Pergamon

Chaos, Solitons & Fractals Vol. 9, No. 7, pp. 1043–1050, 1998

Printed in Great Britain
0960-0779/98 $19.00 + 0.00

S0960-0779(97)00079-9

Scale relativity and Quantization of the Planetary System Around the Pulsar PSR B1257+12

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(Accepted 14 March 1997)

Abstract—The theory of scale relativity suggests that, at very large time scales, classical laws should be replaced by a non-deterministic, quantum-like description. The theory gives up individual, localized trajectories, but it yields the probability density of families of virtual trajectories, as deduced from a generalized Schrödinger equation. In this paper, we compare the system of three planets recently discovered around the pulsar PSR B1257+12 to the predictions of the theory. We expect the planet periods to be quantized according to the formula $T_n = 2\pi G M_{\mathrm{PSR}} (\sqrt{n^2 + n/2}/\mathscr{w}_0)^3$, where $\mathscr{w}_0$ is a multiple of 144 km/s (as determined from the Solar System and from galactic and extragalactic data). The observed periods confirm the theory with a remarkably high precision (relative differences of some 10^{-4}) and in a highly significant way (probability 3×10^{-5}), allowing us to successfully check second and third order terms. We finally predict the periods of possible additional planets in this system, in particular $T_2 = 1.958$ days.

1. INTRODUCTION

The theory of scale-relativity [1, 2] allows one to refound standard quantum mechanics on first principles, namely, on the generalization of Einstein's principle of relativity to scale transformations of resolutions [1, 3, 4], but it can also be applied to very large space–time scales. Giving up the hypothesis of differentiability of the space–time continuum implies that space–time becomes fractal, i.e. explicitly resolution-dependent [1, 5, 6], not only at small scales, but also at large scales [1, 7]. Moreover, one can demonstrate that the trajectories of a strongly chaotic system beyond its horizon of predictability have the same geometrical properties as the geodesics of a non-differentiable, fractal space–(time). This led us to suggest that, on very large time scales, chaotic systems must be described by a new, quantum-like theory, since the classical equations become unusable for $\Delta t \gg \tau$ (where $1/\tau$ is the Lyapunov exponent).

Our macroscopic theory, though it shares with standard quantum mechanics some of its mathematical tools (complex wavefunction, generalized Schrödinger equation), differs from it in an essential way by its context. Its physical interpretation need not make use of the various aspects of measurement theory, in particular the collapse of wavefunctions. The appearance of peaks of probability density is simply interpreted as a tendency for the system to make structures.

We have demonstrated in detail elsewhere [1, 2, 5] that the effects of the non-differentiable and fractal geometry of space time can be described in the simplest case in terms of a scale-covariant, complex time-derivative:

$$\frac{\not{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \mathscr{V} \cdot \nabla - \mathrm{i}\mathscr{D}\Delta \tag{1}$$

where $\mathscr{V} = \not{d}x/\mathrm{d}t$, and where $\mathscr{D}$ is a new fundamental parameter which characterizes the new scale laws. The whole of classical mechanics can be reformulated in terms of this scale-covariant derivative, and transformed into quantum-like mechanics. Introducing a wavefunction ψ which is nothing but another expression for the action $\psi = e^{i\mathscr{S}/2m\mathscr{D}}$, Newton's equation of dynamics, $m\mathrm{d}\mathscr{V}/\mathrm{d}t = -\nabla\phi$, can easily be integrated in terms of a generalized Schrödinger-like equation [1]:

$$\mathscr{D}^2\Delta\psi + \mathrm{i}\mathscr{D}\frac{\partial}{\partial t}\psi - \frac{\Phi}{2m}\psi = 0. \tag{2}$$

Born's interpretation of $\rho = \psi\psi^{\dagger}$ as the density of probability to find the particle at a given position is ensured from the very beginning of the construction: indeed, one can show that the expectations, which are initially defined on the infinite set of geodesics, can finally be taken using ρ as probability density. This is confirmed by the fact that the imaginary part of eqn (2) is the equation of continuity $\partial\rho/\partial t + \mathrm{div}(\rho V) = 0$, (where V is the real part of our complex velocity $\mathscr{V}$). A more detailed account of the theory of scale relativity and of its implications can be found in Ref. [1], and in the recent review paper [2].

The theory has already been successfully applied to the Solar System [1, 8], and to the recently discovered planets around solar-like stars [9]. The three planets around the pulsar PSR B1257+12 [10] deserve a special study, since they allow checking of the theory on a full system other than the Solar one, independently of the star mass. We shall see that our predictions are confirmed on this system with such a precision that second and third order terms can be successfully checked.

2. APPLICATION OF SCALE-RELATIVITY TO THE KEPLER PROBLEM

The simplest case to which our formalism can be applied is the Kepler gravitational problem [1, 6, 11]. The probability amplitude of a test particle moving in a central gravitational potential is given in our theory by the stationary equation (valid on very large time scales):

$$2\mathscr{D}^2\Delta\psi + \left[\frac{E}{m} + \frac{GM}{r}\right]\psi = 0. \tag{3}$$

This equation can be applied not only to the distribution of planets in a planetary system (treated as test particles of mass m in the field of the central star of mass M), but also, more generally, to binary systems (double stars, binary galaxies, etc.), in terms of total mass $M = m_1 + m_2$ and reduced mass $m = m_1 m_2/(m_1 + m_2)$.

We can be even more specific about the value of our only free parameter $\mathscr{D}$. The equivalence principle implies that eqn (3) must not depend on the inertial mass of the test particle, so that $\mathscr{D}$ is independent of m. In order to derive its form in the macroscopic, gravitational case considered here, recall that the scale-covariant derivative has been constructed from two contributions, fractal fluctuations and breaking of local time reversibility, described in terms of complex numbers [1, 2]. If we now include only this last contribution, we can define an incomplete covariant derivative $\tilde{\mathrm{d}}/\mathrm{d}t$, in terms of which the equation of fractal 'free' motion takes the form of Newton's equation of dynamics $\tilde{\mathrm{d}}\mathscr{V}/\mathrm{d}t = \mathrm{i}\mathscr{D}\Delta\mathscr{V}$. In this equation, the effect of the fractal fluctuation is now expressed in terms of a complex 'fractal force'

$$\mathscr{F} = \mathrm{i}m\mathscr{D}\Delta\mathscr{V}. \tag{4}$$

In the situation considered here, the fluctuation remains of purely gravitational origin, so that we expect this force to be proportional to the product mM, where M is the total mass acting on mass m. Then $\mathscr{D} \propto M$, and can be written in terms of a new constant w having the dimension of a velocity:

$$\mathscr{D}=\frac{GM}{2w}. \tag{5}$$

Let us now look for the solutions of eqn (3). It is similar to the Schrödinger equation for the hydrogen atom, up to the substitution $\hbar/2m \to \mathscr{D}$, $e^2 \to GmM$, so that the natural unit of length (which corresponds to the Bohr radius) is:

$$a_0=\frac{4\mathscr{D}^2}{GM}=\frac{GM}{w^2}. \tag{6}$$

Energy must be quantized as $E=-0.5mw^2/n^2$, with $n=1, 2, 3, \ldots$, and angular momenta as $L_z=2m\mathscr{D}\ell$, with $\ell=0, 1, \ldots, n-1$. It is remarkable that, unlike in standard quantum mechanics, E/m and L_z/m are 'quantized' rather than E and L. The average distance a to the central star and the eccentricity e are given, in terms of the quantum numbers n and ℓ, by the relations $a_{n\ell}=[\frac{3}{2}n^2-\frac{1}{2}\ell(\ell+1)]a_0$ and $e^2=1-\ell(\ell+1)/n(n-1)$.

Our first prediction is therefore that circular orbits ($\ell=n-1$) are highly probable for small values of n, since the next to circular orbit ($\ell=n-2$) is predicted to be very eccentric, and thus presumably unstable when several planets are present. This prediction is well verified in our own solar system [1, 8]. In the system around PSR B1257+12, the very low observed eccentricities ($e_A=0$; $e_B=0.0182$; $e_C=0.0264$ [10]) confirm this prediction. We shall then only consider circular orbits in what follows. In this case, the probability distribution is written as:

$$P_n(r) \propto r^{2n} e^{-2r/na}. \tag{7}$$

The mean distance and the peak distance of this probability distribution are respectively:

$$a_n(\text{mean})=(n^2+n/2)a_0;\, a_n(\text{peak})=n^2 a_0. \tag{8}$$

We finally find that velocities in Keplerian gravitational systems must be quantized according to:

$$v_n=\frac{w}{\sqrt{n^2+n/2}}\ (\text{mean});\ v_n=\frac{w}{n}\ (\text{probability peak}) \tag{9}$$

where w is a multiple or submultiple of some fundamental constant w_0 (see Tab. 1 and Refs [9, 12]).

For the time being the constant w_0 is not predicted theoretically and must be determined from observational data. The observed structures of the Solar System, concerning eccentricities, angular momenta, distances of planets [1, 6, 11] and mass distribution [8], are found to be in very good agreement with the predictions of the theory. The inner Solar System yields:

$$w_0=144.3 \pm 1.2\,\text{km/s}. \tag{10}$$

The constant of the outer system can be determined from the fact that the inner system in its whole achieves its orbital $n_1=1$ (Table 1 [8, 11]). The peak of probability of this orbital must then correspond to the peak of the mass distribution in the inner system, which is achieved by the Earth. Since the Earth ranks $n_2=5$ in the inner system, we expect the outer Solar System to be governed by a constant $w(\text{out}) \approx w_0(\text{in})/5.24$, as actually observed Tab. 1. Toward the smaller scales, one finds that the Sun radius also satisfies our law, since it is given by the peak of the orbital $n_3=1$, $w_3=3w_0$ (indeed, the Kepler velocity at the distance from the center given by the Sun's radius is $436.8\,\text{km/s}=3\times145.6\,\text{km/s}$). Such a result can also be expected from the theory, since the internal structure equation of stars can be quantized according to the same rules [12], and the solutions matched to the exterior solutions. We shall see below that the PSR B1257+12 system is governed by the same constant $3w_0$.

Table 1. Data for the planets of the inner and outer Solar System, and for the recently discovered planets around nearby stars (from Ref. [9]). The semi-major axes are given in AU. In column 6, the rank is given by $144/v$ for the inner System and for exoplanets, and by $144/5.24v$ for the outer Solar System. In the last column we compute $w_0 = nv$ in the inner Solar System and for extrasolar planets, and $w_0 = 5.24nv$ in the outer Solar System. (This is due to the fact that the inner System on the whole achieves the orbital $n = 1$ of the outer System: since the Earth, which is the mass peak of the inner System, ranks $n = 5$, a factor $\sqrt{(5^2+5/2)} \approx 5.24$ is applied to the calculation). In the absence of quantization, the values of δn in column 7 would be uniformly distributed between -0.5 and $+0.5$

Star/planet	Star mass	Period (year)	1/2 Major axis	v (km/s)	$144/v$*	n	δn	w_0 (km/s)
Sun/Mercury	1	0.24085	0.387	47.87	3.01	3	+0.01	143.6
Sun/Venus	1	0.61521	0.723	35.06	4.11	4	+0.11	140.2
Sun/Earth	1	1.00004	1.000	29.81	4.83	5	−0.17	149.0
Sun/Mars	1	1.88089	1.524	24.14	5.96	6	−0.04	144.8
Sun/Ceres	1	4.61	2.77	17.92	8.04	8	+0.04	143.4
Sun/Cybeles	1	6.35	3.43	16.09	8.95	9	−0.05	144.8
Sun/Jupiter	1	11.86	5.20	13.07	2.10	2	+0.10	137.2
Sun/Saturn	1	29.46	9.57	9.63	2.85	3	−0.15	151.6
Sun/Uranus	1	84.01	19.28	6.81	4.03	4	+0.03	143.0
Sun/Neptune	1	164.8	30.14	5.42	5.05	5	+0.05	142.3
Sun/Pluton	1	248.5	39.88	4.75	5.78	6	−0.22	149.6
51 Peg B	1.10	0.01158	0.053	137	1.05	1	+0.05	137
47 UMa B	1.05	3.020	2.12	20.9	6.90	7	−0.10	146
70 Vir B	1.12	0.3195	0.485	45.3	3.18	3	+0.18	136
HD114762 B	1.0	0.230	0.376	48.65	2.96	3	−0.04	146
Prox Cen B	0.11	0.211	0.170	24.0	6.00	6	+0.00	144
55 Cnc B	0.8	0.04041	0.11	81.6	1.79	2	−0.21	163
τ Boo B	1.2	0.00907	0.046	152	0.95	1	−0.05	152
υ And B	1.2	0.0126	0.058	136	1.06	1	+0.06	136

Our theory has been successfully checked on the planets recently discovered around solar-type stars [9]. We have found from eight new extra-solar planets (Table 1):

$$w_0 = 143.9 \pm 3.1 \text{ km/s}. \tag{11}$$

The theory also applies to the velocity quantization effect discovered 20 years ago by Tifft [13], and since confirmed by several authors [13–17]. Tifft found that the distribution of the velocity difference between the two members of binary galaxies shows well-defined and statistically significant peaks, when studied using high precision redshift measurements. The reported values are 144, 72, 36, and 24 km/s, quite in agreement with our predicted n^{-1} quantization law (Fig. 1) and with the value of w_0 as it is observed in the Solar System and in extra-solar planetary systems.

All these results allow us to predict that the planetary system recently discovered around the pulsar PSR B1257+12 must also be quantized in terms of $\tilde{w}_0 = kw_0$, with $w_0 = 144.70 \pm 0.55$ km/s (average of the various determinations quoted in Ref. [9]).

3. APPLICATION TO THE SYSTEM AROUND PSR B1257+12

The discovery by Wolszczan and Frail [18] and Wolszczan [10] of three planets around the pulsar PSR B1257+12 offers a unique opportunity to test our prediction outside our solar system. Indeed, (i) this system can be expected to be a purely gravitational system, unperturbed by other forces, so that one can hope to see our predictions verified with high precision; (ii) the periods are known with high accuracy (one sigma relative uncertainty of respectively 600, 5 and 7 ppm); (iii) orbitals can exist very close to the neutron star thanks to its compactness, so that the velocity in the 'fundamental' orbital could reach several times the 'unit' 144 km/s.

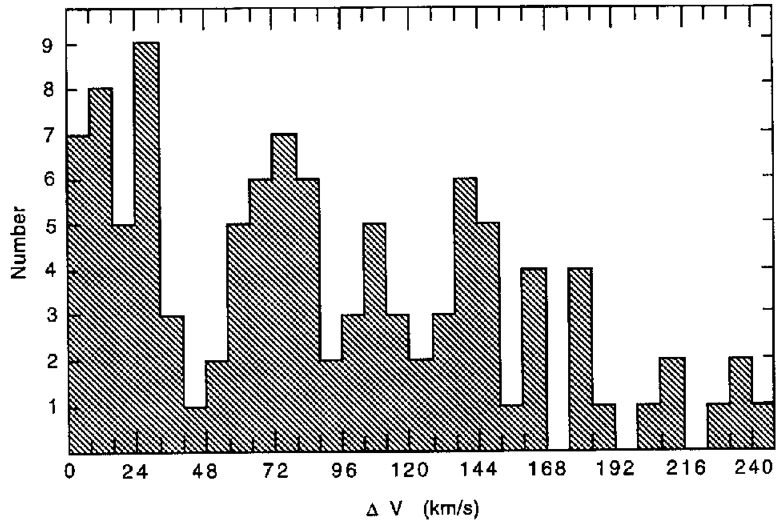

Fig. 1. Histogram of velocity differences for the Schneider–Salpeter (1992) sample of 107 isolated binary galaxies with high precision 21 cm redshifts, from their table 1. The higher probability of the values 72 km/s and 144 km/s (and possibly 24 km/s) is clearly apparent and can be shown to be statistically significant [12].

It seems likely that the planets were formed from a circumpulsar disk created from the remains of the pulsar's binary stellar companion [10]. In our theory, the probability density distribution of matter in the disk must be given locally by eqn (7). The conservation of energy implies that the final position of the planets formed from this distribution must be given with high precision by the mean distance formula $a_n = a_0(n^2+n/2)$, i.e. $\sqrt{(a_n)} \approx \sqrt{(a_0)}\,(n+1/4)$, with $a_0 = GM_{\mathrm{PSR}}/\varpi_0^2$, and $\varpi_0 = 144k$ km/s, with n and k integers. Let us compare the data with these theoretical predictions (Table 2).

(i) Since, from Kepler's third law, $\sqrt{a} = M^{1/6}T^{1/3}$ (in Solar System units, respectively AU, $M_\odot$ and years) $\approx \sqrt{a_0}(n+1/4)$, we expect the $T^{1/3}$ *differences* between the three planets to be quantized in terms of integers (up to very small differences). This first prediction is very well verified, since

$$\frac{T_{\mathrm{B}}^{1/3} - T_{\mathrm{A}}^{1/3}}{T_{\mathrm{C}}^{1/3} - T_{\mathrm{B}}^{1/3}} = 1.984 \tag{12}$$

that differs from $\Delta n = 2$ by only 0.016. This implies that the three planets must rank as $n_{\mathrm{A}} = n_{\mathrm{B}} - 2$, n_{B}, $n_{\mathrm{C}} = n_{\mathrm{B}} + 1$. We have assumed here for simplicity that $n_{\mathrm{C}} - n_{\mathrm{B}} = 1$, but any other choice would be equivalent (it would multiply k by an integer).

(ii) We thus obtain the two following equations:

Table 2. Planetary companions to PSR B1257+12 (observational data from Ref. [10]). In column 2, the periods of A and B are predicted from their ratio with $T_{\mathrm{C}} = T_8$. The distances and velocities are calculated for a pulsar mass 1.46 $M_\odot$. In columns 6 and 7, $\varpi = (n^2+n/2)^{1/2} v_n$ and $w_0 = \varpi/3$.

Planet	n	Predicted period (day)	Observed period (day)	Semi-major axis (AU)	v (km/s)	ϖ_0	w_0
A	5	25.26	25.34	0.1916	82.30	431.6	143.9
B	7	66.63	66.54	0.3646	59.66	432.2	144.1
C	8	(98.22)	98.22	0.4727	52.39	432.0	144.0

$$T_A^{2/3}/|T_C^{2/3} = [n_B^2 - \tfrac{7}{2}n_B + 3]/[n_B^2 + \tfrac{5}{2}n_B + \tfrac{3}{2}] \tag{13a}$$

$$T_B^{2/3}/|T_C^{2/3} = [n_B^2 + \tfrac{1}{2}n_B]/[n_B^2 + \tfrac{5}{2}n_B + \tfrac{3}{2}]. \tag{13b}$$

Considering period *ratios* allows us to eliminate the unknown pulsar mass. We find, respectively, from eqs. (13a, 13b):

$$n_B\,(A,C) = 7.016;\; n_B\,(B,C) = 6.971. \tag{14}$$

The quantization is once again confirmed with a remarkable precision, yielding $n_A = 5$, $n_B = 7$ and $n_C = 8$ (see Fig. 2).

(iii) We can go even further. The precision reached by this system is so good that we can expect to be able to make the difference between the 'mean' and 'peak' formulae (this means to test for second order terms in the theory). We find:

$$\frac{T_A^{1/3}}{T_C^{1/3}} = 0.63666 \text{ while } \frac{\sqrt{5^2+5/2}}{\sqrt{8^2+8/2}} = 0.63593 \text{ and } \frac{5}{8} = 0.625 \tag{15a}$$

$$\frac{T_B^{1/3}}{T_C^{1/3}} = 0.87827 \text{ while } \frac{\sqrt{7^2+7/2}}{\sqrt{8^2+8/2}} = 0.87866 \text{ and } \frac{7}{8} = 0.875. \tag{15b}$$

Even though the agreement with the 'peak' formula could already be considered as excellent (relative differences of respectively 0.018 and 0.004), the agreement with our more precise expectation, the 'mean' formula, is more than 10 times better (respectively 0.0011 and −0.0004).

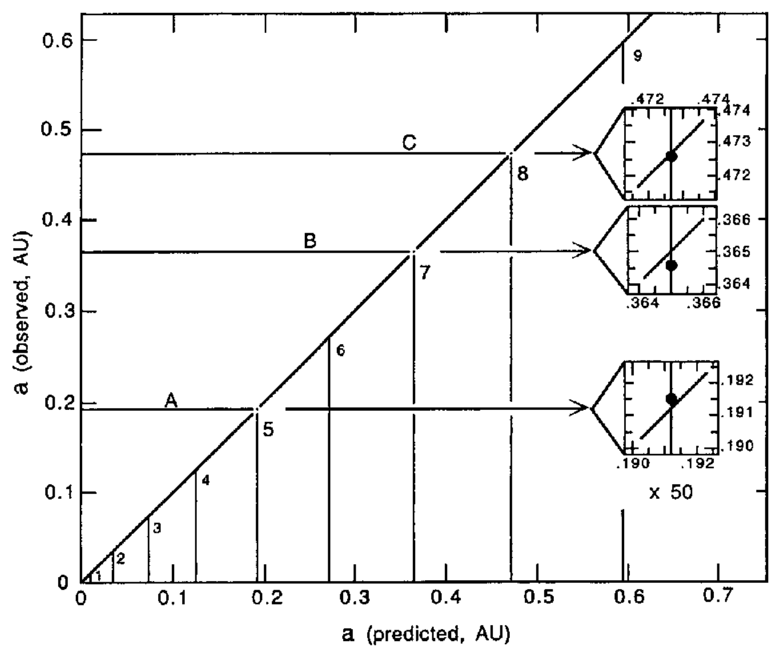

Fig. 2. Comparison between theoretical prediction and observations for the three planets around the pulsar PSR B1257+12, assuming a pulsar mass of 1.46 $M_\odot$. The agreement is far better than the resolution of the diagram (and independent of the choice of the pulsar mass): the four points (the star and the three planets) are aligned with differences smaller than the thickness of the line. We have included three insets enlarged by a factor of ≈50 to show the small residual differences.

Actually, the PSR B1257+12 pulsar planetary system tests our formula even up to the third order. Let us express our theoretical expectation as an expansion in terms of $(1/n)$. It is written as $a_n/a_0 = n^2(b_1 + b_2/n + b_3/n^2 + b_4/n^3 + \ldots)$, with $b_1 = 1$, $b_2 = 1/2$ and $b_j = 0$ for $j \geqslant 3$. To third order, the relative correction is $\approx b_3\,(1/n_i^3 - 1/n_j^3)$. The difference $1/n_i^3 - 1/n_j^3$ remains 10 times larger than the observed residuals in eqn (14), so that we get $b_3 = 0.0 \pm 0.1$. Finally, we need to go to order four to find corrections of the order of the residuals.

(iv) All these results are completely independent of the (unknown) pulsar mass. We shall now see that this mass can be inferred from our theory. Let us first show that our last prediction, i.e. $\tilde{w}_0 = 144k$ km/s, is well verified by any standard value of the neutron star mass. The pulsar mass is given by:

$$M = \frac{\tilde{w}^3}{2\pi G}\,\frac{T_n}{(n^2 + n/2)^{3/2}}. \tag{16}$$

Assuming a standard neutron star mass of $1.4 \pm 0.1\,M_\odot$, we get a confirmation of our last quantization law:

$$k = \frac{\tilde{w}_0}{144} = 2.96 \pm 0.07 \tag{17}$$

within 0.04 of the quantized value $k = 3$. We can now compute the pulsar mass. From $w_0 = 144.7 \pm 0.6$ km/s, and taking $k = 3$ strictly, we obtain:

$$M_{\mathrm{PSR}} = 1.48 \pm 0.02\,M_{ERROR}. \tag{18}$$

Let us conclude this section by an estimate of the probability of getting such an observed configuration by chance. The observed ratios $(T_A/T_C)^{1/3}$ and $(T_B/T_C)^{1/3}$ fall within $\pm 7.3 \times 10^{-4}$ and $\pm 3.9 \times 10^{-4}$ of two of the predicted quantized ratios. The number of possible configurations is $C_{n_m}^3 = n_m\,(n_m - 1)\,(n_m - 2)/6$, where n_m is the maximal reasonable value for the quantum number n. Taking $n_m = 10$ yields a probability $P = C_{10}^3 \times (1.46 \times 10^{-3}) \times (7.8 \times 10^{-4}) = 1.4 \times 10^{-4}$. Also accounting for the fact that k falls within 0.1 of an integer using standard neutron star masses, we find a highly significant total probability $P = 3 \times 10^{-5}$. Even with the choice $n_m = 20$, one would still get the significant result $P = 3 \times 10^{-4}$.

4. CONCLUSION

The theory is falsifiable, since it allows us to make predictions about possible additional planets in this system. The period of A could already have been predicted after the discovery [18] of only B and C. Indeed, the periods published in that reference were $T_B = 66.6$ days and $T_C = 98.2$ days, from which $T_5 = 25.3$ days is predicted. The agreement with observation is remarkable, since the third planet has been found at a period $T_A = 25.34$ days [10]. Such a prediction could possibly be reproduced then in a totally blind way. The periods predicted for $n = 1$ to 10 from their ratio with the observed period of planet C are as follows (in days).

1	2	3	4	5	6	7	8	9	10
0.322	1.958	5.960	13.38	25.26	42.66	66.63	98.22	138.5	188.5

A more complete discussion of the implications of these results will be presented elsewhere [12]. Let us simply finally remark that the universality of the quantization, especially of the numerical value of the constant w_0, leads us to conclude that the source of the fundamental structuring chaos cannot be the fluctuations due to the environment of each particular system considered. We interpret this universality by the fact that the underlying geometry of space–time is fractal, not only in the microscopic domain, but also at large space–time scales, and plays the role of a structuring field.

Acknowledgements—I am grateful to Dr. E. Gérard for having drawn this pulsar system to my attention, and to Drs G. Schumacher and E. T. Lefèvre for stimulating discussions.

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