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Scale relativity and quantization of the solar system

Orbit quantization of the planet's satellites

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Abstract. In a first paper Nottale, Schumacher and Gay have given the bases of the Scale Relativity theory applied to the gravitational field, which leads to the quantization of the solar system. In the present paper we show that one more class of objects of the solar system satisfy the rule of quantization, this class including the main satellites and rings of the outer planets. We also give a classification of the satellites by rank, showing that one can predict the existence of certain orbits that are not occupied, or whose objects are not yet discovered.

Key words: chaos – gravitation – planets and satellites: general – solar system: general

1. Introduction

Scale Relativity theory is an extension of Einstein's principle of relativity, applied to scale laws. By giving up the differentiability of space-time coordinates at very large time-scale, one can describe the solar system in terms of fractal trajectories governed by a Schrödinger-like equation. The theory is due to L. Nottale (see for example (Nottale, 1993), (Nottale, 1996a), and (Nottale, 1997a)). In a previous paper (Nottale et al., 1997) we have shown how the theory can be applied to the solar system, and how the orbits of the planets are quantized. In the present paper we consider that not only the orbits of the planets are quantized, but also the orbits of their main satellites and maybe the rings.

Lets consider a gravitational system with a central mass (Kepler problem) and an orbiting body, and consider the case of almost circular orbits. This is the case of most planets and satellites in the solar system. We have shown that the radius of the orbits are quantized, and that their distribution is given by:

$$\sqrt{\frac{a}{M}} = \sqrt{(n^2 + \frac{n}{2})\frac{a_0}{M}} \simeq (n + \frac{1}{4})\frac{\sqrt{G}}{w_0} \tag{1}$$

where a is the semi-major axis of the orbit, M the mass of the central object, G the gravitational constant, w_0 a constant having the dimension of a velocity, and n is the rank of the orbit. This relationship is valid in the framework of a theory of formation of a planetary system. The matter fills the orbitals with time, and then the planet, or the satellite, is formed by accretion at the mean distance given by 1(see (Nottale et al., 1997)).

In the first part of this paper, we expose the method to determine the rank of the satellites, and we study the statistical significance of the results. In a second part, the results are given for every planetary system. In a third part we show the classification by rank of the main satellites of the whole solar system, suggesting that some orbits are unoccupied, or not yet discovered.

2. Method

2.1. Determination of the rank of a body

We have built a software which automatically determines the best rank for each object of a given satellite system. The different steps of the process are the following, for a system of N bodies:

1. we first normalize the values of $\sqrt{a}$, by dividing by the largest $\sqrt{a}$
2. we then set a largest possible rank m
3. we take all slopes d of the lines $\sqrt{a} = d\sqrt{n^2 + \frac{n}{2}}$ between 0 and 1 with steps of 10^{-6} (see Fig. 1)
4. for each of our N bodies, we choose the rank n_i (less than m) so that $s_i^2 = (\sqrt{a_i} - d\sqrt{n_i^2 + \frac{n_i}{2}})^2$ is minimum
5. we compute $\chi^2 = \sum_{i=1}^{N} s_i^2$, and finally we get a value of d which minimizes χ^2, and this value is associated with a given configuration of the rank n_i of the bodies
6. we now repeat the steps 2 to 5 in order to get the best configurations for every value of the largest possible rank (in practice we ranged m from N to $3N + 1$, see comment).

Comment:

It is obvious that, if we take m larger and larger, it will be very easy to achieve a very good fit for our law. But, in practice, we see that generally at some value of m, the value of χ^2 suddenly decreases by a factor 10, and then stays stable for larger values

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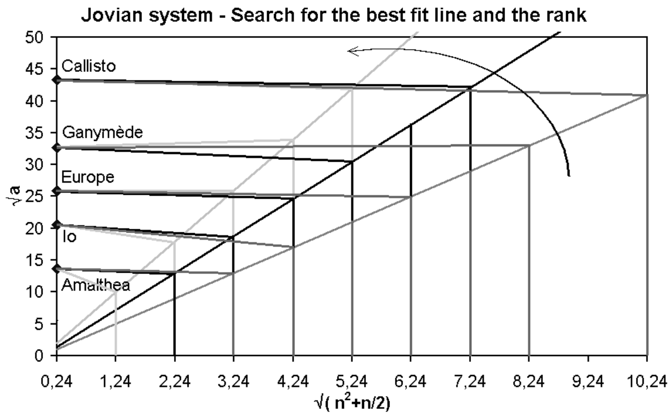

Fig. 1. Example of the technique to determine the rank applied to the satellite system of Jupiter, with N=5. The semi-major axis a is given in 10^3 km.

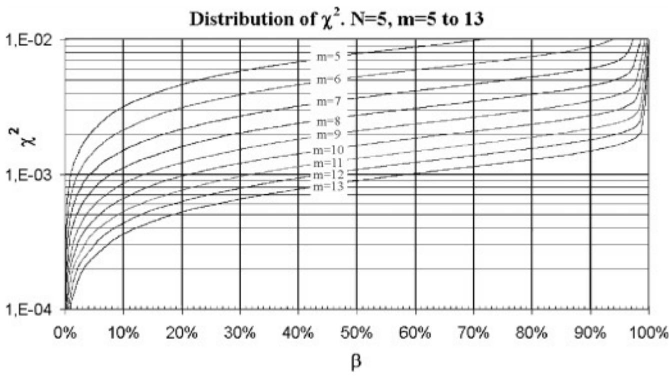

Fig. 2. Distribution of χ^2 giving percentage of random systems, for the number of bodies $N = 5$, and the maximum rank m ranging from 5 to 9.

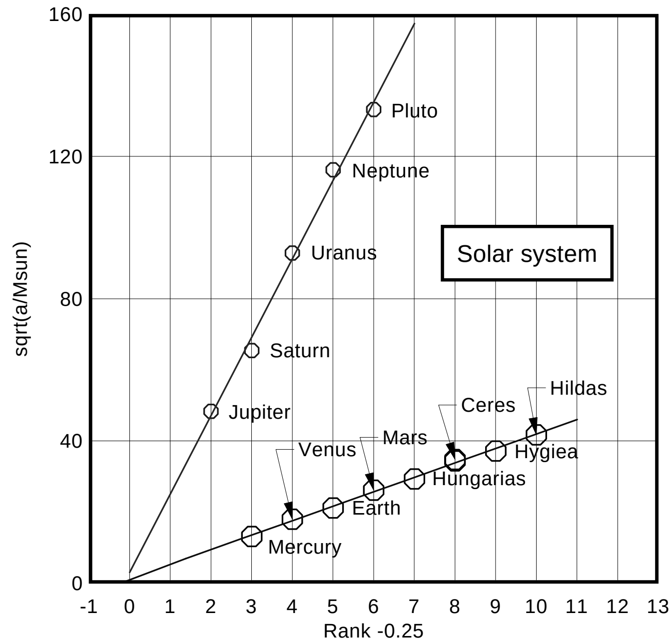

Fig. 3. Solar system with the main asteroid belts. M is in Earth masses, an a in km.

Table 1. Statistical significance and correlation coefficient for the ranks attributed to the elements of the solar and the planetary systems

System	N	M	α %	γ	σ %
Outer solar sys.	5	6	92.9	0.9967	2.0
Inner solar sys.	4	6	91.0	0.9981	2.8
Titius Bode				0.9637	118
Jupiter	5	7	96.2	0.9985	2.2
Saturn ext.	2	5		0.9993	2.6
Saturn int.	6	12	99.2	0.9998	1.0
Uranus ext.	5	9	90.1	0.9987	1.75
Uranus int.	12	29		0.9995	
Neptune ext.	2	2	87	0.9997	3.12
Neptune int.	3	12	96.7	0.9993	1.46
PSR B1257+12	3	8	99.9	0.999999	0.0307

of m. We call this phenomenon convergence, and we define M as the value of m achieving convergence.

Fig. 1 gives a graphical description of the method, applied to the Jupiter system.

2.2. Computing the statistical significance of the result

It is important to know whether the results obtained by the previous method are statistically significant, or if we could fit any set of values, not involving any physics, with our law. In order to check this, we have made simulations with 10000 random systems having N bodies and for a maximum of m available positions. In Fig. 2 we show the distribution of χ^2 for random systems of $N = 5$ bodies as a function m. Analogous distributions were obtained for other values of N. For example, for Jupiter, $N = 5$ and $M = 7$, and we see that only $\beta = 4\%$ of random systems have a better χ^2 that the χ^2 we have found for the Jupiter system.

Table 1 gives the statistical significance for the different planetary systems. The significance is defined as $\alpha = 100 - \beta$. We give also the correlation coefficient γ of the regression line fitting the data points as defined in step 3 of the software process, and the origin corresponding to $x = -0.25$ and $y = 0$. The origin is given by equation 1 for $n = 0$.

In the following pictures, the regression line has been calculated with the data represented by circles. The data represented by other symbols are added to the picture just for information, but are not taken into account for the computation.

Finally, we compute also a quantity called normalized standard deviation σ, which is the standard deviation divided by the mean value:

$$\sigma = \frac{\sqrt{\frac{1}{n-1}\sum_i (Ov_i - Tv_i)^2}}{\frac{1}{n}\sum_i Ov_i} \tag{2}$$

Ov_i is the observed value of the semi-major axis, Tv_i is the corresponding value given by the theory, and n is the number of values.

As a comparison, we have also considered the Titius-Bode law applied to the solar system (see for example (Nieto, 1972)):

$$r_n = 0.4 + 0.3.2^n \tag{3}$$

It gives the distance r of a planet versus the rank n. The law does not fit well the observed data for Neptune and Pluto, and the normalized standard deviation looks very bad.

Finally, we give the statistical significance for the system made of 3 planets found around pulsar PSR B1257+12 (see (Nottale, 1996b)). The good results we obtain with our method are probably due to the fact that the law underlying this method originates in a real theory (Scale Relativity), whereas other methods, like the law of Titius-Bode, are only empirical approaches.
(see also (Neuhauser and Feitzinger, 1986) and
(Dubrulle B., 1996)).

Fig. 3 shows the solar system, which is divided in inner system and outer system. The objects for the inner solar system are: Mercury, Venus, the Earth, Mars, and the main mass peaks of the asteroid belts: Hungarias, Ceres, Hygeia and Hildas. The maximum of the mass distribution of the inner system, close to the position of the earth, fits well with the rank $n = 1$ of the outer system. This suggests that the inner system is a sub-system of the outer one, consistent with the fragmentation process proposed in (Nottale et al., 1997). It is important to note that, on this type of diagram, the horizontal correlation does not have any significance since it results from our construction. Only the vertical discrepancies to the straight line have a significance.

3. Quantization of the satellite orbits

We have applied this method to the different satellite systems of the solar system. We have used only data concerning the most massive objects, having accurate and reliable parameters. The orbital data used in this paper are extracted from a compilation by Calvin J. Hamilton (Hamilton, 1997). The reader can also consult the following references:
(J. K. Beatty, 1990), (Henbest, 1992), (Simon, 1992),
(Thomas et al., 1983) and (L. A. Soderblom, 1982) .

3.1. Jupiter system

The Fig. 4 shows the quantization of the main satellites of Jupiter, obtained by applying our method, the central mass M in this case being the mass of Jupiter. For all the following pictures, on the vertical axis, a is given in km, and M is given in terrestrial masses.

We have taken into account only the satellites of Jupiter having a mass more than 10^{18} kg: Amalthea, Io, Europa, Ganymede and Callisto. We have also reported the position of the equatorial radius of the planet.

3.2. Saturn system

The Fig. 5 shows the quantization of the orbits of the main satellites of Saturn, the central mass being the mass of Saturn.

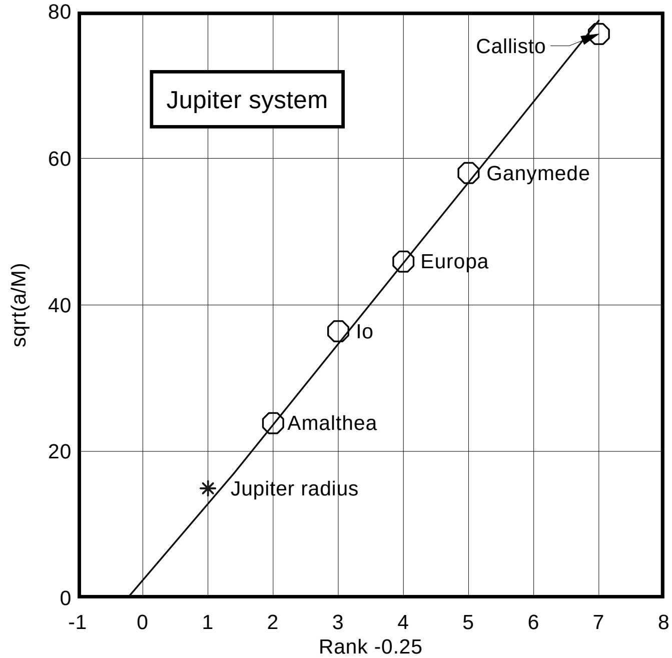

Fig. 4. Main satellites of Jupiter. M is in Earth masses, an a in km.

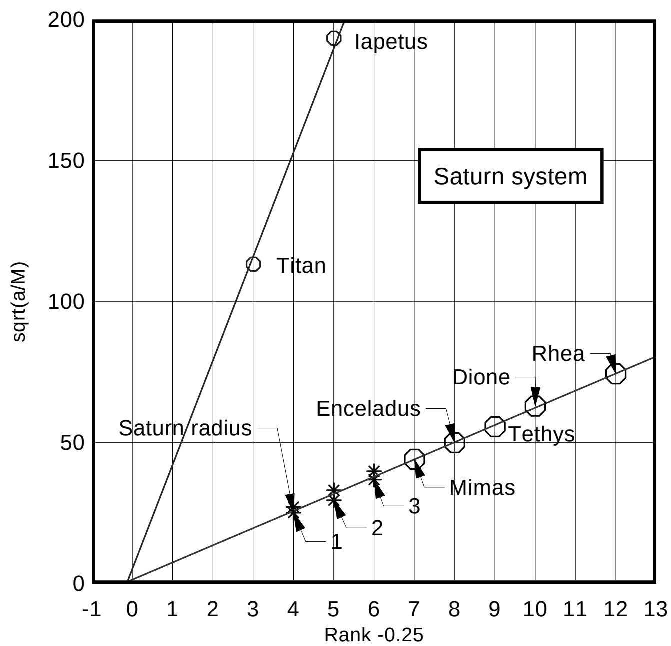

Fig. 5. Main satellites and rings of Saturn.

It appears that we have to classify Saturn's satellites in two systems: an inner and an outer system. Saturn is a complex system with rings and bodies orbiting some times at the same distance from the planet. On Fig. 5, for the inner system, the label Tethys refers to Tethys, Telesto and Calypso. The label Dione includes Dione and Helene.

As an indication, we show also the position of a set including the D ring and the radius of Saturn itself, labeled 1 on the figure, a set including the B and the C ring labeled 2. The set labeled 3 includes the A ring, Pan, Atlas, Prometheus, Pandora,

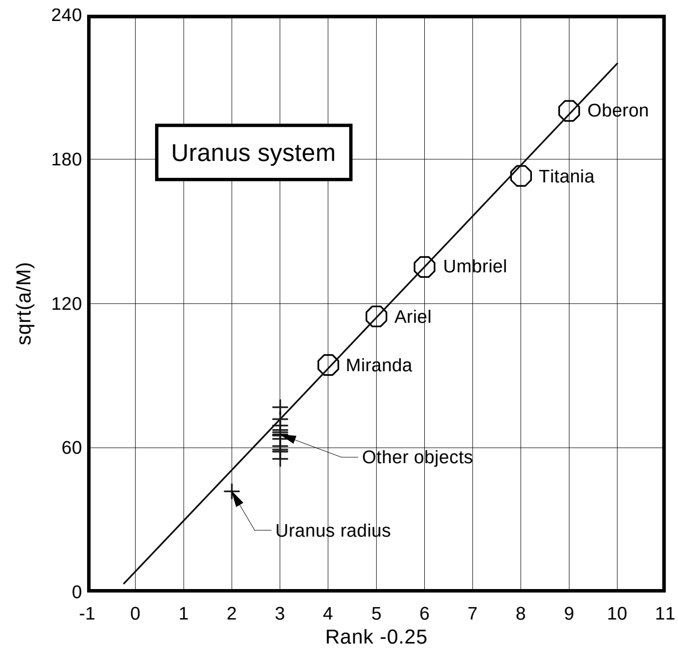

Fig. 6. Main satellites and rings of Uranus.

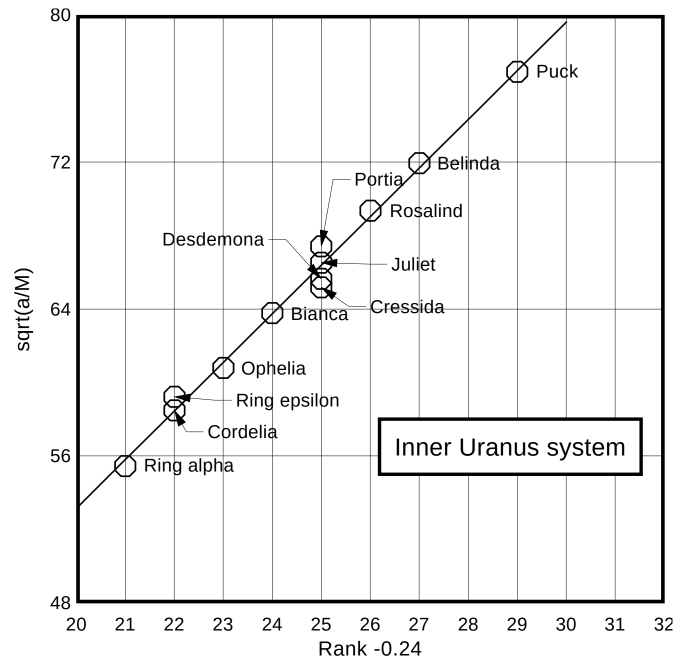

Fig. 7. Possible inner system of Uranus.

Epimetheus and Janus. For the outer system, we have not plotted the position of Hyperion which has a perturbed orbit due to the short time-scale resonance with Titan. One can also notice that the most heavy object of the inner system is Rhea, which corresponds roughly to the rank $n = 2$ of the outer system. This suggests that the inner system is maybe a sub-system obtained by the fragmentation process from the outer system.

3.3. Uranus system

The Fig. 6 shows the quantization of the main satellites of Uranus, the central mass being the mass of Uranus.

The satellites of Uranus considered here are Miranda, Ariel, Umbriel, Titania and Oberon. The data labeled "other objects" are a set including the rings α and ε, and the moons Cordelia, Ophelia, Bianca, Cressida, Desdemona, Juliet, Portia, Rosalind, Belinda and Puck. All these objects have orbits that confer them a rank around n=3 in the diagram. But they could also be represented as an internal system, as shown in Fig. 7. But this last representation is questionable, in spite of the good correlation coefficient, because the values of rank are too high. That is why we show this picture only tentatively. The fact that these inner satellites of Uranus seem to be spread around $n = 3$ of the outer system, shows that there is a need for a second-order theory treating this system not as a simple 2 body problem, but also taking into account some other effects (tides, mutual effects...)

Two new satellites have recently been discovered around Uranus (Gladman, 1997). But for the moment no orbital parameters are available, and we only have some positions . Therefore we are not able to add data concerning these satellites to our diagram.

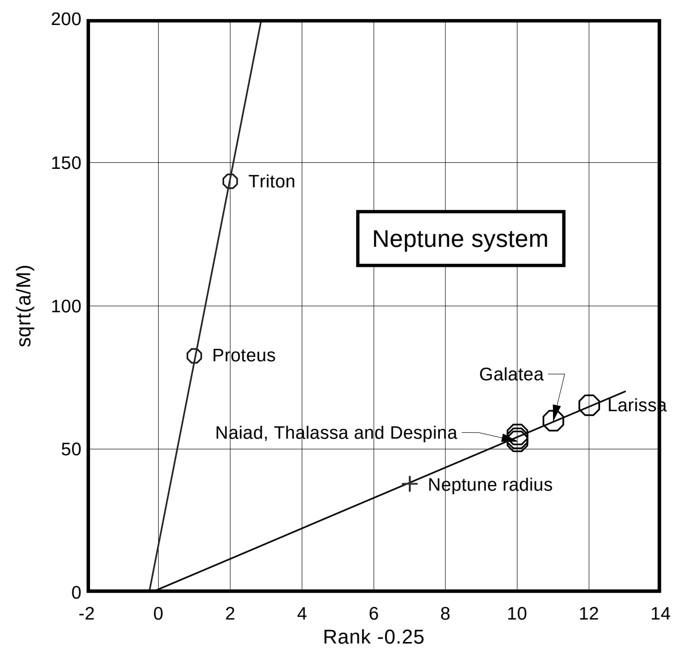

Fig. 8. A possible representation of the satellite system of Neptune.

3.4. Neptune system

Neptune is the most distant planet of the solar system for which we know the existence of satellites. Pluto-Charon looks more like a binary object. That is probably the reason why we know only its most distant satellites. We find here also high values of the ranks n, but as for Saturn and Uranus, it appears that there are two systems. We have computed the statistical significance only for the inner system that involves a set including Despina, Naiad and Thalassa, and Galatea and Larissa. Fig. 8 show a possible

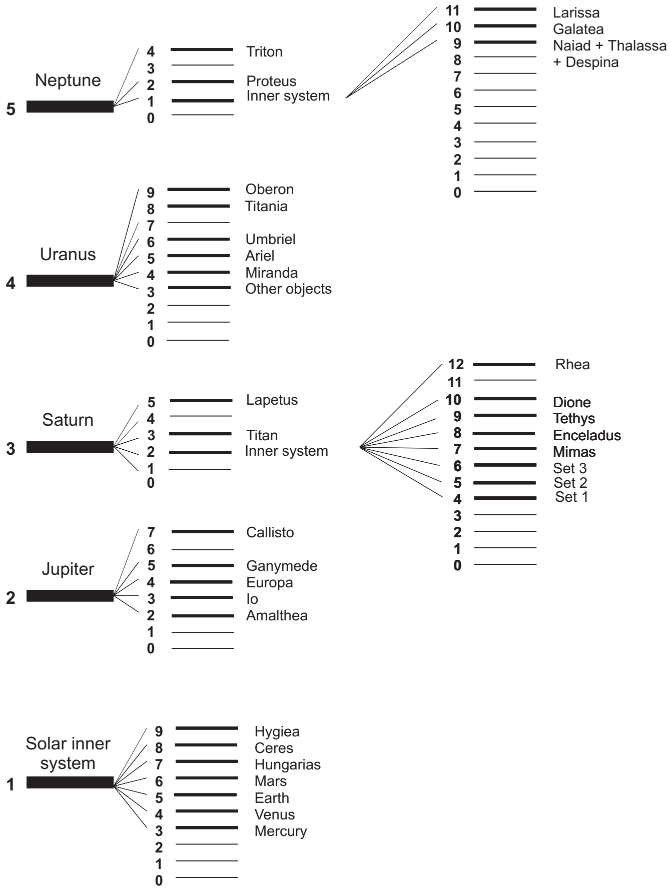

Fig. 9. Classification of the solar system by rank.

Table A1. Orbital parameters

Name	distance km	Name	distance km
Jupiter		**Uranus**	
radius	71,492	radius	25,559
Amalthea	181,300	Ring α	44,720
Io	421,600	Ring ε	51,190
Europa	670,900	Cordelia	49,750
Ganymede	1,070,000	Ophelia	53,760
Callisto	1,883,000	Bianca	59,160
Saturn		Cressida	61,770
radius	60,268	Desdemona	62,660
D ring	70,500	Juliet	64,360
C ring	83,250	Portia	66,100
B ring	104,750	Rosalind	69,930
A ring	129,500	Belinda	75,260
Pan	133,583	Puck	86,010
Atlas	137,640	Miranda	129,780
Prometheus	139,350	Ariel	191,240
Pandora	141,700	Umbriel	265,970
Epimetheus	151,422	Titania	435,840
Janus	151,472	Oberon	582,600
Mimas	185,520	**Neptune**	
Enceladus	238,020	radius	24,764
Tethys	294,660	Naiad	48,000
Telesto	294,660	Thalassa	50,000
Calypso	294,660	Despina	52,500
Dione	377,400	Galatea	62,000
Helene	377,400	Larissa	73,600
Rhea	527,040	Proteus	117,600
Titan	1,221,850	Triton	354,800
Iapetus	3,561,300		

representation of the system, the outer system involving only two objects: Proteus and Triton.

4. Classification

Finally, one can give a hierarchical classification of all the objects of the solar system we have considered here (Fig. 9). We assume that the outer planetary system (Jupiter, Saturn...) constitute the basic distribution of objects in the solar system. The level 1 of this system is split in a sub-system made of the inner planets (Mercury, Venus...). Each outer planet has also its own sub-system of satellites. But in the case of Saturn, Uranus and Neptune, the level 1 is again split in a sub-sub-system. This classification is consistent with the model of fragmentation for the formation of the solar system described in (Nottale et al., 1997)

5. Conclusion

With the present work we have tried to see if there is some consistency between the data of the solar system and the theory of Scale Relativity. Surprisingly, one finds almost always a good fit, except maybe for Neptune where there is a lack of data for close objects. This analysis reinforces then the confidence one can have in Nottale's description of large scale gravitational systems in terms of quantified systems. But the method also allows predictions: some levels are unoccupied. It means that either, for some unknown reason, there are no objects on the corresponding orbits, or they have not yet been discovered. Further analysis should take into account short time-scale resonances, and be extended to second order terms linked with the 3-body problem.

Notice that, on level 1, there is never any object. The most we have are objects in fragmented sublevels in case of Saturn, Uranus, Neptune, and for the outer solar system. Notice also that for every planetary system, the radius of the central object has a value wich seems also to fit in a specific rank.

It is also remarkable to notice that the orbital velocity corresponding to the rank $n = 1$ of the inner solar system is equal to 144 km/s. It is the same value as the one found by Tifft in the double galaxies (Tifft, 1977). This point has already been approached by Nottale in his last theoretical work (Nottale, 1997b).

Acknowledgements. This work has been supported by CNRS. We would like to thank Dr. Laurent Nottale for helpful discussions. We would also like to thank Dr. Calvin Hamilton for allowing us the use of his data.

Appendix

Table A1 gives the orbital parameters we have taken for this paper, extracted from reference (Hamilton, 1997). For the rings, the distance indicated in the table is the distance of the middle of the ring

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