Journée des Thèses - 03 Juillet 2014 - Meudon

SIMULATING CLUSTERING DARK ENERGY COSMOLOGIES

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cnrs dépasser les frontières

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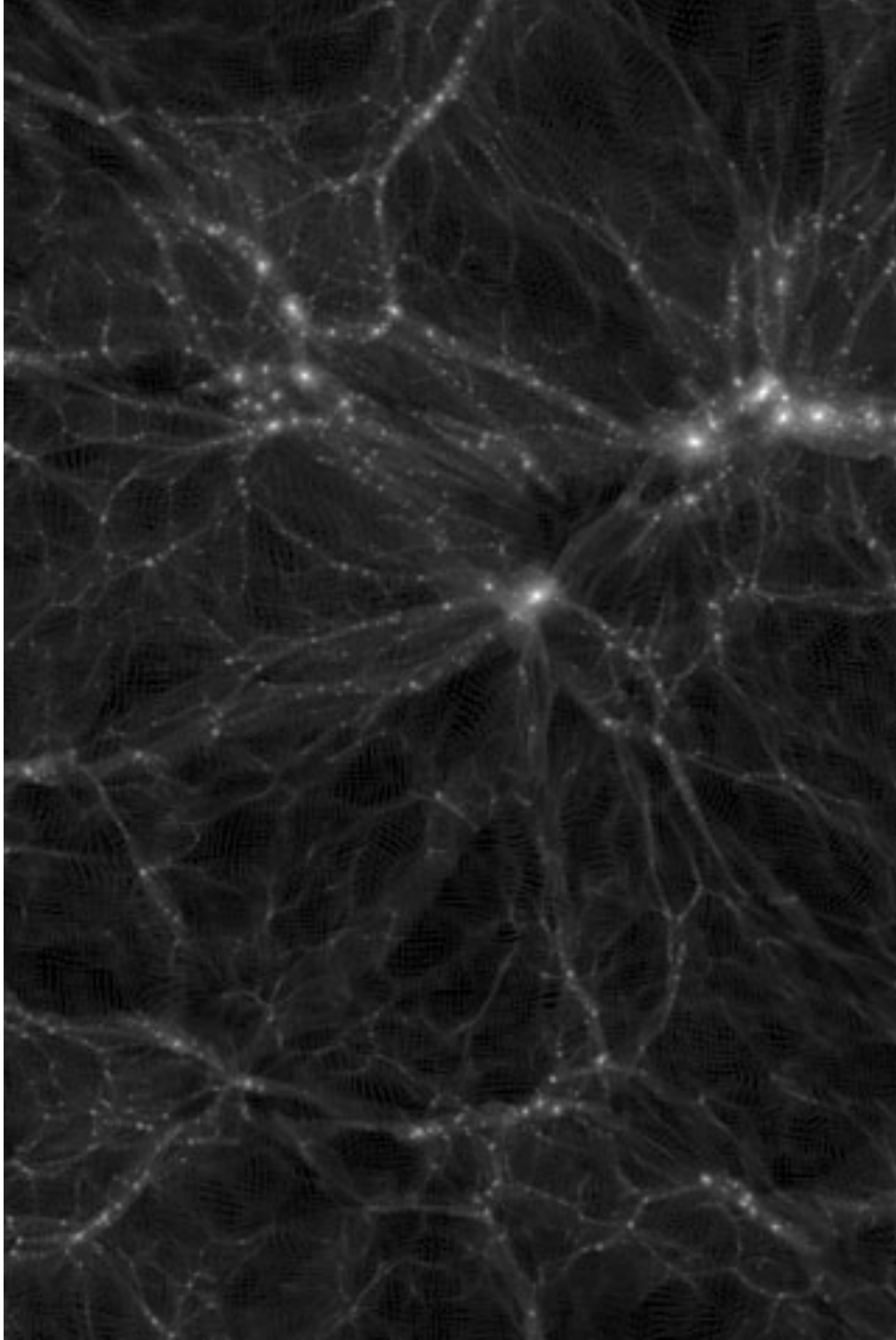
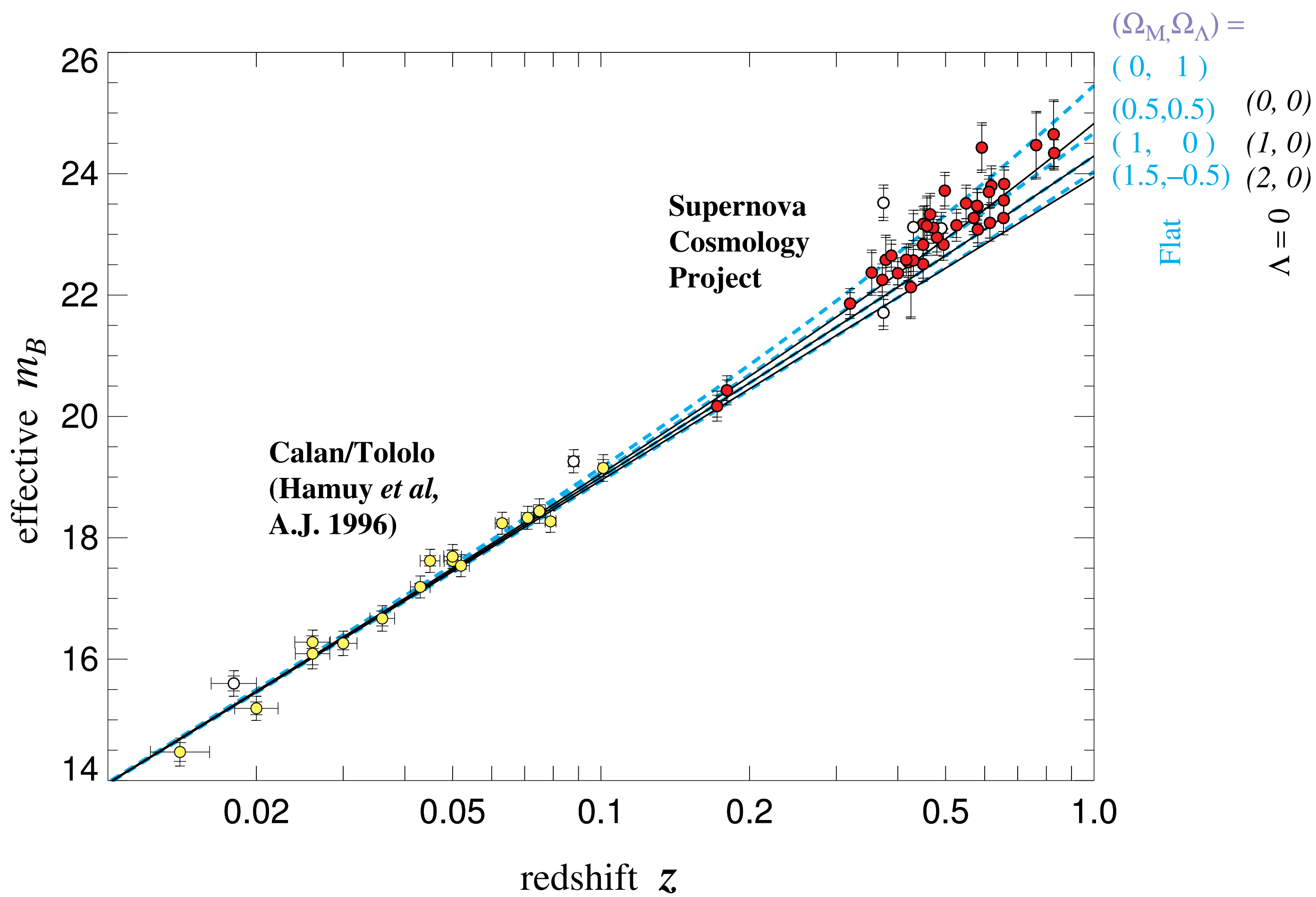
EDECS
Exploring Dark Energy Cosmic Structures



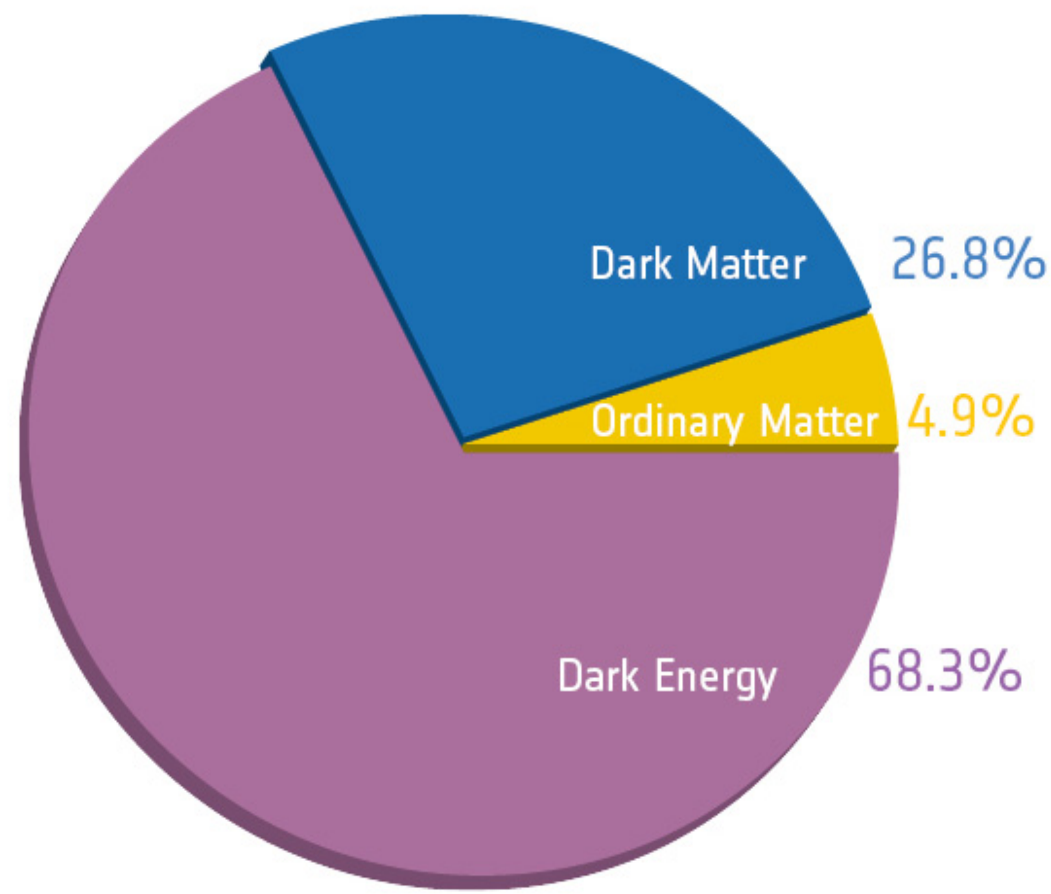
Motivation: Dark Energy



Perlmutter et al. 1999

$$\ddot{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3\,p}{c^2}\right)a$$

$$\dot{a}^2 + k\,c^2 = \frac{8\pi G}{3}\rho\,a^2$$



Planck collaboration 2014

Clustering Dark Energy

$$\begin{cases} \frac{\partial \rho}{\partial \tau} + 3\mathcal{H}(\rho + p) + \vec{\nabla} \cdot [(\rho + p)\vec{v}] = 0 \\ \frac{\partial \vec{v}}{\partial \tau} + \mathcal{H}\vec{v} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho+p}\left(\vec{\nabla}p + \vec{v}\,\frac{\partial p}{\partial \tau}\right) - \vec{\nabla}\Phi \end{cases}$$

Sefusatti and Vernizzi 2011

$$\rho = \bar{\rho} + \delta\rho$$

Background Perturbations

EoS $p = \bar{\rho}(w + c_s^2 \delta)$

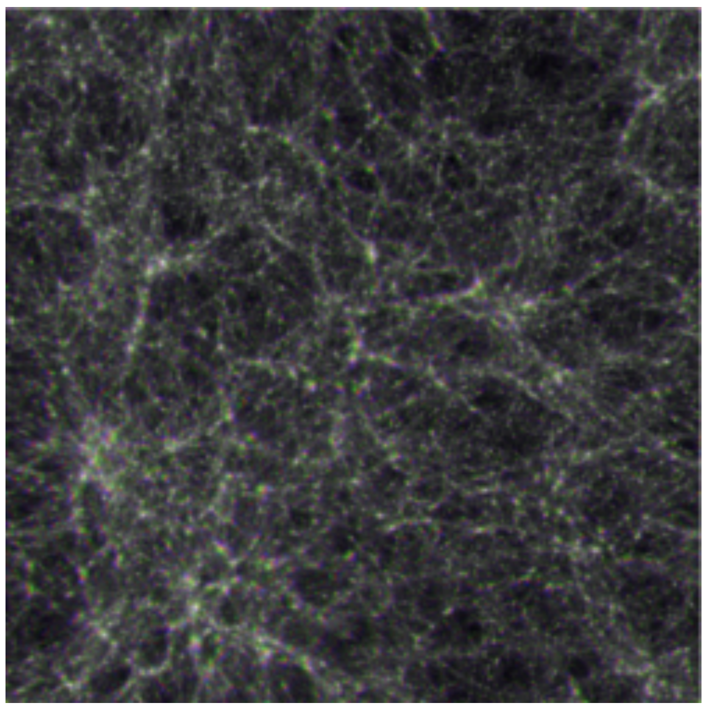
$\boxed{c_s^2 = 1}$ $\longleftrightarrow$ $\boxed{c_s^2 = 0}$

Quintessence:
perturbations are allowed to grow only at the horizon scale

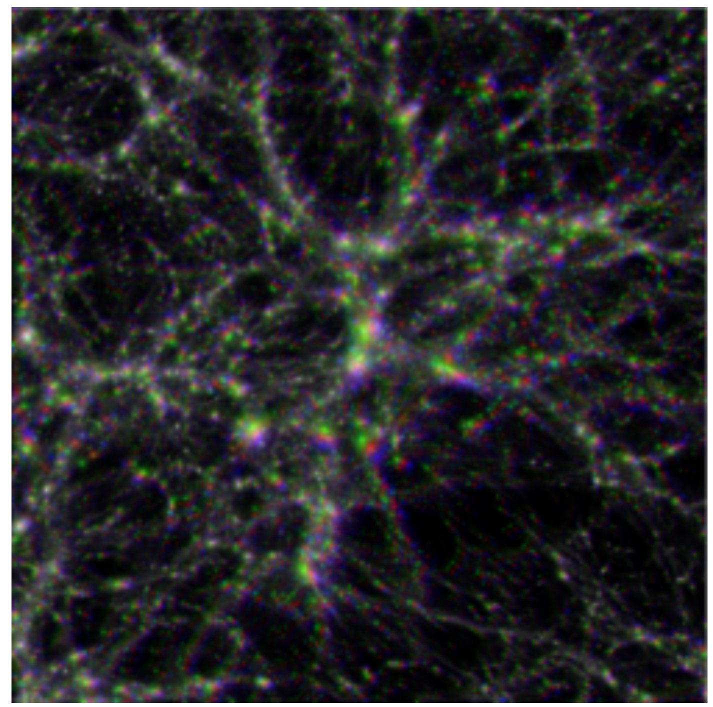
K-essence or DE interactions:
perturbations are allowed to collapse on all scales

Clustering Dark Energy

$$H^2(z) = H_0^2 \left[\Omega_r^{(0)} (1+z)^4 + \Omega_m^{(0)} (1+z)^3 + \Omega_{DE}^{(0)} \exp\left(\int_0^z \frac{3(1+w)}{1+z} dz \right) + \Omega_K^{(0)} (1+z)^2 \right]$$



162 Mpc/h



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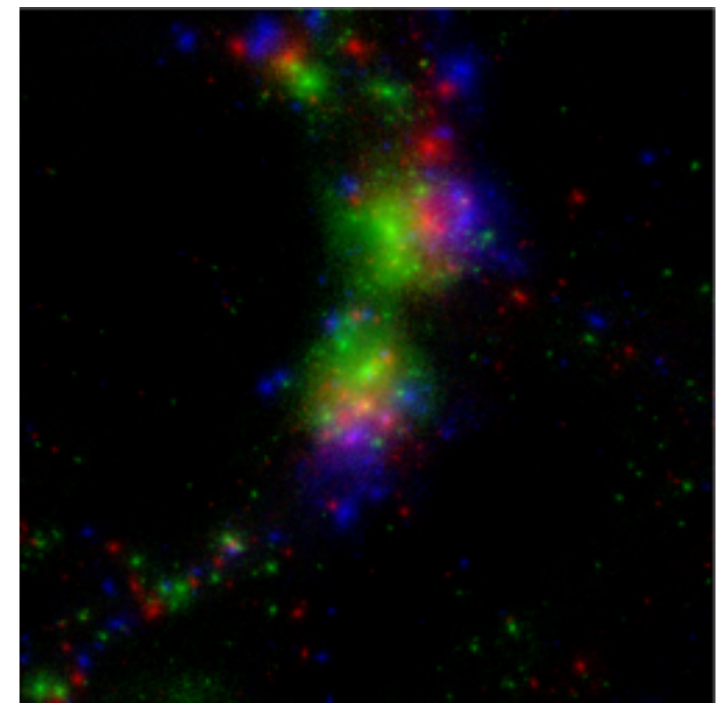


6 Mpc/h

Alimi et al. 2009

Goal: hi-resolution cosmological simulations of Dark Matter + Dark Energy perturbations (+ baryons)

Euler equations in quasi-conservative form

Dark Energy:

$$\left\{ \begin{array}{l} \frac{\partial \Delta}{\partial t} + (1 + c_s^2)\vec{\nabla} \cdot (\Delta \vec{v}) = \boxed{-3a^2 E(a)(c_s^2 - w)(\Delta - 1 - w)} \\ \frac{\partial \Delta \vec{v}}{\partial t} + \vec{\nabla} \cdot [\Delta \vec{v} \otimes \vec{v}] = -\frac{1}{1+c_s^2}\vec{\nabla} P + \boxed{3wa^2 E(a) \Delta \vec{v}} - \Delta \vec{\nabla} \Phi \end{array} \right.$$

$$\Delta = 1 + w + (1 + c_s^2)\delta \,, \quad P = c_s^2 \Delta \,, \quad E(a) = H(a)/H_0$$

Baryonic fluid:

$$\left\{ \begin{array}{l} \frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \\ \frac{\partial \rho \vec{v}}{\partial t} + \vec{\nabla} \cdot [\rho \vec{v} \otimes \vec{v}] = -\vec{\nabla} P - \rho \vec{\nabla} \Phi \end{array} \right.$$

Numerical methods

1D

$$\frac{\partial U}{\partial t} + \frac{\partial F(U)}{\partial x} = S(U)$$

Source splitting

$$U_i^{n*} = U_i^n + \frac{\Delta t}{2} S(U_i^n)$$

$$U_i^{n**} = U_i^{n*} + \frac{\Delta t}{\Delta x}(F_{i-1/2} - F_{i+1/2})$$

$$U_i^{n+1} = U_i^{n**} + \frac{\Delta t}{2} S(U_i^{n**})$$

Godunov method

$$F_{i+1/2} = F(U_{i+1/2}(0))$$

where $U_{i+1/2}(0)$ is the exact similarity solution U(x/t) of the Riemann problem:

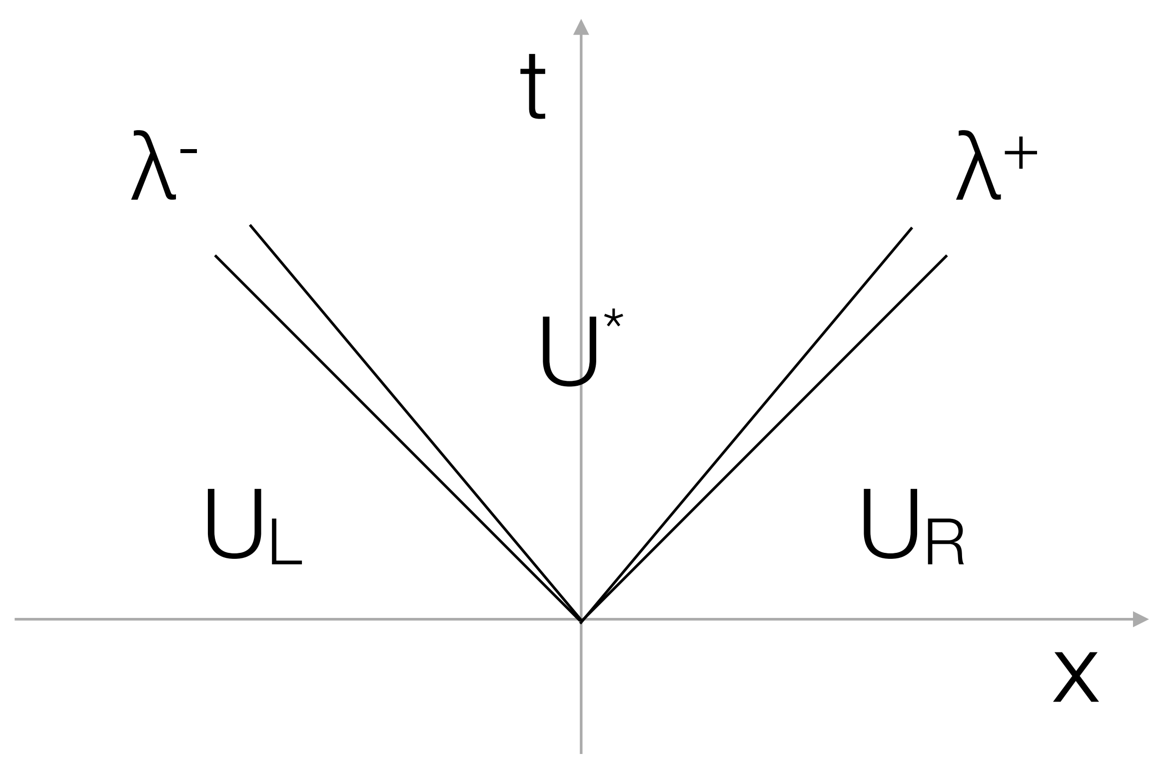
$$\begin{cases} \dfrac{\partial U}{\partial t} + \dfrac{\partial F(U)}{\partial x} = 0 \\ U(x,0) = \begin{cases} U_L & \text{if } x < 0 \\ U_R & \text{if } x > 0 \end{cases} \end{cases}$$

Riemann Problem

$$\begin{cases} \dfrac{\partial U}{\partial t} + \dfrac{\partial F(U)}{\partial x} = 0 \\ U(x,0) = \begin{cases} U_L & \text{if } x < 0 \\ U_R & \text{if } x > 0 \end{cases} \end{cases}$$

$$U = (U_1, U_2) = (\Delta, \Delta v)$$

$$F(U) = \left((1+c_s^2)U_2, \frac{c_s^2}{(1+c_s^2)} U_1 + \frac{U_2^2}{U_1} \right)$$



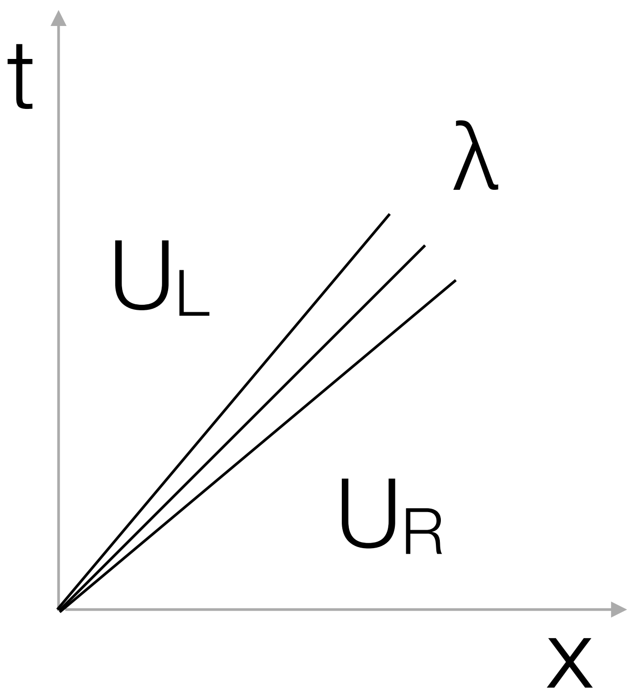
$$\lambda_\pm = v \pm c_s \sqrt{1 - v^2}$$

$$K_\pm = \left(1 + c_s^2, \lambda_\pm\right)$$

Genuinely non linear

Rarefaction: Riemann Invariants

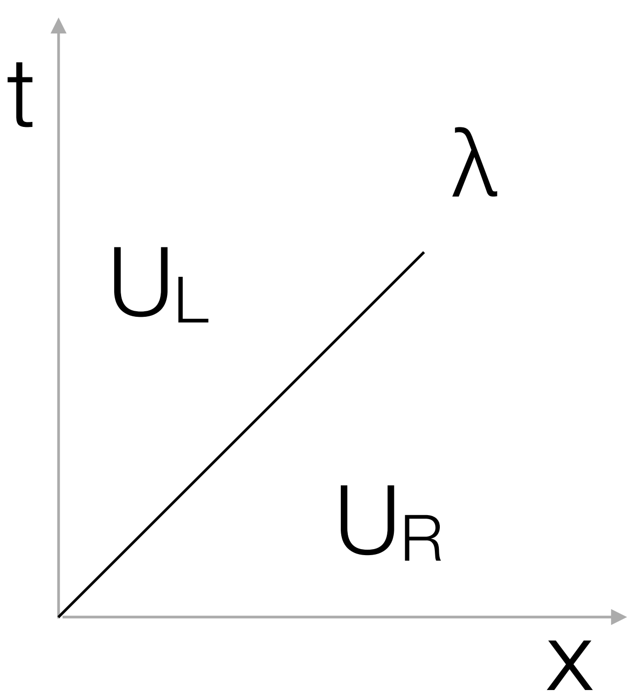
Shock: Rankine-Hugoniot conditions



Rarefaction: Riemann Invariants

$$\frac{\mathrm{d}U_1}{1+c_s^2} = \frac{\mathrm{d}U_2}{\lambda_\pm}$$

Shock: Rankine-Hugoniot conditions



$$\begin{cases} (1+c_s^2)(\Delta_L v_L - \Delta_R v_R) = S(\Delta_L - \Delta_R) \\ \dfrac{c_s^2}{1+c_s^2}(\Delta_L - \Delta_R) + (\Delta_L v_L^2 - \Delta_R v_R^2) = S(\Delta_L v_L - \Delta_R v_R) \end{cases}$$

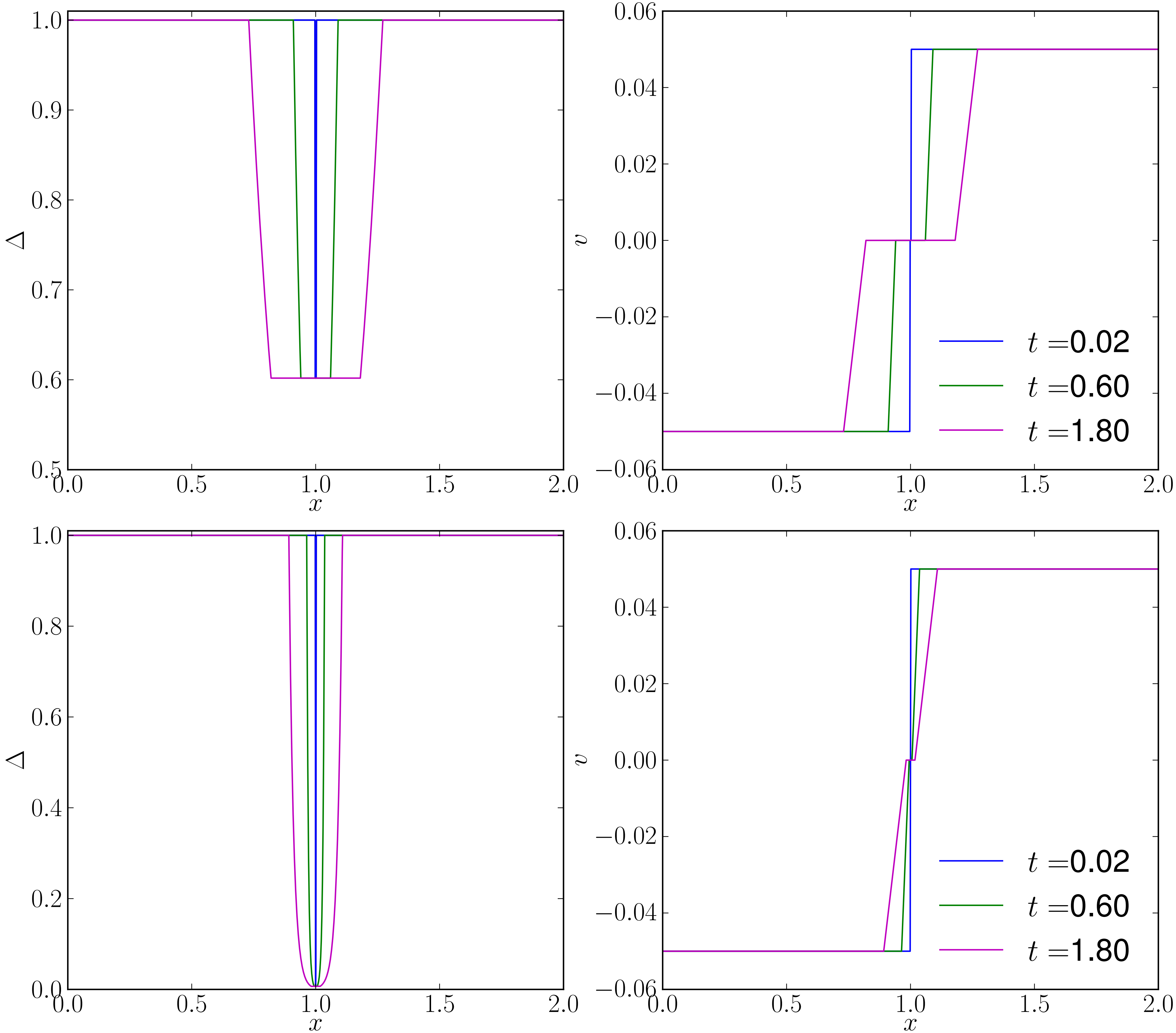
Results - exact solution of the Riemann problem

Test	Δ	v	Δ	v
1. Rarefaction - rarefaction	1	-0.05	1	0.05
2. Rarefaction - shock	0.1	0	1	0
3. Shock - rarefaction	1	0	0.1	0
4. Shock - shock	1	0.05	1	-0.05

Rarefaction - Rarefaction

c_s=0.1

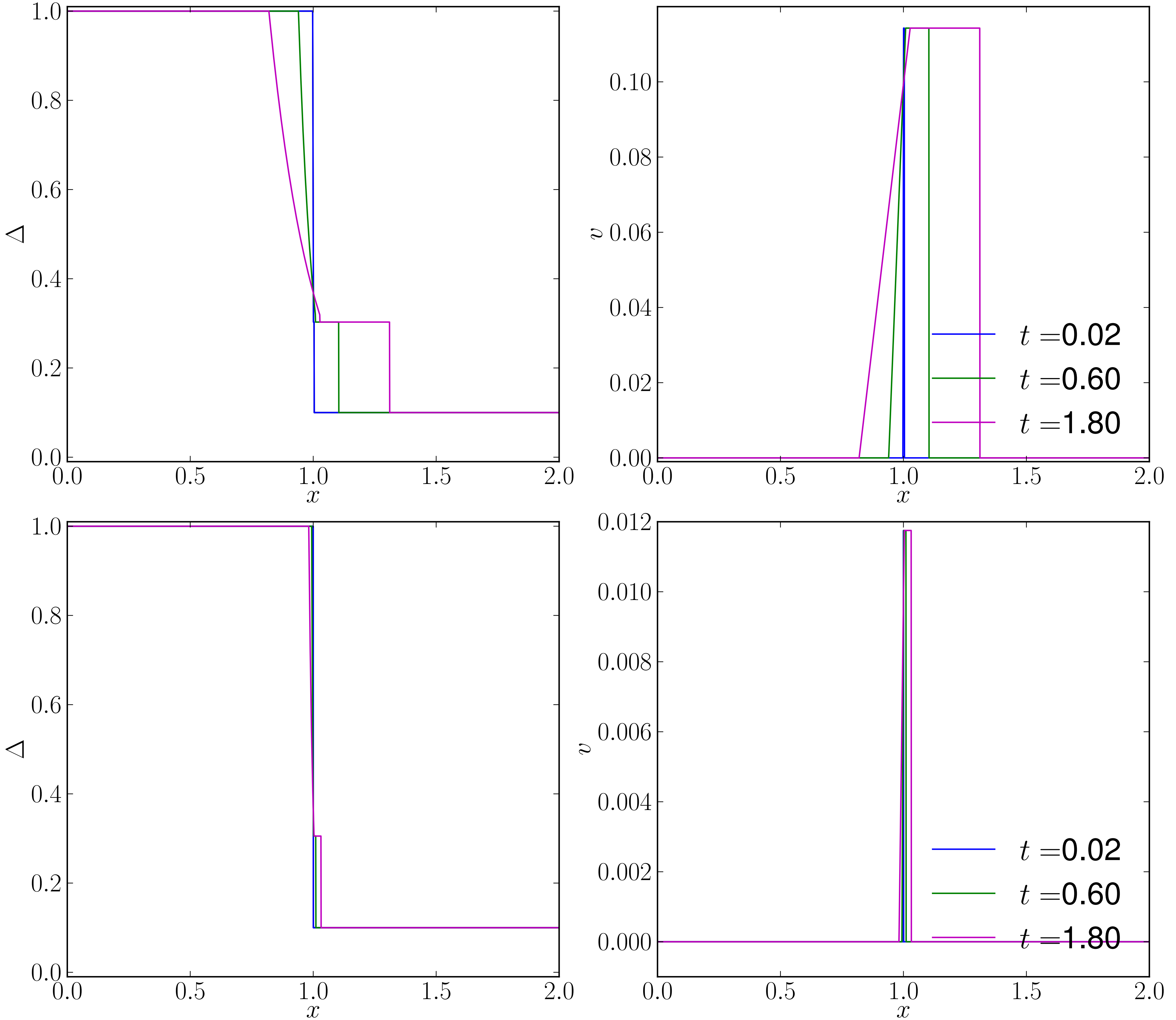
c_s=0.01



Rarefaction - Shock

c_S=0.1

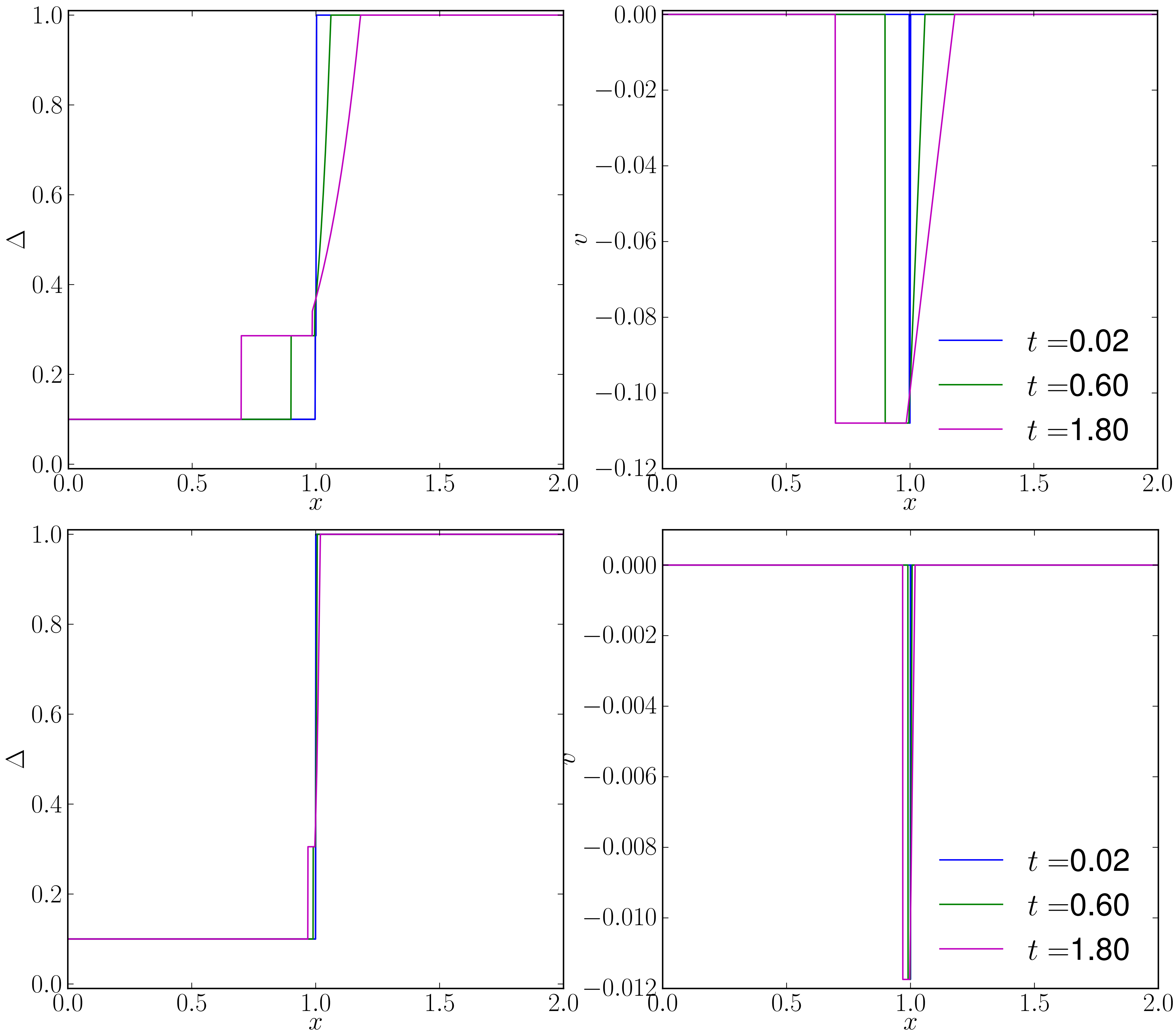
c_S=0.01



Shock - Rarefaction

c_S=0.1

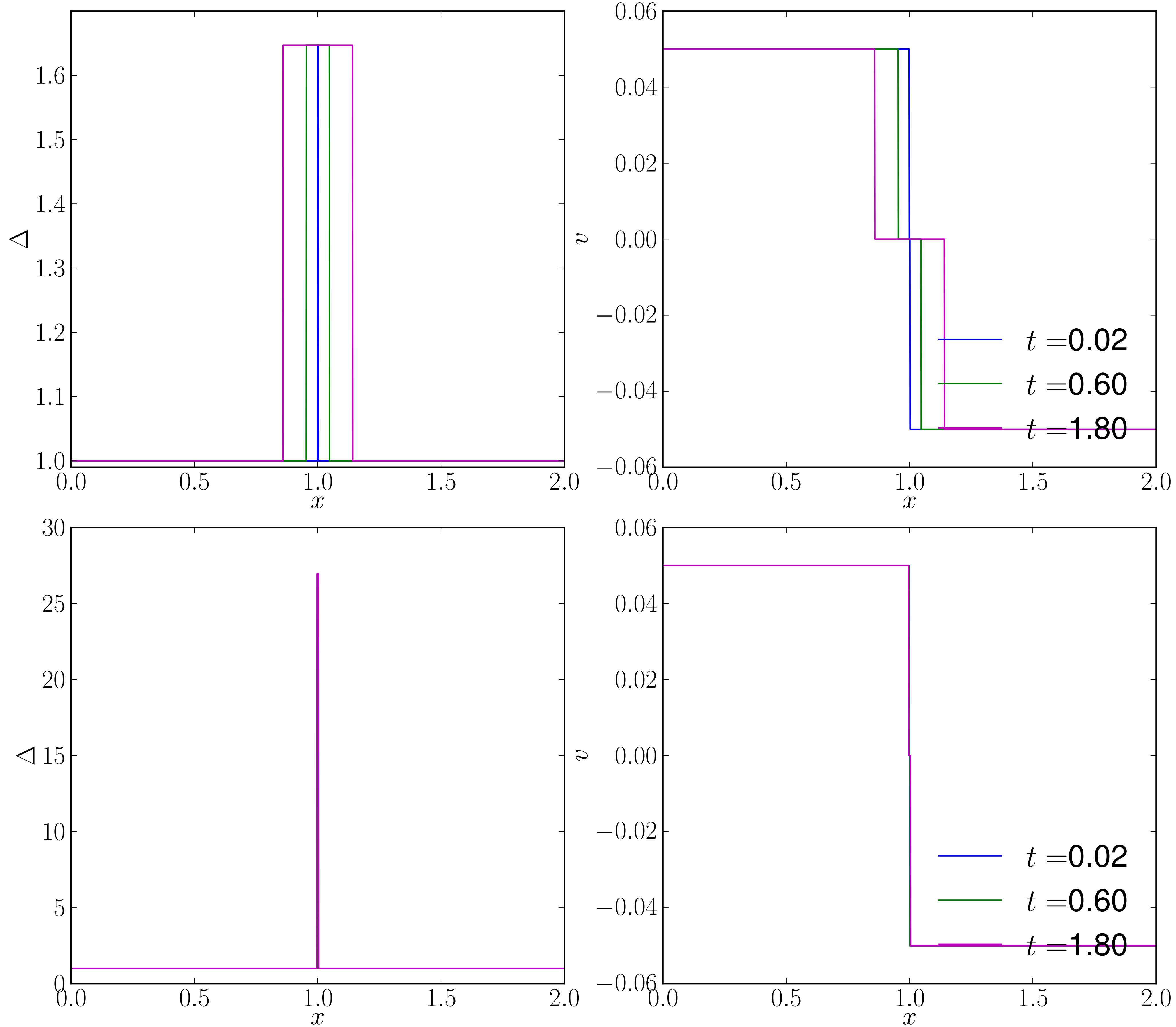
c_S=0.01



Shock - Shock

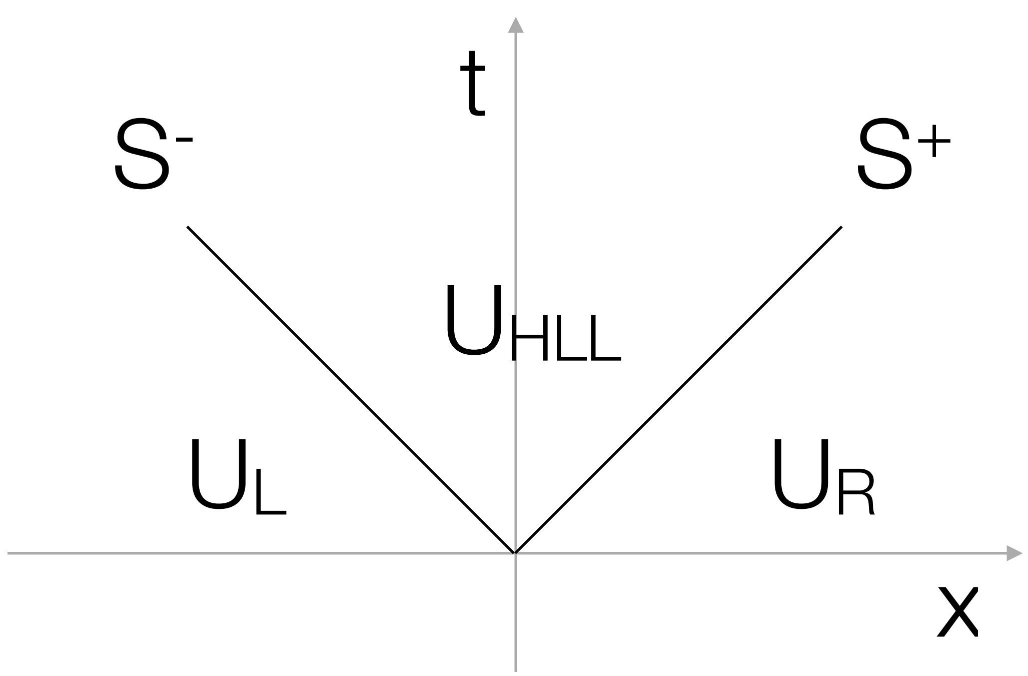
c_S=0.1

c_S=0.01



Approximate Riemann Solvers

HLL



$$F_{HLL} = \frac{S^+F_L - S^-F_R + S^-S^+(U_R - U_L)}{S^+ - S^-}$$

Acoustic

Characteristic equations

$$\frac{\mathrm{d}\Delta}{\Delta} + \frac{1+c_s^2}{c_s}\frac{\mathrm{d}v}{\sqrt{1-v^2}+vc_s} \quad \text{for } \lambda_+$$

$$\frac{\mathrm{d}\Delta}{\Delta} - \frac{1+c_s^2}{c_s}\frac{\mathrm{d}v}{\sqrt{1-v^2}-vc_s} \quad \text{for } \lambda_-$$

$$C_\pm = \frac{1+c_s^2}{c_s}\frac{1}{\sqrt{1-v^2}\pm vc_s}$$

Future prospects

- Testing Riemann Solvers
- Implement Riemann Solvers in Ramses
- First cosmological runs

Thank you!