# Black hole horizons

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# Outline


1. Concept of black hole and event horizon

2. Quasi-local horizons

3. Astrophysical black holes

4. The near-future observations of black holes

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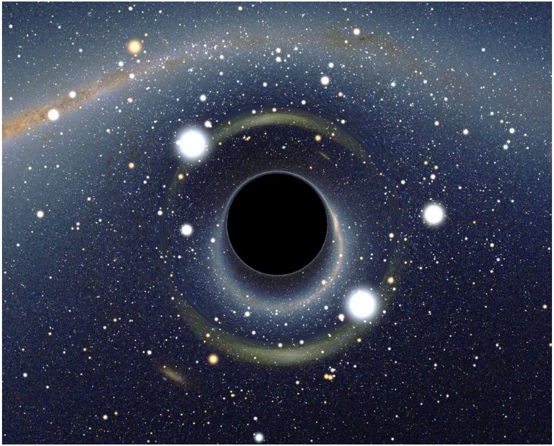
# What is a black hole ?



[Alain Riazuelo, 2007]

... for the layman:

A **black hole** is a region of spacetime from which nothing, not even light, can escape.

The (immaterial) boundary between the black hole interior and the rest of the Universe is called the **event horizon**.

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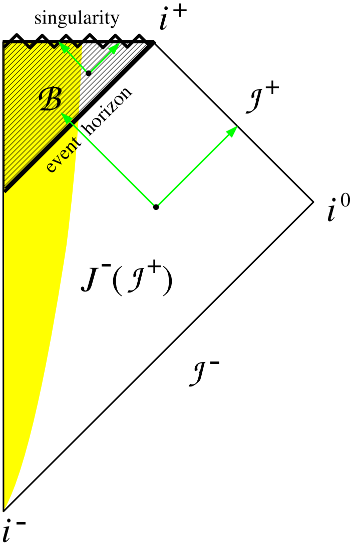


... for the mathematical physicist:

**black hole:** $\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)$

i.e. the region of spacetime where light rays cannot escape to infinity

- $(\mathscr{M}, \boldsymbol{g})$ = asymptotically flat manifold
- $\mathscr{I}^+$ = future null infinity
- $J^-(\mathscr{I}^+)$ = causal past of $\mathscr{I}^+$

**event horizon:** $\mathcal{H} := \partial J^-(\mathscr{I}^+)$ (boundary of $J^-(\mathscr{I}^+)$)

$\mathcal{H}$ smooth $\Longrightarrow$ $\mathcal{H}$ null hypersurface

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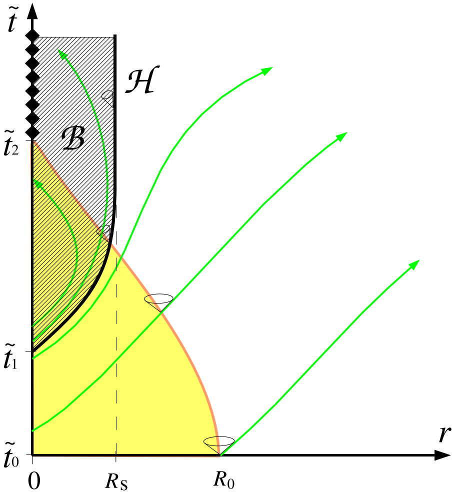


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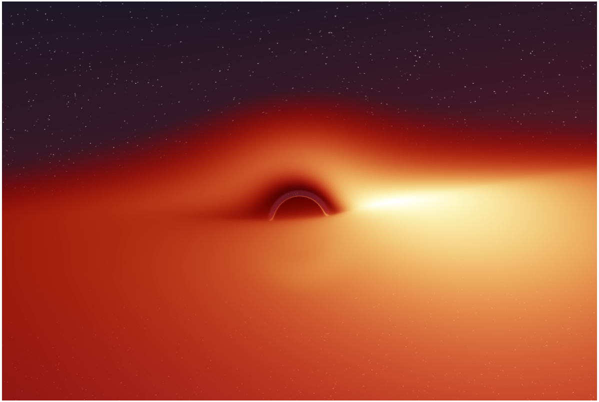
... for the astrophysicist: a very deep gravitational potential well

Release of potential gravitational energy by **accretion** on a black hole: up to 42% of the mass-energy $mc^2$ of accreted matter !

NB: thermonuclear reactions release less than 1% $mc^2$



Matter falling in a black hole forms an **accretion disk** [Lynden-Bell (1969), Shakura & Sunayev (1973)]

[J.-A. Marck (1996)]

# Main properties of black holes (1/3)

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- Viewed by a distant observer, the horizon approach is perceived with an infinite redshift, or equivalently, by an infinite time dilation
- A black hole is not an infinitely dense object: on the contrary it is made of vacuum (except maybe at the singularity); black holes can form in spacetimes empty of any matter, by collapse of gravitational wave packets.

# Main properties of black holes (2/3)

## Uniqueness theorem

(Dorochkevitch, Novikov & Zeldovitch 1965, Israel 1967, Carter 1971, Hawking 1972) :
A black hole in equilibrium is necessarily a Kerr-Newmann black hole, which is a vacuum solution of Einstein described by only three parameters:

- the total mass $M$
- the total angular momentum $J$
- the total electric charge $Q$

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

- $Q = 0$ and $J = 0$ : Schwarzschild solution (1916)
- $Q = 0$ : Kerr solution (1963)

# Main properties of black holes (3/3)

- The mass $M$ is not a measure of the "matter amount" inside the black hole, but rather a parameter characterizing the external gravitational field; it is measurable from the orbital period of a test particle in circular orbit around the black hole and far from it *(Kepler's third law)*.

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- The radius of a black hole is not a well defined concept: it *does not* correspond to some distance between the black hole "centre" (the singularity) and the event horizon. A well defined quantity is the area of the event horizon, $A$.
  The radius can be then defined from it: for a Schwarzschild black hole:

$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left( \frac{M}{M_\odot} \right) \text{ km}$$

# Other theoretical aspects

- The four laws of black hole dynamics
- Quantum properties (Bekenstein entropy, Hawking radiation)
- Black holes in higher dimensions

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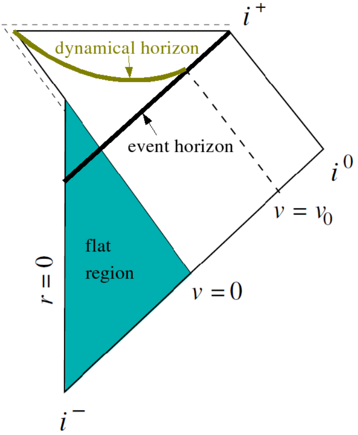


Example of event horizon in a **flat** region of spacetime:

Vaidya metric, describing incoming radiation from infinity:

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dv\,dr + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

with
$m(v) = 0$ for $v < 0$
$dm/dv > 0$ for $0 \leq v \leq v_0$
$m(v) = M_0$ for $v > v_0$

[Ashtekar & Krishnan, LRR **7**, 10 (2004)]

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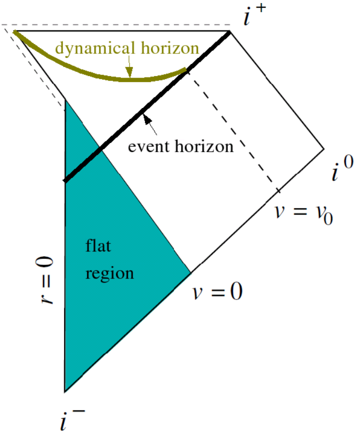


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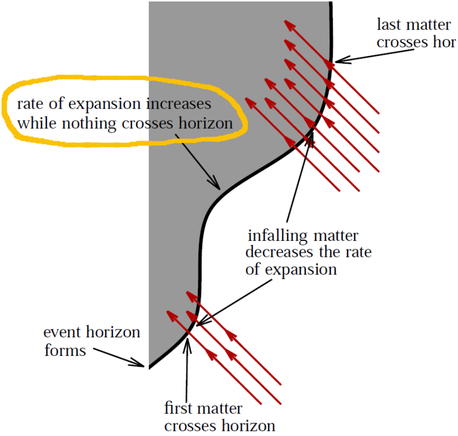
$\Rightarrow$ no local physical experiment whatsoever can locate the event horizon

# Another non-local feature: teleological nature of event horizons

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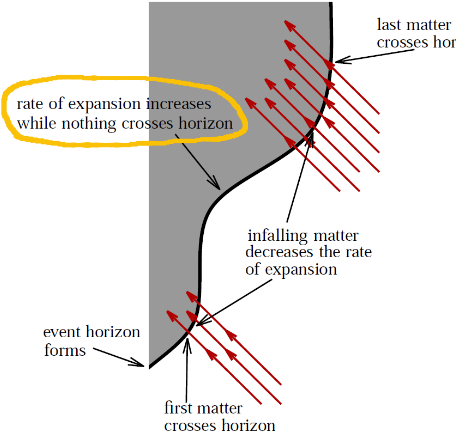


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To deal with black holes as ordinary physical objects, a **local** definition would be desirable
→ quantum gravity, numerical relativity

# Local characterizations of black holes

**New paradigm** for the theoretical approach to black holes: instead of *event horizons*, black holes are described by

- trapping horizons (Hayward 1994)
- isolated horizons (Ashtekar et al. 1999)
- dynamical horizons (Ashtekar and Krishnan 2002)
- slowly evolving horizons (Booth and Fairhurst 2004)

All these concepts are **local** and are based on the notion of trapped surfaces

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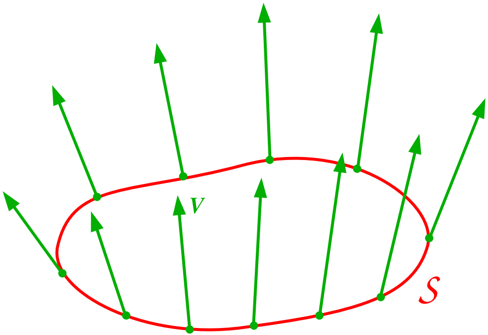


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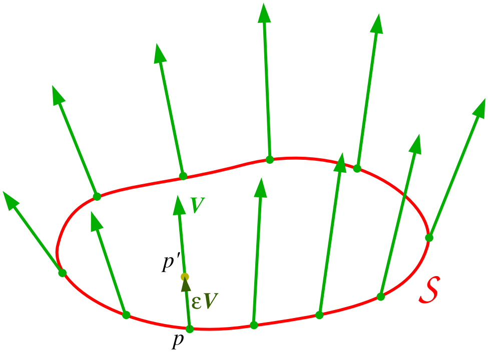


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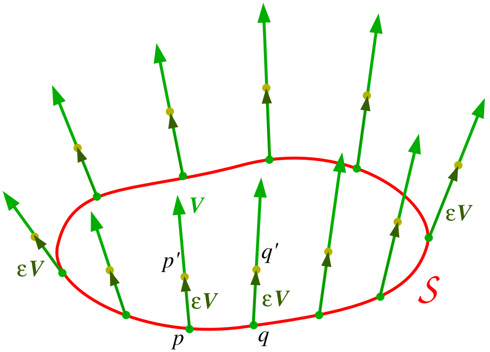


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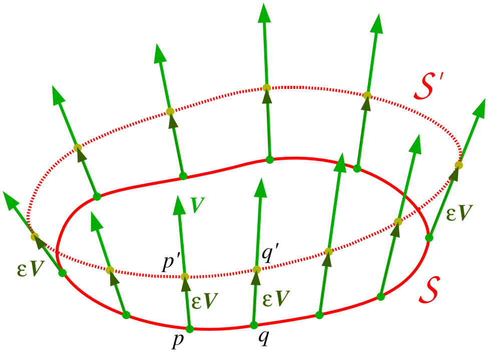


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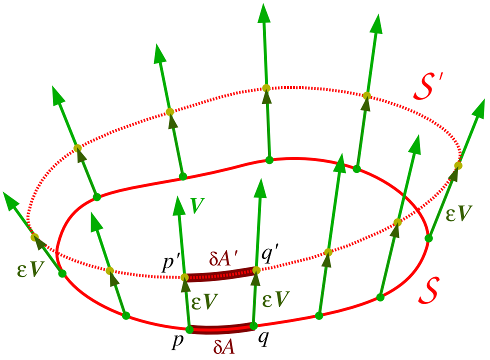


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At each point, the **expansion of $\mathcal{S}$ along $\boldsymbol{v}$** is defined from the relative change in the area element $\delta A$:

$$\theta^{(\boldsymbol{v})} := \lim_{\varepsilon \to 0} \frac{1}{\varepsilon} \frac{\delta A' - \delta A}{\delta A} = \mathcal{L}_{\boldsymbol{v}} \ln \sqrt{q} = q^{\mu\nu} \nabla_\mu v_\nu$$

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## 2/ The definition

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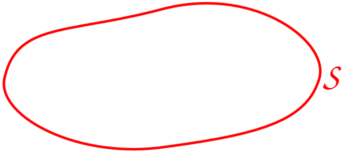

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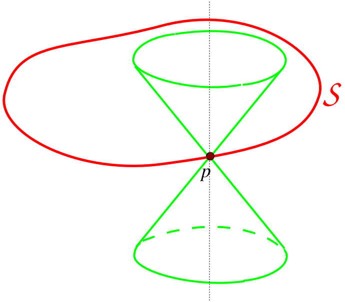


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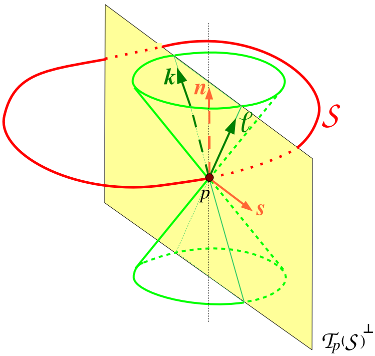


Being spacelike, $\mathscr{S}$ lies outside the light cone

$\exists$ two future-directed null directions orthogonal to $\mathscr{S}$:

$\boldsymbol{\ell}$ = outgoing, expansion $\theta^{(\boldsymbol{\ell})}$

$\boldsymbol{k}$ = ingoing, expansion $\theta^{(\boldsymbol{k})}$

In flat space, $\theta^{(\boldsymbol{k})} < 0$ and $\theta^{(\boldsymbol{\ell})} > 0$

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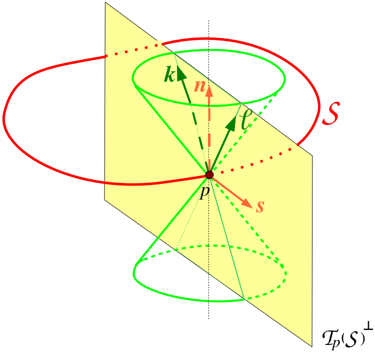


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$\mathscr{S}$ is trapped $\iff$ $\theta^{(\boldsymbol{k})} < 0$ and $\theta^{(\boldsymbol{\ell})} < 0$ [Penrose 1965]

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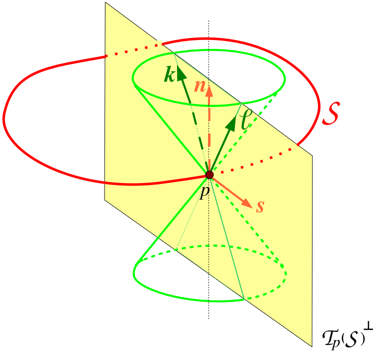


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*trapped surface* = **quasi-local** concept characterizing very strong gravitational fields

# Link with apparent horizons

A closed spacelike 2-surface $\mathscr{S}$ is said to be outer trapped (resp. marginally outer trapped (MOTS)) iff [Hawking & Ellis 1973]

- the notions of *interior* and *exterior* of $\mathscr{S}$ can be defined (for instance spacetime asymptotically flat) $\Rightarrow$ $\boldsymbol{\ell}$ is chosen to be the *outgoing* null normal and $\boldsymbol{k}$ to be the *ingoing* one
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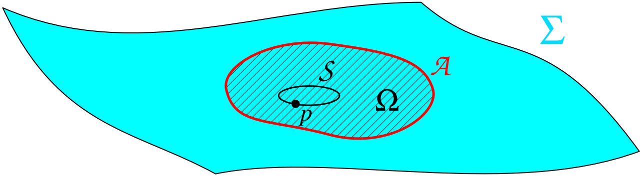



$\Sigma$: spacelike hypersurface extending to spatial infinity (Cauchy surface)

outer trapped region of $\Sigma$ : $\Omega$ = set of points $p \in \Sigma$ through which there is a outer trapped surface $\mathscr{S}$ lying in $\Sigma$

apparent horizon in $\Sigma$: $\mathcal{A}$ = connected component of the boundary of $\Omega$

*Proposition* [Hawking & Ellis 1973]: $\mathcal{A}$ smooth $\Longrightarrow$ $\mathcal{A}$ is a MOTS

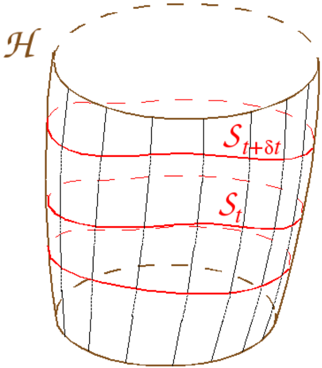
# Connection with singularities and black holes

*Proposition* [Penrose (1965)]:
provided that the weak energy condition holds,
$\exists$ a trapped surface $\mathscr{S} \Longrightarrow \exists$ a singularity in $(\mathscr{M}, \boldsymbol{g})$ (in the form of a future inextendible null geodesic)

*Proposition* [Hawking & Ellis (1973)]:
provided that the cosmic censorship conjecture holds,
$\exists$ a trapped surface $\mathscr{S} \Longrightarrow \exists$ a black hole $\mathcal{B}$ and $\mathscr{S} \subset \mathcal{B}$

# Local definitions of "black holes"

A hypersurface $\mathcal{H}$ of $(\mathscr{M}, \boldsymbol{g})$ is said to be




- a **future outer trapping horizon (FOTH)** iff
  (i) $\mathcal{H}$ foliated by marginally trapped 2-surfaces
  ($\theta^{(\boldsymbol{k})} < 0$ and $\theta^{(\boldsymbol{\ell})} = 0$)
  (ii) $\mathcal{L}_k \, \theta^{(\boldsymbol{\ell})} < 0$ (locally outermost trapped surf.)
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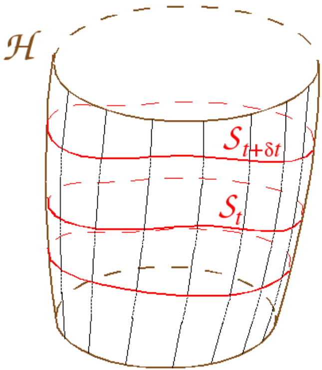


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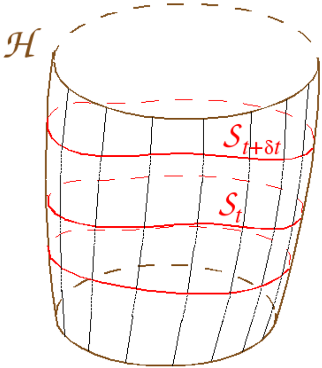


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- a **non-expanding horizon (NEH)** iff
  (i) $\mathcal{H}$ is null (null normal $\boldsymbol{\ell}$)
  (ii) $\theta^{(\boldsymbol{\ell})} = 0$ [Hájíček (1973)]

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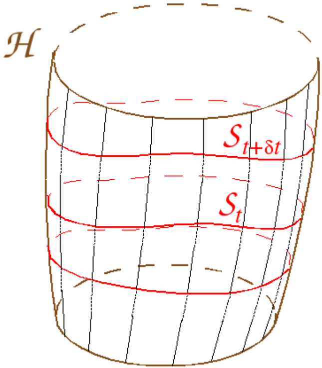


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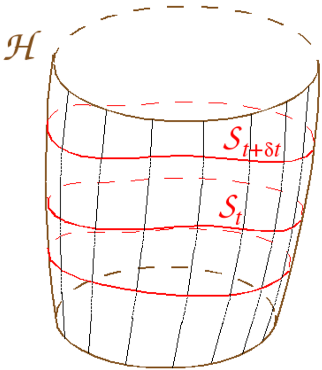
  (i) $\mathcal{H}$ foliated by marginally trapped 2-surfaces ($\theta^{(\boldsymbol{k})} < 0$ and $\theta^{(\boldsymbol{\ell})} = 0$)
  (ii) $\mathcal{L}_k \, \theta^{(\boldsymbol{\ell})} < 0$ (locally outermost trapped surf.)
  [Hayward, PRD **49**, 6467 (1994)]
- a **dynamical horizon (DH)** iff
  (i) $\mathcal{H}$ foliated by marginally trapped 2-surfaces
  (ii) $\mathcal{H}$ spacelike
  [Ashtekar & Krishnan, PRL **89** 261101 (2002)]
- a **non-expanding horizon (NEH)** iff
  (i) $\mathcal{H}$ is null (null normal $\boldsymbol{\ell}$)
  (ii) $\theta^{(\boldsymbol{\ell})} = 0$ [Hájíček (1973)]
- an **isolated horizon (IH)** iff
  (i) $\mathcal{H}$ is a non-expanding horizon
  (ii) $\mathcal{H}$'s full geometry is not evolving along the null generators: $[\mathcal{L}_{\ell} \, , \hat{\boldsymbol{\nabla}}] = 0$
  [Ashtekar, Beetle & Fairhurst, CQG **16**, L1 (1999)]

# Local definitions of "black holes"

A hypersurface $\mathcal{H}$ of $(\mathscr{M}, \boldsymbol{g})$ is said to be




BH in equilibrium = IH
(e.g. Kerr)
BH out of equilibrium = DH
generic BH = FOTH

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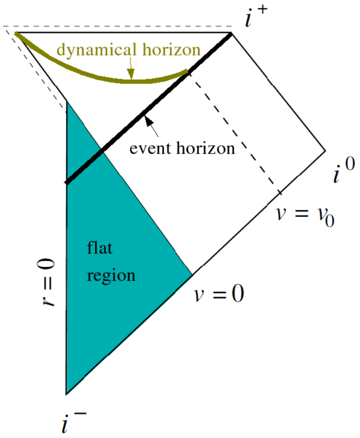
# Example: Vaidya spacetime




[Ashtekar & Krishnan, LRR **7**, 10 (2004)]

- The event horizon crosses the flat region
- The dynamical horizon lies entirely outside the flat region

# Dynamics of the quasi-local horizons

The *trapping horizons* and *dynamical horizons* have their **own dynamics**, ruled by Einstein equations.
In particular, one can establish for them

- existence and (partial) uniqueness theorems
  [Andersson, Mars & Simon, PRL **95**, 111102 (2005)],
  [Ashtekar & Galloway, Adv. Theor. Math. Phys. **9**, 1 (2005)]
- first and second laws of black hole mechanics
  [Ashtekar & Krishnan, PRD **68**, 104030 (2003)], [Hayward, PRD **70**, 104027 (2004)]
- a viscous fluid bubble analogy ("membrane paradigm", as for the event horizon)
  [EG, PRD **72**, 104007 (2005)], [EG & Jaramillo, PRD **74**, 087502 (2006)], [Jaramillo, arXiv:1309.6593 (2013)],

For a review see [Jaramillo, arXiv:1108.2408 (2011)]

# Outline


1. Concept of black hole and event horizon
2. Quasi-local horizons
3. Astrophysical black holes
4. The near-future observations of black holes

# Known black holes

Three kinds of black holes are known in the Universe:

- **Stellar black holes**: supernova remnants:
  $M \sim 10 - 30\ M_\odot$ and $R \sim 30 - 90\ \mathrm{km}$

  example: Cyg X-1 : $M = 15\ M_\odot$ and $R = 45\ \mathrm{km}$

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- **Supermassive black holes**, in galactic nuclei:
  $M \sim 10^5 - 10^{10}\ M_\odot$ and $R \sim 3 \times 10^5\ \mathrm{km} - 200\ \mathrm{UA}$

  example: Sgr A* : $M = 4.3 \times 10^6\ M_\odot$ and
  $R = 13 \times 10^6\ \mathrm{km} = 18\, R_\odot = 0.09\ \mathrm{UA} = \frac{1}{4} \times$ radius of Mercury's orbit

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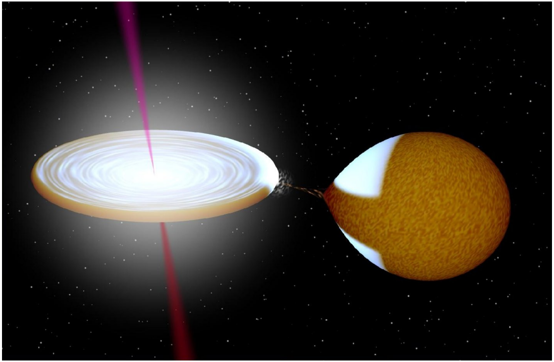
- **Intermediate mass black holes**, as ultra-luminous X-ray sources (?):
  $M \sim 10^2 - 10^4 \, M_\odot$ and $R \sim 300$ km $- 3 \times 10^4$ km

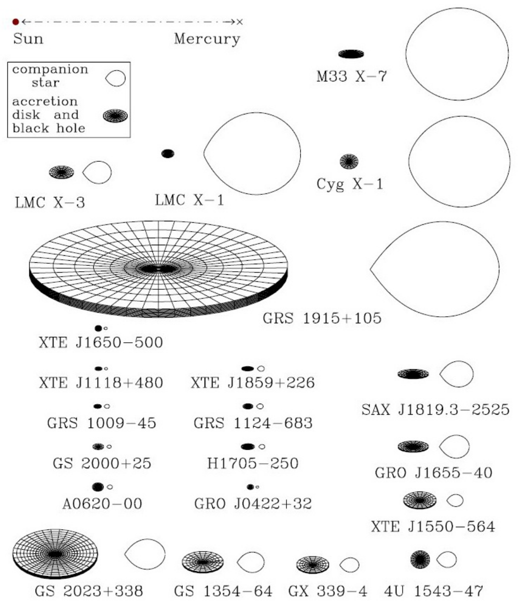
  example: ESO 243-49 HLX-1 : $M > 500 \, M_\odot$ and $R > 1500$ km

# Stellar black holes in X-ray binaries



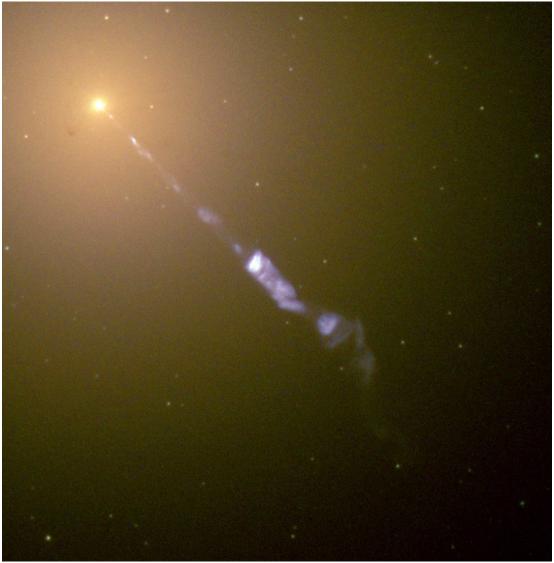
$\sim$ 20 identified stellar black holes in our galaxy

# Stellar black holes in X-ray binaries




[McClintock et al. (2011)]

# Supermassive black holes in active galactic nuclei (AGN)

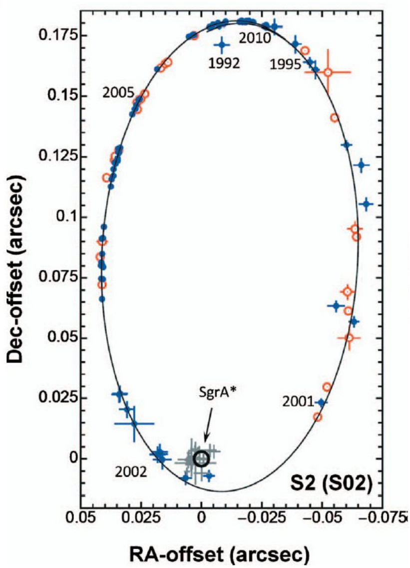


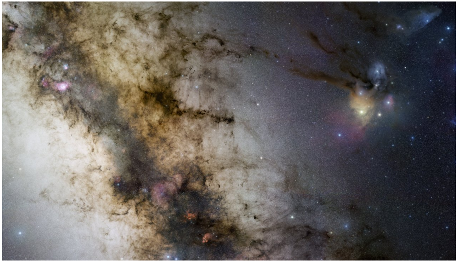
Jet emitted by the nucleus of the giant elliptic galaxy M87, at the centre of Virgo cluster [HST]

$M_{\rm BH} = 3 \times 10^9 \, M_\odot$

$V_{\rm jet} \simeq 0.99 \, c$

# The black hole at the centre of our galaxy: Sgr A*



[ESO (2009)]

Determination of the mass of Sgr A* black hole by stellar dynamics:

$$M_{\rm BH} = 4.3 \times 10^6 \, M_\odot$$

← Orbit of the star S2 around Sgr A*
$P = 16\,{\rm yr}$, $r_{\rm per} = 120\,{\rm UA} = 1400\,R_{\rm S}$,
$V_{\rm per} = 0.02\,c$
[Genzel, Einsenhauer & Gillessen, RMP **82**, 3121 (2010)]

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# Can we see a black hole from the Earth ?

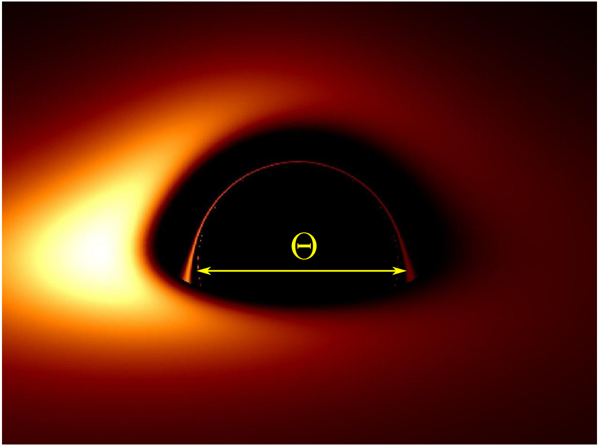


Image of a thin accretion disk around a Schwarzschild BH
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Angular diameter of the event horizon of a Schwarzschild BH of mass $M$ seen from a distance $d$:

$$\Theta = 6\sqrt{3}\,\frac{GM}{c^2 d} \simeq 2.60\,\frac{2R_S}{d}$$

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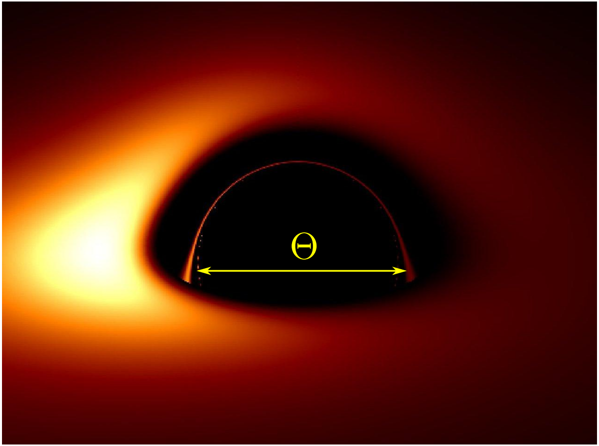


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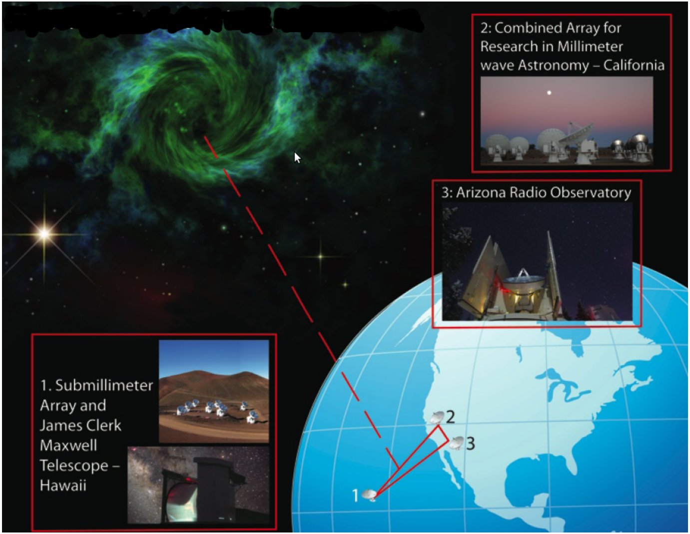
Largest black holes in the Earth's sky:

**Sgr A*** : $\Theta = 53\ \mu\mathrm{as}$
**M87** : $\Theta = 21\ \mu\mathrm{as}$
**M31** : $\Theta = 20\ \mu\mathrm{as}$

*Remark:* black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$

# The solution to reach the $\mu$as regime: interferometry !




Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

Existing American VLBI network [Doeleman et al. 2011]

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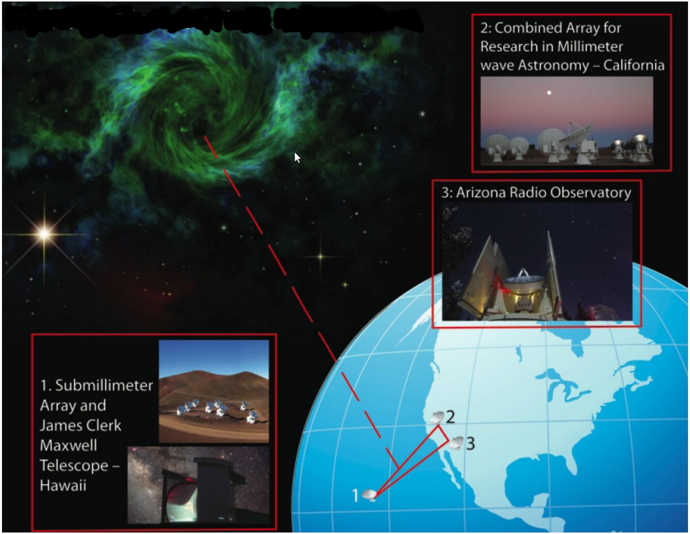



Existing American VLBI network [Doeleman et al. 2011]

Very Large Baseline Interferometry (VLBI) in (sub)millimeter waves

The best result so far: VLBI observations at $1.3$ mm have shown that the size of the emitting region in Sgr A* is only $37\ \mu\mathrm{as}$

[Doeleman et al., Nature **455**, 78 (2008)]

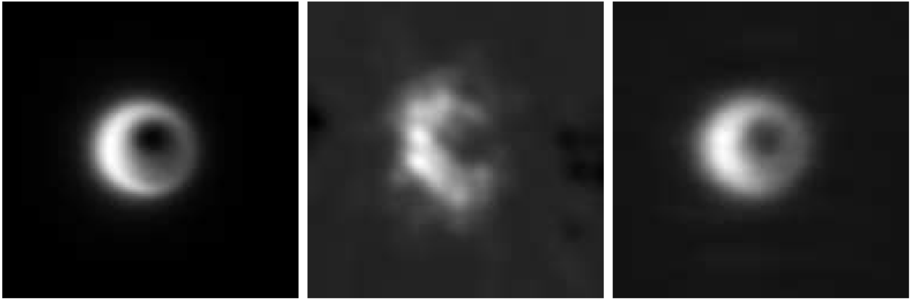
# The near future: the Event Horizon Telescope

To go further:

- shorten the wavelength: $1.3 \text{ mm} \to 0.8 \text{ mm}$
- increase the number of stations; in particular add ALMA

Atacama Large Millimeter Array (ALMA)
part of the Event Horizon Telescope (EHT) to be completed by 2020

# The near future: the Event Horizon Telescope



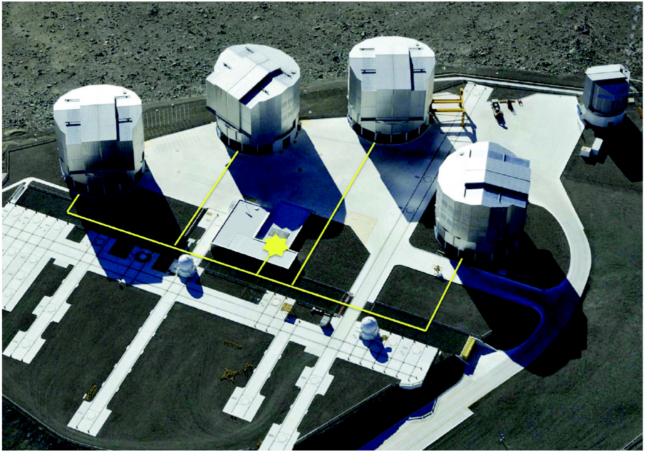
Simulations of VLBI observations of Sgr A* at $\lambda = 0.8$ mm

*left:* perfect image, *centre:* 7 stations ($\sim 2015$), *right:* 13 stations ($\sim 2020$)

$$a = 0, \; i = 30^\circ$$

[Fish & Doeleman, Proc. IAU Symp 261 (2010)]

# Near-infrared optical interferometry: GRAVITY



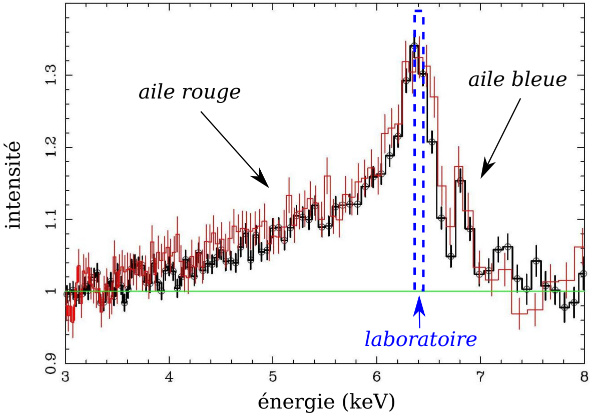
[Gillessen et al. 2010]

GRAVITY instrument at VLTI (2015)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes) $\Longrightarrow$ astrometric precision of $10\ \mu\mathrm{as}$

# X-ray observations (Athena)

## The accretion disk as a spacetime probe



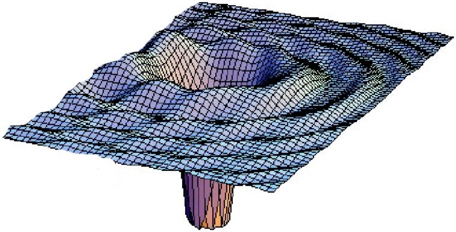

K$\alpha$ line in the nucleus of the galaxy MCG-6-30-15 observed by XMM-Newton (red) and Suzaku (black) (adapted from [Miller (2007)])

**K$\alpha$ line:** X fluorescence line of Fe atoms in the accretion disk (the Fe atoms are excited by the X-ray emitted from the plasma corona surrounding the disk)

Redshift $\Rightarrow$ time dilatation

*Athena scientific theme selected for ESA L2 mission*

# Another way to "see" BHs: gravitational waves



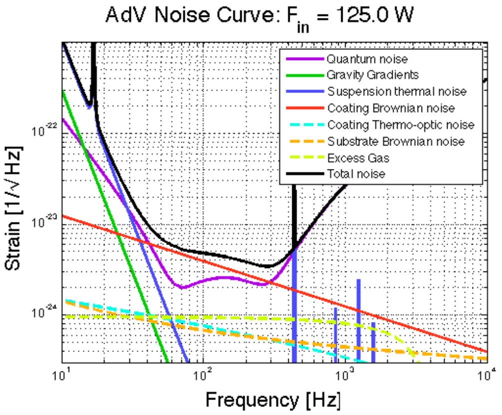
Link between black holes and gravitational waves:
Black holes and gravitational waves are both spacetime distortions:

- extreme distortions (black holes)
- small distortions (gravitational waves)

In particular, black holes and gravitational waves are both vacuum solutions of general relativity equations (Einstein equations)

# Advanced VIRGO

Advanced VIRGO: dual recycled (power + signal) interferometer with laser power $\sim$ 125 W

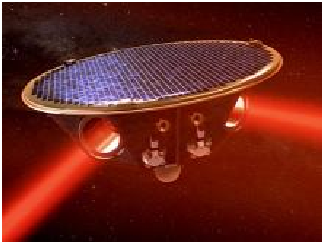


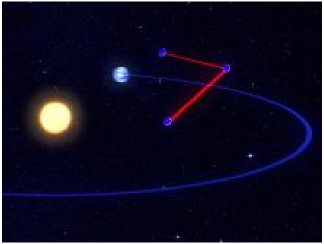

[CNRS/INFN/NIKHEF]

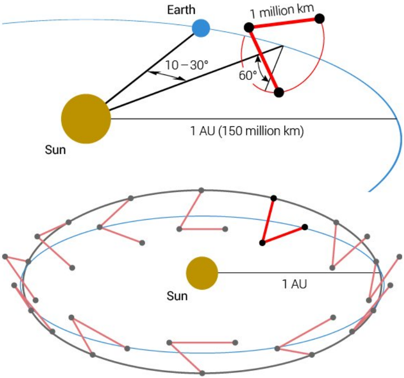
- VIRGO+ decommissioned in Nov. 2011
- Construction of Advanced VIRGO underway
- First lock in 2015
- Sensitivity $\sim$ 10 $\times$ VIRGO
- $\Longrightarrow$ explored Universe volume $10^3$ times larger !

# eLISA

Gravitational wave detector in space $\Longrightarrow$ low frequency range: $[10^{-3}, 10^{-1}]$ Hz








[http://www.elisascience.org/]

- eLISA scientific theme selected in Nov. 2013 for ESA L3 mission $\Longrightarrow$ launch in 2028
- LISA Pathfinder to be launched in 2015