Gravitational waves: a brief overview

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Outline

1. A short introduction to gravitational waves

2. The current observational status

3. The near-future projects

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Spacetime dynamics

- Special relativity: metric tensor $\boldsymbol{g}$ = fixed bilinear form on the vector space $\sim \mathbb{R}^4$ associated with the spacetime affine space
- General relativity: metric tensor $\boldsymbol{g}$ = field of bilinear forms: $\boldsymbol{g} = \boldsymbol{g}(p)$

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Einstein equation : $\boxed{\boldsymbol{R} - \frac{1}{2} R\, \boldsymbol{g} = \frac{8\pi G}{c^4} \boldsymbol{T}}$

- $\boldsymbol{R}$ = Ricci tensor = symmetric bilinear form = trace of *curvature tensor* (Riemann tensor) : "$\boldsymbol{R} \sim \boldsymbol{g}\, \partial^2 \boldsymbol{g} + \boldsymbol{g}\, \partial \boldsymbol{g}\, \partial \boldsymbol{g}$"
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    - $E = \boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{u}})$ is the energy density of matter as measured by an observer $\mathcal{O}$ of 4-velocity $\vec{\boldsymbol{u}}$
    - $p_i = -\boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{e}}_i)$ component i of the matter momentum density as measured by $\mathcal{O}$ in the direction $\vec{\boldsymbol{e}}_i$
    - $S_{ij} = \boldsymbol{T}(\vec{\boldsymbol{e}}_i, \vec{\boldsymbol{e}}_j)$ component i of the force exerted by matter on the unit surface normal to $\vec{\boldsymbol{e}}_j$

Comparing Newtonian and relativistic gravitation theories

Newtonian gravitation:

fundamental equation: Poisson equation for the gravitational potential Φ:

$$\Delta\Phi = 4\pi G\rho$$

- scalar equation
- linear equation
- elliptic equation (⇒ instantaneous propagation)
- only source: mass density ρ

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$$\boldsymbol{R}(\boldsymbol{g}) - \frac{1}{2} R(\boldsymbol{g})\,\boldsymbol{g} = \frac{8\pi G}{c^4}\boldsymbol{T}$$

- tensorial equation (10 scalar equations)
- non-linear equation
- propagation at finite speed (c)
- source: energy-momentum of matter and electromagnetic field

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Remark: for a weak gravitational field, one of the 10 components of Einstein equation reduces to the Poisson equation (and the other 9 reduced to $0 = 0$).

What is a strong gravitational field ?

Relativity parameter or compactness parameter of a self-gravitating body of mass M and mean radius R:

$$\Xi = \frac{GM}{c^2 R} \sim \frac{|E_{\rm grav}|}{Mc^2} \sim \frac{|\Phi_{\rm surf}|}{c^2} \sim \frac{v_{\rm esc}^2}{c^2}$$

- $E_{\rm grav}$: gravitational potential energy[1]
- $\Phi_{\rm surf}$: gravitational potential at the surface of the body
- $v_{\rm esc}$: escape velocity from the body's surface[2]

	Earth	Sun	white dwarf	neutron star	black hole
Ξ	10^{-10}	10^{-6}	10^{-3}	0.2	1

if $\Xi \gtrsim 0.1$, general relativity must be employed to describe the body (**compact object**)

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[1] for a homogeneous ball: $E_{\rm grav} = -\frac{3}{5}\frac{GM^2}{R}$

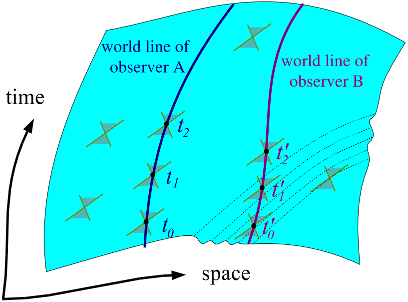
[2] for a spherically symmetric body: $v_{\rm esc} = \sqrt{\frac{2GM}{R}}$

Gravitational waves

Linearization of Einstein equation in weak field: $\boldsymbol{g} = \boldsymbol{\eta} + \boldsymbol{h}$,
$\boldsymbol{\eta}$ = Minkowski metric[3]

$$\Longrightarrow \text{wave equation: } \boxed{\square \bar{\boldsymbol{h}} = -\frac{16\pi G}{c^4}\boldsymbol{T}} \qquad \text{(Lorenz gauge)}$$

with $\square = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\bar{\boldsymbol{h}} = \boldsymbol{h} - \frac{1}{2} h\, \boldsymbol{\eta}$ and $h = \mathrm{Trace}(\boldsymbol{h})$.



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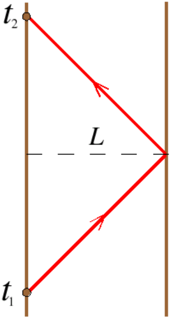
[3] $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ en Cartesian coordinates

Measurable effects of a gravitational wave passage

Measure of the distance L between two free masses by a “radar” method:

$$L = \frac{1}{2}\, c(t_2 - t_1)$$



Variation of length L when a gravitational wave passes by:

$$\boxed{\delta L \simeq h\, L}$$

$h =$ amplitude of the gravitational wave

In practice, h is so small that our senses are not sensitive to it:
for the most important **astrophysical sources:** $h \sim 10^{-21}$!!!

Gravitational wave emission

- For a weakly relativistic source: quadrupole formula:

$$\boxed{h^{\rm TT}_{ij}(t,\vec{\boldsymbol{x}}) = \frac{2G}{c^4 r}\left[P_i^{\ k} P_j^{\ l} - \frac{1}{2} P_{ij} P^{kl}\right] \ddot{Q}_{ij}\left(t - \frac{r}{c}\right)}$$

  - r : distance to the source
  - $P_{ij} = \delta_{ij} - x^i x^j / r^2$: transverse projector
  - $Q_{ij}(t) := \int_{\text{source}} \rho(t,\vec{\boldsymbol{x}}) \left(x^i x^j - \frac{1}{3}\vec{\boldsymbol{x}}\cdot\vec{\boldsymbol{x}}\,\delta_{ij}\right) d^3\vec{\boldsymbol{x}}$: mass quadrupole
- GW luminosity :

$$\boxed{L \sim \frac{c^5}{G}\, s^2\, \boldsymbol{\Xi}^2 \left(\frac{v}{c}\right)^6}$$

  - s : asymmetry factor ($s = 0$ fpr spherical symmetry)
  - $\boldsymbol{\Xi} := GM/(c^2 R)$: compactness parameter
  - v : characteristic velocity of matter in the source

NB: $c^5/G \simeq 4\ 10^{52}\ \mathrm{W}$!

Generation of gravitational waves in the lab

In the 19th Century, Hertz has demonstrated the existence of electromagnetic waves by producing them in his laboratory.
Is the same experiment possible for gravitational waves ?

- electromagnetic waves: produced by the acceleration of *electric charges*
- gravitational waves: produced by the acceleration of *masses*

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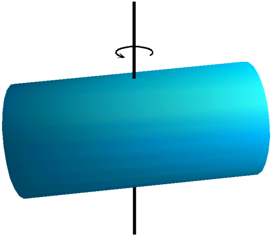
A simple mean of providing a constant acceleration to some material: make it *rotate*



Steel cylinder: diameter = 1 m, length = 20 m, mass = 490 t, rotating at 28 rad/s (break-up limit)
$\Longrightarrow$ emitted energy in gravitational waves per unit of time: 2×10^{-29} W !

$\Longrightarrow$ No hope of detection !

Generation of gravitational waves by astrophysical sources

Emitted energy per unit of time in the form of gravitational waves:

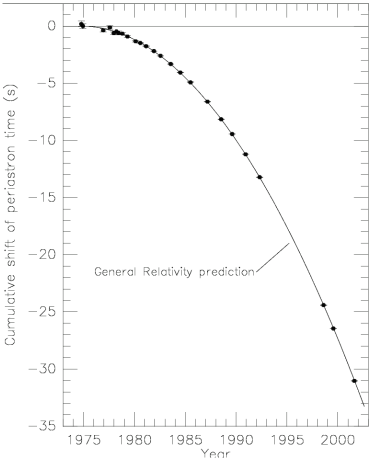
$$\boxed{L \sim \frac{c^5}{G}\, s^2 \left(\frac{v}{c}\right)^6 \boldsymbol{\Xi}^2}$$

gravitational luminosity

- G : Newton's constant → *gravitation*
- c : velocity of light → *relativity*
- s : asymmetry factor: $s = 0$ if spherical symmetry
- v : characteristic speed of motions inside the source
- $\boldsymbol{\Xi}$: *compactness parameter* or *relativity parameter*

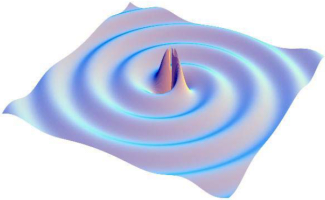
Only compact objects are good emitters of gravitational waves

Gravitational waves do exist !



[Weisber & Taylor (2002)]

Emission of gravitational waves by the neutron star binary system PSR B1913+16 (*binary pulsar*)



← Observed decay of the orbital period $P = 7\,\mathrm{h}\,45\,\mathrm{min}$ of the binary pulsar PSR B1913+16 produced by the *reaction to gravitational radiation*

$\Longrightarrow$ coalescence in 140 millions year.

Nobel Prize in Physics to R. Hulse & J. Taylor (1993)

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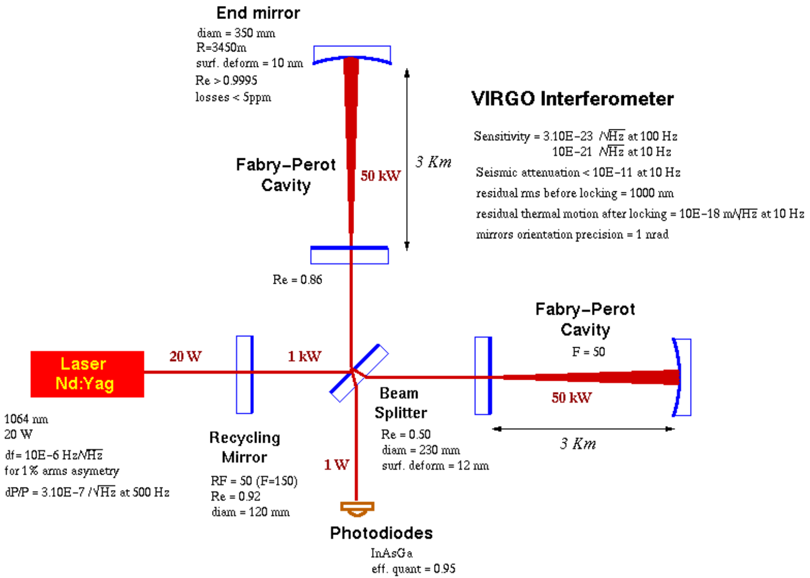
Road map of Pisa neighbourhood (Italy)...

VIRGO: a giant Michelson interferometer...

Gravitational wave detector VIRGO in Cascina, near Pisa (Italy) [CNRS/INFN]

Optical scheme of the VIRGO interferometer



End mirror

diam = 350 mm
R=3450m
surf. deform = 10 nm
Re > 0.9995
losses < 5ppm

Fabry−Perot Cavity

Re = 0.86

VIRGO Interferometer

Sensitivity = 3.10E−23 $/\sqrt{\mathrm{Hz}}$ at 100 Hz
10E−21 $/\sqrt{\mathrm{Hz}}$ at 10 Hz
Seismic attenuation < 10E−11 at 10 Hz
residual rms before locking = 1000 nm
residual thermal motion after locking = 10E−18 $\mathrm{m}/\sqrt{\mathrm{Hz}}$ at 10 Hz
mirrors orientation precision = 1 nrad

Fabry−Perot Cavity

F = 50

Laser Nd:Yag

1064 nm
20 W
df= 10E−6 $\mathrm{Hz}/\sqrt{\mathrm{Hz}}$
for 1% arms asymetry
dP/P = 3.10E−7 $/\sqrt{\mathrm{Hz}}$ at 500 Hz

Recycling Mirror

RF = 50 (F=150)
Re = 0.92
diam = 120 mm

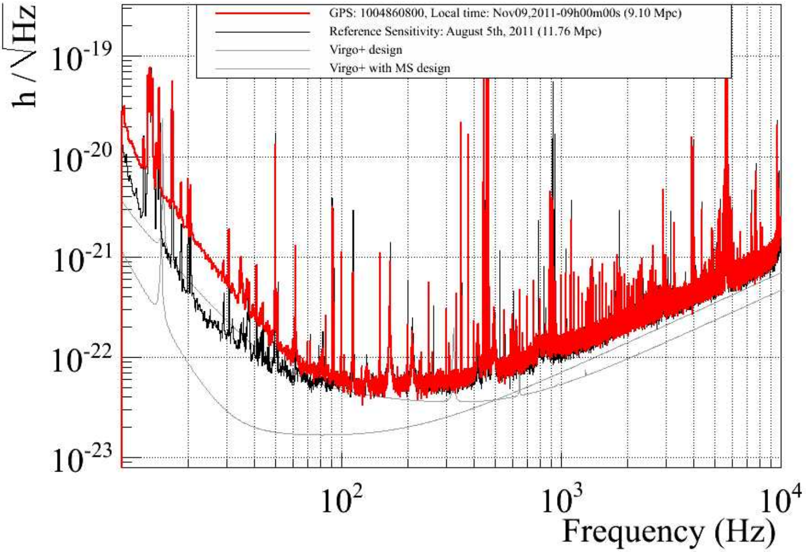
Beam Splitter

Re = 0.50
diam = 230 mm
surf. deform = 12 nm

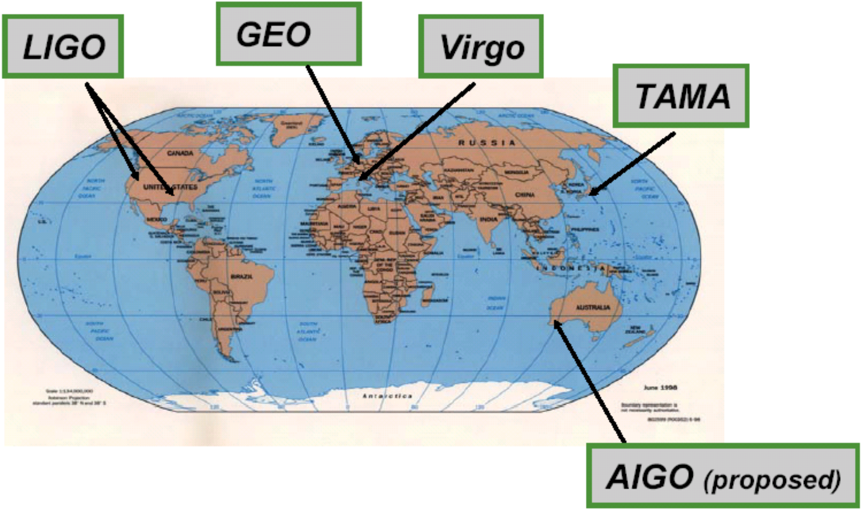
Photodiodes

InAsGa
eff. quant = 0.95

VIRGO sensitivity curve



Other interferometric detectors operating in the world



Continuous gravitational-wave signal from pulsars

Amplitude of GW emission for a triaxial ellipticity ϵ, rotation period P, distance r and moment of inertia I:

$$\boxed{h_0 = 4.21 \times 10^{-24} \ \left(\frac{\mathrm{ms}}{P}\right)^2 \left(\frac{\mathrm{kpc}}{r}\right) \left(\frac{I}{10^{38}\ \mathrm{kg\,m^2}}\right) \left(\frac{\epsilon}{10^{-6}}\right)}$$

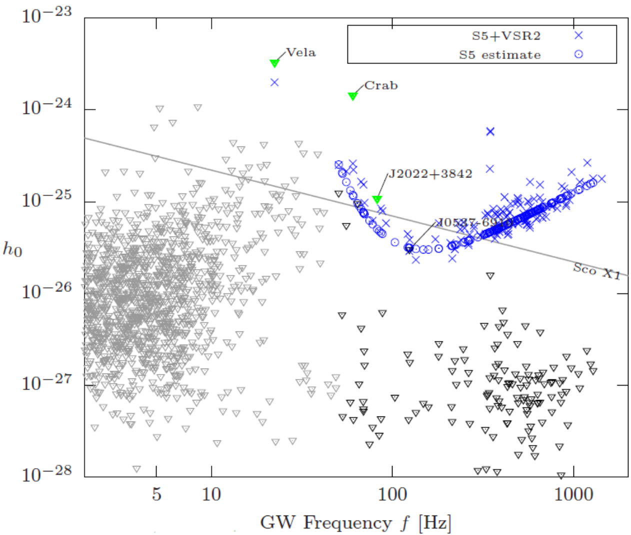
Crab: $h_0 \simeq 2 \times 10^{-27} \left(\frac{\epsilon}{10^{-6}}\right)$ Vela: $h_0 \simeq 1 \times 10^{-27} \left(\frac{\epsilon}{10^{-6}}\right)$

Spindown limit: upper limit on h_0 or ϵ by assuming that the observed $\dot{P}$ is entirely due to GW emission

Crab: $\max h_0 = 1.4 \times 10^{-24}$, Vela: $\max h_0 = 3.3 \times 10^{-24}$

Continuous gravitational-wave signal from pulsars

Results from LIGO and VIRGO: known pulsars



LIGO S5:
Nov. 2005 → Oct 2007

VIRGO VSR2:
Jul. 2009 → Jan. 2010

Spindown limit beaten for 2 pulsars:

Crab and Vela

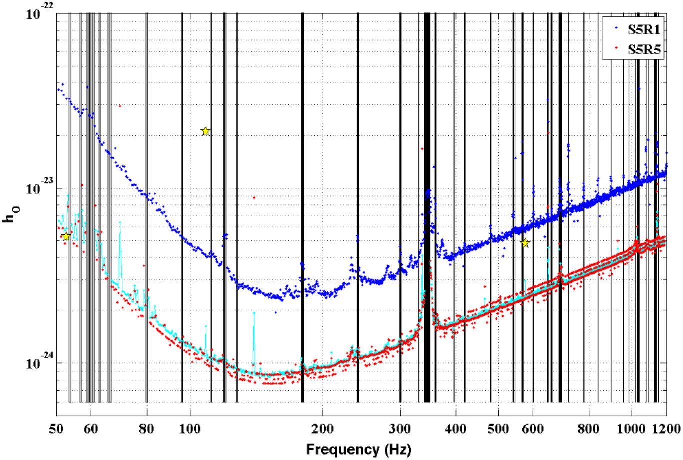
Crab: $\dot{E}_{\rm GW} < 0.02\,\dot{E}_{\rm rot}$

[Abbott et al., ApJ **713**, 671 (2010)] [Abadie et al., ApJ **737**, 93 (2011)]

Figure by Reinhard Prix (AEI)

Continuous gravitational-wave signal from pulsars

Results from LIGO and VIRGO: blind search



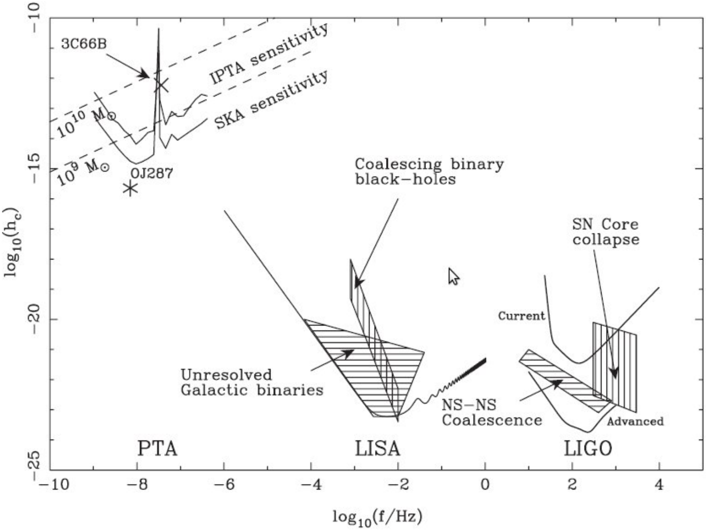
[Aasi et al., PRD **87**, 042001 (2013)]

Einstein@Home analysis of LIGO S5 data

Einstein@Home:
∼ 200,000 volunteers
∼ 750,000 host machines
∼ 25,000 CPU-years

LIGO S5:
Nov. 2005 → Oct 2007

International Pulsar Timing Array (IPTA)



cf. talks by A. Sesana, A. Petiteau et A. Lassus

[Hobbs et al., CQG **27**, 084013 (2010)]

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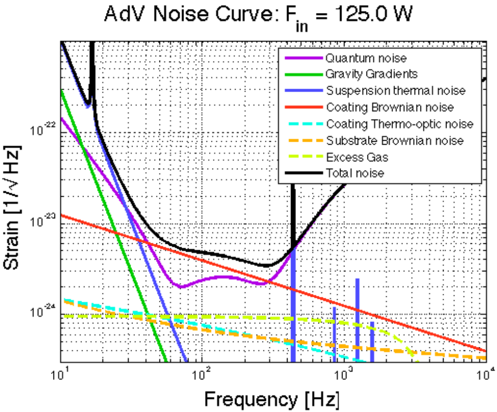
Advanced VIRGO

Advanced VIRGO: dual recycled (power + signal) interferometer with laser power ~ 125 W

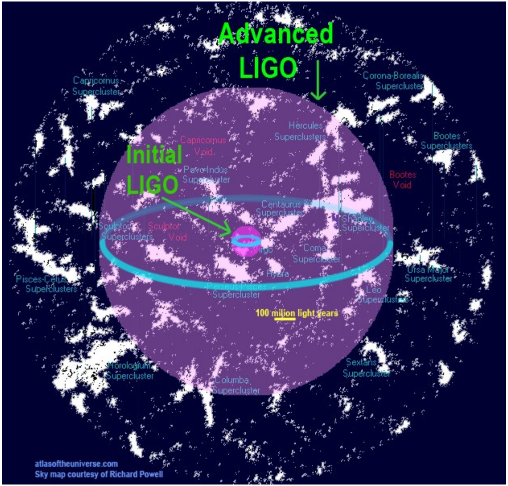


[CNRS/INFN/NIKHEF]

- VIRGO+ decommissioned in Nov. 2011
- Construction of Advanced VIRGO underway
- First lock in 2015
- Sensitivity $\sim 10 \times$ VIRGO
- $\Longrightarrow$ explored Universe volume 10^3 times larger !

Advanced LIGO

Advanced LIGO: dual recycled (power + signal) interferometer with laser power $\sim$ 200 W and better seismic insulation



- LIGO Livingston decommissioned in 2011
- LIGO Hanford decommissioned in 2012
- Advanced LIGO optical cavities locked in summer 2012
- Advanced LIGO in operation by 2014
- Sensitivity $\sim 10 \times$ LIGO
- $\Longrightarrow$ explored Universe volume 10^3 times larger !

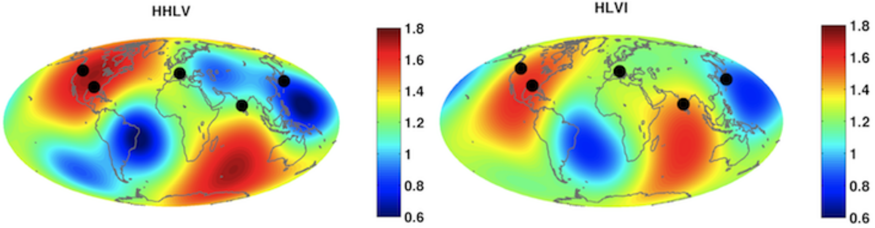
[Advanced LIGO, NSF]

IndIGO / LIGO-India

Project under consideration of the science funding agencies in India and USA:

Move one Advanced LIGO detector from Hanford to India

$\Longrightarrow$ better sky coverage:

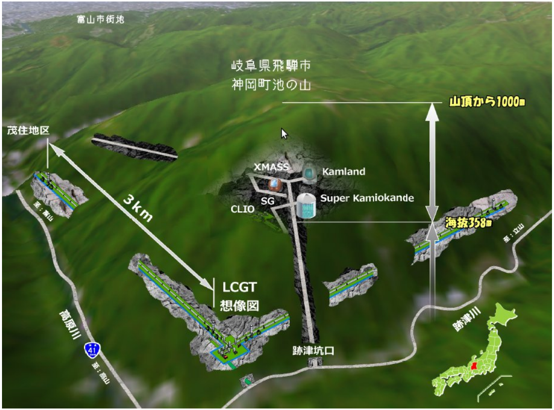


[IndIGO]

Schedule: start of LIGO-India science run: 2020

KAGRA

3-km cryogenic interferometric detector at Kamioka (Japan)

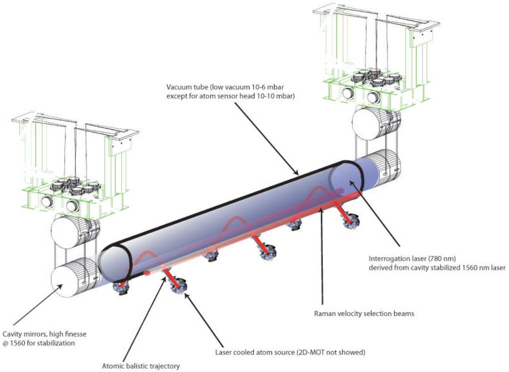


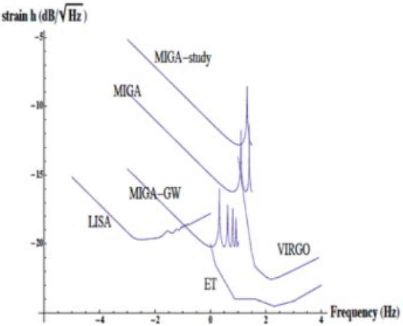
[ICRR GW group, U. Tokyo]

- Construction started in 2012 (tunnel excavation)
- Start of observations: ∼ 2019

Détection par interféromètre à ondes de matière: MIGA

MIGA : *Matter wave - laser Interferometry Gravitation Antenna*[4] : EquipEx sélectionné en 2011, sera implémenté au Laboratoire Souterrain à Bas Bruit (plateau d'Albion, Vaucluse), avec participation du SYRTE





Chaîne d'interféromètres à ondes de matière (accéléromètres) dans une cavité optique: mesure de l'accélération différentielle entre les interféromètres atomiques $\Longrightarrow$ détection des ondes gravitationnelles

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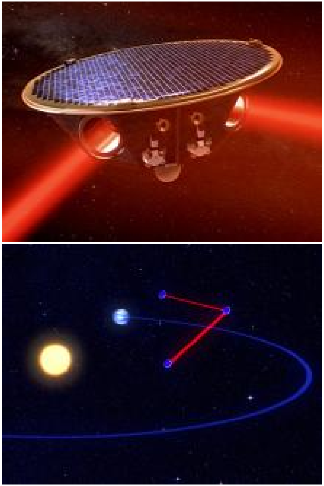
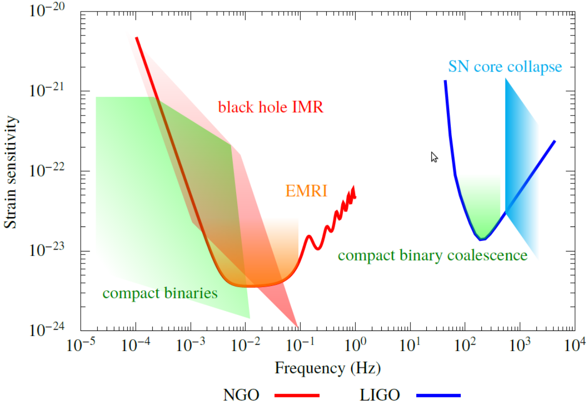
[4] http://sites.google.com/site/migaproject/

eLISA

Gravitational wave detector in space

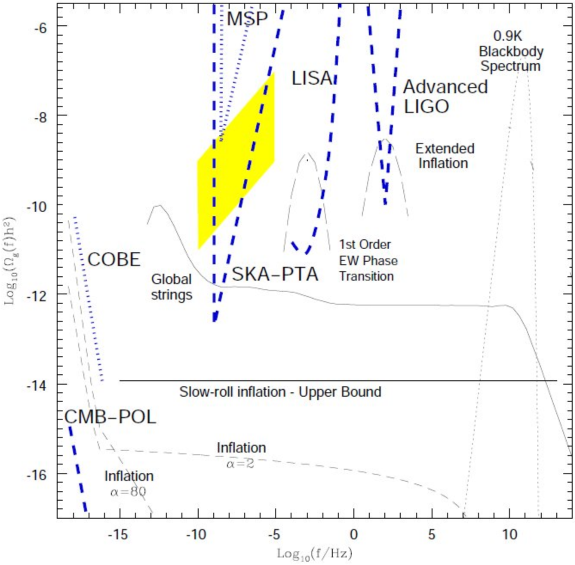


[eLISA / NGO]



- Selection in 2013 ? (ESA L2 mission)
- LISA Pathfinder to be launched in 2015

International Pulsar Timing Array with SKA



[Lorimer, LLR **11**, 8 (2008)]