Spacetimes characterizations

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Outline

Sgr A*

Main observational facts

- Radio source
- Distance : $R_0 = 8.33 \pm 0.35\ \mathrm{kpc}$
- Mass : $(4.31 \pm 0.736) \times 10^6\ M_\odot$

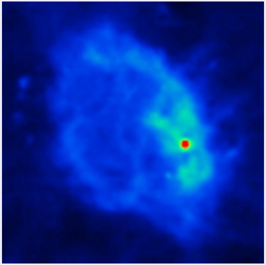


Figure: SgrA Est for $\lambda = 20$ cm

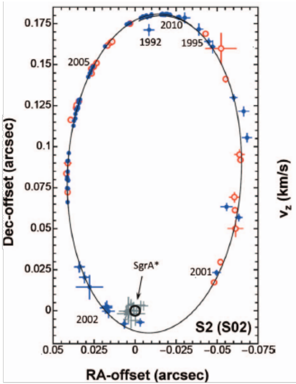


Figure: Orbit of S2 around Sgr A*

Sgr A* : Kerr Black Hole versus Boson Star

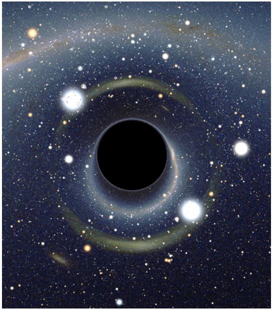


Figure: Image of a Schwarzschild Black Hole

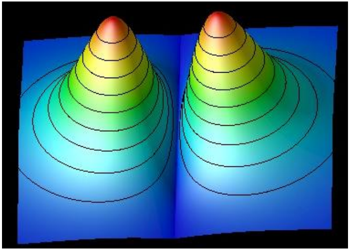


Figure: Rotating Boson Star

These 2 models are compatible with the observations of Sgr A*

... but hopefully it won't be the case very long thanks to new observations

New observations in the near future

Stellar orbits with GRAVITY instrument

- Optical interferometry in the near-infrared
- astrometric precision of 10 μas on each orbit

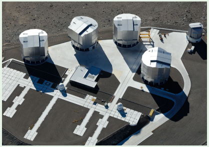


Figure: Four 8 m telescopes at VLT (Chile)

Images of BH shadow with Event Horizon Telescope

- (Sub)mm VLBI all over the world
- angular resolution expected : 1 μas !

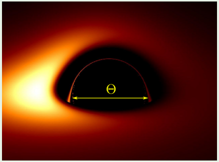
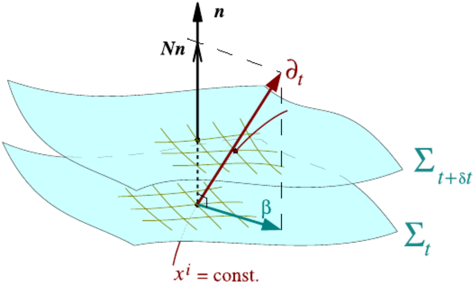


Figure: Accretion disk around a Schwarzschild BH
$\Theta_{SgrA*} = 53\ \mu$as
[Vincent et al., 2011]

3+1 Splitting of Spacetime



- Unit normal & lapse : $n = -N\nabla t$
- Shift : β
- Spatial metric : γ_{ij}

3+1 metric :

$$g_{\alpha\beta}dx^\alpha dx^\beta = -N^2 dt^2 + \gamma_{ij}\left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$$

Quasi-isotropic coordinates

Case of axisymmetric and stationary spacetimes

Quasi-isotropic coordinates

$(x^\alpha) = (t, r, \theta, \varphi)$

- Metric in Quasi-isotropic coordinates :

$$g_{\alpha\beta}dx^\alpha dx^\beta = -N^2 dt^2 + A^2\left(dr^2 + r^2 d\theta^2\right) + B^2 r^2 \sin^2\theta \left(d\varphi - N^\varphi dt\right)^2$$

Link with 3+1

- Lapse : $N = N$
- Shift : $\beta = (0, 0, -N^\varphi)$
- Spatial metric $\gamma_{ij} = \begin{pmatrix} A^2 & 0 & 0 \\ 0 & A^2 r^2 & 0 \\ 0 & 0 & B^2 r^2 \sin^2\theta \end{pmatrix}$

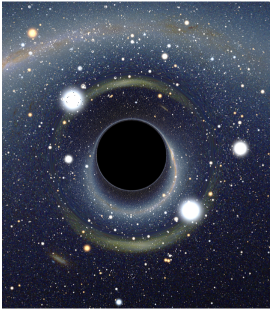
Kerr metric

Kerr Black Hole

Solution of the Einstein equations in vacuum :

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} = 0$$

characterized by the mass M and the kerr parameter a
$a = 0$: Schwarzschild spacetime



In QI coordinates

- Analytic expressions for : N, N^{φ}, A, and B

Boson Star : field equations

Boson Star : gravitationally bound state of a complex scalar field

ϕ which is solution of the following system

- Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}$$

$$T_{\mu\nu} = \frac{1}{2}\left[\nabla_\mu \bar{\phi} \nabla_\nu \phi + \nabla_\mu \phi \nabla_\nu \bar{\phi}\right] - \frac{1}{2} g_{\mu\nu}\left[g^{\gamma\delta} \nabla_\gamma \bar{\phi} \nabla_\delta \phi + V\left(\mid \phi \mid^2\right)\right]$$

- Klein Gordon equation

$$\nabla_\mu \nabla^\mu \phi = \frac{\mathrm{d}V}{\mathrm{d} \mid \phi \mid^2} \phi$$

Here we consider “mini” boson stars with $V\left(\mid \phi \mid^2\right) = \frac{m^2}{\hbar^2} \mid \phi \mid^2$

Boson Star : numerical metric

Assumptions

- Ansatz for the field ϕ

$$\phi = \phi_0\left(r,\theta\right)e^{i(\omega t - k\varphi)}$$

with $\phi_0\left(r,\theta\right)$ a real function, $\omega \in \mathbb{R}$ and k is an integer.

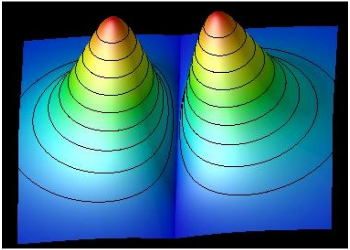


Figure: Numerical solutions found with Kadath

Pointy Petal orbits

Idea : Find different timelike geodesics for the 2 spacetimes

It has been done for a specific class of zero angular momentum orbits using GYOTO ray-tracing code

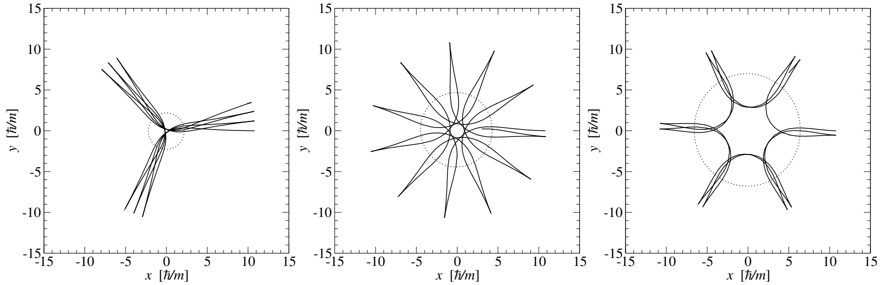


Figure: Orbit of a $\ell = 0$ test particle in the equatorial plane of a boson star with $\omega = 0.8\ m/\hbar$ and $k = 1, 2, 3$

Spacetime Simon Tensor

- Definition : $\boxed{S_{\alpha\beta\nu} = 4\xi^{\mu}\xi^{\rho}\mathcal{C}_{\mu\alpha\rho[\beta}\,\sigma_{\nu]} + \gamma_{\alpha[\beta}\,\mathcal{C}_{\nu]\mu\rho\delta}\mathcal{F}^{\rho\delta}\xi^{\mu}}$

where ξ^{μ} is a Killing vector

$$\mathcal{C}_{\alpha\beta\gamma\delta} = C_{\alpha\beta\gamma\delta} + \frac{1}{2} i\eta_{\gamma\delta\rho\sigma} C_{\alpha\beta}^{\ \ \ \rho\sigma}$$

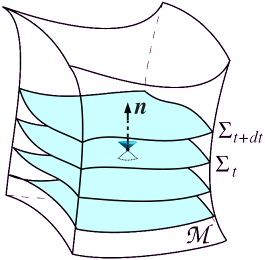
where $\eta_{\gamma\delta\rho\sigma}$ is the volume form, $C_{\alpha\beta\gamma\delta}$ the Weyl tensor and

$$\begin{aligned} \mathcal{F}_{\alpha\beta} &= \nabla_{\alpha}\xi_{\beta} + \frac{i}{2}\eta_{\alpha\beta\lambda\mu}\nabla^{\lambda}\xi^{\mu} \\ \sigma_{\mu} &= 2\xi^{\alpha}\mathcal{F}_{\alpha\mu} \\ \gamma_{\alpha\beta} &= -\xi_{\rho}\xi^{\rho}g_{\alpha\beta} + \xi_{\alpha}\xi_{\beta} \end{aligned}$$

$S_{\alpha\beta\nu}$ is antisymmetric in its last 2 indices

- Property : $S_{\alpha\beta\nu} = 0$ for Kerr spacetime !

3+1 Decomposition of the Spacetime Simon Tensor

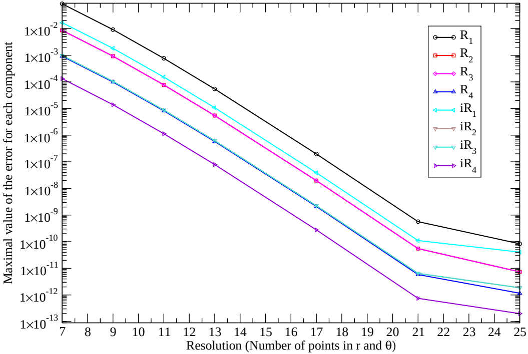


$$\mathrm{Re}\left(S_{\alpha\beta\nu}\right) \Rightarrow \begin{cases} \text{Totally spatial component} & = R_1 \\ n_\nu \ \text{component} & = R_2 \\ n_\alpha \ \text{component} & = R_3 \\ n_\nu n_\alpha \ \text{component} & = R_4 \end{cases}$$

$$\mathrm{Im}\left(S_{\alpha\beta\nu}\right) \Rightarrow \begin{cases} \text{Totally spatial component} & = iR_1 \\ n_\nu \ \text{component} & = iR_2 \\ n_\alpha \ \text{component} & = iR_3 \\ n_\nu n_\alpha \ \text{component} & = iR_4 \end{cases}$$

$\Rightarrow$ 8 components computed by Kadath

Spacetime Simon Tensor for Kerr spacetime



Conclusion

The Spacetime Simon tensor is zero for Kerr.

Spacetime Simon Tensor for Boson Star spacetime

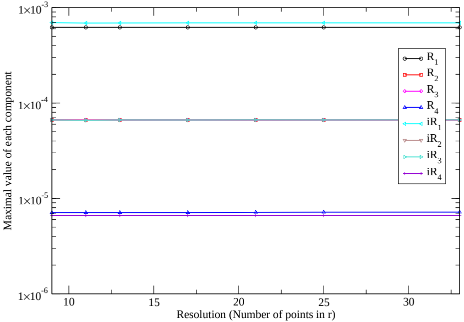


Figure: Boson star with $k = 1$ &$\omega = 0.70\ m/\hbar$

Conclusion : The Spacetime Simon tensor is not zero for Boson Stars !

Spacetime Simon Tensor for Boson Star spacetime

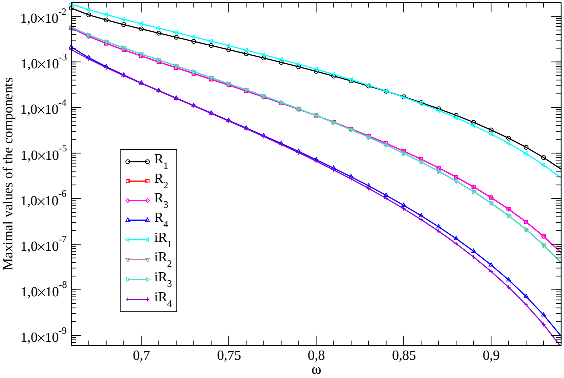


Figure: Family of boson stars with $k = 1$

Conclusion

Comparison to Kerr spacetime

- Generalize these comparison techniques for many different spacetimes
- Find the right model for Sgr A*