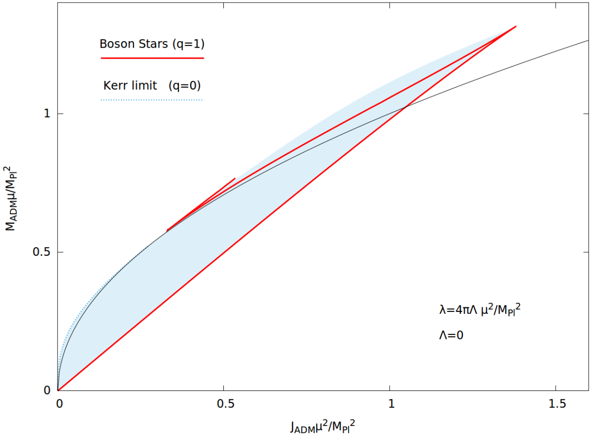
Hairy black holes, w vs. M



J vs. M, ADM quantities



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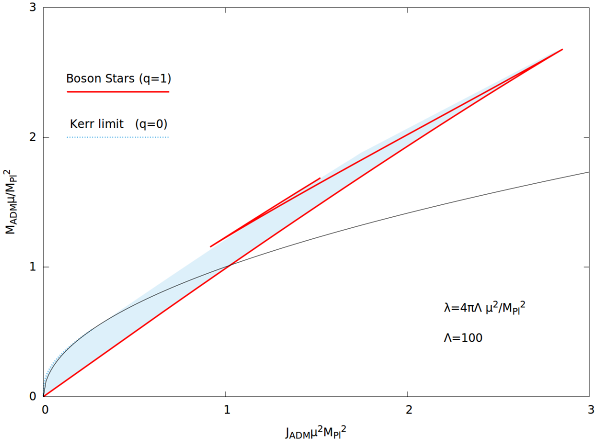

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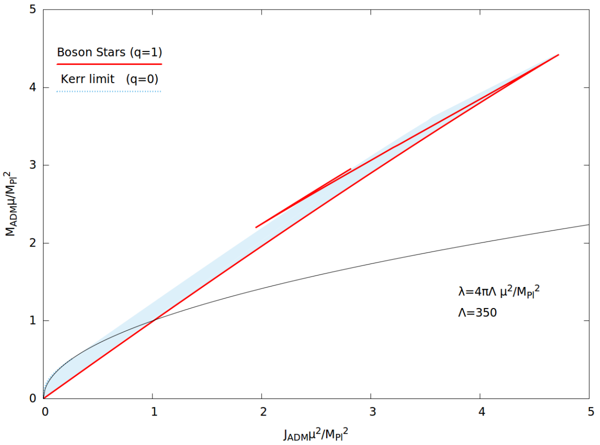

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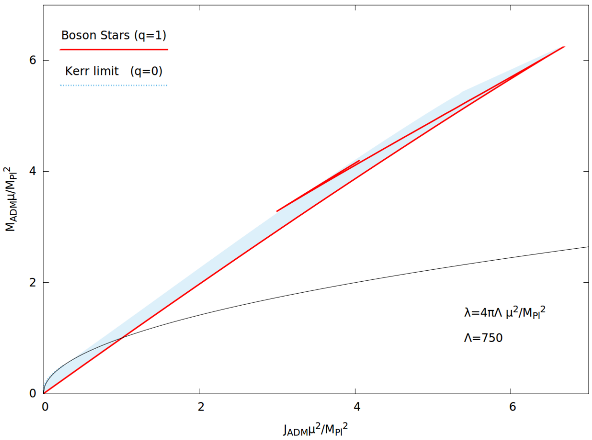

J vs. M, horizon quantities

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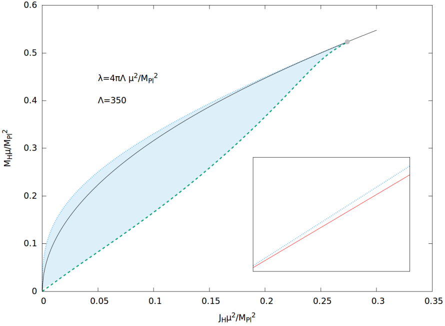
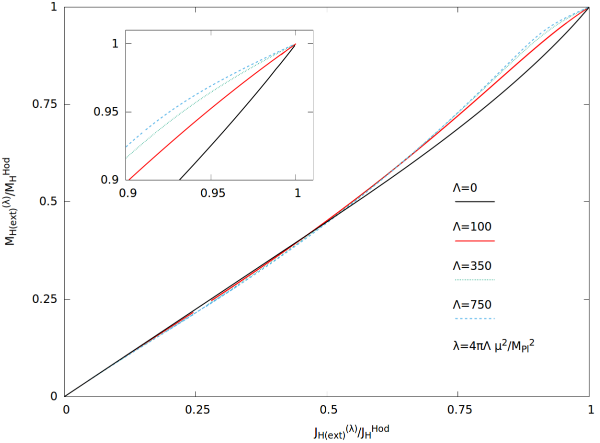
- No matter how strong the self-interaction coupling is, the maximal horizon mass and angular momentum are obtained at the “Hod point”.



- Notice that the extremal HBH curves approach the “Hod point” from below.



A hexic potential

- Let us now consider a potential of the form

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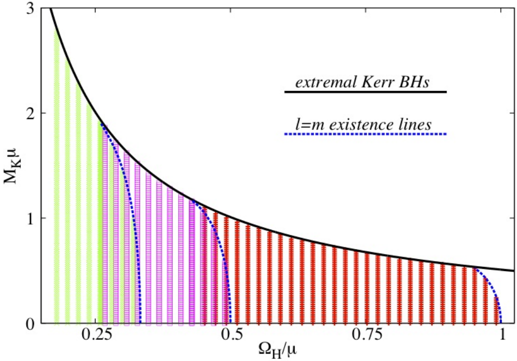
- We fix $\mu^2 = 1.1$, $\beta = 2.0$ and $\gamma = 1.0$ for all solutions to follow.

Q-clouds

- The 1D existence line of the previous hairy black holes is now expanded into a 2D existence plane.



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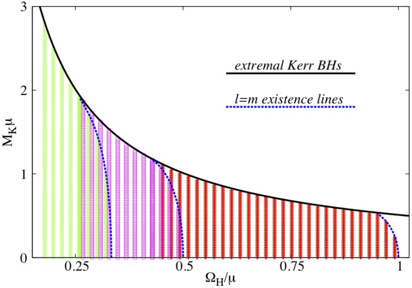
- For $m\Omega_H^{extremal}/\mu \leq w/\mu < 1$, Q-clouds arise from flat spacetime Q-balls and end on the previous existance line.



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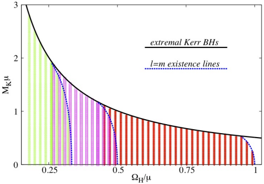
- For $m\Omega_H^{min} \leq w < m\Omega_H^{extremal}$, any Kerr black hole is allowed as a background for a Q-cloud. Even extremal Kerr black holes.



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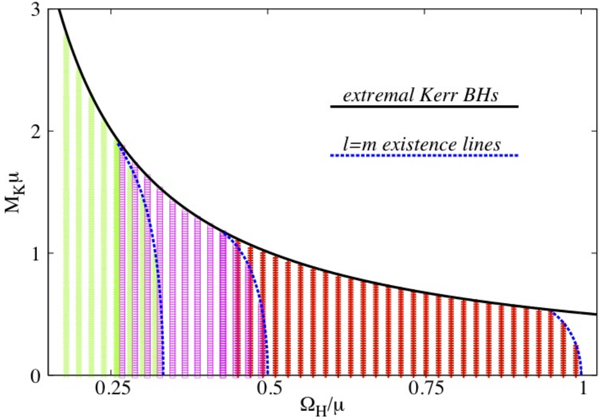
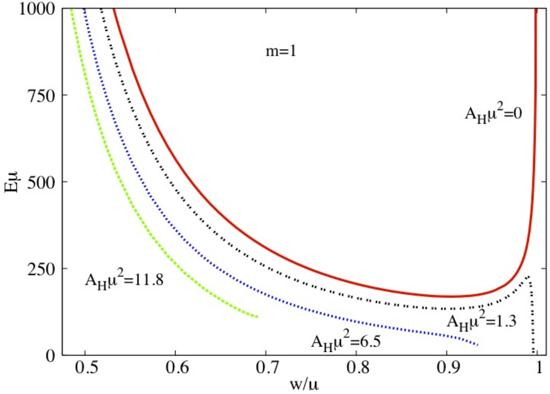

extremal Kerr BHs
l=m existence lines
3
2
1
0
0.25
0.5
0.75
1
$M_K\mu$
Ω_H/μ

Q-clouds: energy diagram

- Let us look at the energy of these Q-clouds for a fixed area horizon of the black hole



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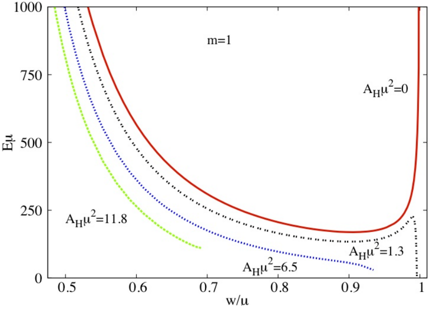

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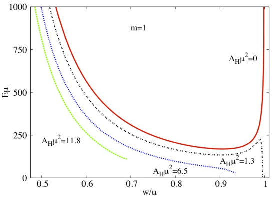

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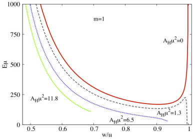

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- For a small black hole, Q-clouds approach a critical configuration with zero global charges
- While for a large black hole, they end at a critical configuration with non-zero energy and angular momentum. The corresponding Kerr black holes have $T_H = 0$.

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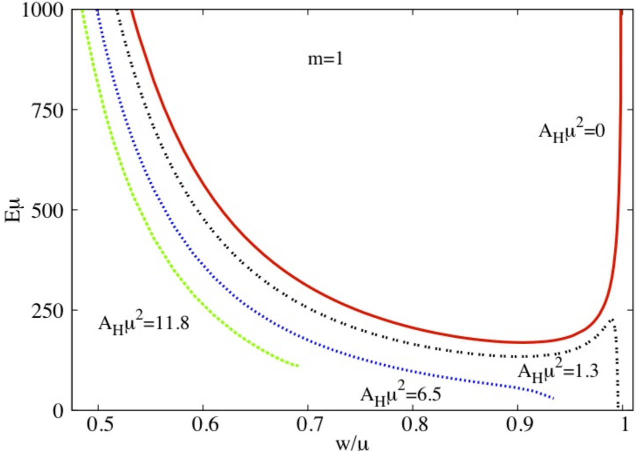

From Q-balls to Self-interacting boson stars

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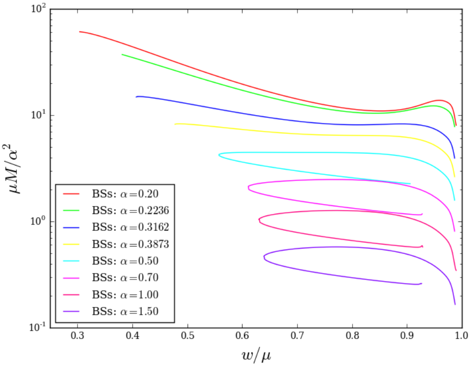
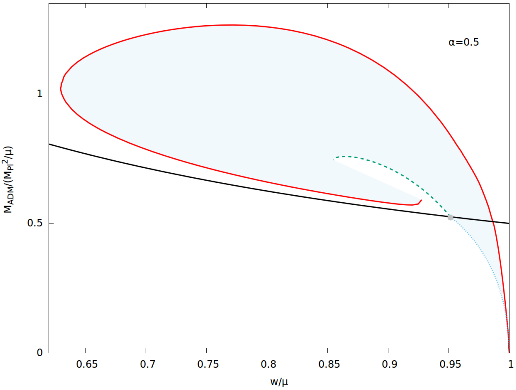


Hairy black holes

- We have filled in the envelope with hairy black holes for some values of the gravitational coupling.



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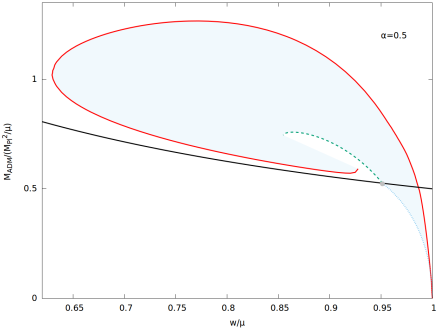
- The shape seems to follow closely that of the mini-boson stars.



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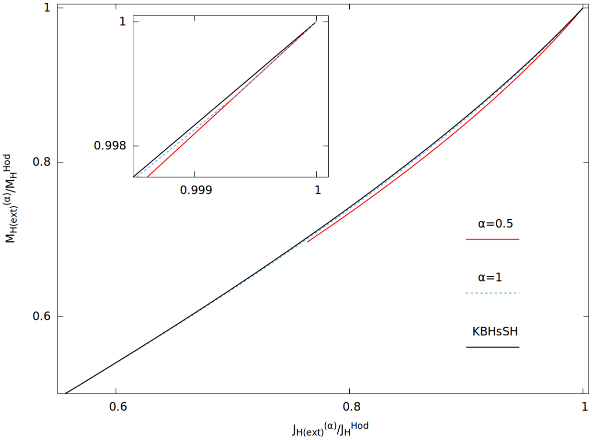

Summary

- When adding self-interactions to boson stars, one can obtain considerably higher masses.
- In terms of ADM quantities, the same holds for KBHsSH while the horizon quantities of the solutions change very little.
- In particular, the maximal horizon quantities are attained at the "Hod point" (which is independent of the self-interaction coupling), and thus these self-interacting KBHsSH are "hairier but not heavier".
- If these results hold for general scalar field models, KBHsSH with astrophysically interesting horizon masses require ultra-light scalar fields.