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Scale Relativity and Structuration of the Universe.

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Abstract. Some cosmological consequences of the principle of scale relativity are briefly considered in this contribution. The general form of linear scale-laws satisfying the above principle takes the form of the Lorentz group. This allows one to solve the horizon/causality problem without inflation, and implies that the fractal dimension of structures in the universe must vary with scale, in a way that depends on the value of the cosmological constant. This theory explains the observed value of the exponent of the correlation function, γ =1.8, predicts that γ must decrease for increasing scales, and accounts for the transition to uniformity at large scales. It allows a measurement of the cosmological constant that agrees with an independant estimate from the vacuum energy density.

The principle of scale relativity is an extension of Einstein's principle of relativity that takes into account the transformation of reference systems not only under space-time displacements (as in motion relativity), but also under dilations of space-time resolutions (scale laws). When applied to the cosmological domain, the theory of scale relativity leads to several new proposals and predictions.

In scale relativity, we first substitute to the "Galilean-like" laws of dilation $ln\rho'' = ln\rho + ln\rho'$ the more general Lorentzian law [1,2]:

$$ln\rho'' = \frac{ln\rho + ln\rho'}{1 + ln\rho \; ln\rho' \, / \, \mathbb{C}^2} \quad . \tag{1}$$

Under this form the scale relativity symmetry remains unbroken. Such a law corresponds, at the present epoch, only to the null mass limit. It is expected to apply in a universal way during the very first instants of the universe. This law assumes that, at very high energy, no static scale and no space or time unit can be defined, so that only pure contractions and dilations have physical meaning. (One could object that the Planck scale can always be used as unit, but, as we shall recall hereafter, it plays a special role in scale relativity). In Eq. 1, there appears a universal purely numerical constant $\mathbb{C} = ln\mathbb{K}$. As we shall see, the value of $\mathbb{K}$ is of the order of 5 x 10^{60}: its emergence yields an explanation to the Eddington-Dirac large number "coincidences" [3].

Now, pure scale relativity is broken in microphysics by the mass of elementary particles, i.e., by the emergence of their de Broglie length:

$$\lambda_{dB} = \hbar \ / \ mv \tag{2}$$

and in macrophysics by the emergence of static structures (galaxies, groups, cluster cores) of typical sizes:

$$\lambda_g \approx \frac{1}{3} Gm \ / <v^2> \tag{3}$$

The effect of these two symmetry breakings is to separate the scale axis into three domains, a quantum (scale-dependent), a classical (scale-independent) and a cosmological (scale-dependent) domain (see Fig. 1).

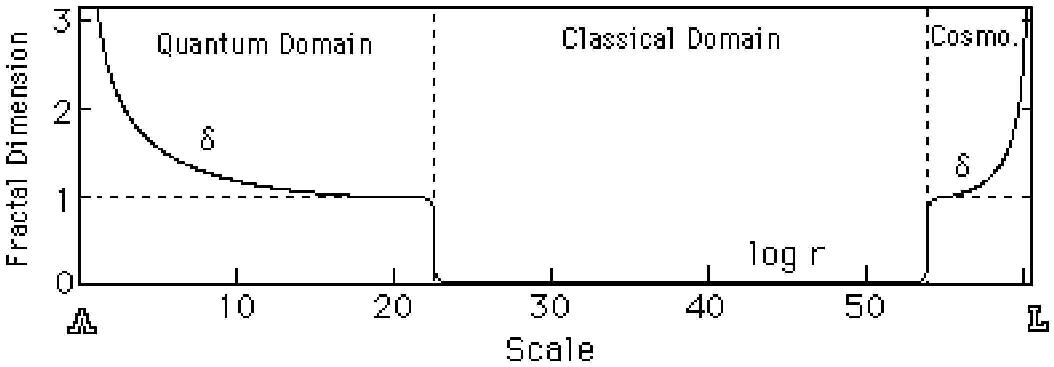

Figure 1. Schematic representation of the three, quantum, classical and cosmological domains of the present era, in terms of logarithm of resolution, and of the variation of the fractal dimension in these domains, in the case of scale-relativistic (Lorentzian) scale laws.

The consequence is that, in the two scale-dependent domains, the static scales can be taken as reference, so that one do not deal any longer with pure dilation laws, but with new laws involving dimensioned space and time intervals:

$$ln(\lambda''/\lambda) = \frac{ln(\lambda'/\lambda) + ln\rho}{1 + ln(\lambda'/\lambda)\ ln\rho \ / \ ln^2(\mathbb{A}/\lambda)} \tag{4}$$

In this new dilation law, the symmetry breaking has substituted a length-time scale that is invariant under dilations to the invariant dilation of Eq. 1. In the microphysical domain, this scale is naturally identified with the Planck scale, $\mathbb{A} = (\hbar G / c^3)^{1/2}$, that now becomes impassable and plays the physical role that was previously devoted to the zero point [1,2]. In the cosmological domain, the invariant scale is identified with the scale of the cosmological constant, $\mathbb{L} = \Lambda^{-1/2}$ [4].

We shall in this contribution briefly consider the various consequences and predictions of the new theory in the cosmological domain. A more detailed account can be found in Refs. [3,4].

Horizon / causality problem:

In the theory of scale relativity, the standard laws of dilations currently used up to now are shown to be low energy (i.e., large length-time scale) approximations of more general laws that take a Lorentzian form (see Eq. 1). One can indeed demonstrate [1,2] that the general solution to the special relativity problem (i.e., find the laws of transformation of coordinates that are linear and satisfy the principle of relativity) is the Lorentz group. This result, which was known to apply to motion laws [2,5], applies also to scale laws (i.e., contraction and dilations of resolutions).

The horizon / causality problem is simply solved in this framework without needing an inflation phase, thanks to the new role played by the Planck lenth-time scale. It is identified with a limiting scale, invariant under dilations. This implies a causal connection of all points of the universe at the Planck epoch. The light cones flare when $t \to \Lambda/c$ and finally always cross themselves (see Fig. 2 and Ref. 4]).

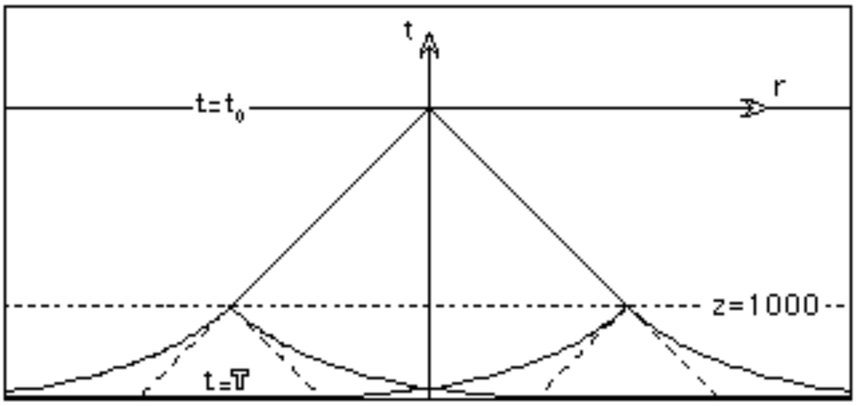

Figure 2. Illustration of the flare of light cones in scale-relativity, allowing causal connection of any couple of points in the universe (from Fig.7.1 of Ref [4]).

Cosmological constant and vacuum energy density.

One of the most difficult open questions in present cosmology is the problem of the vacuum energy density and its manifestation as an effective cosmological constant [6,7]. Scale relativity solves this problem and connect it to Dirac's large number hypothesis.

The first step toward our solution consists in considering the vacuum as fractal, (i.e., explicitly scale dependent). As a consequence, the Planck value of the vacuum energy density (that gave rise to the 10^{120} discrepancy with observational limits) is relevant only at the Planck scale, and becomes irrelevant at the cosmological scale. We expect the vacuum energy density to be solution of a scale (renormalisation group-like) differential equation [2,3,8]:

$$d\rho/d\ln r \;=\; \Gamma(\rho) = a + b\,\rho + \mathrm{O}(\rho^2)\ , \tag{5}$$

where ρ has been normalized to its Planck value, so that it is always <1, allowing the Taylor expansion of $\Gamma(\rho)$. This equation is solved as:

$$\rho = \rho_c \ \ [1 + (r_o/r)^{-b}] \ \ . \tag{6}$$

We recover the well known combination of a fractal, power law behavior at small scales (here), and of scale-independence at large scale, with a fractal/non-fractal transition about some scale r_o that comes out as an integration constant [2, 8-10].

The second step toward a solution is to realize that, when considering the various field contributions to the vacuum density, we may always chose $<E> = 0$ (i.e., renormalize the energy density of the vacuum). But consider now the gravitational self-energy of vacuum fluctuations [11]. It writes:

$$E_g = \frac{G}{c^4} \frac{<E^2>}{r} \ \ . \tag{7}$$

The Heisenberg relations prevent from making $<E^2> = 0$, so that this gravitational self-energy *cannot* vanish. With $<E^2>^{1/2} = \hbar c / r$, we obtain the asymptotic high energy behavior:

$$\rho_g = \ \rho_{\mathbb{P}} \left(\frac{\mathbb{\Lambda}}{r}\right)^6 \ \ , \tag{8}$$

where $\rho_{\mathbb{P}}$ is the Planck energy density. From this equation we can make the identification $-b = 6$. We are now able to demonstrate one of Dirac's large number relations, and to write it in terms of invariant quantities (i.e., we do not need varying constants to implement it in this form).

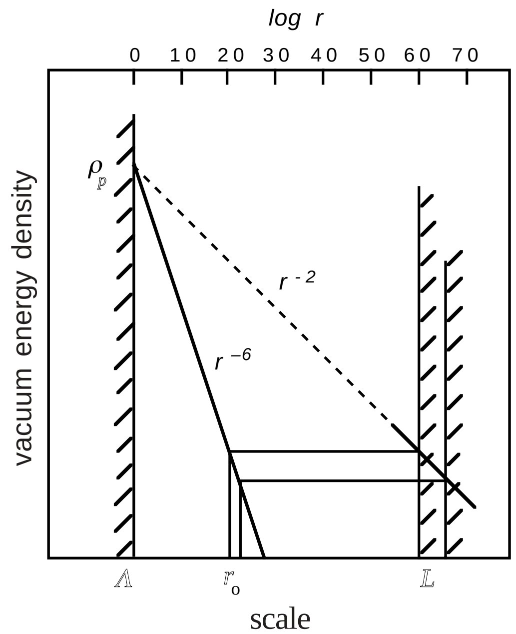

Figure 3. Variation of the gravitational self-energy density of vacuum fluctuations from the Planck length-scale to the cosmological scale $\mathbb{L} = \Lambda^{-1/2}$ (see text).

Indeed, introducing our maximal scale-relativistic length scale $\mathbb{L} = \Lambda^{-1/2}$, we get the relation:

$$\mathbb{K} \ = \ \mathbb{L} \ / \ \mathbb{\Lambda} \ = (r_o/\mathbb{\Lambda})^3 = (m_{\mathbb{P}}/m_o)^3 \ \ , \tag{9}$$

where r_o is the Compton length of the typical particle mass m_o. Then the power 3 in Dirac's relation is understood as coming from the power 6 of the gravitational self-energy of vacuum fluctuations and of the power 2 that relies the invariant impassable scale $\mathbb{L}$ to the cosmological constant,following the relation $\Lambda = 1/\mathbb{L}^2$ (see Fig. 3).

Now a complete solution to the problem would be reached only provided the transition scale r_o be known. Consider the possible range for the cosmological constant: Ω_Λ is observationnally upper bounded by $\Omega_\Lambda < 1$. Concerning lower bounds, one may remark that any value smaller than ≈ 0.01 would be indistinguishable from zero, since it would loose any effect on the determination of the comological evolution. It is remarkable that the corresponding range for $\mathbb{K}$, $3 \times 10^{60} - 3\times 10^{61}$, yields a very small range for m_o, namely 40 – 85 MeV. This short interval is known to contain several important scales of particle physics: the classical radius of the electron, that yields the e^+e^- annihilation cross section at the energy of the electron mass and corresponds to an energy 70.02 MeV; the effective mass of quarks in the lightest meson, $m_{\pi/2}$ = 69.78 MeV; the QCD scale for 6 quark flavours, Λ_{QCD} = 66±10 MeV; the diameter of nucleons, that corresponds to an energy 2 x 64 MeV. Then we can make the conjecture that the present value of the cosmological constant has been fixed at the end of the quark-hadron transition, so that the transition scale r_o is nothing but this particular scale (Fig. 3). Along such lines, we get:

$$\mathbb{K} = (5.3 \pm 2)\ 10^{60} \ , \tag{10}$$

corresponding to $\Lambda = 1.36\ 10^{-56}$ cm^{-2} and to $\Omega_\Lambda = 0.36\ h^{-2}$. Such a value of Λ would solve the age problem. Indeed the age of the universe becomes larger than 13 Gyr (in agreement with globular clusters) provided $h < 0.75$ in the flat case ($\Omega_{tot} = 1$), and $h < 0.85$ if $\Omega_{tot} < 1$.

Note that all the above calculation is made in the framework of Galilean scale laws. The passage to Lorentzian laws would change only the domain of very high energies, and thus would not affect our result, since it depends essentially on what happens at scale r_o.

Slope of the autocorrelation function.

It has been observed for long that the autocorrelation functions of various classes of extragalactic objects (from galaxies to superclusters) were characterized by a power law variation in function of scale, with an apparently universel index γ =1.8, smaller than the value $\gamma = 2$ expected from the simplest models of hierarchical formation. The theory of scale relativity brings a simple solution to this problem. The value $\gamma = 2$ is nothing but what is expected in the framework of Galilean scale laws: it is the manifestation of a fractal dimension $\delta = 3 - \gamma = 1$. In scale relativity, one must jump to Lorentzian laws at large scales in order to ensure scale-covariance. The fractal dimension now becomes itself scale-varying and depends on the cosmological constant $\Lambda = 1/\mathbb{L}^2$ as [4]:

$$\delta = \delta(r) = \frac{1}{\sqrt{1 - ln^2(r/\lambda_g) \ / \ ln^2(\mathbb{L}/\lambda_g)}} \ , \tag{11}$$

where λ_g is the typical static radius of the objects considered (10 kpc for giant galaxies, 100 to 300 kpc for clusters...). Several consequences and new predictions arise from this formula.

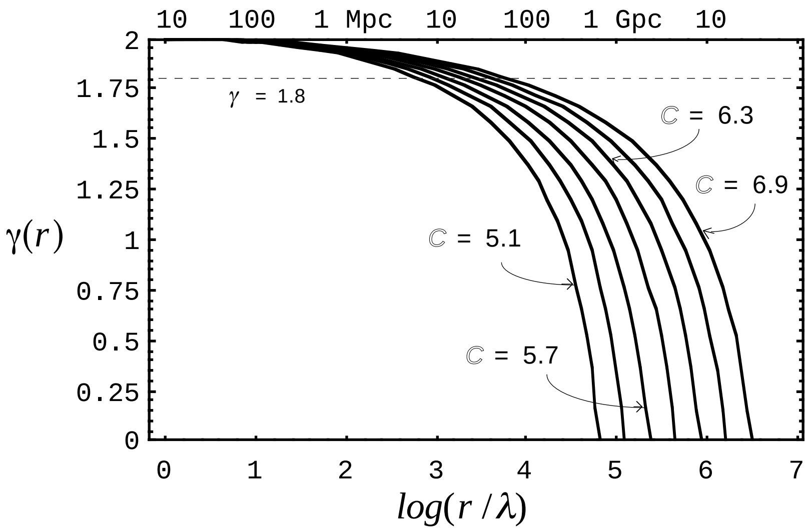

Figure 4. Variation with scale of the power of the galaxy-galaxy autocorrelation function in scale-relativity, for various values of the constant $\mathbb{C} = log(\mathbb{L}/\lambda)$.

First we expect $\gamma = 2$ at small scales. Several observations confirm this prediction: the flat rotation curves of galaxies imply halos in which mass varies as $M(r) = r^{\delta}$, with $\delta = 1$; Vader and Sandage [12] have found an autocorrelation of dwarf galaxies at small scales (10–200 kpc) caracterized by a power $\gamma \approx 2.2$; the analysis of the CfA survey by Davies and Peebles [13] shows that, apart from fluctuations coming from deconvolution, the average γ is 2 between 10 and 300 kpc, while it reaches its value 1.8 only between 1 and 10 Mpc (see their Fig.).

We predict a value of 1.8 at a scale of ≈10 Mpc. Conversely, this becomes a direct measurement of the cosmological constant. We have indeed plotted in Fig. 4 the function $\gamma(r) = 3 - \delta(r)$ for various values of $\mathbb{C} = log(\mathbb{L}/\lambda_g)$, from $\mathbb{C} = 5.1$ (i.e., $\mathbb{K} = 2.3 \times 10^{60}$) to $\mathbb{C} = 6.9$ ($\mathbb{K} = 1.4 \times 10^{62}$). The best fit of the observed value of γ (1.8 at 10 Mpc) is obtained for $\mathbb{C} = 5.4$–5.7, i.e., $\mathbb{L} = 2.5$–5 Gpc, $\mathbb{K} = 4.4$–9.3×10^{60}, $\Lambda = 1.9$–0.18×10^{-56} (i.e. $\Omega_\Lambda = 0.77$–0.18 for $H_0 = 80$ km.s^{-1}.Mpc^{-1}): these values are in good agreement with our previous estimate from the vacuum energy density. This new determination is expected to be highly improved in the near future. Indeed we predict a fast variation of γ at large scales: it must fall to a value of 1.4–1.5 at a scale of 100 Mpc for galaxies. Some recent results [14] seem to confirm such a prediction.

The transition to uniformity ($\gamma = 0$, $\delta = 3$) is reached only at very large scales (> 1 Gpc). This seems to be confirmed by the recent suggestion that the COBE map remains characterized by a low fractal dimension $\delta = 1.43 \pm 0.07$. [15].

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Notes added to this pdf version (12 October 2004)

*A misprint in Eq. (11) of the published version has been corrected.

*Reference 3: Nottale, L., 1996, *Chaos, Solitons and Fractals*, **7**, 877-938.