Gravitational Waves from Coalescing Binary Black Holes and Neutron Stars

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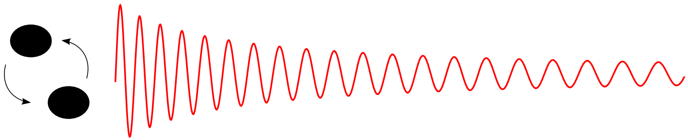

(LIGO, Virgo,
eLISA, ET,...)

Revisiting the Classical Tests of General Relativity with Compact Binaries

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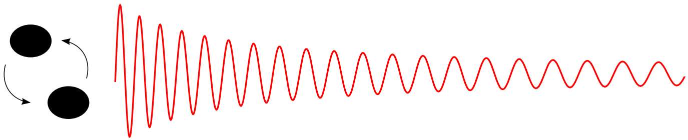

(LIGO, Virgo,
eLISA, ET,...)

Outline

① Astrophysical motivation

② Gravitational wave source modelling

③ Gravitational redshift effect

④ Geodetic spin precession

⑤ Relativistic periastron advance

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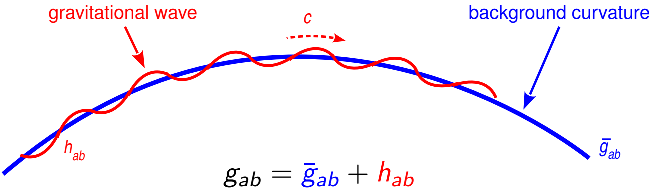
What is a gravitational wave?

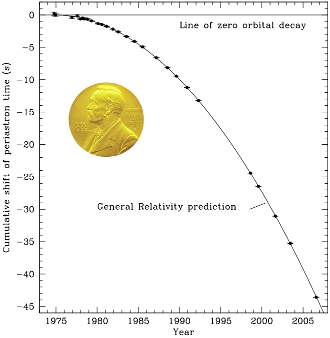
A gravitational wave is a perturbation in the curvature of spacetime that propagates at the vacuum speed of light



$$g_{ab} = \bar{g}_{ab} + h_{ab}$$

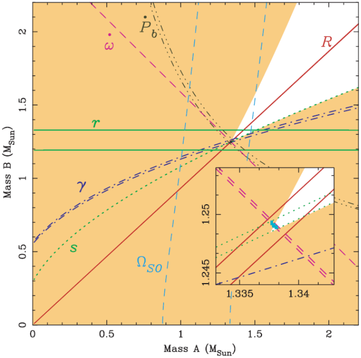
Key prediction of Einstein's theory of General Relativity

Indirect evidence of gravitational waves



Binary pulsar PSR B1913+16

[Hulse & Taylor, ApJ (1975)]

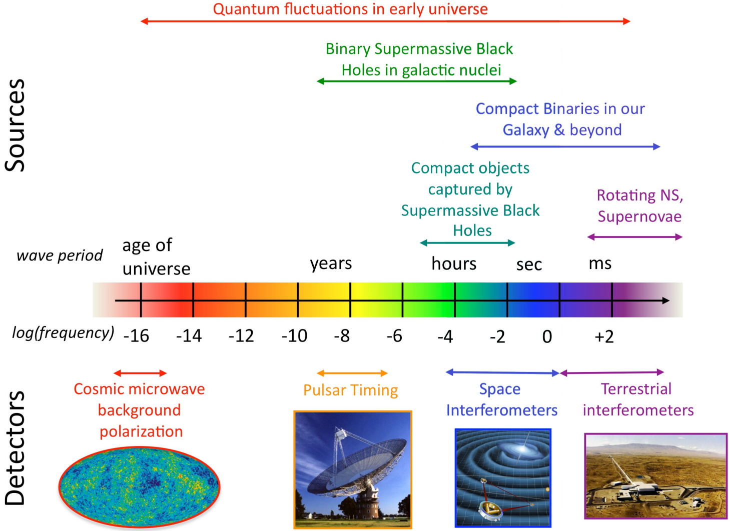


Double pulsar PSR J0737-3039

[Burgay et al., Nature (2003)]

Orbital decay due to GW emission confirmed at the **0.1% level**

The gravitational wave spectrum



Sources

Quantum fluctuations in early universe

Binary Supermassive Black Holes in galactic nuclei

Compact Binaries in our Galaxy & beyond

Compact objects captured by Supermassive Black Holes

Rotating NS, Supernovae

Detectors

Cosmic microwave background polarization

Pulsar Timing

Space Interferometers

Terrestrial interferometers

Science with gravitational wave observations

Fundamental physics

- Precision tests of GR in the non-linear regime
- Existence of black holes — cosmic censorship
- Dark energy equation of state $w = p/\rho$

Astrophysics

- Formation and evolution of compact binaries
- Origin and mechanism of γ-ray bursts
- Internal structure of neutron stars

Cosmology

- Cosmography and independent measure of H_0
- Origin and growth of supermassive black holes
- Phase transitions in the early Universe

Ground-based laser interferometric detectors

Virgo (Cascina, Italy)

LIGO (Livingston, USA)

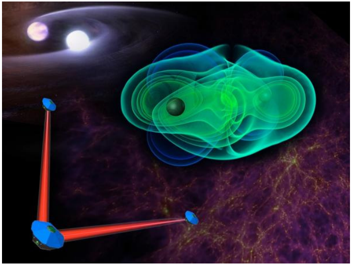
- 6 science runs and ~ 80 publications over 2003–2012
- No direct detection but stringent upper limits, e.g.
  - Ellipticity of Crab pulsar $< 10^{-4}$ [Abbott et al., ApJ (2008)]
  - Energy density of GW stochastic background $< 6.9 \times 10^{-6}$ around 100 Hz [Abbott et al., Nature (2009)]

A worldwide network of GW observatories

- Ongoing upgrade to Advanced LIGO/Virgo, KAGRA in Japan
- 2nd generation detectors: sensitivity $\times 10 \Rightarrow$ event rates $\times 10^3$
- Beginning of gravitational wave astronomy $\sim$ 2015-2020

eLISA: a gravitational wave antenna in space

LISA Pathfinder



eLISA/NGO

- LISA Pathfinder scheduled for launch in 2015
- "Gravitational Universe" science theme was selected by ESA for the L3 mission with a nominal launch in 2034
- If LISA Pathfinder is successful then eLISA should fly!

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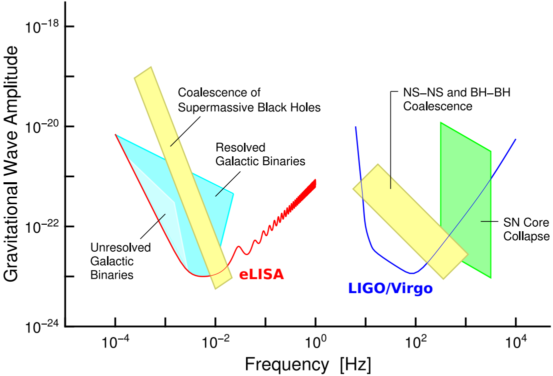
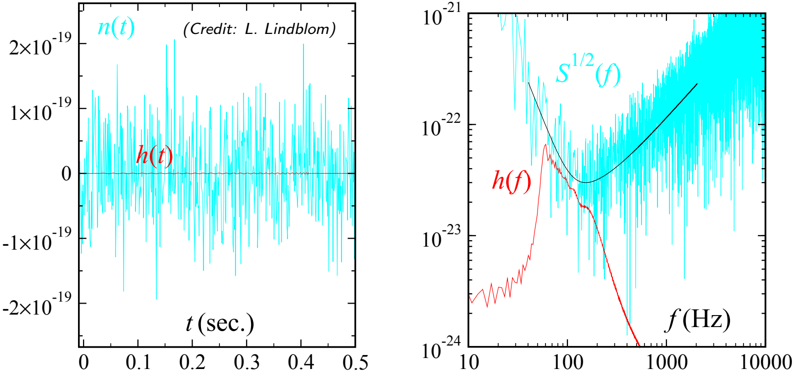
Main sources of GW for LIGO/Virgo and eLISA



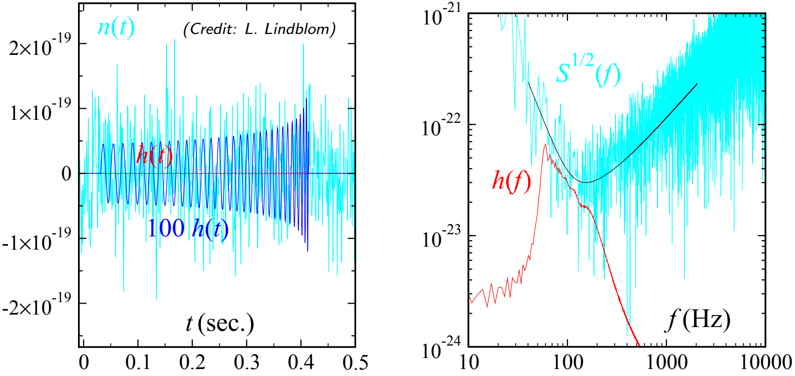
- Binary neutron stars ($2\times\sim 1.4M_\odot$)
- Stellar mass black hole binaries ($2\times\sim 10M_\odot$)
- Supermassive black hole binaries ($2\times\sim 10^6M_\odot$)
- Extreme mass ratio inspirals ($\sim 10M_\odot+\sim 10^6M_\odot$)

Need for highly accurate template waveforms



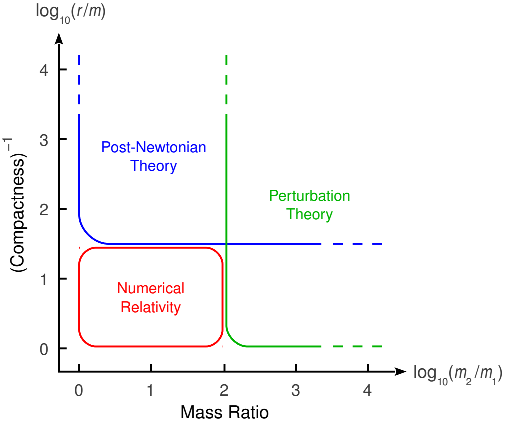
If the expected signal is *known in advance* then $n(t)$ can be filtered and $h(t)$ recovered by matched filtering ⟶ **template waveforms**

Need for highly accurate template waveforms

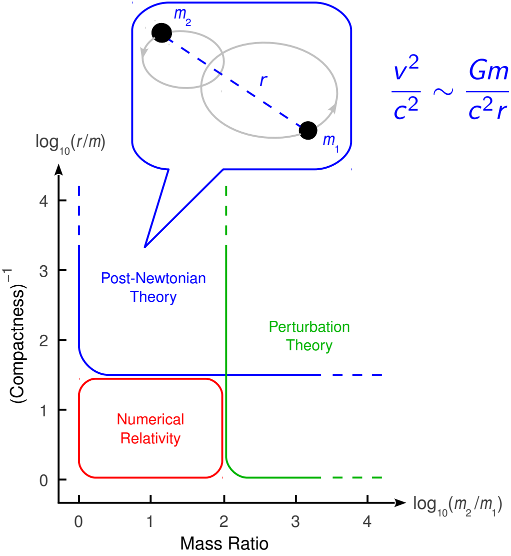


If the expected signal is *known in advance* then $n(t)$ can be filtered and $h(t)$ recovered by matched filtering ⟶ **template waveforms**

Methods to compute GW templates for compact binaries



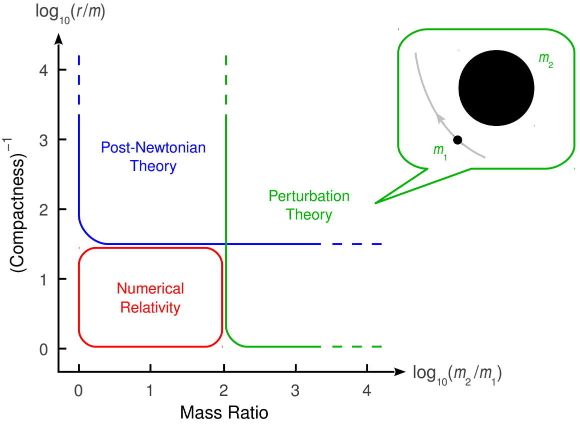
Methods to compute GW templates for compact binaries



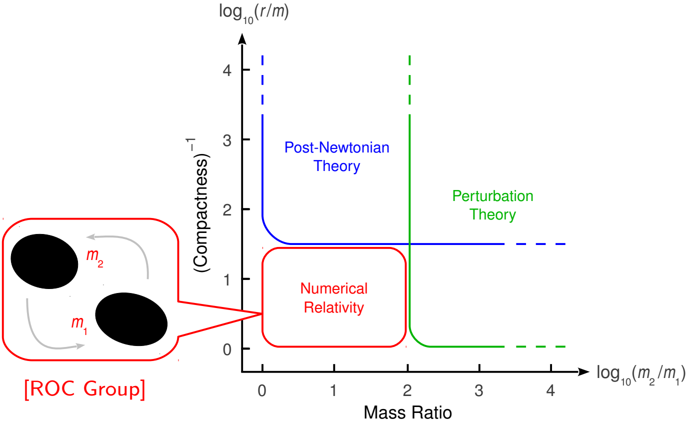
$$\frac{v^2}{c^2} \sim \frac{Gm}{c^2 r} \ll 1$$

Methods to compute GW templates for compact binaries

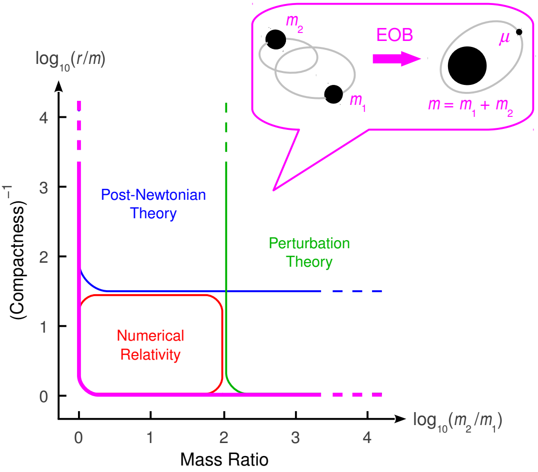
$$q \equiv \frac{m_1}{m_2} \ll 1$$



Methods to compute GW templates for compact binaries



Methods to compute GW templates for compact binaries



Comparing the predictions from these various methods

Why?

- Independent checks of long and complicated calculations
- Identify domains of validity of approximation schemes
- Extract information inaccessible to other methods
- Develop a universal model for compact binaries

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How?

- ✗ Use the same coordinate system in all calculations
- ✓ Using coordinate-invariant relationships

Comparing the predictions from these various methods

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How?

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- ✓ Using coordinate-invariant relationships

What?

- Gravitational waveforms at null infinity
- Conservative effects on the orbital dynamics

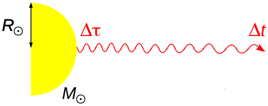
Comparing the predictions from these various methods

Paper	**Year**	**Methods**
Boyle, Brown *et al.*	2007	NR/PN
Detweiler	2008	BHP/PN
Blanchet, Detweiler *et al.*	2010a	BHP/PN
Blanchet, Detweiler *et al.*	2010b	BHP/PN
Damour	2010	BHP/EOB
Barack, Damour, Sago	2010	BHP/EOB
Lousto, Nakano *et al.*	2010	NR/BHP
Sperhake, Cardoso *et al.*	2011	NR/BHP
Le Tiec, Mroué *et al.*	2011	NR/BHP/PN/EOB
Damour, Nagar *et al.*	2012	NR/EOB
Le Tiec, Barausse, Buonanno	2012	NR/BHP/PN/EOB
Akcay, Barack *et al.*	2012	BHP/EOB
Nagar	2013	NR/BHP
Hinderer, Buonanno *et al.*	2013	NR/EOB
Le Tiec, Buonanno *et al.*	2013	NR/BHP/PN
Dolan, Warburton *et al.*	2013	BHP/PN

Revisiting the classical tests of General Relativity

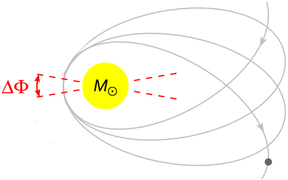
- Gravitational redshift of light

$$\frac{\Delta\tau}{\Delta t} = 1 - \frac{G}{c^2}\frac{M_\odot}{R_\odot}$$



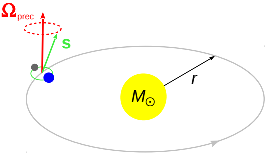
- Perihelion advance of Mercury

$$\Delta\Phi = \frac{G}{c^2}\frac{6\pi M_\odot}{a\,(1-e^2)}$$



- Precession of Earth-Moon spin

$$\boldsymbol{\Omega}_{\text{prec}} = \frac{G}{c^2}\,\mathbf{v}\times\boldsymbol{\nabla}\left(\frac{3M_\odot}{2r}\right)$$



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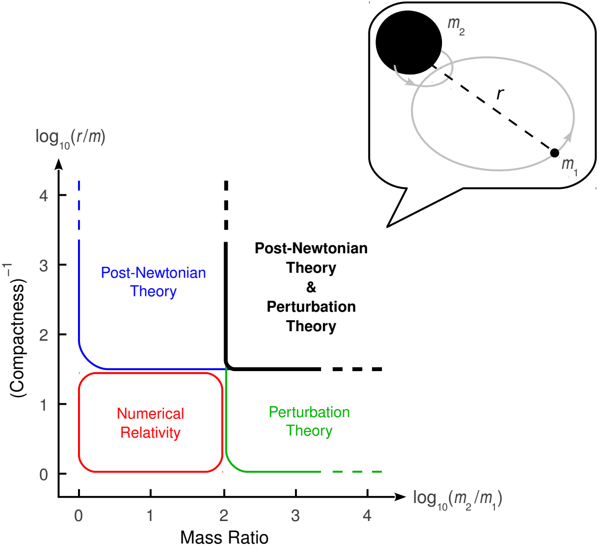
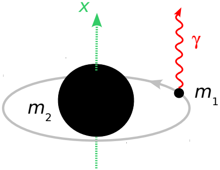
Post-Newtonian expansions and black hole perturbations



The "redshift observable" for circular orbits

- It measures the redshift of light emitted from the point particle:

$$\frac{\mathcal{E}_{\text{obs}}}{\mathcal{E}_{\text{em}}} = \frac{(p^a u_a)_{\text{obs}}}{(p^a u_a)_{\text{em}}} = z$$

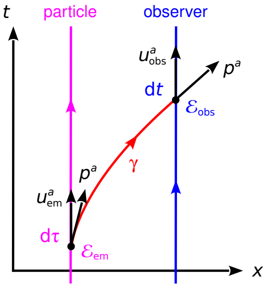


- It is a constant of the motion associated with the helical Killing field:

$$z = -k^a u_a$$

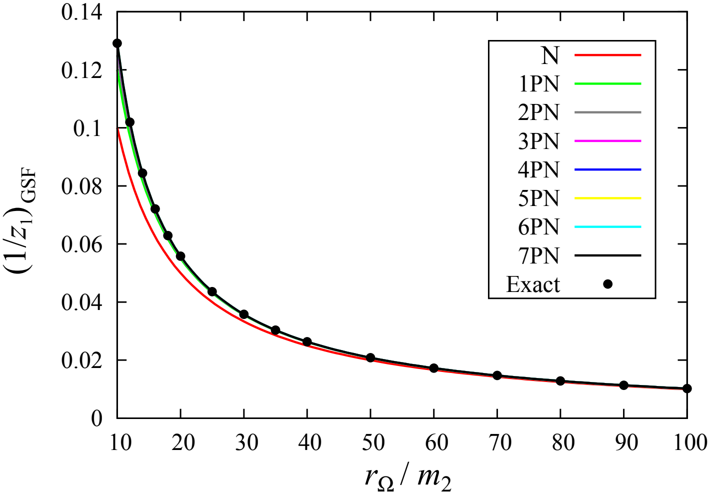
- In coordinates adapted to the symmetry:

$$z = \frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{1}{u^t}$$



Redshift observable *vs* orbital separation

[Blanchet, Detweiler, Le Tiec & Whiting, PRD (2010)]



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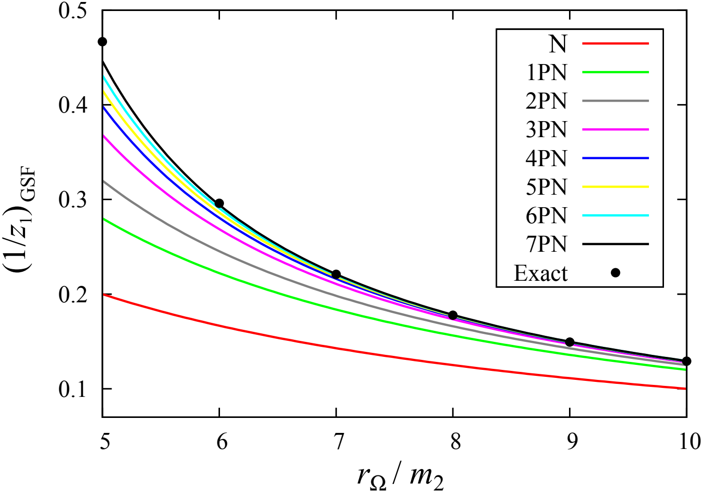

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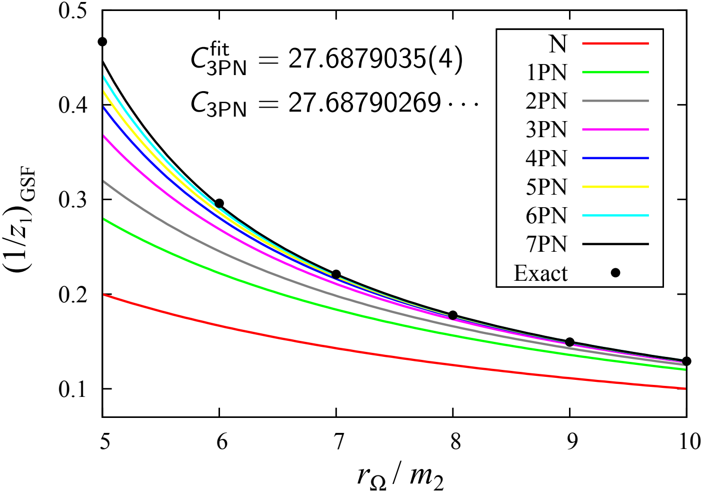

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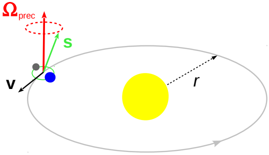
De Sitter's spin precession, or geodetic effect

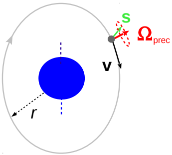
- In 1916 de Sitter showed that, to leading order, a system's spin **s** precesses at

$$\boldsymbol{\Omega}_{\text{prec}} \simeq \frac{3}{2}\,\mathbf{v}\times\boldsymbol{\nabla}\Phi\,, \quad \Phi \equiv \frac{GM}{c^2 r} \ll 1$$

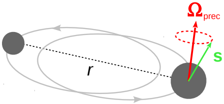
- Precession of Earth-Moon spin axis of ∼ 1.9" /cent. confirmed using LLR data



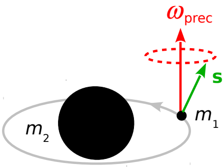
- Precession of test gyro on polar Earth orbit of ∼ 6.6" /yr confirmed by GPB



- Geodetic spin precession of ∼ 5°/yr measured in the double pulsar



Geodetic precession in black hole binaries



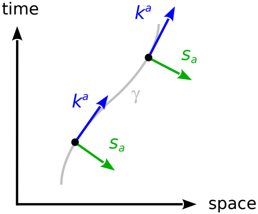
- A test spin s_a is parallel-transported along a geodesic γ with unit tangent u^a:

$$u^b \nabla_b s_a = 0 \quad \Longleftrightarrow \quad \frac{\mathrm{d}\mathbf{s}}{\mathrm{d}\tau} = \omega_{\text{prec}} \times \mathbf{s}$$

- For a circular orbit with a helical Killing field k^a such that $k^a|_\gamma = u^a$,

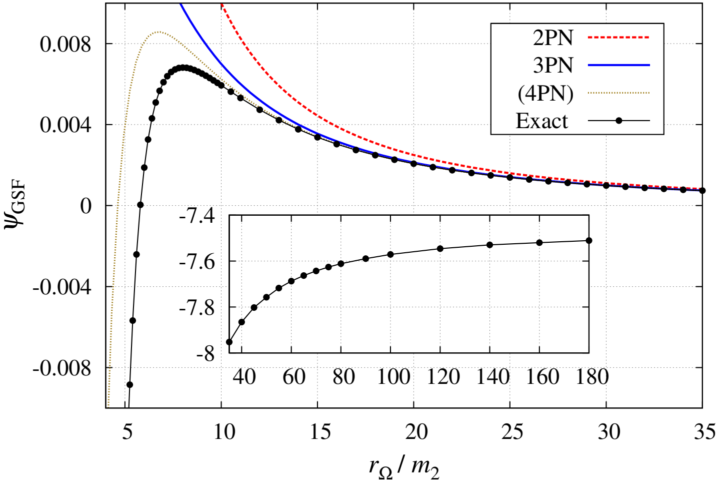
$$\omega^2_{\text{prec}} = \frac{1}{2} \left(\nabla_a k_b \nabla^a k^b \right) |_\gamma$$

- Compute $\psi \equiv 1 - \omega_{\text{prec}} / u^\phi$, the angle of spin precession per radian of revolution



Precession angle *vs* orbital separation

[Dolan, Warburton, Harte, Le Tiec, Wardell & Barack (2013)]



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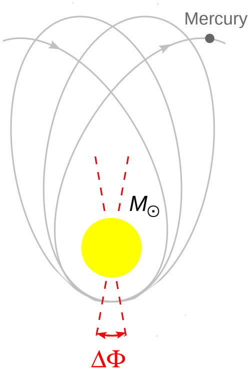
Relativistic perihelion advance of Mercury

- Observed anomalous advance of Mercury's perihelion of ∼ 43"/cent.
- Accounted for by the leading-order relativistic angular advance per orbit

$$\Delta\Phi = \frac{6\pi G M_\odot}{c^2 a\,(1-e^2)}$$

- Periastron advance of ∼ 4°/yr now measured in binary pulsars



Periastron advance in black hole binaries

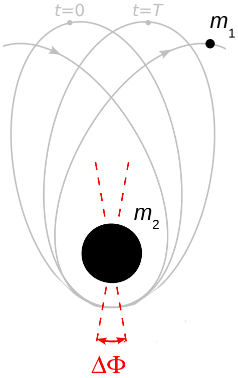
- Generic non-circular orbit parametrized by the two frequencies

$$\Omega_r = \frac{2\pi}{T}\,, \quad \Omega_\varphi = \frac{1}{T}\int_0^T \dot{\varphi}(t)\,\mathrm{d}t$$

- Periastron advance per radial period

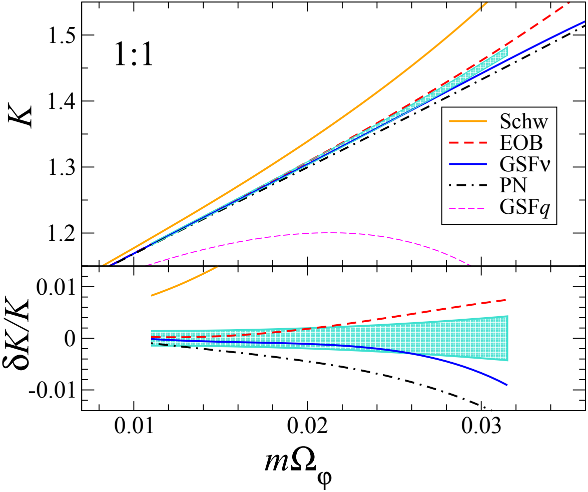
$$K \equiv \frac{\Omega_\varphi}{\Omega_r} = 1 + \frac{\Delta\Phi}{2\pi}$$

- In the circular orbit limit $e \to 0$, the relation $K(\Omega_\varphi)$ is coordinate-invariant



Periastron advance *vs* orbital frequency

[Le Tiec, Mroué *et al.*, PRL (2011)]



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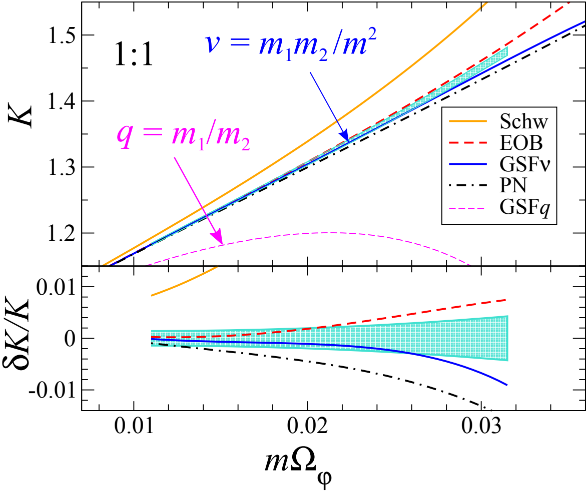
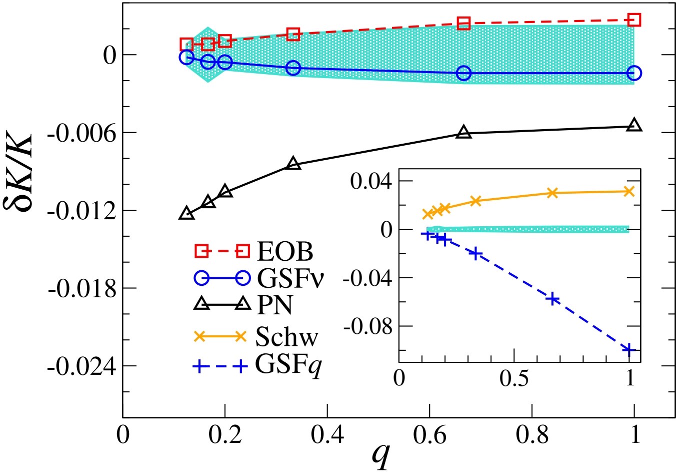


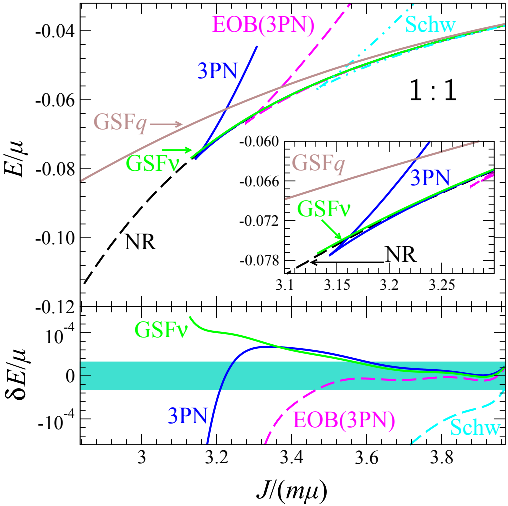
Periastron advance *vs* mass ratio

[Le Tiec, Mroué *et al.*, PRL (2011)]

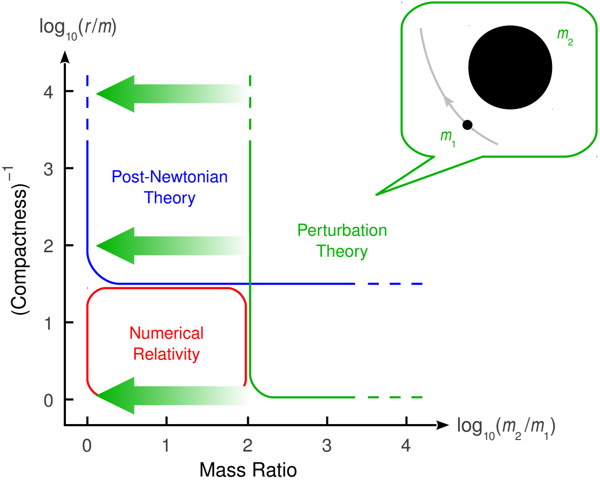


Binding energy *vs* angular momentum

[Le Tiec, Barausse & Buonanno, PRL (2012)]



Using perturbation theory for comparable-mass binaries



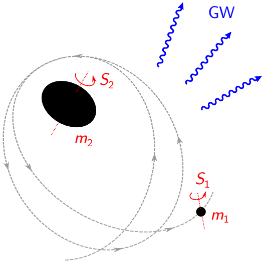
Using perturbation theory for comparable-mass binaries

Why?

- Results valid in the weak-field and strong-field regimes
- Solve “only” linear partial differential equations
- Results may be valid for all mass ratios

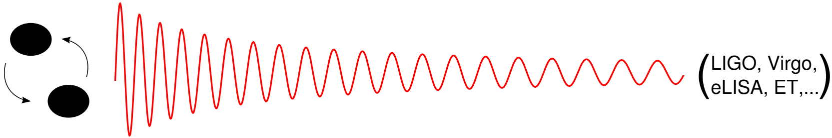
How?

- Comparisons with NR
- Inclusion of spin effects
- Extension of the first law
- Computation of GW fluxes



Summary and prospects

- Observing GWs will open a new window on the Universe
- Highly accurate template waveforms are a prerequisite for doing science with GW observations
- It is crucial to compare the predictions from PN theory, perturbation theory and numerical relativity
- Perturbation theory may prove useful to build templates for comparable-mass binaries (LIGO/Virgo and eLISA)



The dark matter problem in astrophysics

Model of dipolar dark matter [Blanchet & Le Tiec, PRD (2008; 2009)]

- General Relativity + modified dark matter
- Equivalent to ΛCDM model on cosmological scales
- Reproduces phenomenology of MOND at galactic scale

Predictions and comparisons to observations

- Non-gaussianity in cosmic microwave background → Planck
  [Blanchet, Langlois, Le Tiec & Marsat, JCAP (2013)]
- Stochastic background of gravitational waves → LIGO, eLISA
  [Birnbaum, Gerling-Dunsmore & Le Tiec (work in progress)]
- Intensive numerical simulations to study growth of structures in non-linear regime → 2dF, SDSS [With COS group at LUTh?]