Black holes and tests of gravitation

Éric Gourgoulhon

Laboratoire Univers et Théories (LUTH)
CNRS / Observatoire de Paris / Université Paris Diderot
92190 Meudon, France

eric.gourgoulhon@obspm.fr

http://luth.obspm.fr/~luthier/gourgoulhon/

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Outline

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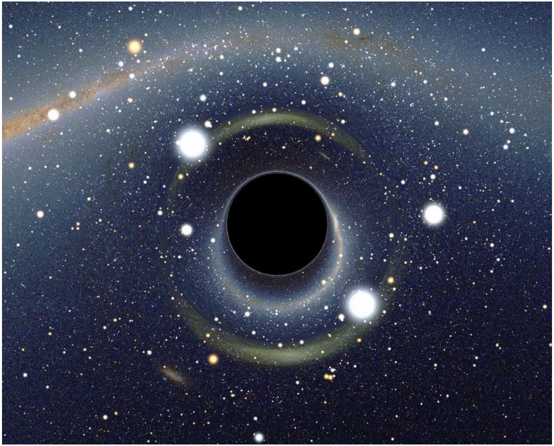
1. Black holes in general relativity

2. Alternatives to the Kerr black hole

What is a black hole ?



[Alain Riazuelo, 2007]

... in a few words:

A **black hole** is a region of spacetime from which nothing, not even light, can escape.

The (immaterial) boundary between the black hole interior and the rest of the Universe is called the **event horizon**.

The "no-hair" theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a black hole in equilibrium in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is a vacuum solution of Einstein described by only three parameters:

- the total mass M
- the total angular momentum J
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

Astrophysical black holes have to be electrically neutral:

- $Q = 0$: Kerr solution (1963)

The Kerr solution

Roy Kerr (1963)

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -\left(1-\frac{2GMr}{c^2\rho^2}\right)c^2\mathrm{d}t^2 - \frac{4GMar\sin^2\theta}{c^2\rho^2}\,c\,\mathrm{d}t\,\mathrm{d}\varphi + \frac{\rho^2}{\Delta}\,\mathrm{d}r^2 + \rho^2\mathrm{d}\theta^2 + \left(r^2+a^2+\frac{2GMa^2r\sin^2\theta}{c^2\rho^2}\right)\sin^2\theta\,\mathrm{d}\varphi^2$$

where

$$\rho^2 := r^2 + a^2\cos^2\theta, \qquad \Delta := r^2 - \frac{2GM}{c^2}r + a^2, \qquad a := \frac{J}{cM}$$

Event horizon (black hole) $\iff |a| \leq \frac{GM}{c^2}$

Schwarzschild subcase ($a = 0$):

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -\left(1-\frac{2GM}{c^2r}\right)c^2\mathrm{d}t^2 + \left(1-\frac{2GM}{c^2r}\right)^{-1}\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\varphi^2\right)$$

The black hole parameters

- The mass M is not some measure of the "matter amount" inside the black hole, but rather a parameter characterizing the external gravitational field; it is measurable from the orbital period of a test particle in circular orbit around the black hole and far from it *(Kepler's third law)*.

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Remark: the radius of a black hole is not a well defined concept: it *does not* correspond to some distance between the black hole "centre" (the singularity) and the event horizon. A well defined quantity is the area of the event horizon, A. The radius can be then defined from it: for a Schwarzschild black hole:

$$R := \sqrt{\frac{A}{4\pi}} = \frac{2GM}{c^2} \simeq 3 \left(\frac{M}{M_\odot} \right) \text{ km}$$

Why is the Kerr metric special ?

Spherically symmetric (non-rotating) case:

Birkhoff theorem

Within 4-dimensional general relativity, the spacetime outside any spherically symmetric body is described by Schwarzschild metric

$\Longrightarrow$ No possibility to distinguish a non-rotating black hole from a non-rotating dark star by monitoring orbital motion or fitting accretion disk spectra

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Rotating axisymmetric case:

No Birkhoff theorem

Moreover, no "reasonable" matter source has ever been found for the Kerr metric (the only known source consists of two counter-rotating thin disks of collisionless particles [Bicak & Ledvinka, PRL **71**, 1669 (1993)])

$\Longrightarrow$ The Kerr metric is specific to rotating black holes (in 4-dimensional general relativity)

Lowest order no-hair theorem: quadrupole moment

Asymptotic expansion (large r) of the metric in terms of multipole moments $(\mathcal{M}_k, \mathcal{J}_k)_{k\in\mathbb{N}}$ [Geroch (1970), Hansen (1974)]:

- $\mathcal{M}_k$: mass 2^k-pole moment
- $\mathcal{J}_k$: angular momentum 2^k-pole moment

$\Longrightarrow$ For the Kerr metric, all the multipole moments are determined by (M, a):

- $\mathcal{M}_0 = M$
- $\mathcal{J}_1 = aM = J/c$
- $\boxed{\mathcal{M}_2 = -a^2 M = -\frac{J^2}{c^2 M}}$ $(*)$ $\leftarrow$ mass quadrupole moment
- $\mathcal{J}_3 = -a^3 M$
- $\mathcal{M}_4 = a^4 M$
- $\cdots$

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Measuring the three quantities M, J, $\mathcal{M}_2$ provides a compatibility test w.r.t. the Kerr metric, by checking $(*)$

Outline

Theoretical alternatives to the Kerr black hole

Within general relativity

The compact object is not a black hole but

- boson stars (cf. Claire Somé's talk)
- gravastar
- dark stars
- ...

Beyond general relativity

The compact object is a black hole but in a theory that differs from GR:

- Einstein-Gauss-Bonnet with dilaton
- Chern-Simons gravity
- Hořava-Lifshitz gravity
- Einstein-Yang-Mills
- ...

Alternative theories of gravity

Class of metric theories of gravity, described by the action

$$S = S_{\rm grav} + S_{\rm mat}(\boldsymbol{g}, \Psi_1, \Psi_2, \ldots)$$

$\boldsymbol{g}$: spacetime metric, $\Psi_1, \Psi_2, \ldots$: matter fields

$\Longrightarrow$ test particles follow geodesics of $\boldsymbol{g}$

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General relativity:

$$S_{\rm grav} = \frac{1}{16\pi G} \int R \sqrt{-g}\, \mathrm{d}^4 x \qquad \text{(Einstein-Hilbert action)}$$

R: scalar curvature of metric $\boldsymbol{g}$: $R := g^{\mu\nu} R^{\sigma}{}_{\mu\sigma\nu}$
$R^{\alpha}{}_{\beta\mu\nu}$: Riemann curvature tensor of $\boldsymbol{g}$

Scalar-tensor theories

Gravity action depends on a scalar field ϕ in addition to the spacetime metric $\boldsymbol{g}$:

$$S_{\rm grav} = S_{\rm grav}(\boldsymbol{g}, \phi) = \frac{1}{16\pi G} \int \left[\phi R - \frac{\omega(\phi)}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \phi^2 V \right] \sqrt{-g}\, \mathrm{d}^4 x$$

Special case: Jordan-Fierz-Brans-Dicke theory: $\omega(\phi) = \mathrm{const}$

No-hair theorem: for a *real* scalar field ϕ, the only black hole solution is Kerr

However, for *complex* scalar fields, hairy black hole solutions exist [Herdeiro & Radu, arXiv:1403.2757 (2014)]

Einstein-Gauss-Bonnet with dilaton

Gravity action is quadratic in the curvature:

$$S_{\rm grav} = \frac{1}{16\pi G}\int \Big[R + e^{\gamma\phi}\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right) - \frac{\beta}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2V(\phi)\right)\Big]\sqrt{-g}\,\mathrm{d}^4x$$

Low energy expansion of string theory

Chern-Simons gravity

Gravity action is quadratic in the curvature:

$$S_{\rm grav} = \frac{1}{16\pi G}\int \left[R + \frac{\alpha}{4}\phi R^*_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - \frac{\beta}{2}\left(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + 2V(\phi)\right)\right]\sqrt{-g}\,\mathrm{d}^4x$$

Low energy expansion of string theory or loop quantum gravity

How to test the alternatives ?

Search for

- orbital motion (stellar orbits, hot spot in accretion structure) deviating from Kerr timelike geodesics (cf. talks by C. Somé, F. Vincent and T. Paumard)
  $\rightarrow$ GRAVITY (cf. talk by F. Eisenhauer)
- accretion disk spectra different from those arising in Kerr metric
  $\rightarrow$ X-ray observatories
- images of the black hole shadow different from that of a Kerr black hole
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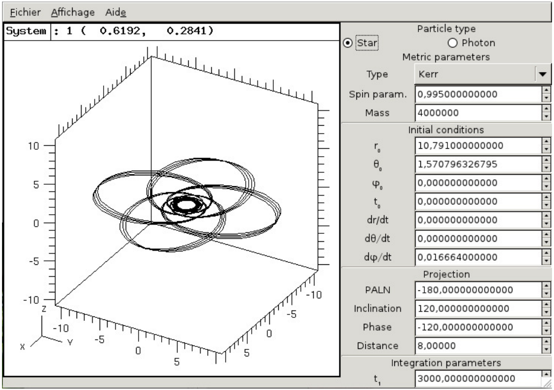
Need for a good and versatile geodesic integrator
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) *in any kind of metric*

Gyoto code

developed by F. Vincent and T. Paumard



- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick interface
- Free software (GPL) : http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]