NEUTRON STARS AS DENSE MATTER LABORATORIES

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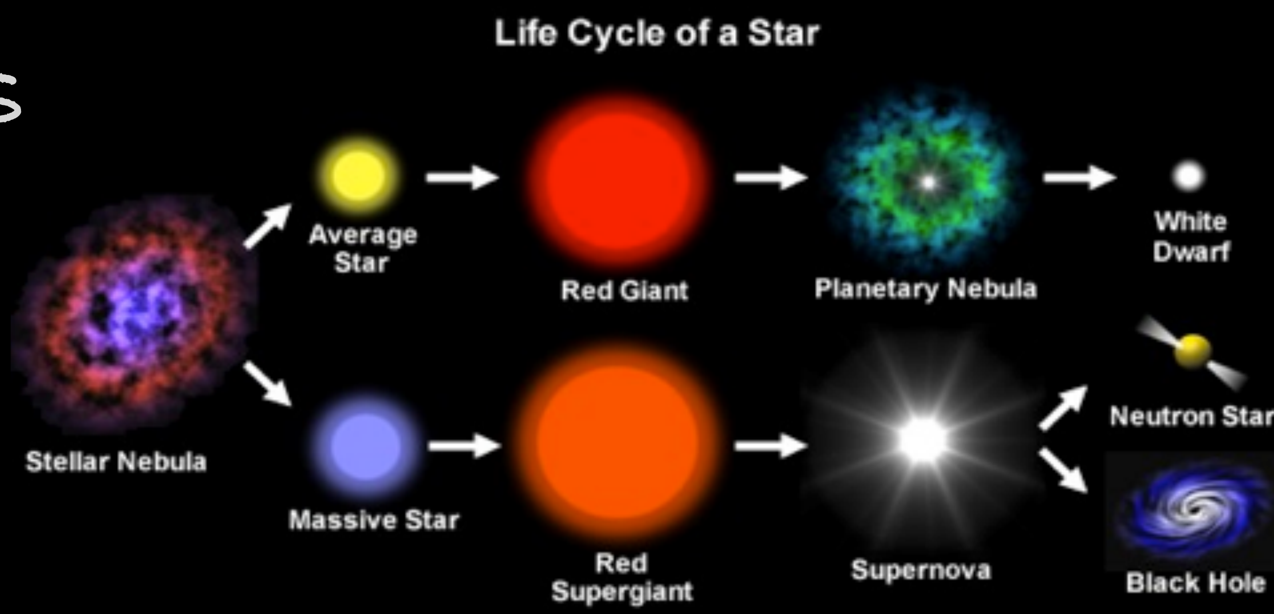
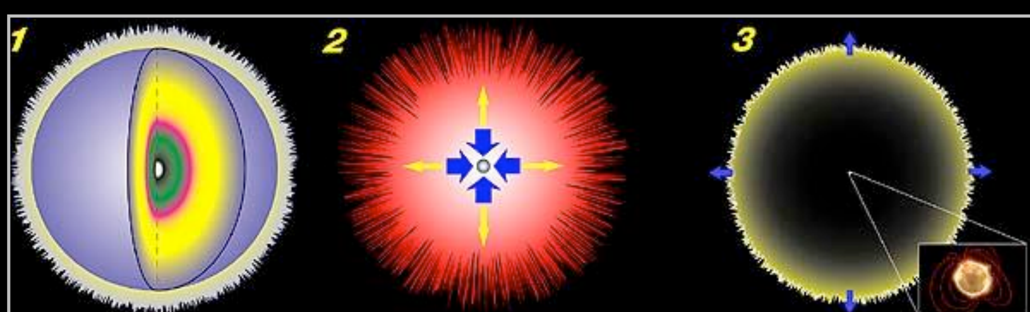
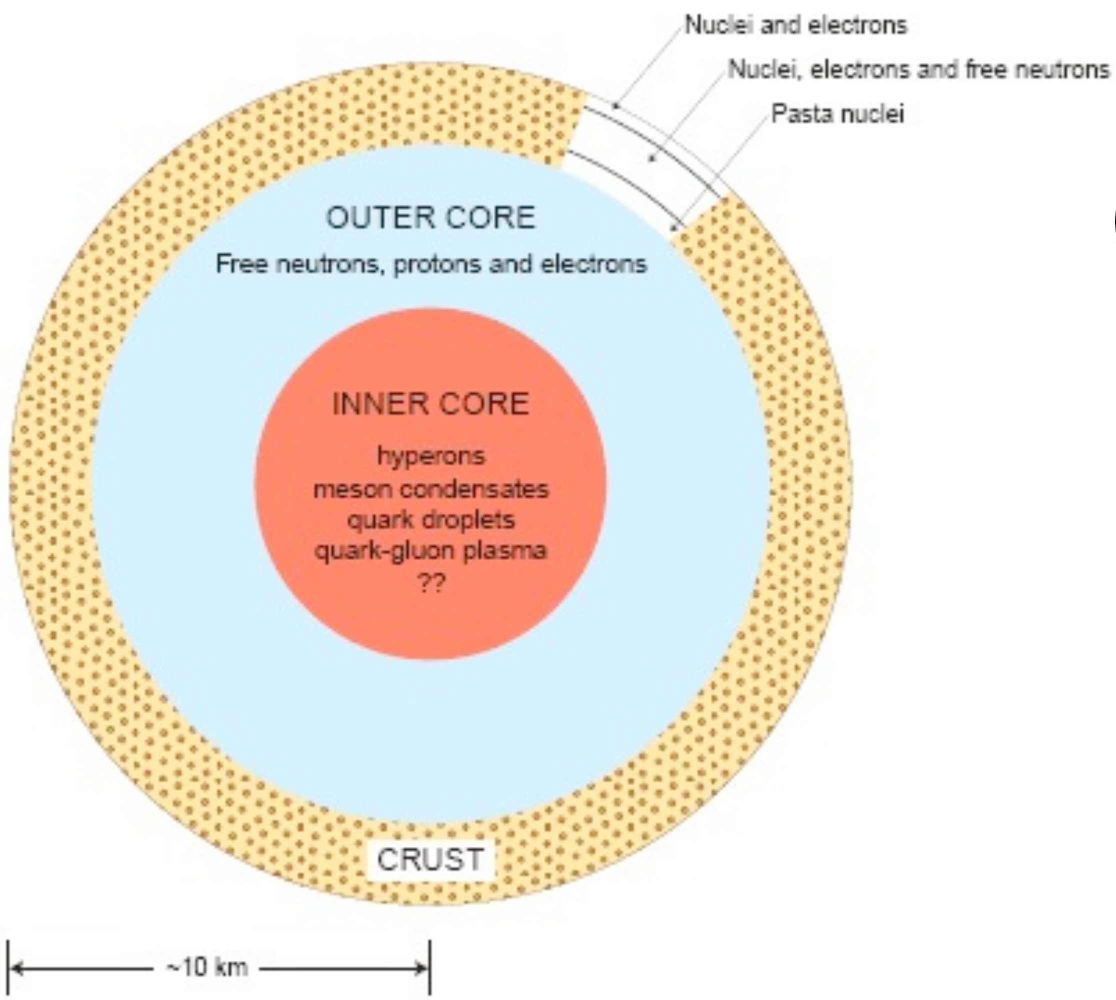
DENSE MATTER IN ASTROPHYSICS

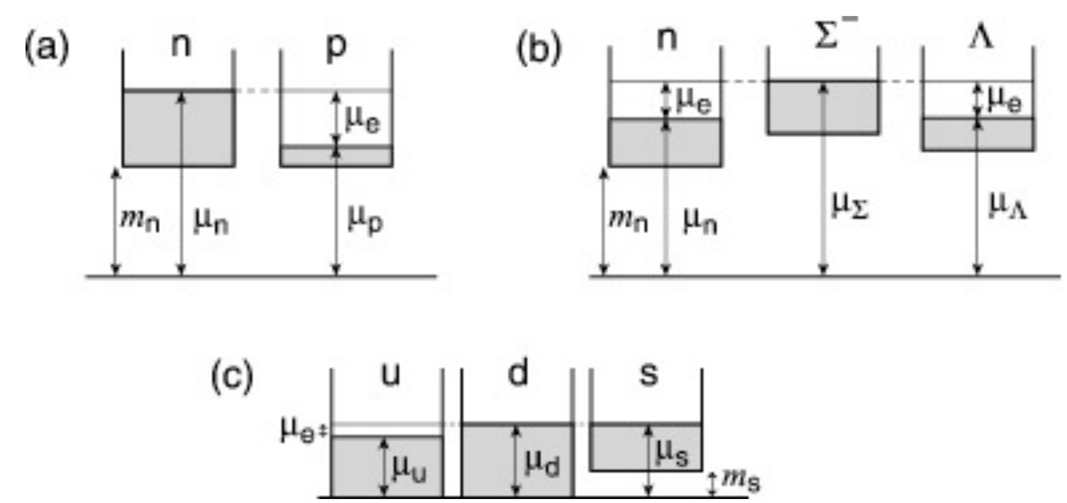




- *Produced in supernova explosions (Type II)*
- *Compact massive objects, $M \sim 1\text{-}2\ M_{solar}$, $R \sim 10$ km*
- *Normal stars eventually exhaust their nuclear fuel and collapse under gravity.*
- *Collapse is halted at nuclear density.*
- *A compact star forms, and the outer layers are blown off as a supernova.*

NEUTRON STAR STRUCTURE

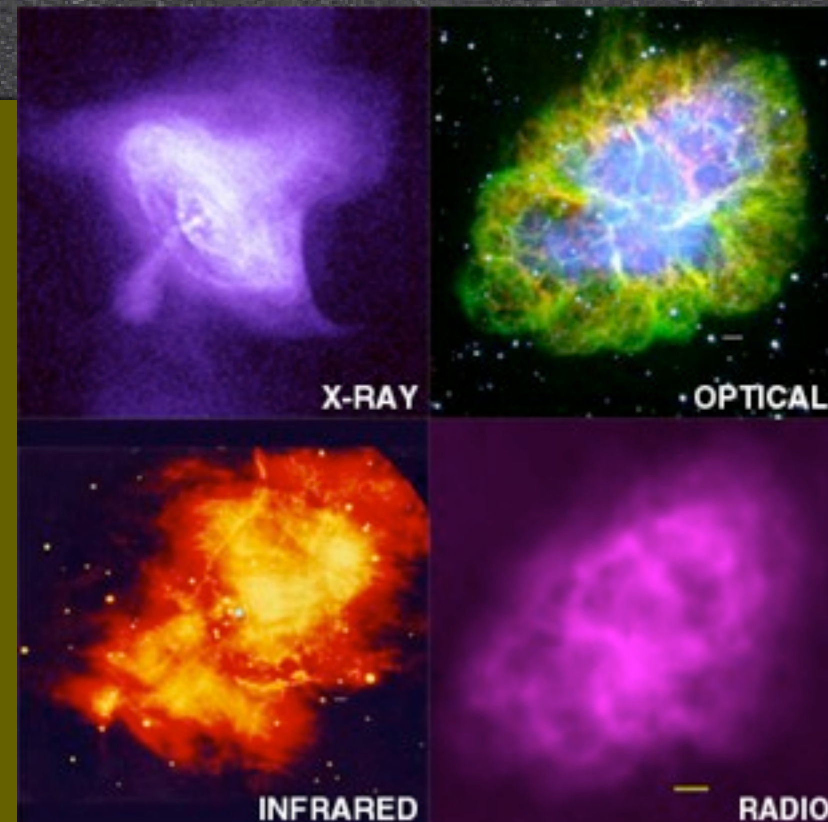


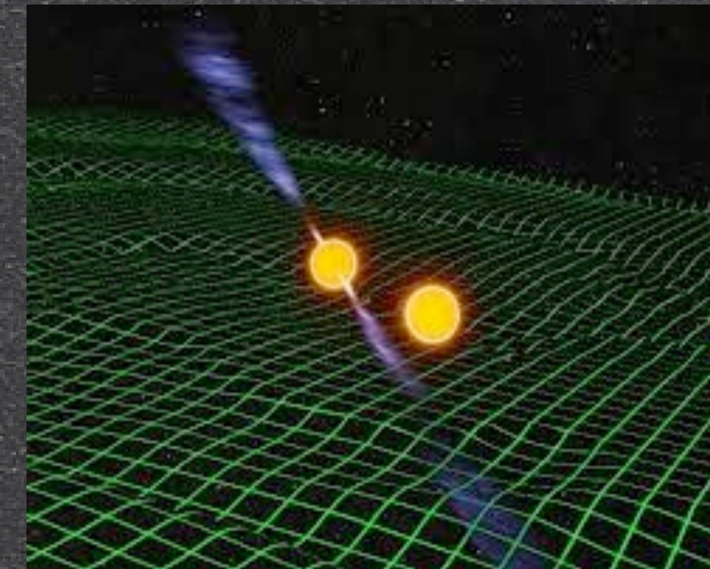


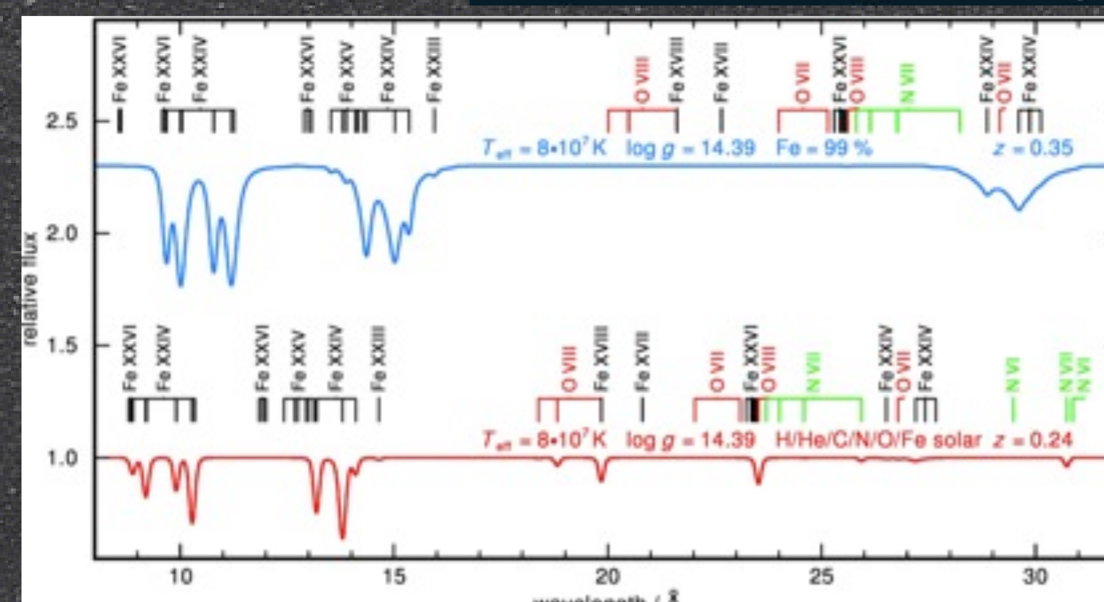
- *Compact massive objects, $M \sim 1\text{-}2\ M_{solar}$, $R \sim 10$ km*

Astrophysical Observables

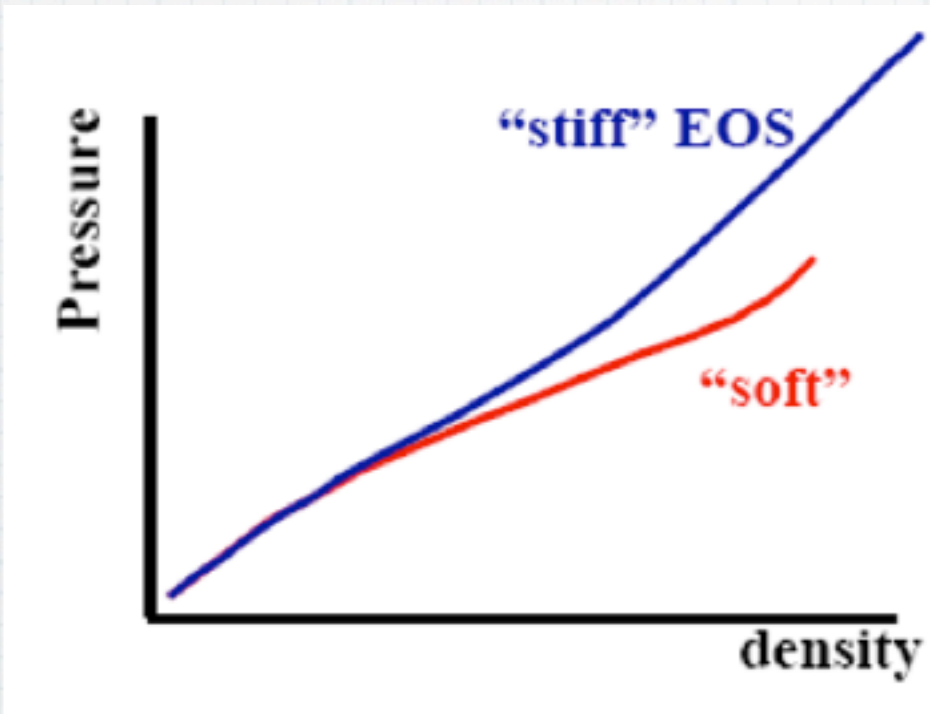
- *spin frequency*
- *Mass*
- *Radius*
- *moment of inertia*
- *gravitational redshift*
- *cooling*

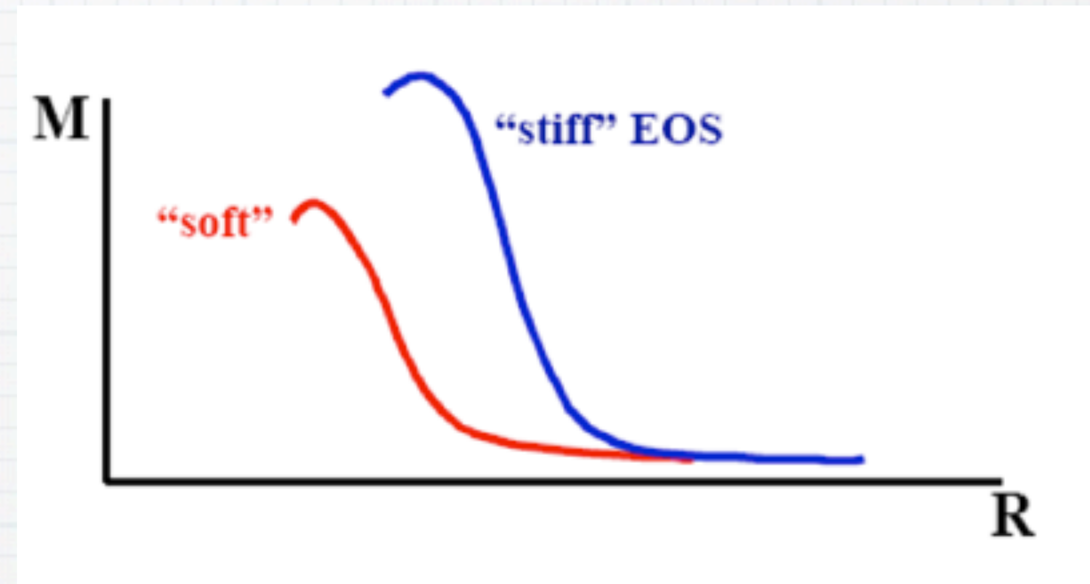






Equation of state (EoS)



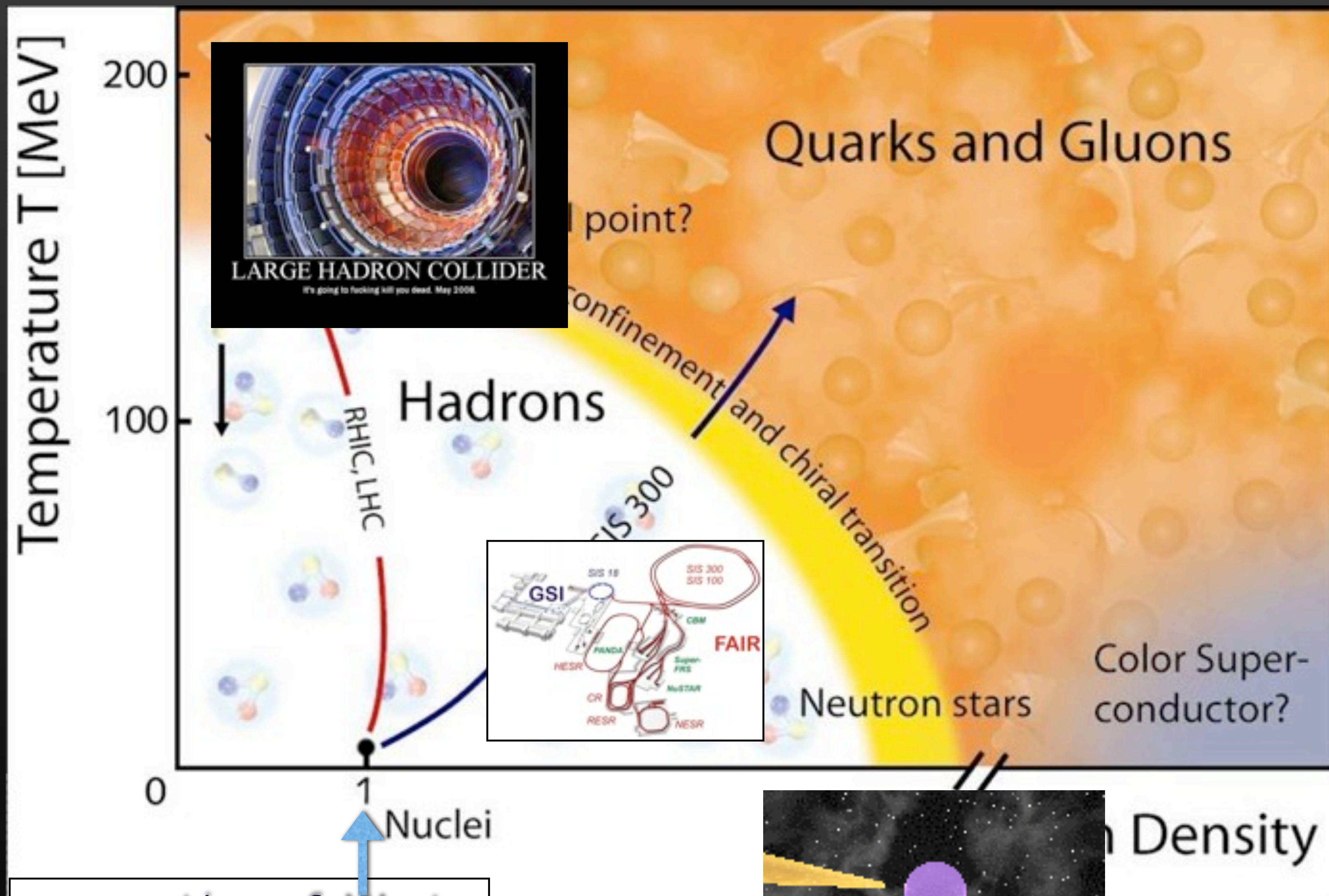


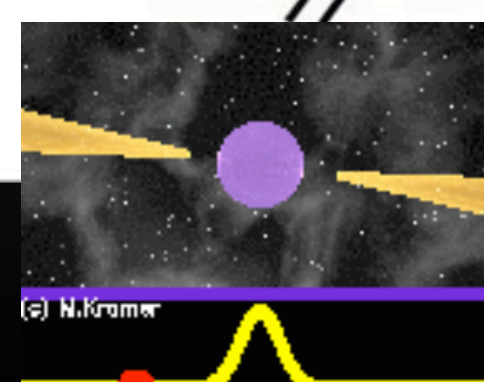
Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m + 4\pi p r^3)(\epsilon + p)}{r(r - 2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$$

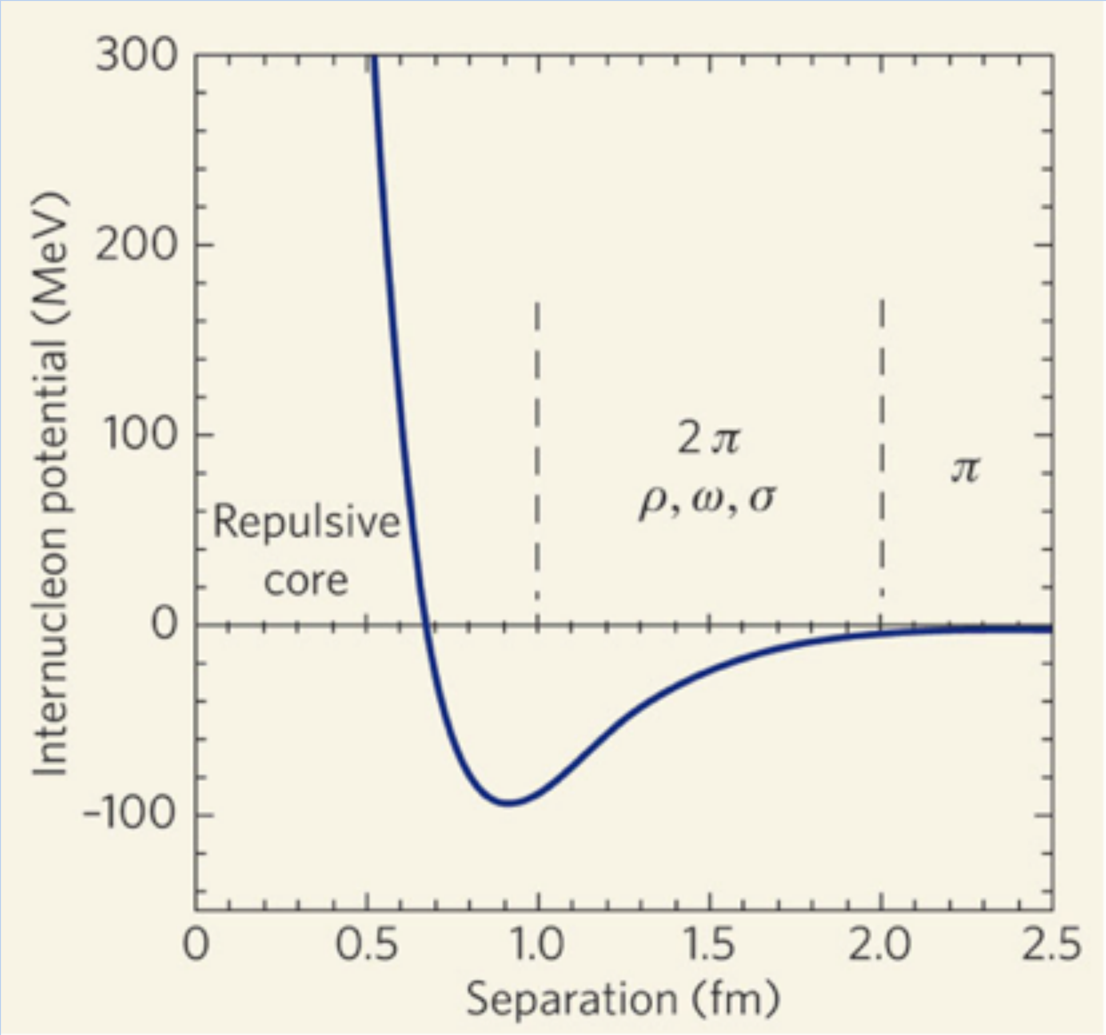
Cold and dense nuclear matter





properties of NM at saturation n_0:
B/A, m*/m, K, a_{sym}

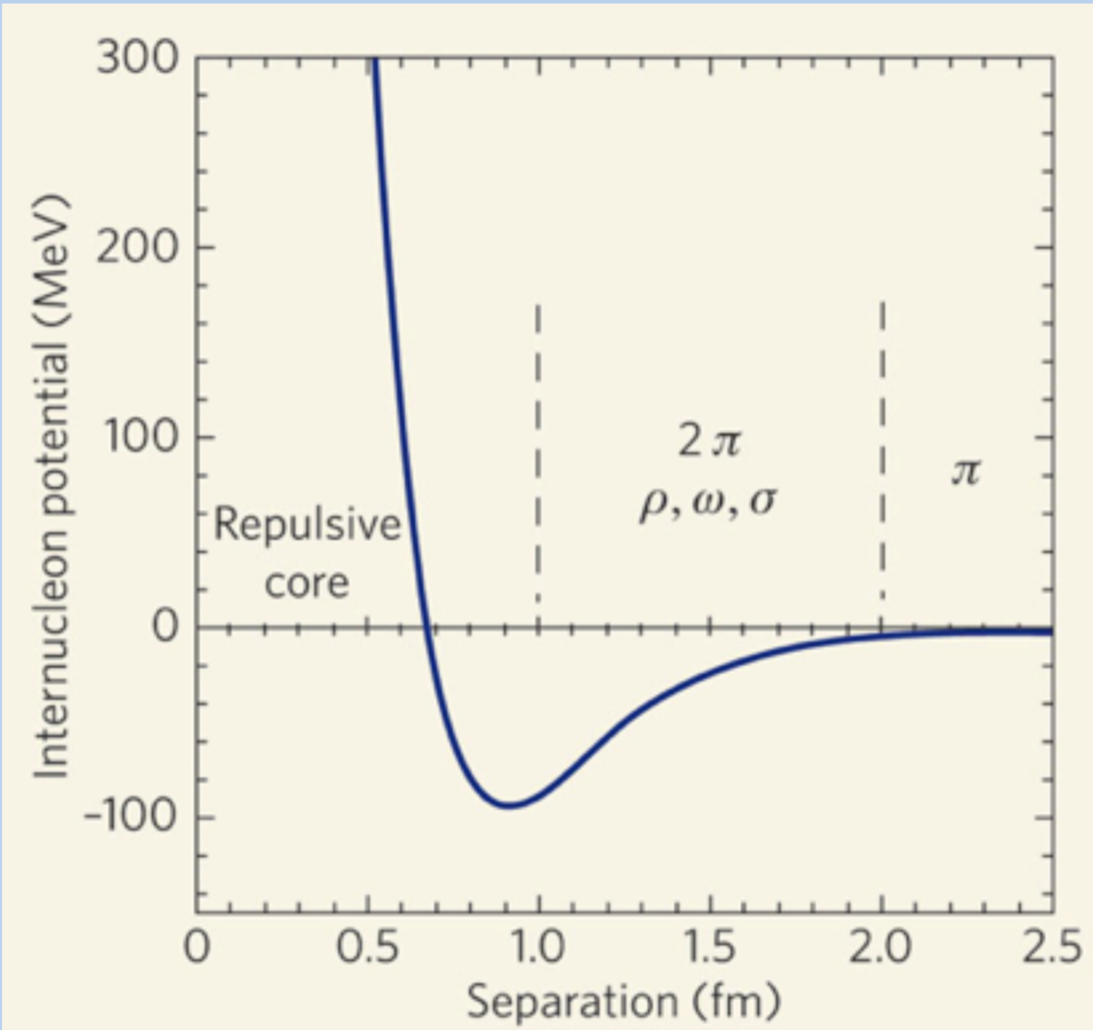
Relativistic Mean Field Model



Basic assumptions for describing nuclear and hypernuclear properties:

- *nucleons and hyperons interact through meson exchange*
- *assume that only low spin, isospin is needed (from OBEP)*
- *natural parity:* $\Pi = (-1)^J \rightarrow \pi$*-meson vanishes*
  T, J^{Π}: $\sigma(0, 0+)$, $\omega(0, 1-)$, $\rho(1, 1-)$
  [$\delta(1, 0+)$: *not needed*]
- σ*-meson: mimics attractive potential nonlinearities of the* σ*-meson-field: needed for a correct compression modulus of nuclear matter*
- ω*-meson: repulsive part of the potential*
- ρ*-meson: isospin dependent part of the potential*

Relativistic Mean Field Model



J. Schaffner and I. N. Mishustin, PRC 53, 1416 (1996)

THE HADRONIC PHASE

$$
\begin{aligned}
\mathcal{L} = & \sum_B \bar{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu \\
& -\frac{1}{2} g_{\rho B}\gamma_\mu \vec{\tau}_B \cdot \vec{\rho^\mu})\psi_B \\
& +\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma{}^2\sigma^2\right) - U(\sigma) \\
& -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega{}^2 \omega_\mu\omega^\mu \\
& -\frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2} m_\rho{}^2 \rho_\mu \cdot \rho^\mu + \mathcal{L}_{YY} \\
& + \sum_{e^-,\mu^-} \bar{\psi}_\lambda \left(i\gamma_\mu\partial^\mu - m\right)\psi_\lambda .
\end{aligned}
$$

where, $U(\sigma) = \frac{1}{3} b m_N (g_{\sigma N}\sigma)^3 + \frac{1}{4} c (g_{\sigma N}\sigma)^4$.

Hyperon-Hyperon interaction:

$$
\begin{aligned}
\mathcal{L}_{YY} = & \sum_B \bar{\psi}_B \left(g_{\sigma^* B}\sigma^* - g_{\phi B}\gamma_\mu\phi^\mu\right)\psi_B \\
& +\frac{1}{2}\left(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}\right) \\
& -\frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi_\mu\phi^\mu .
\end{aligned}
$$

Nucleon-meson coupling constants

THE HADRONIC PHASE

$$\begin{aligned}\mathcal{L} = & \sum_B \bar{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu \omega^\mu \\ & -\frac{1}{2} g_{\rho B}\gamma_\mu \vec{\tau}_B \cdot \vec{\rho^\mu})\psi_B \\ & +\frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma - m_\sigma{}^2 \sigma^2\right) - U(\sigma) \\ & -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega{}^2 \omega_\mu \omega^\mu \\ & -\frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2} m_\rho{}^2 \rho_\mu \cdot \rho^\mu + \mathcal{L}_{YY} \\ & + \sum_{e^-, \mu^-} \bar{\psi}_\lambda \left(i\gamma_\mu \partial^\mu - m\right)\psi_\lambda .\end{aligned}$$

where, $U(\sigma) = \frac{1}{3} b m_N (g_{\sigma N}\sigma)^3 + \frac{1}{4} c\,(g_{\sigma N}\sigma)^4$.

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- *Fit to nuclear matter (bulk properties)*
- *5 coupling constants*
- *5 properties of nuclear matter at saturation :*
- *saturation density $n_0 = 0.16\ fm^{-3}$,*
  binding energy B/A = -16.3 MeV,
  asymmetry energy a_{sym} = 32.5 MeV
  effective mass m/m = 0.55-0.8*
  incompressibility K= 200-300 MeV

Hyperon-meson coupling constants

THE HADRONIC PHASE

$$
\begin{aligned}
\mathcal{L} = & \sum_B \bar{\psi}_B (i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu \\
& -\frac{1}{2} g_{\rho B}\gamma_\mu \vec{\tau}_B \cdot \vec{\rho^\mu})\psi_B \\
& +\frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma{}^2\sigma^2\right) - U(\sigma) \\
& -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2} m_\omega{}^2\omega_\mu\omega^\mu \\
& -\frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2} m_\rho{}^2\rho_\mu\cdot\rho^\mu + \mathcal{L}_{YY} \\
& + \sum_{e^-,\mu^-} \bar{\psi}_\lambda \left(i\gamma_\mu\partial^\mu - m\right)\psi_\lambda .
\end{aligned}
$$

where, $U(\sigma) = \frac{1}{3} b m_N (g_{\sigma N}\sigma)^3 + \frac{1}{4} c (g_{\sigma N}\sigma)^4$.

Hyperon-Hyperon interaction:

$$
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& +\frac{1}{2}\left(\partial_\mu\sigma^*\partial^\mu\sigma^* - m_{\sigma^*}^2\sigma^{*2}\right) \\
& -\frac{1}{4}\phi_{\mu\nu}\phi^{\mu\nu} + \frac{1}{2} m_\phi^2\phi_\mu\phi^\mu .
\end{aligned}
$$

- *SU(6) relations*

$$
\begin{aligned}
\frac{1}{3} g_{\omega N} = \frac{1}{2} g_{\omega\Lambda} = \frac{1}{2} g_{\omega\Sigma} &= g_{\omega\Xi} \\
g_{\rho N} = \frac{1}{2} g_{\rho\Sigma} &= g_{\rho\Xi} \\
g_{\rho\Lambda} &= 0
\end{aligned}
$$

$$\frac{g_{\phi\Lambda}}{g_{\omega N}} = -\frac{\sqrt{2}}{3}, \quad \frac{g_{\phi\Xi}}{g_{\omega N}} = -\frac{2\sqrt{2}}{3}$$

- *Hypernuclear data: Potential depths of hyperons in nuclear matter* $\quad U_Y^N = g_{\sigma Y}\sigma^{eq} + g_{\omega Y}\omega_0^{eq}$

$$U_\Lambda^N = -30 MeV, U_\Sigma^N = +30 MeV, U_\Xi^N = -18 MeV$$

$$g_{\sigma^* N} = g_{\sigma^* Y} = 0$$

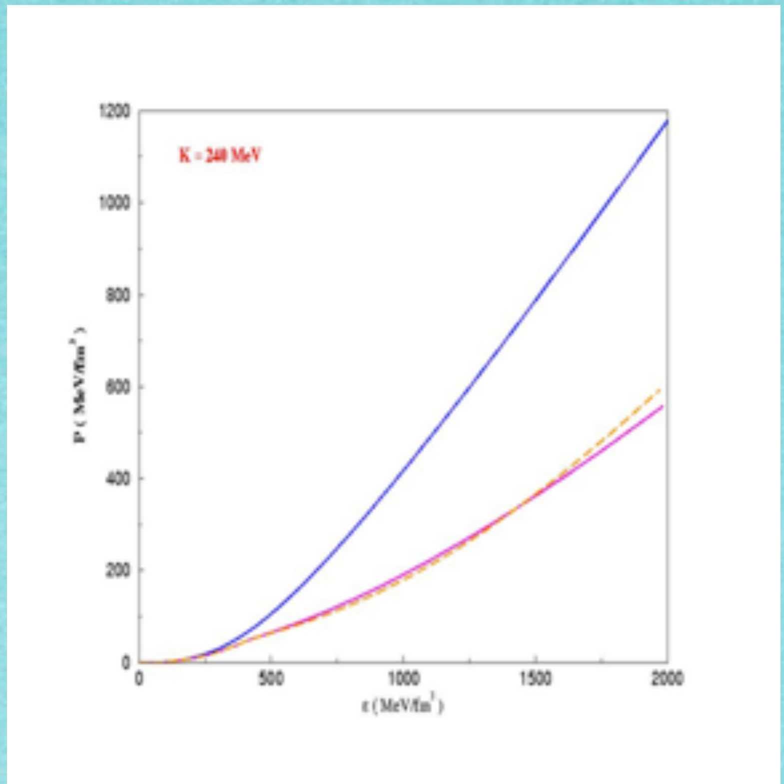
Equation of state

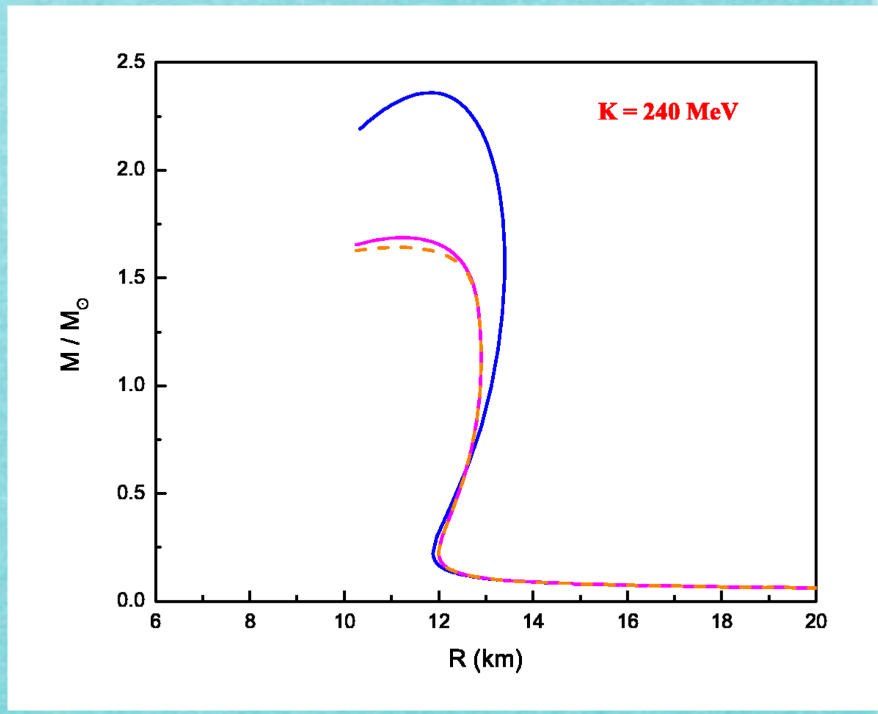
The total energy density, $\varepsilon = \varepsilon_B + \varepsilon_l$

$$\begin{aligned}
\varepsilon \;=\;& \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} \\
&+ \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\
&+ \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_{F_B}}(k^2+m_B^{*2})^{1/2}k^2\ dk \\
&+ \sum_l \frac{1}{\pi^2}\int_0^{K_{F_l}}(k^2+m_l^2)^{1/2}k^2\ dk
\end{aligned}$$

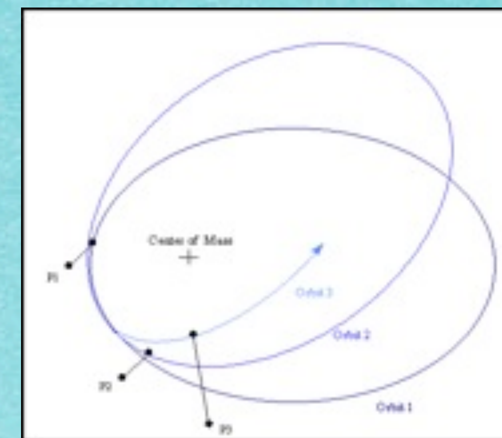
The pressure

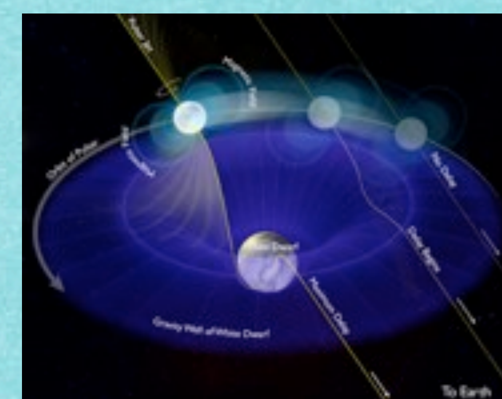
$$\begin{aligned}
P \;=\;& -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
&- \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\phi^2\phi_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\
&+ \frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_{F_B}}\frac{k^4\ dk}{(k^2+m_B^{*2})^{1/2}} \\
&+ \frac{1}{3}\sum_l \frac{1}{\pi^2}\int_0^{K_{F_l}}\frac{k^4\ dk}{(k^2+m_l^2)^{1/2}}\ .
\end{aligned}$$





Constraints from Neutron Star masses :
Relativistic binaries



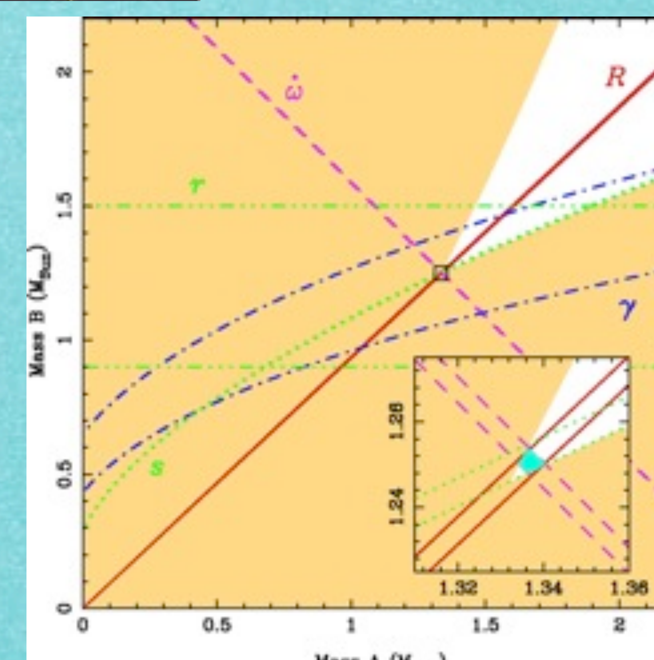


Keplerian parameters

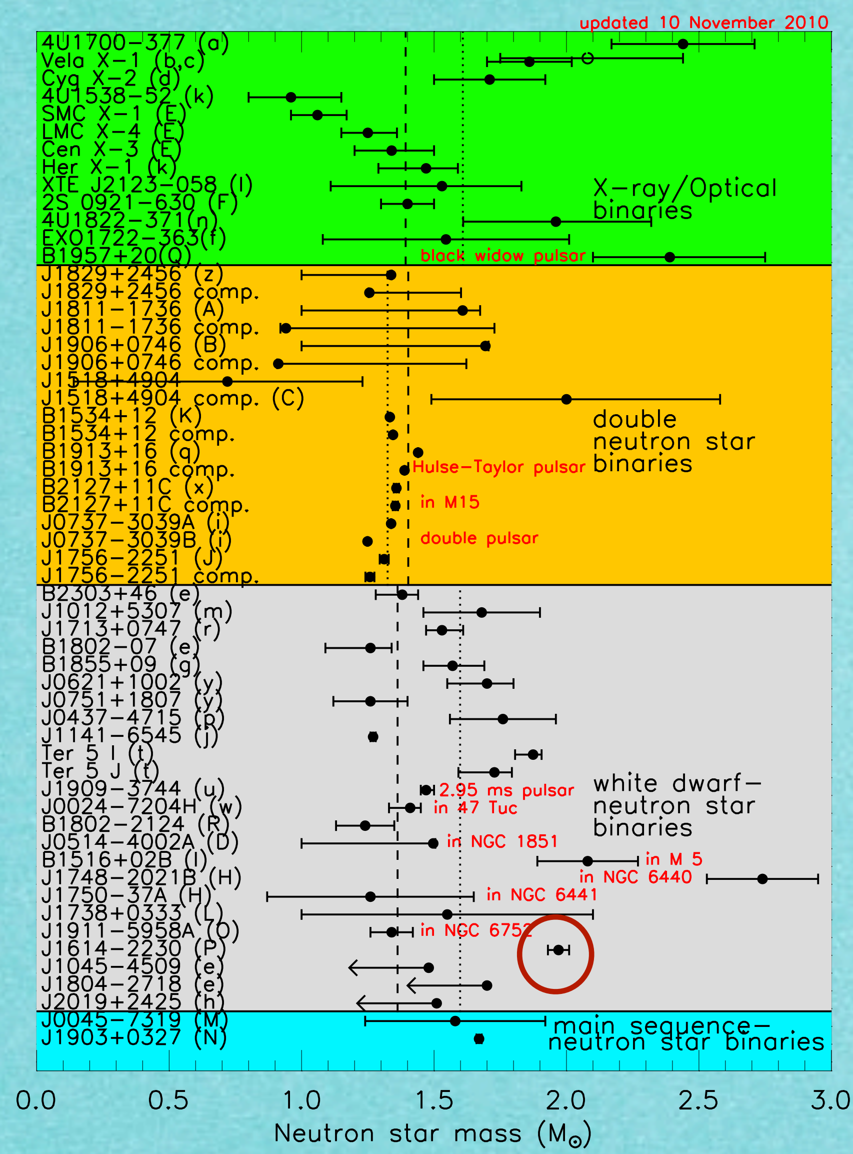
- Orbital period P_b
- Projected semi-major axis $x = (a_p \sin i) / c$
- Orbital eccentricity e
- Longitude of periastron ω
- Epoch of periastron passage T_o

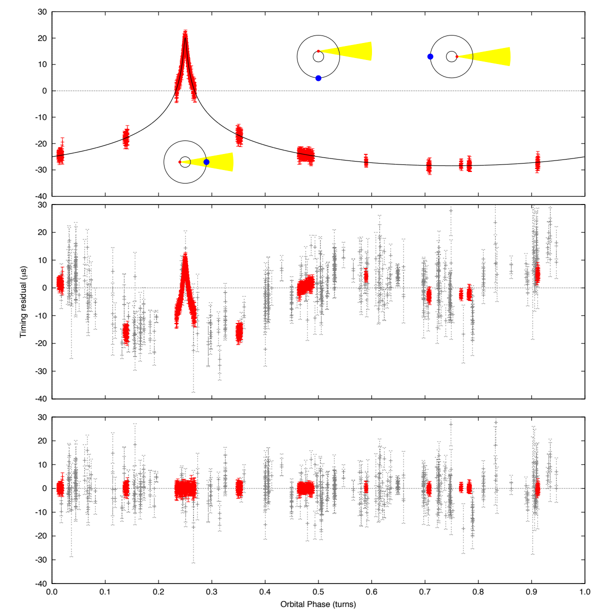
Post-Keplerian Parameters

- *Relativistic advance of periastron* $\dot{\omega}$
- *Gravitational redshift and time dilation* γ
- *Orbital decay in period* $\dot{P}_b$
- *Shapiro time delay (range r and shape s)*



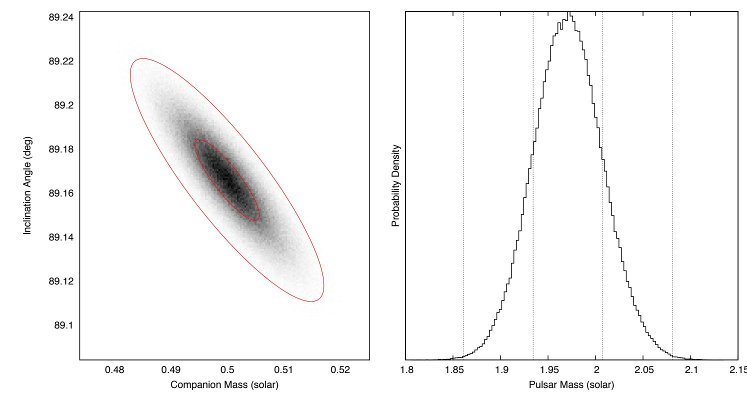
Highest mass measurement : PSR J1614-2230





$1.97 \pm 0.04\ M_\odot$

Timing residual as a function of pulsar's orbital phase



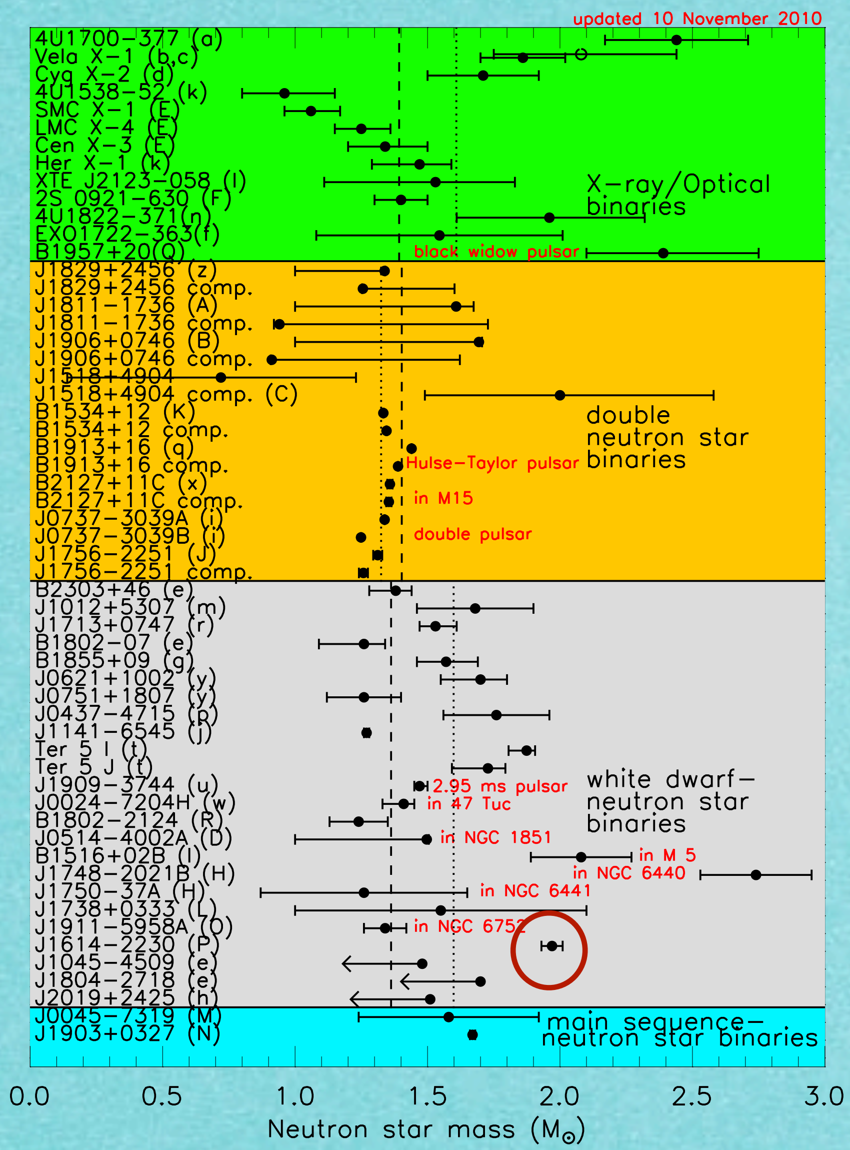
Monte Carlo analysis: Probability density function

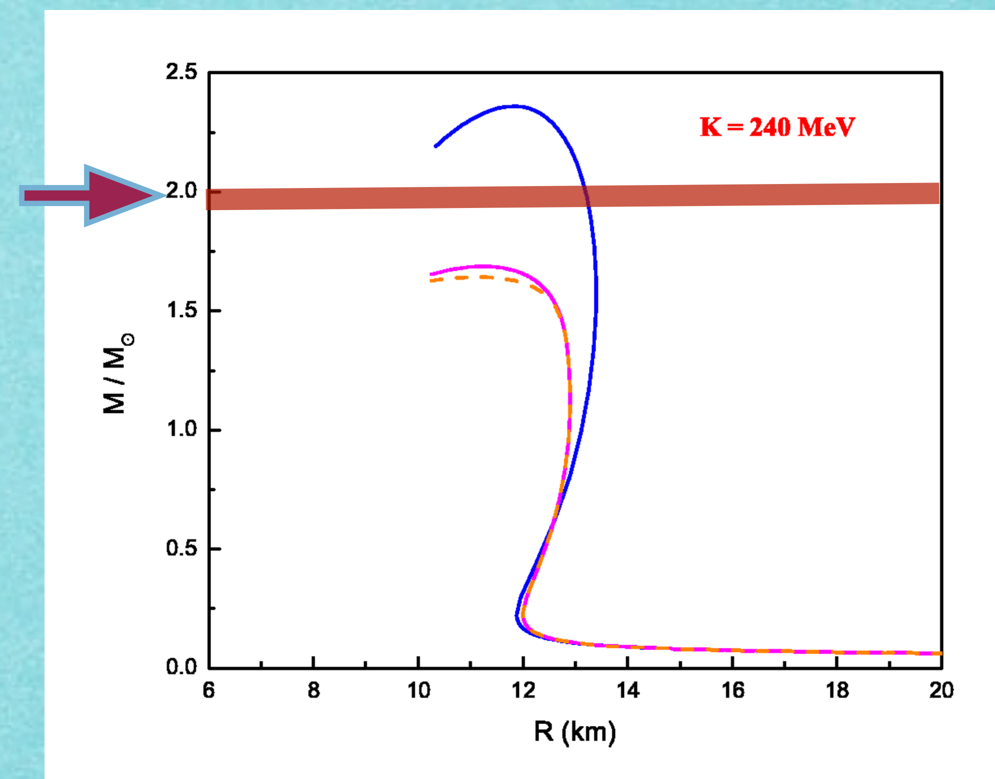
Lattimer and Prakash, arXiv:1012.3208

Demorest et al (Nature 2010)

Constraining the EoS

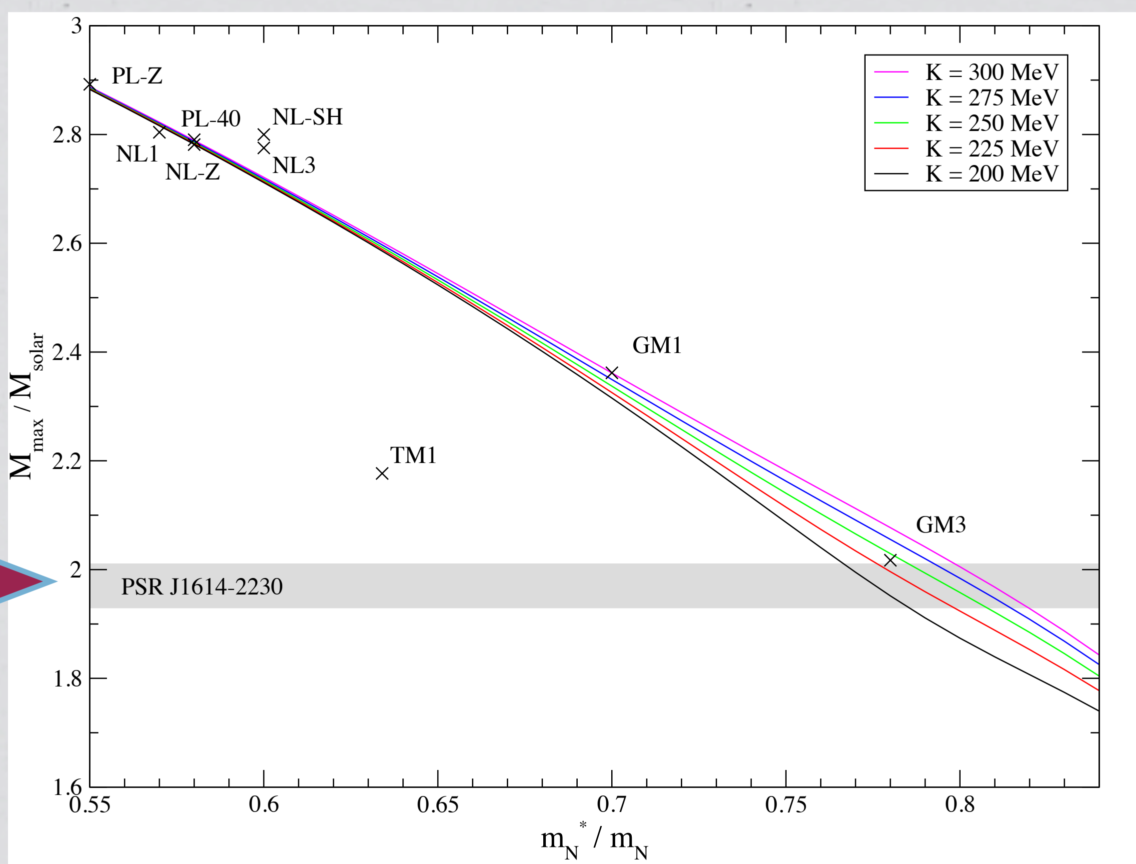
$$\boxed{M^{max}(theo) > M^{max}(obs)}$$





Lattimer and Prakash, arXiv:1012.3208

Compression Modulus and EffectiveMass



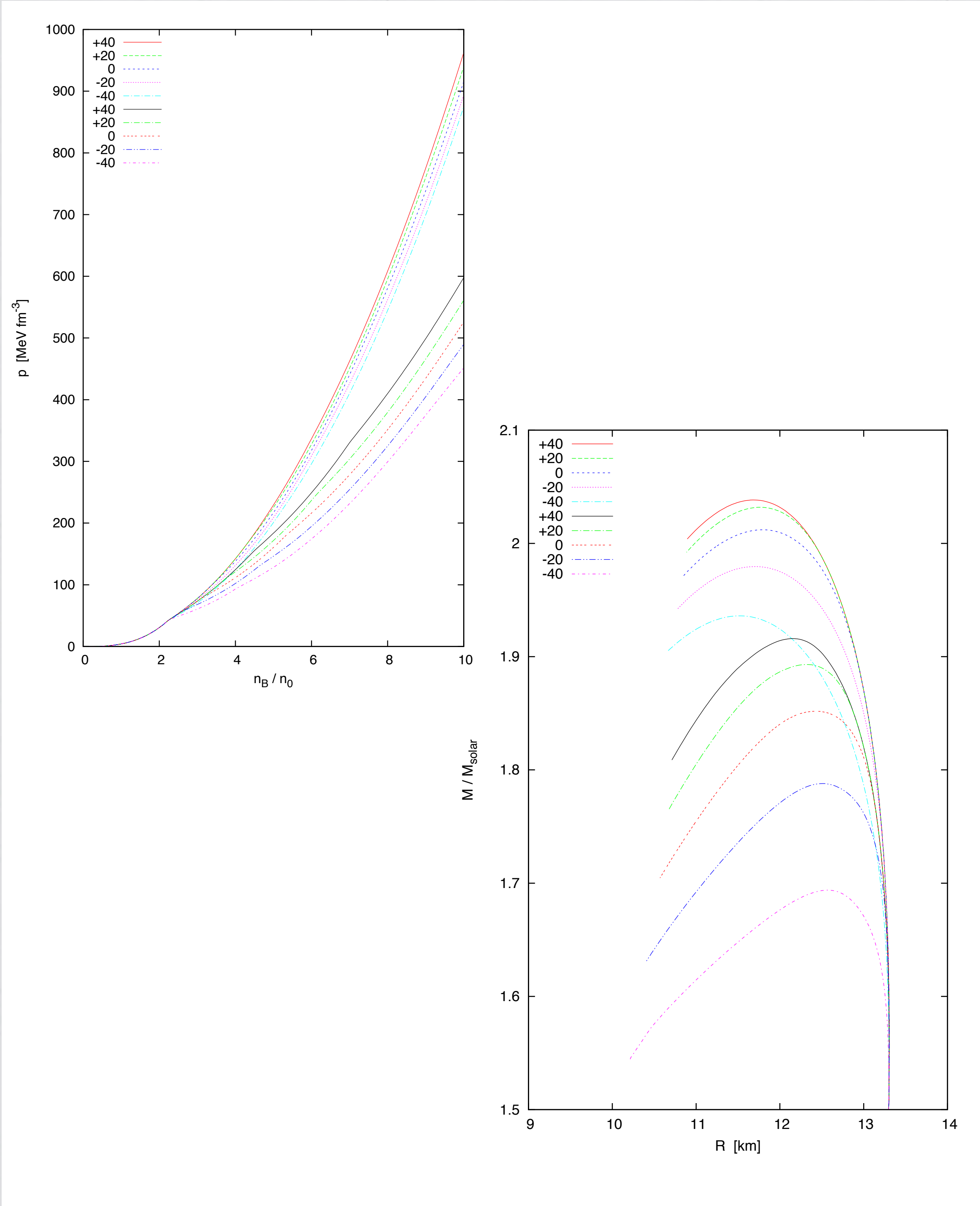
- In the range 200 − 300 MeV, K plays virtually no role for $m^*_N / m_N \leq 0.6$,
  most prominent effect is for $m^*_N / m_N \geq 0.7$

- The most massive stars (up to ~ 2.9 $M_\odot$) are obtained for low effective nucleon masses independently of the compression modulus, while for $m^*_N / m_N \geq 0.78$ we cannot reach the mass limit of $1.97 \pm 0.04 M_\odot$ if the compression modulus is too low.

- K is a good indicator for the stiffness of the nuclear matter EoS at low densities (around saturation), but the high density behavior is much more sensitive to m^*_N / m_N

S. Weissenborn, D.C. and J. Schaffner-Bielich, Nucl. Phys. A 881(2012) 62

Varying the Potential Depths



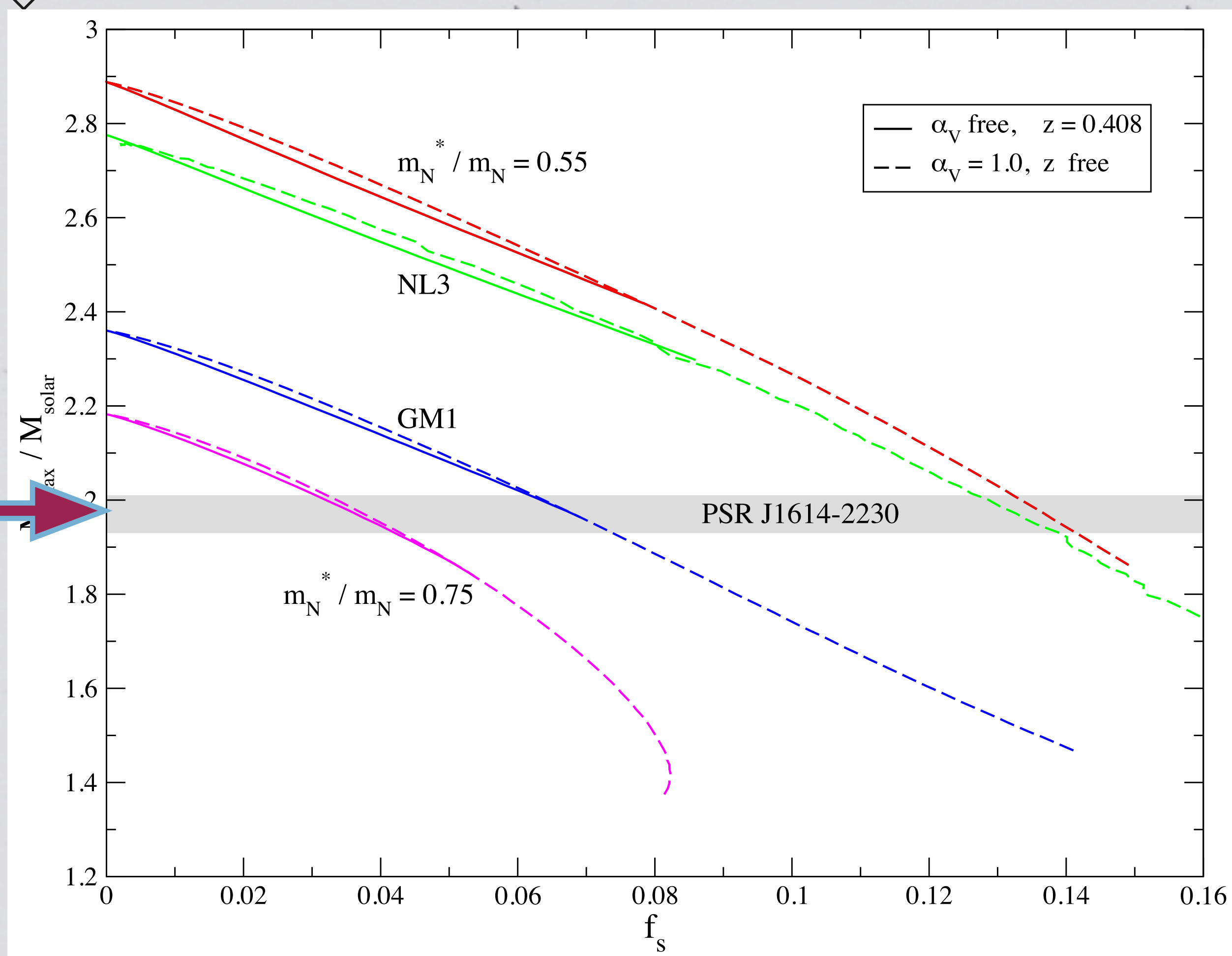
- We varied the coupling strength of scalar meson by changing the potential depths of Σ and Ξ hyperons in nuclear matter, keeping the Λ potential fixed.

- For deeper potential depths, EoS is softer and maximum masses smaller

- The effect of potential depths on EoS is not enough to raise maximum masses above observed value, unless the ϕ meson is included

S. Weissenborn, D.C. and J. Schaffner-Bielich, Nucl. Phys. A 881(2012) 62

Beyond SU(6)



- We plot the maximum masses of neutron stars as a function of strangeness fraction f_s (the number of strange quarks per baryon) for four different equations of state
- On decreasing the strangeness fraction in the core, there is a corresponding increase in the maximum mass of the star.
- At zero strangeness fraction the nucleonic limit is reached, and this corresponds to the highest value of the maximum mass.
- One can predict the maximally allowed strangeness fraction in maximum mass neutron stars.

$$\frac{M_{max}}{M_\odot} = \frac{M_{max}(f_s = 0)}{M_\odot} - c\left(\frac{f_s}{0.1}\right) \ , \ c \approx 0.6 M_\odot$$

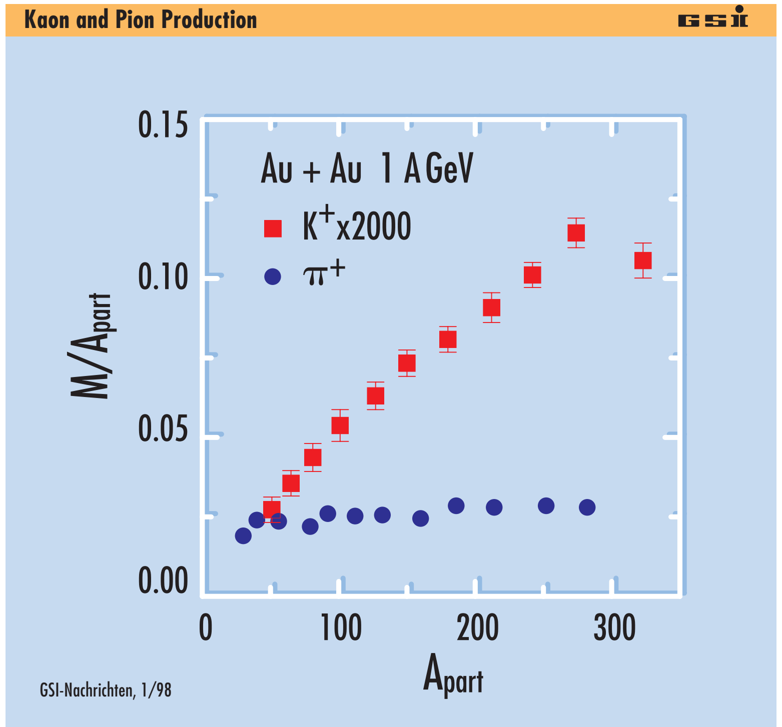
S. Weissenborn, D.C. and J. Schaffner-Bielich, Phys. Rev. C 85 (2012) 065802

K^+ meson production in heavy-ion collisions

KaoS experiment,
GSI Darmstadt

$NN \to N\Delta \to NYK$

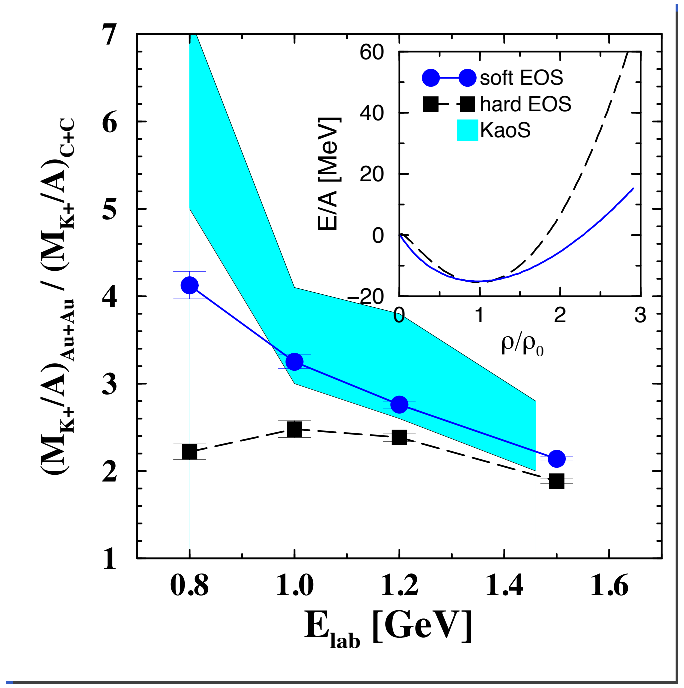
$\quad\quad\quad\quad\quad \hookrightarrow \pi N \to YK$



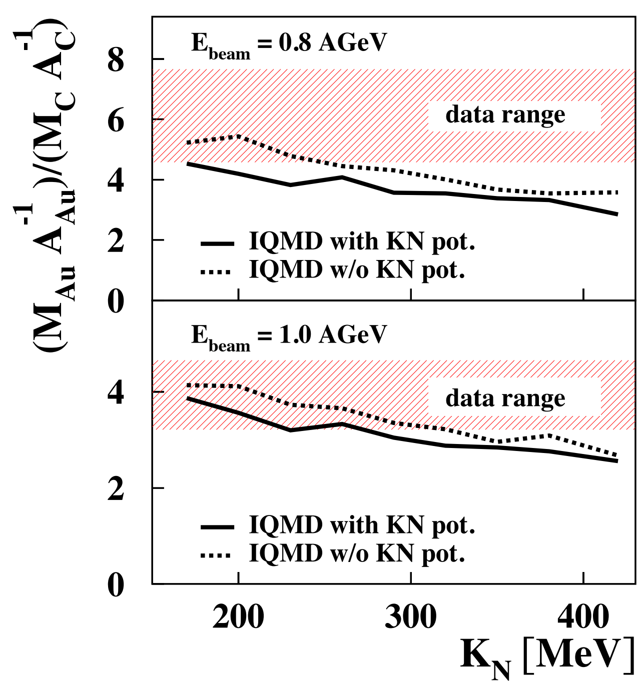
Subthreshold production of K^+ particles

- K^+ particles produced by multiple NN collisions ($NN \to N\Lambda K$, $NN \to NNK\bar{K}$) or secondary collisions ($\pi N \to \Lambda K$, $\pi\Lambda \to N\bar{K}$)
- Nuclear matter compressed up to $\sim$ 2-3 n_0
- Production of K^+ particles sensitive to the nuclear EoS $\Rightarrow$ tool to probe compressibility of nuclear matter at $\sim$ 2-3 n_0

Soft equation of state from heavy-ion data



Sturm et al. (KaoS collaboration), PRL 2001

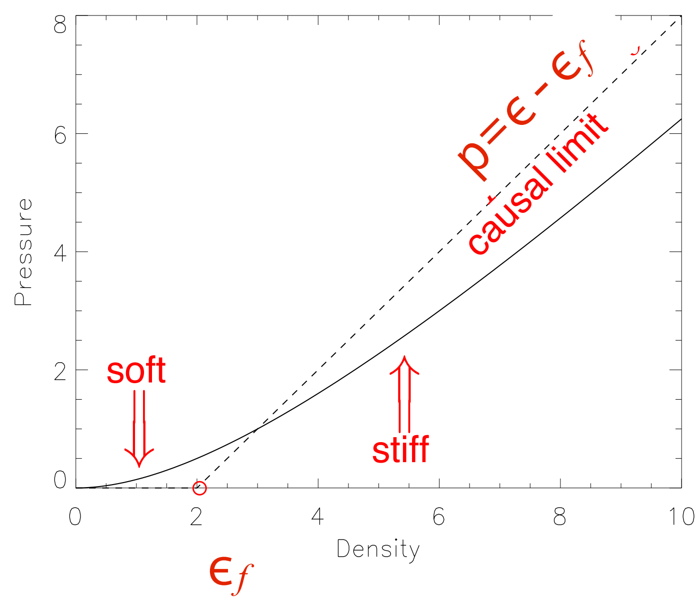


Hartnack, Oeschler, Aichelin, PRL 2006

- *K^+ multiplicity ratio in Au+Au and C+C collisions at 0.8 AGeV and 1.0 AGeV is sensitive to the compression modulus of matter*
- *transport model calculations performed: Skyrme-type nucleon potential with 2BF, 3BF were applied, with parameters to reproduce a soft EoS (with K =200 MeV) and a stiff one (with K =380 MeV).*
- *transport models agree, confirm that matter in the collision zone reaches densities up to 2-3 n_0*
- *only K~ 200 MeV can describe the data (KaoS collaboration, 2007)*

⇒ ***the nuclear EoS is soft!***

EoS soft or stiff?



$$M_{\rm max} = 4.2 M_\odot (\epsilon_0/\epsilon_f)^{1/2}$$

Rhoades & Ruffini (1974)
Hartle (1978)

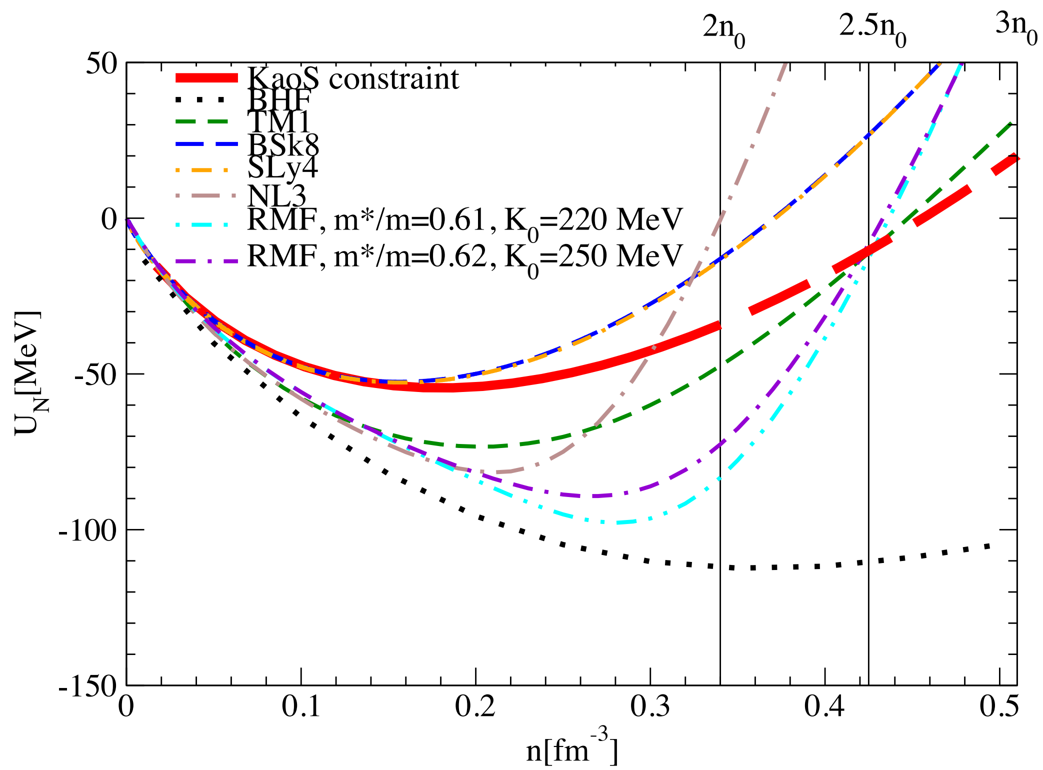
- *Stiffest causal EoS: $p = \epsilon - \epsilon_f$ above the fiducial density ϵ_f*
- *at high densities, smooth transition to the stiffest EoS*
- *gives the highest possible mass of a compact star*
- *At low densities, EoS should satisfy KaoS constraint*

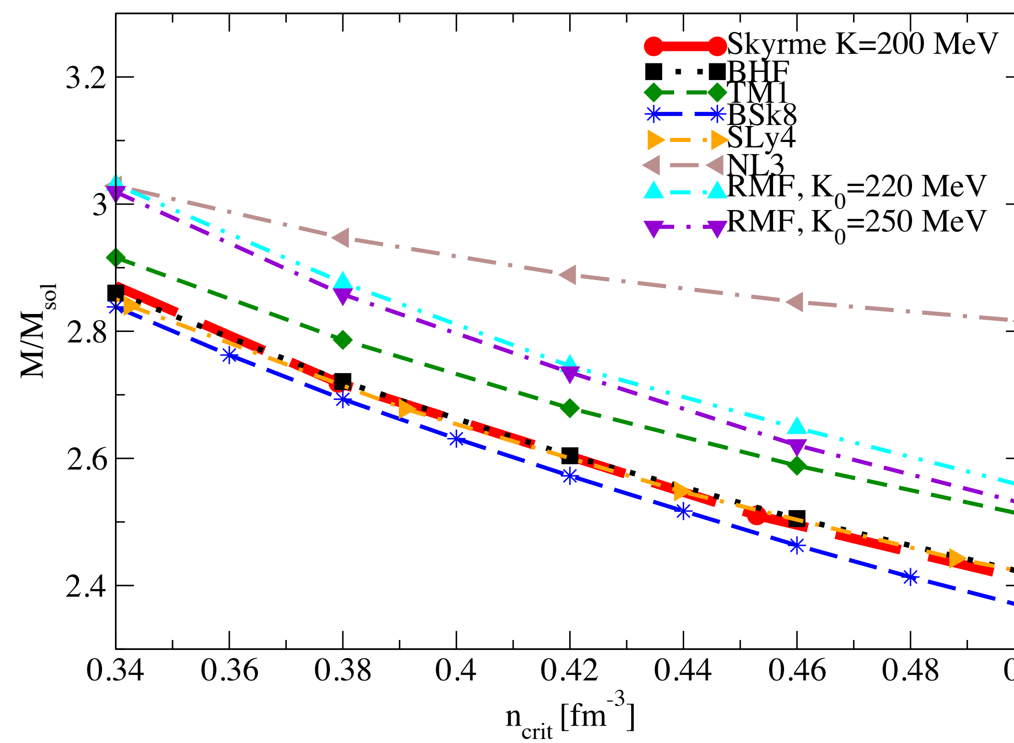
Theoretical Models

- *Brueckner Hartree Fock models (BHF)*
  - realistic N-N interactions

- *Phenomenological models*
  - Skyrme interactions (Bsk8, SLy4)

- *Relativistic Mean Field Models (RMF)*
  - with non-linear interaction of mesons
  - parameter sets with different scalar and vector self-interactions
  - coupling parameters fitted to properties of bulk nuclear matter (GL, GM1) or to properties of nuclei (NL3, TM1, FSUGold)





• *the EoS is chosen so as to obtain a nucleon potential similar to or more attractive than the KaoS constraint within the density limits*

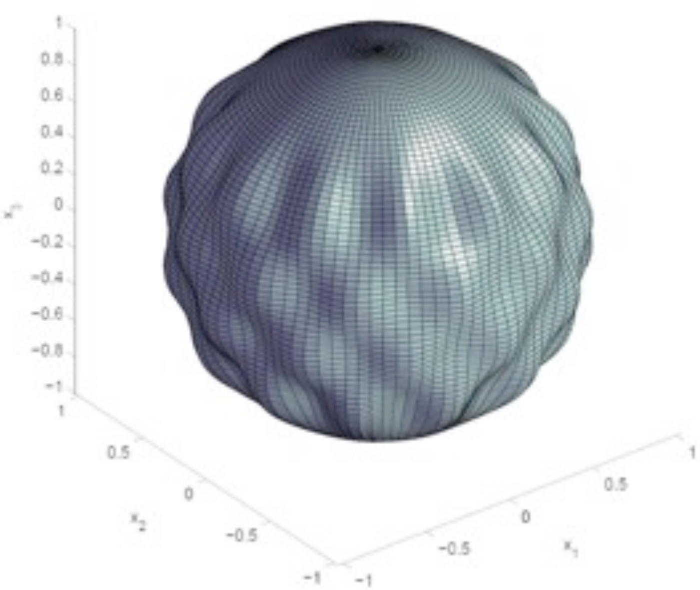
• *highest allowed NS mass 3M_{sol} at n_{crit} ~2 n_0, smaller maximum mass obtained for n_{crit} ~ 3 n_0*

• *higher n_{crit} is, the later is the onset of the stiffest EoS in the star's interior, less mass is supported*

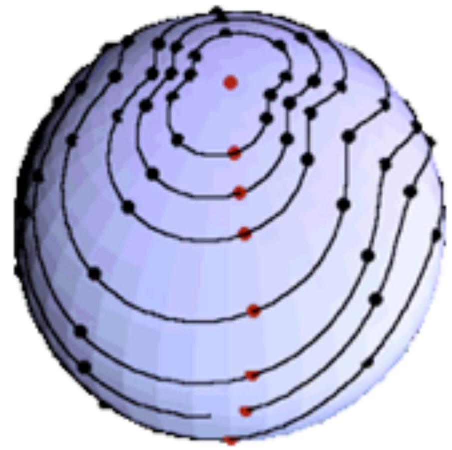
• *a pulsar of 2.7M_{sol} not ruled out by KaoS data, but requires a fiducial density of ~ 2.2 - 2.5 n_0*

I. Sagert, L. Tolos, D. Chatterjee, J. Schaffner-Bielich and C. Sturm, Phys. Rev. C 85 (2012) 065802

Neutron Star Oscillations : Asteroseismology



p-mode

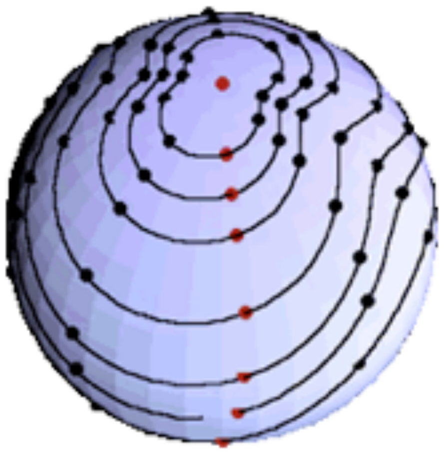


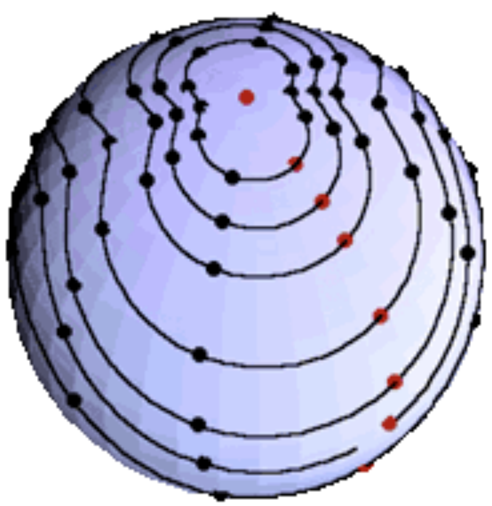
r-mode

Non-radial Oscillations:
f-modes: fundamental
g-modes: byuoyancy
p-modes: pressure
R-modes: Coriolis force
w-modes: space-time

G W detectors

R-modes: probe of NS interior





- *R-modes are generic to all rotating neutron stars*
- *They are unstable by the CFS mechanism: R- mode amplitude grows under the effect of its own gravitational radiation-reaction; possible sources of GW*
- *The instability can be damped by (shear, bulk) viscosity, which depend on the composition of the neutron star interior*
- *Shear viscosity results from momentum transport due to particle scattering; For normal fluids, the main contribution comes from n-n scattering*
- *Bulk viscosity results from variation in pressure and density when the system is driven away from beta equilibrium*
- *timescale associated with growth/dissipation*
  $\tau_{\zeta,\,\eta} \gg \tau_{GW}$: *r-mode unstable, star spins down*
  $\tau_{\zeta,\,\eta} \ll \tau_{GW}$: *r-mode damped, star can spin rapidly*

R-mode instability window

Possible sources of bulk viscosity:

- *Leptonic weak processes*

  direct Urca process:

$$n \to p + e^- + \bar{\nu}_e$$
$$p + e^- \to n + \nu_e$$

  modified Urca process:

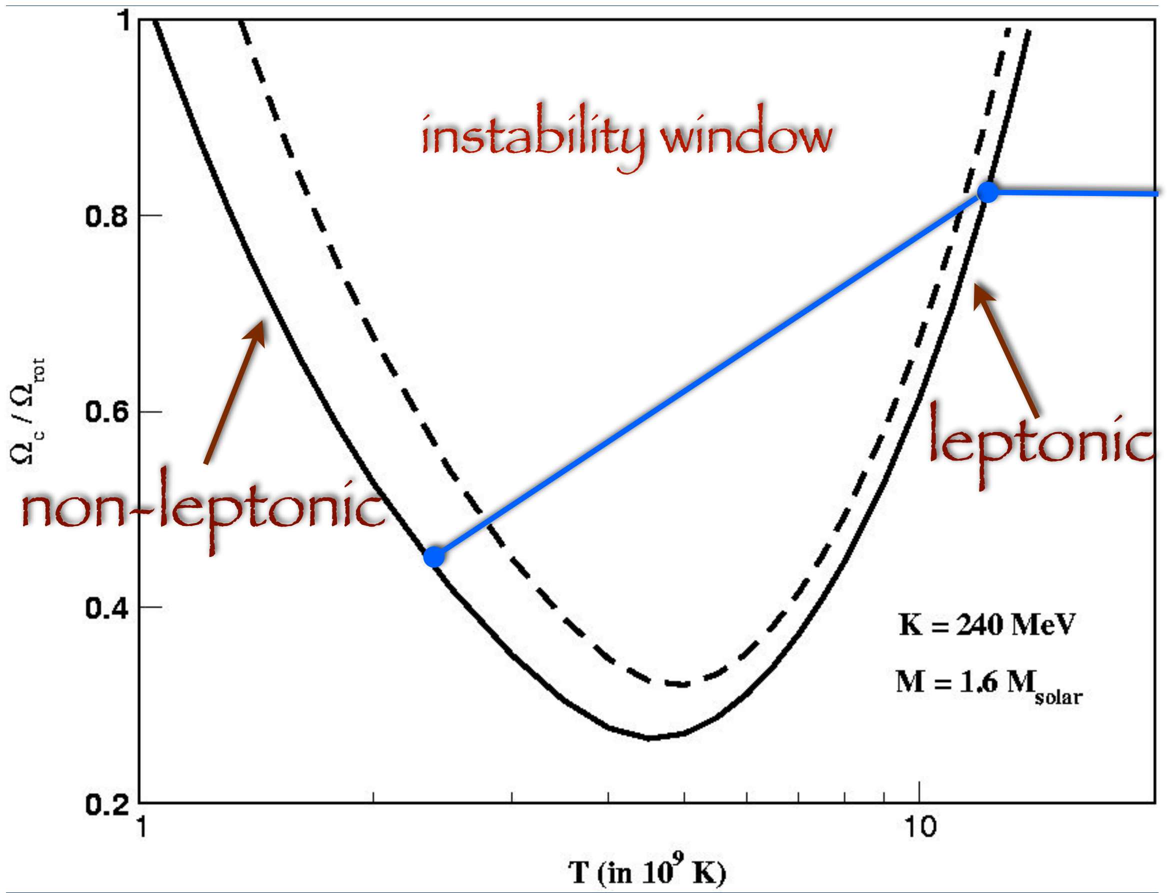
$$n + N \to p + e^- + \bar{\nu}_e + N$$
$$p + e^- + N \to n + \nu_e + N$$

- *Non-leptonic processes involving hyperons, Bose condensates or quarks*

$$n + p \leftrightarrow p + \Lambda$$
$$n + n \leftrightarrow n + \Lambda$$

$$n \to p + K^-$$

$$u + d \leftrightarrow u + s$$



- *r-mode instability damped by leptonic bulk viscosity at high T and non-leptonic bulk viscosity at low T*

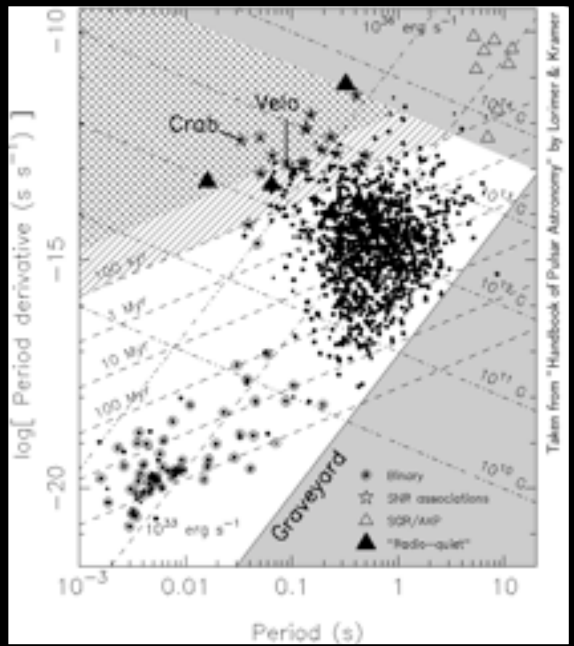
- *In the intermediate T regime, there exists an Instability window*

D.C. and D. Bandyopadhyay, Phys. Rev. D 74 (2006) 023003

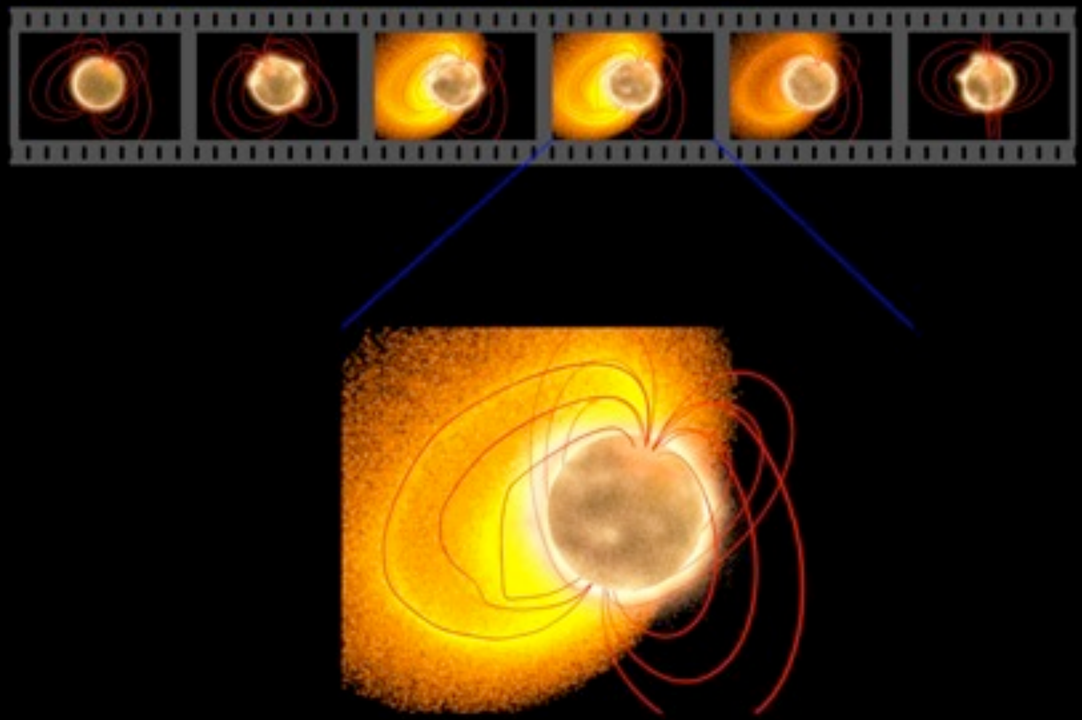
Current Interest: Magnetars

Ultra strong magnetic field $B \sim 10^{15}$ G

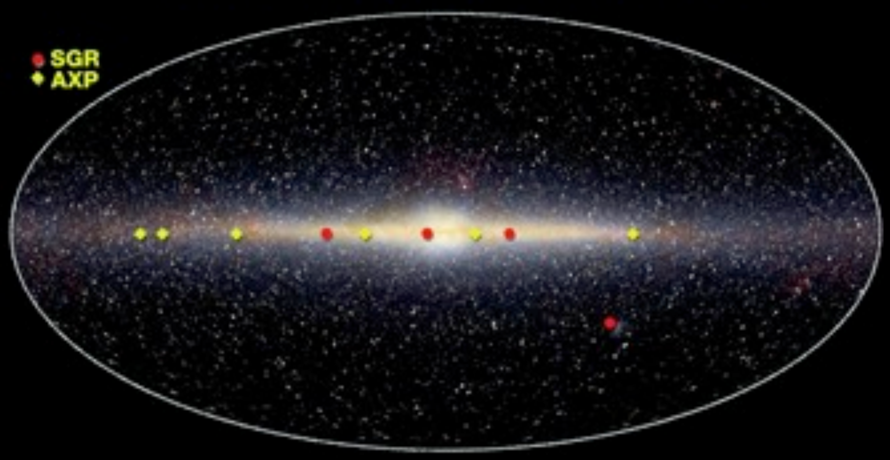
- *Identified with AXPs+SGRs*
- *Slow rotators (Spin period ~ 5-10 s)*
- *Rapid spin-down due to magneto-dipolar losses*
- *Highly magnetized!*
- *Direct measurements of the field (Ibrahim et al.)*
- *Interior field could be much higher!*



Magnetar burst sequence

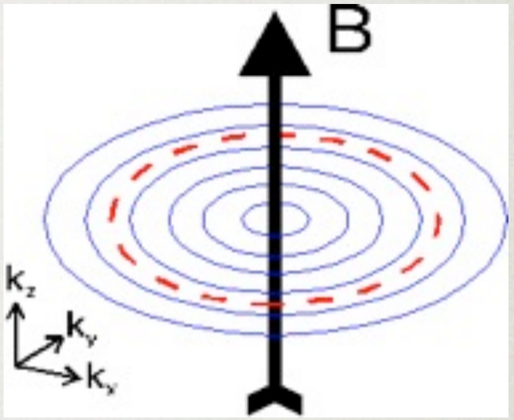


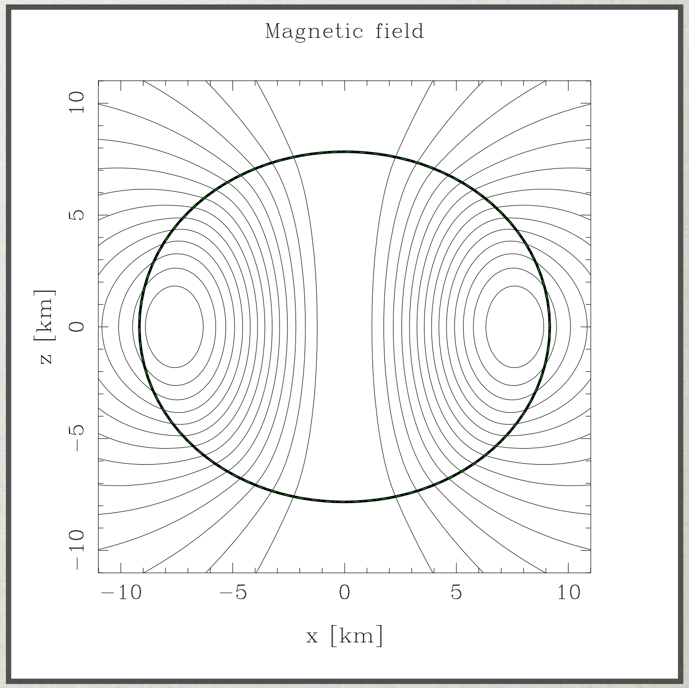
Known magnetar candidates



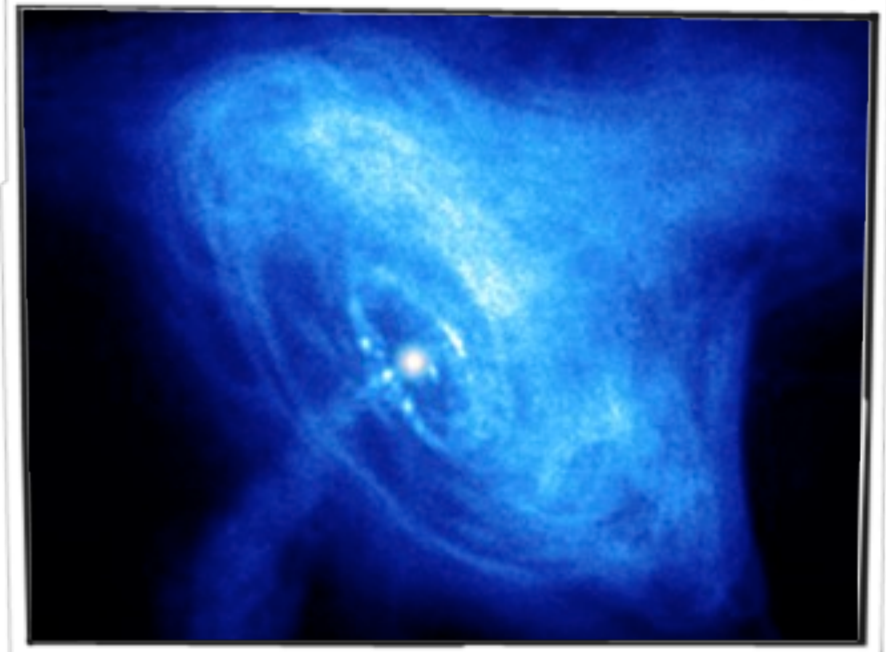
EFFECT OF MAGNETIC FIELD

- *Magnetic field may influence the EoS in 2 ways :*
- *In the presence of the magnetic field, the motion of the charged particles is Landau quantized in the direction perpendicular to the magnetic field.*
- *Background magnetic field breaks the spherical symmetry of the system*
- *This results in pressure anisotropy - the pressure longitudinal to the field is distinct from the pressure transverse to the field*
- *The aim is to calculate stationary and axisymmetric rotating NS models with strong magnetic field within the framework of GR*
- *At large magnetic fields, one should solve Einstein's equations in an axisymmetric metric, which is determined self consistently from the axisymmetric energy-momentum tensor of the star, and solve it numerically*
- *The problem can be formulated on the basis of the 3+1 Formalism (Bonazzolla, Gourgoulhon, Salgado, Marck, 1993)*





Neutron stars are perfect astrophysical laboratories for ..



- EoS of cold and dense matter
- shear and bulk viscosity
- oscillation modes and gravitational waves
- physics in ultra strong magnetic fields
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