Recent progress in the numerical simulations of gravitational wave sources

Éric Gourgoulhon
Laboratoire de l'Univers et de ses Théories (LUTH)
CNRS / Observatoire de Paris
F-92195 Meudon, France

in collaboration with
Silvano Bonazzola, Dorota Gondek-Rosińska, Philippe Grandclément,
François Limousin, Jérôme Novak & Keisuke Taniguchi

Eric.Gourgoulhon@obspm.fr

http://www.luth.obspm.fr

Talk at Journées du GREX 2003, Paris, 8-10 Oct. 2003

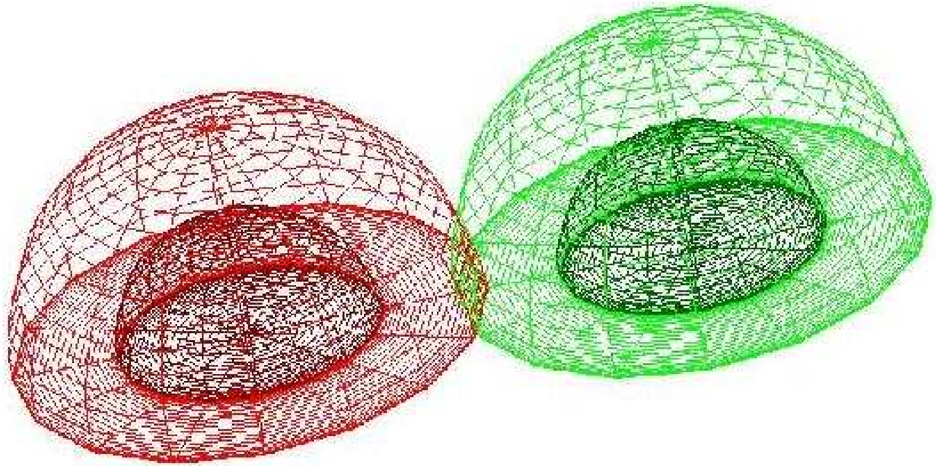
Plan

1. An overview of the numerical techniques employed in our group
2. Triaxial instability of rapidly rotating stars
3. Stellar core collapse (supernovæ)
4. Inspiraling binary neutron stars and black holes
5. Dynamics of the gravitational field

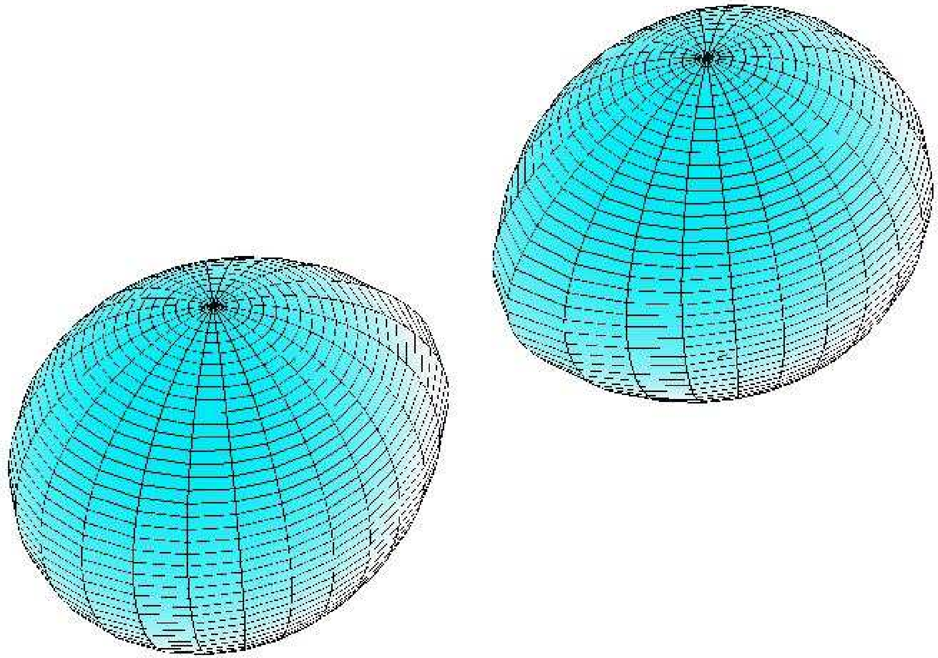
An overview of the numerical techniques employed in Meudon

- Multidomain three-dimensional spectral method
- Spherical-type coordinates (r, θ, φ)
- Expansion functions: r : Chebyshev; θ : cosine/sine or associated Legendre functions; φ : Fourier
- Domains = spherical shells + 1 nucleus (contains $r = 0$)
- Entire space ($\mathbb{R}^3$) covered: compactification of the outermost shell
- Adaptative coordinates : domain decomposition with spherical topology
- Multidomain PDEs: patching method (strong formulation)
- Numerical implementation: C++ codes based on LORENE (http://www.lorene.obspm.fr)

Domain decomposition



Double domain decomposition
for binary systems



Surface fitted coordinates:
$F_0(\theta, \varphi)$ and $G_0(\theta, \varphi)$ chosen so that
$\xi = 1 \Leftrightarrow$ surface of the star

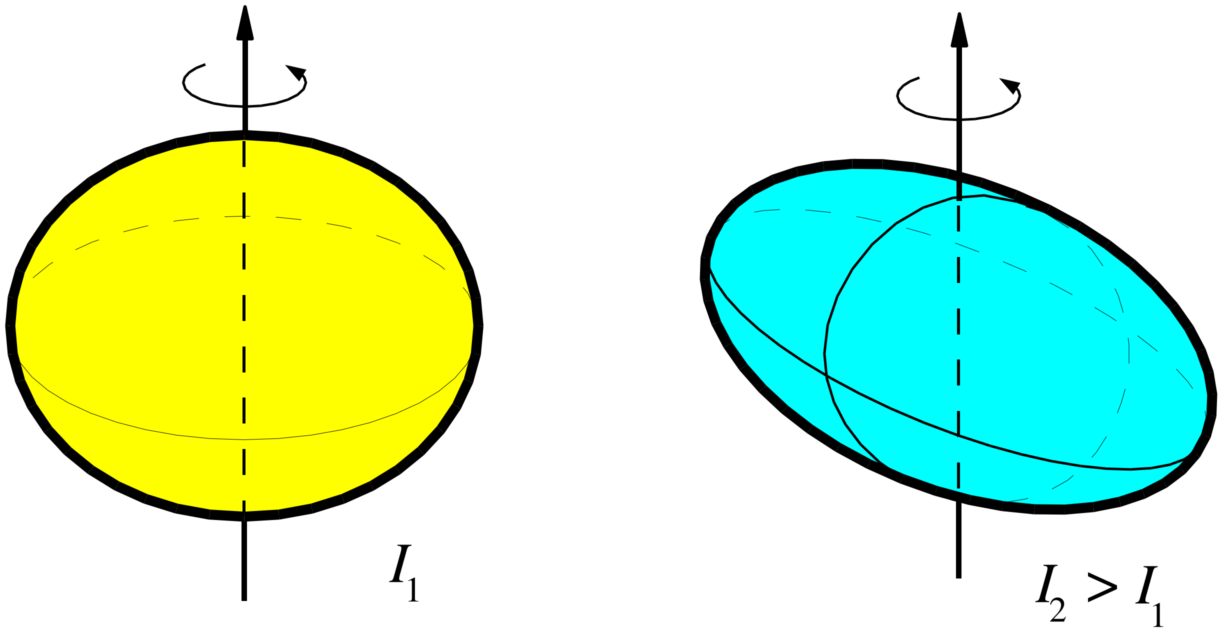
[Bonazzola, Gourgoulhon & Marck, PRD **58**, 104020 (1998)]

Triaxial instability of rapidly rotating neutron stars

Spontaneous symmetry breaking for a self-gravitating fluid body in rigid rotation:

$$E = E_{\rm kin} + E_{\rm grav} + E_{\rm int}$$

with $E_{\rm kin} = J^2/(2I)$ (rigid rotation with total angular momentum J)

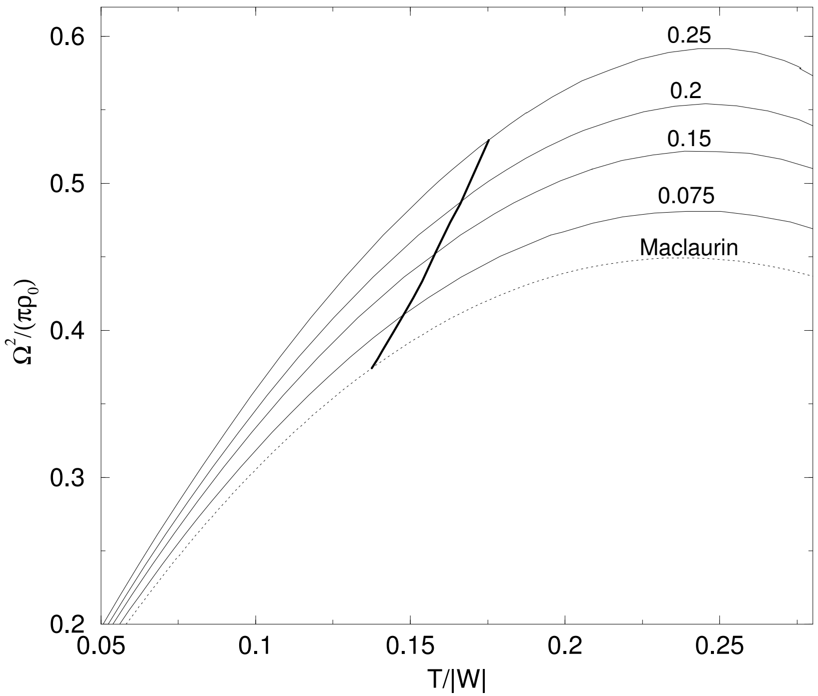


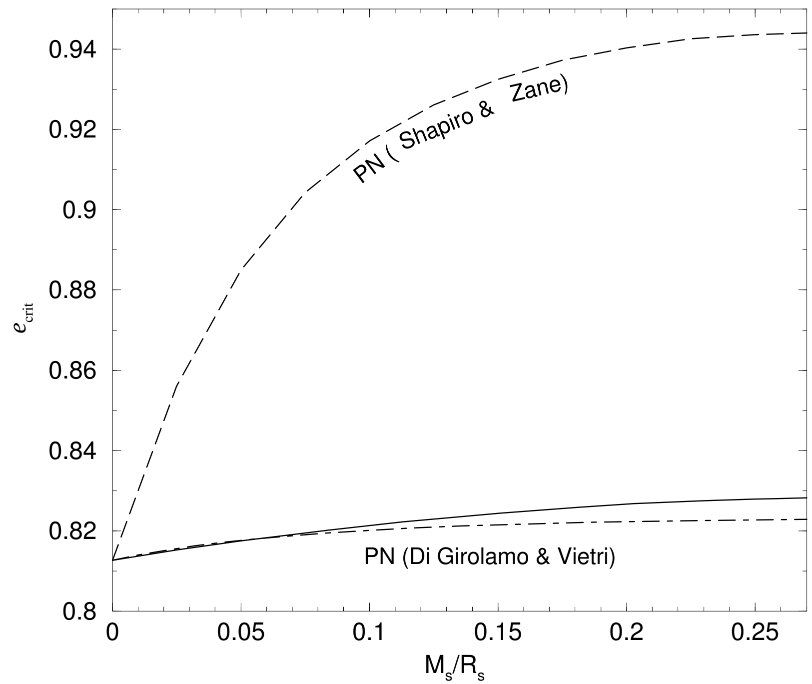
Minimization of E at fixed $J \Longrightarrow$ triaxial configuration when J is large

Physical mechanism of energy dissipation at fixed J: **viscosity**
$\Longrightarrow$ **Astrophysical source of gravitational waves**

Search of the instability point for relativistic bodies

Numerical study based on the helical symmetry
Incompressible fluid: relativistic generalization of the bifurcation
Maclaurin ellipsoid → Jacobi ellipsoid





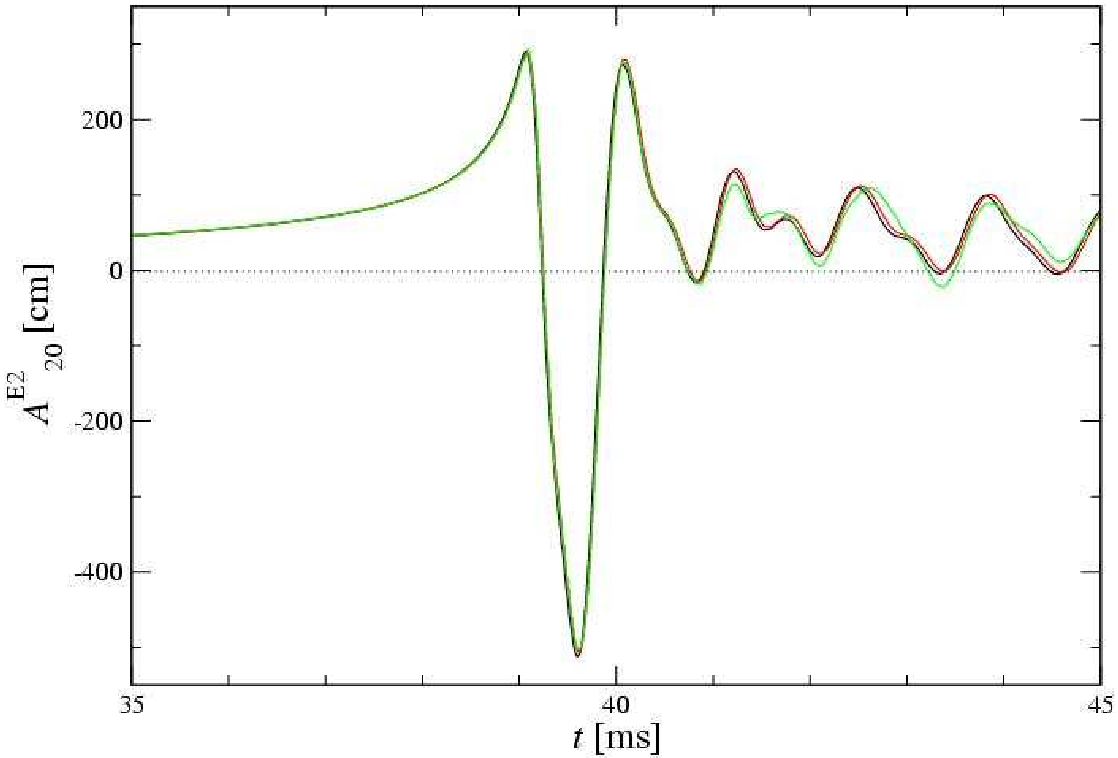
$$\left.\frac{T}{|W|}\right|_{\rm sec} = 0.137526 + 0.148\,\Xi(\Xi+1)$$

[Gondek-Rosińska & Gourgoulhon, PRD **66**, 044021 (2002)]

3-D relativistic supernova computations

New 3-D relativistic code developed by H. Dimmelmeier & J. Novak :

- hydro part: High-resolution shock capturing finite difference methods (developed originaly in Valencia)
- gravitational field part: spectral methods (via LORENE)



Gravitational wave $r\,h_+$ emitted during the collapse and bounce of the supernova iron core: comparison of an axisymmetric computation with the 2-D code of Dimmelmeier, Font & Müller [A&A **393**, 523 (2002)]

[Dimmelmeier & Novak, in preparation]

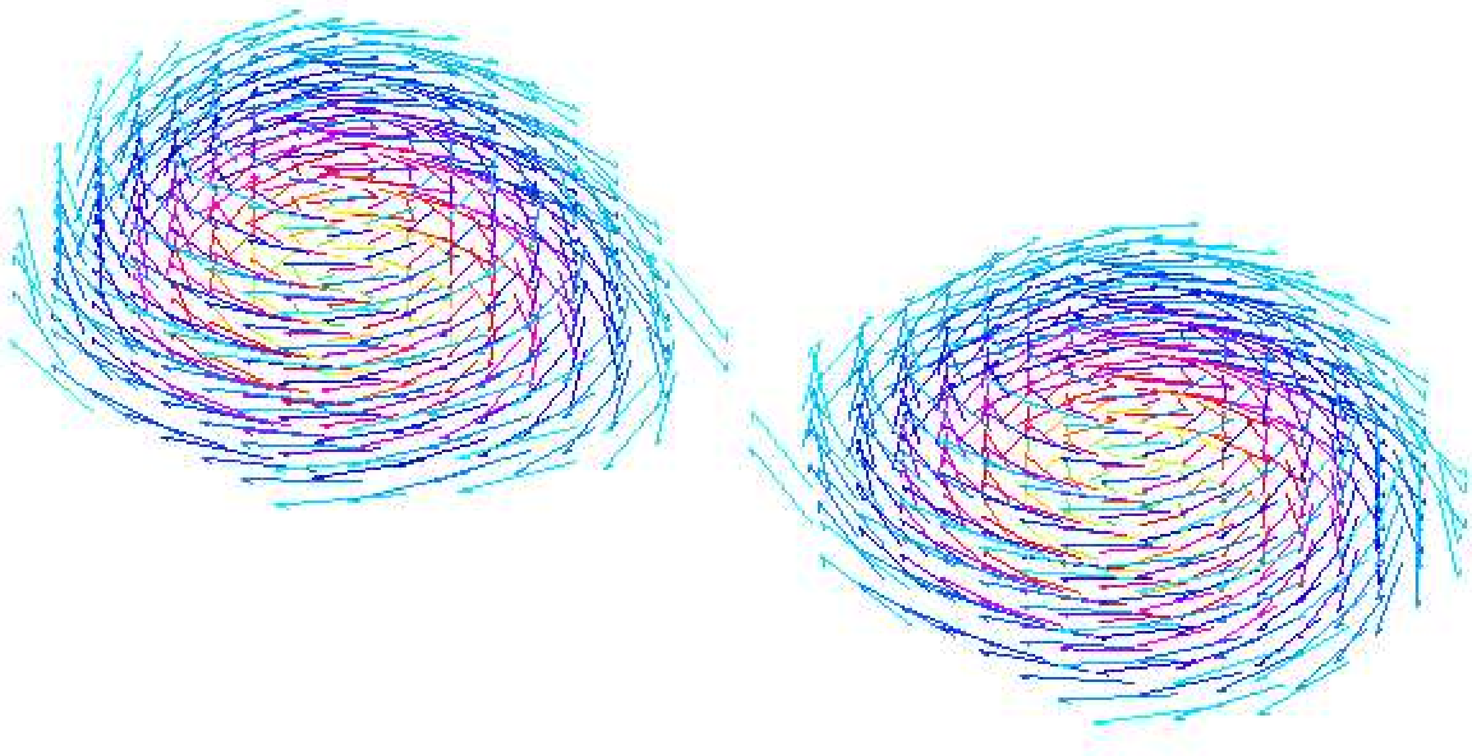
Quasiequilibrium computations of binary NS and BH

Problem treated: Binary black holes or neutron stars in the pre-coalescence stage
⇒ the notion of **orbit** has still some meaning
Basic idea: Construct an approximate, but full spacetime (i.e. **4-dimensional**) representing 2 orbiting compact objects. Previous binary BH numerical treatments: 3-dimensional (initial value problem on a spacelike 3-surface) 4-dimensional approach
⇒ rigorous definition of orbital angular velocity

- Binary NS :
  - ⋆ corotating stars : [Baumgarte et al., PRL **79**, 1182 (1997)], [Baumgarte et al., PRD **57**, 7299 (1998)], [Marronetti, Mathews & Wilson, PRD **58**, 107503 (1998)]
  - ⋆ irrotational stars : [Bonazzola, Gourgoulhon & Marck, PRL **82**, 892 (1999)], [Gourgoulhon et al., PRD **63**, 064029 (2001)], [Marronetti, Mathews & Wilson, PRD **60**, 087301 (2000)], [Uryu & Eriguchi, PRD **61**, 124023 (2000)], [Uryu & Eriguchi, PRD **62**, 104015 (2000)], [Taniguchi & Gourgoulhon, PRD **66**, 104019 (2002)], [Taniguchi & Gourgoulhon, gr-qc/0309045 (2003)]
  - ⋆ arbitrary spins : [Marronetti & Shapiro, gr-qc/0306075]
- Binary BH :
  - ⋆ corotating BH : [Gourgoulhon, Grandclément & Bonazzola, PRD **65**, 044020 (2002)], [Grandclément, Gourgoulhon & Bonazzola, PRD **65**, 044021 (2002)],
  - ⋆ arbitrary spin : [Cook, PRD **65**, 084003 (2002)]

Irrotational binary neutron stars

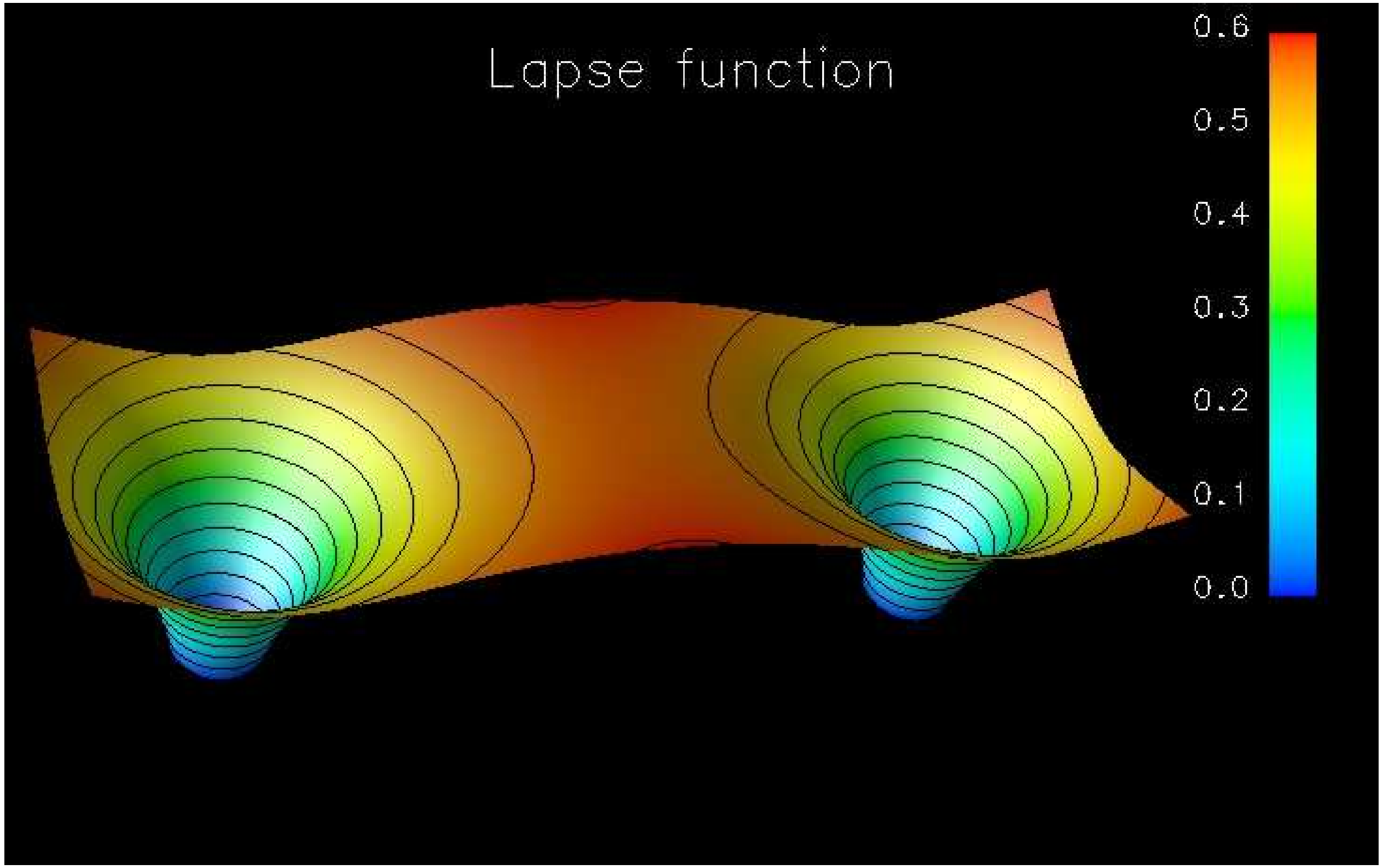
Viscosity of NS matter far too low to ensure synchronization $\Rightarrow$ irrotational motion = much better approximation for slowly rotating NS



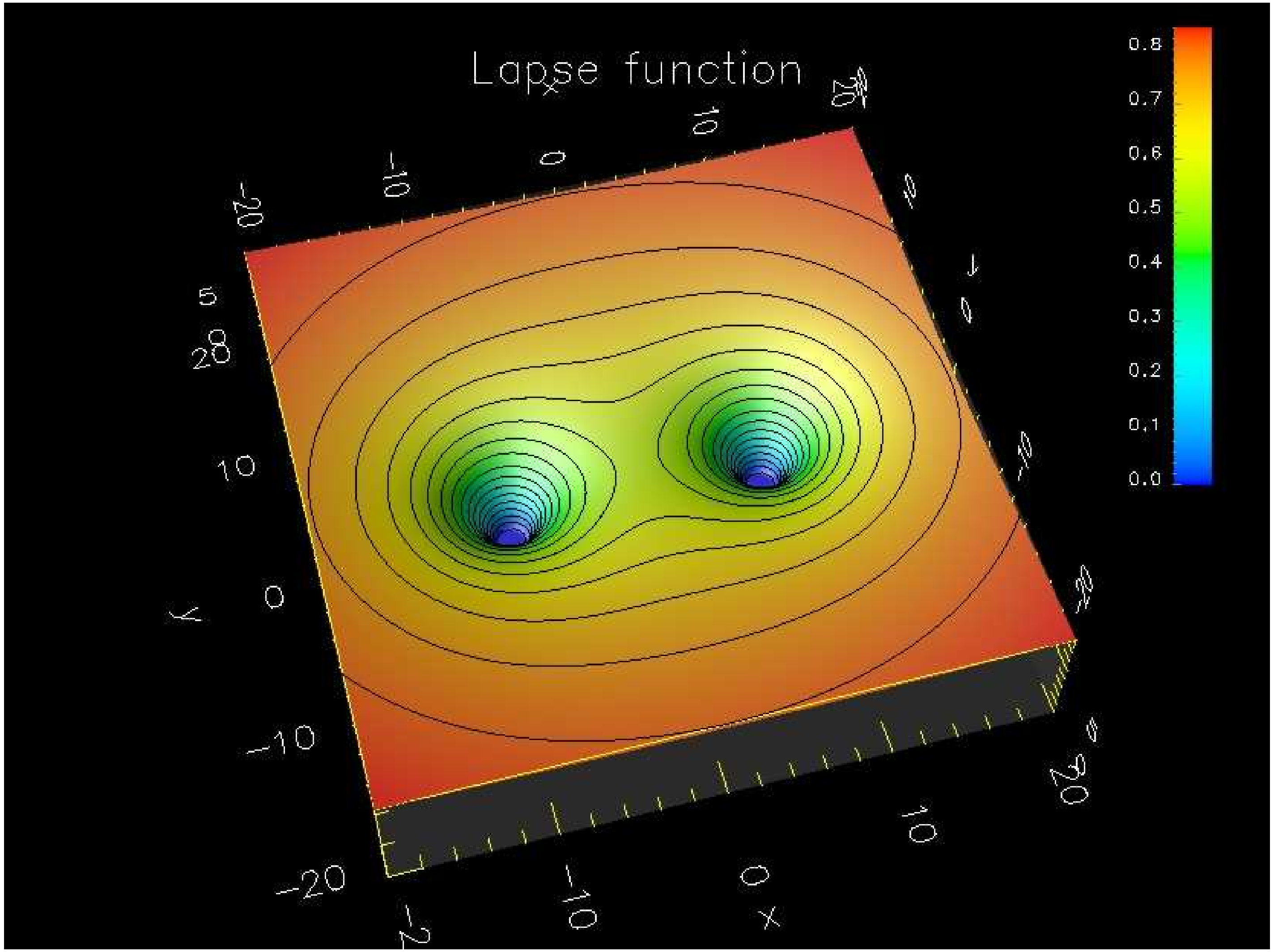
Velocity field w.r.t. co-orbiting frame for irrotational binaries

[from Gourgoulhon, Grandclément, Taniguchi, Marck & Bonazzola, Phys. Rev. D **63**, 064029 (2001)]

Binary black hole in circular orbit



[Grandclément, Gourgoulhon, Bonazzola, PRD **65**, 044021 (2002)]



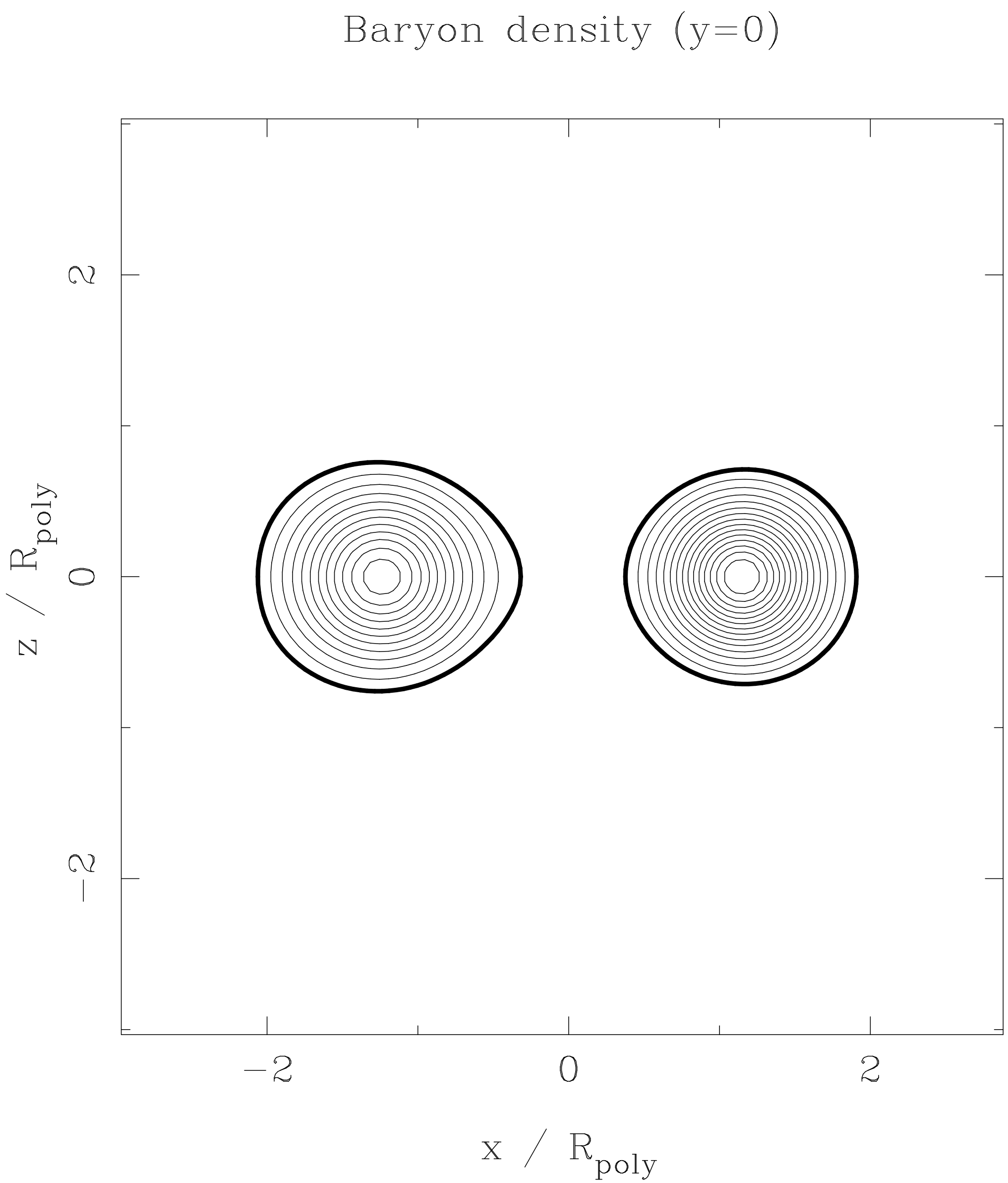
[Grandclément, Gourgoulhon, Bonazzola, PRD **65**, 044021 (2002)]

End point of the inspiral

The quasi-equilibrium inspiral ends either

- by an **orbital instability**, located at the minimum of the system ADM mass [Friedman, Uryu & Shibata, PRD **65**, 064035 (2002)]
- by a **tidal disruption** of the lightest star (mass-shedding limit)

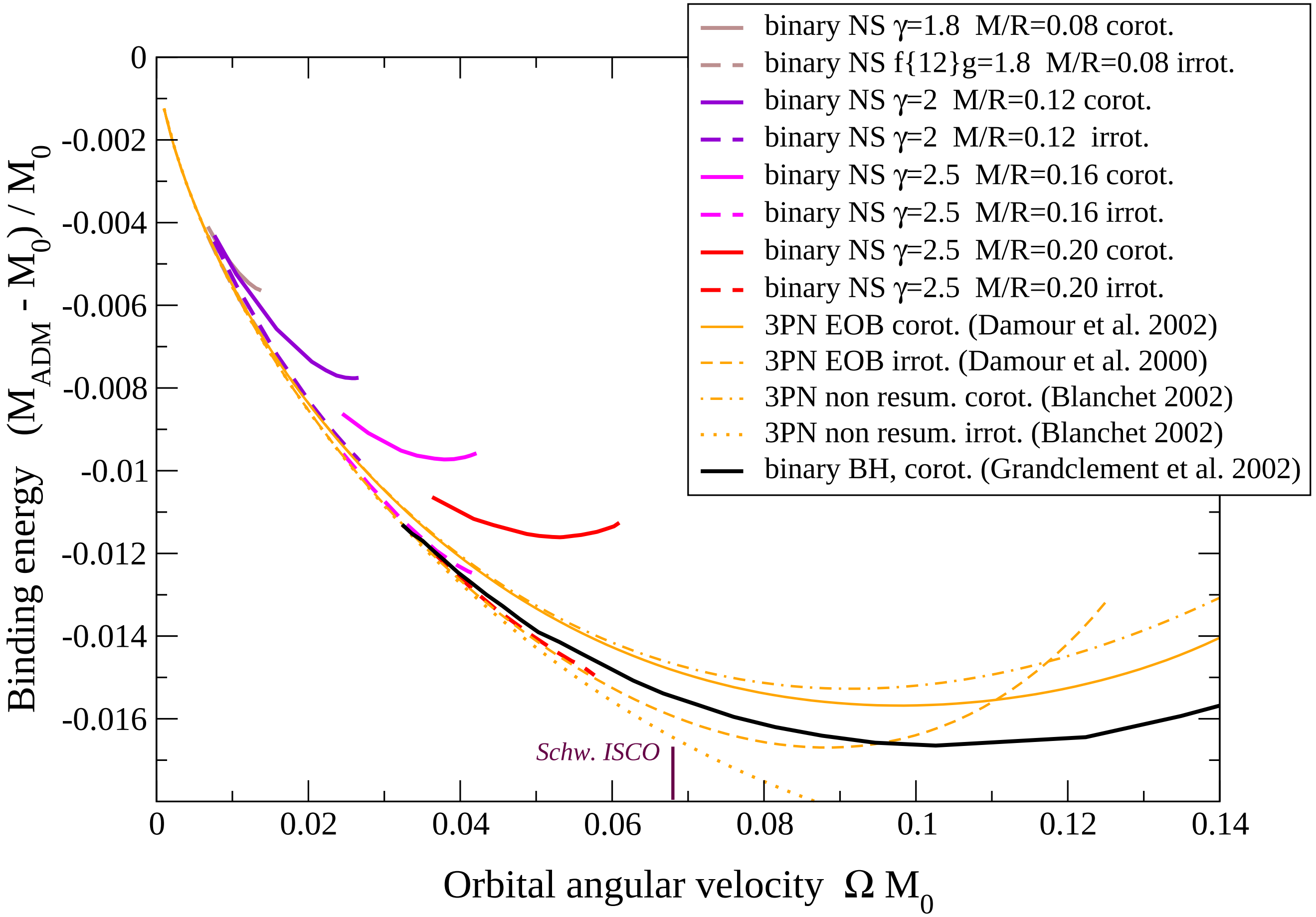
Mass-shedding configuration



Isocontour of baryon density for an irrotational binary system constructed upon a polytropic EOS with $\gamma = 2$. The compactness of the left star is $M/R = 0.14$ and that of the right star is $M/R = 0.16$

[Taniguchi & Gourgoulhon, PRD **66**, 104019 (2002)]

Evolutionary sequences of binary compact objects



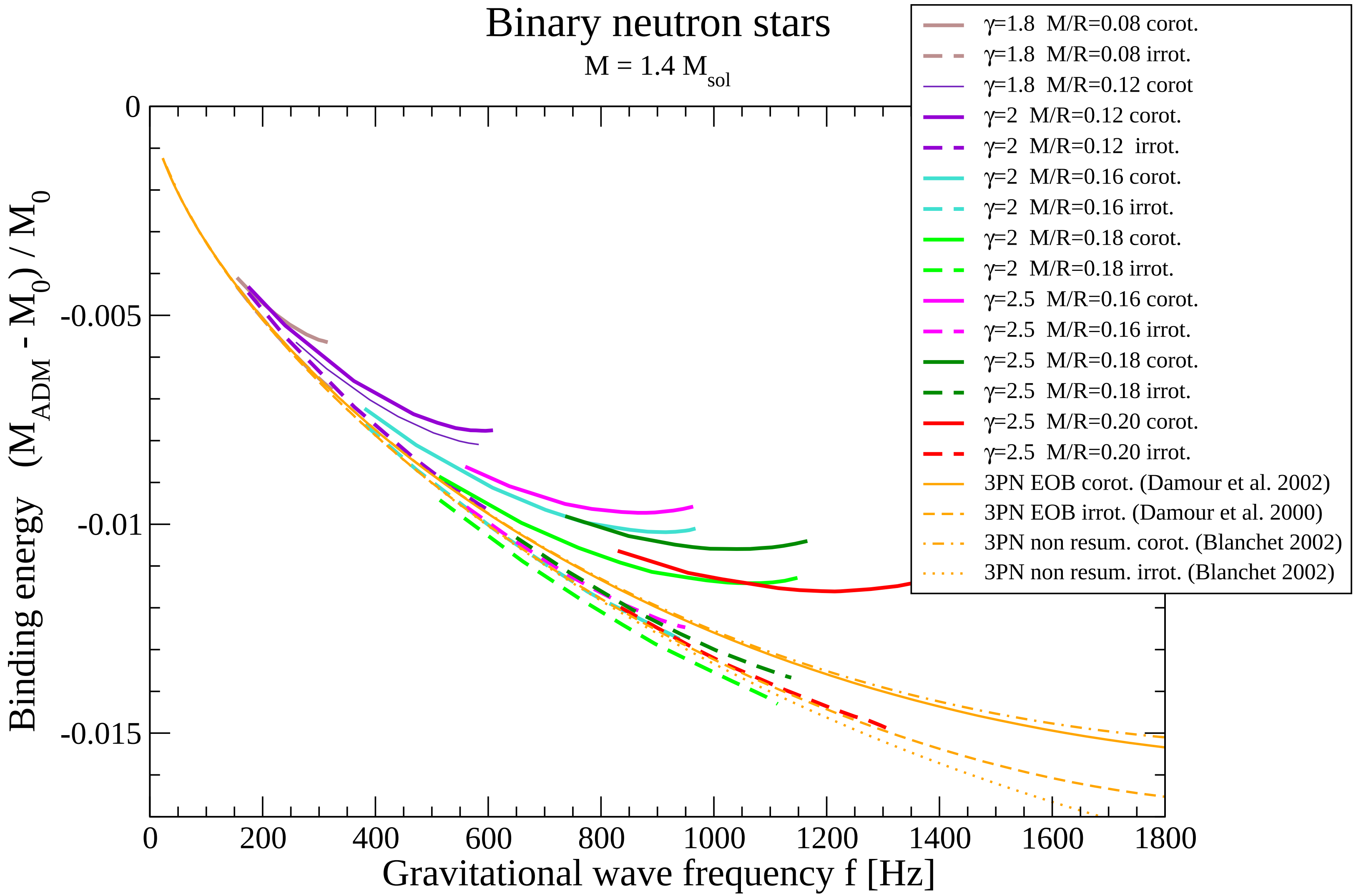
Gravitational wave frequency:

$$f = 320 \, \frac{\Omega M_0}{0.1} \, \frac{20\, M_\odot}{M_0} \ \mathrm{Hz}$$

$$= 1140 \, \frac{\Omega M_0}{0.05} \, \frac{2.8\, M_\odot}{M_0} \ \mathrm{Hz}$$

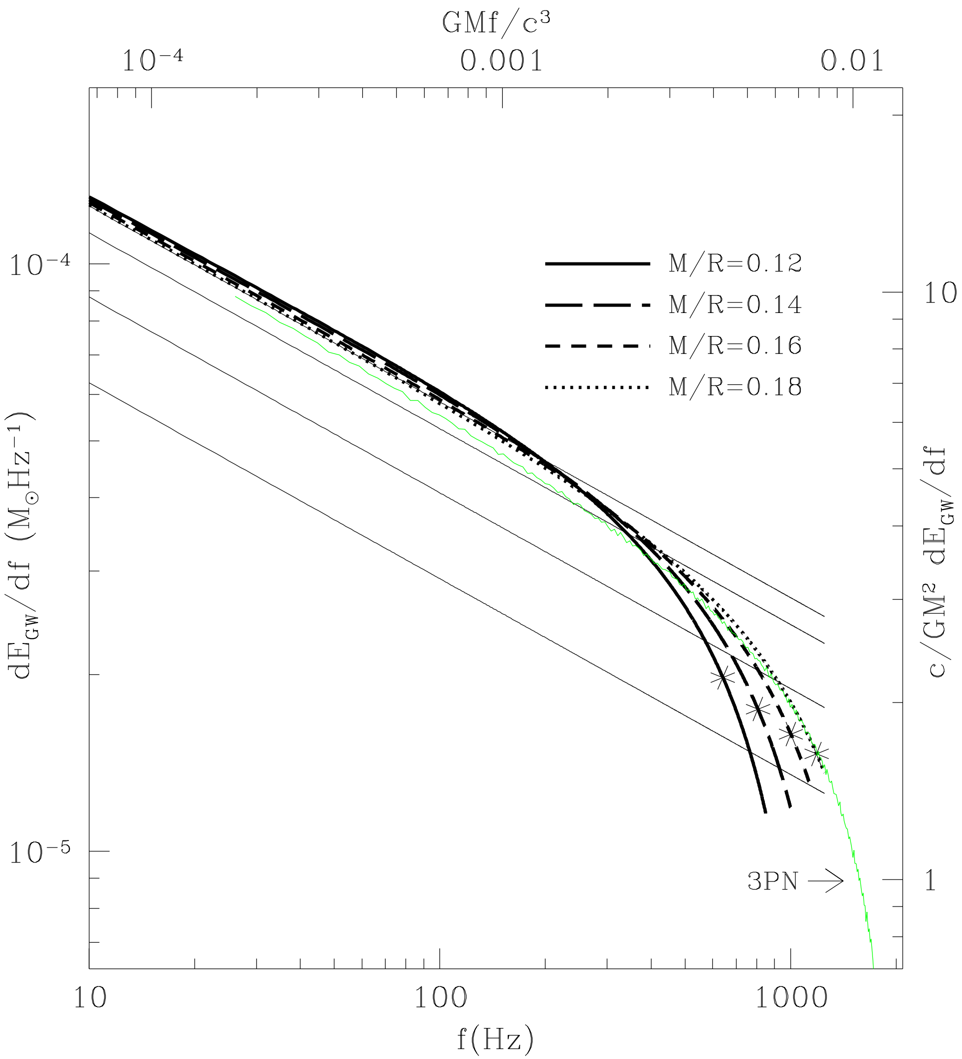
[Taniguchi & Gourgoulhon, PRD in press, gr-qc/0309045 (2003)]

Inspiral end point and neutron star properties



[Taniguchi & Gourgoulhon, PRD in press, gr-qc/0309045 (2003)]

Measuring the neutron-star radius from the GW spectrum



Energy spectrum of GW along four sequences of irrotational binary $1.35\,M_\odot$ neutron stars, constructed upon a polytropic $\gamma = 2$ EOS.

Fit to the numerical results :

$$M_{\rm ADM} = 2.7\,M_\odot - k_{\rm N} f^{2/3} + k_1 f + k_2 f^2$$

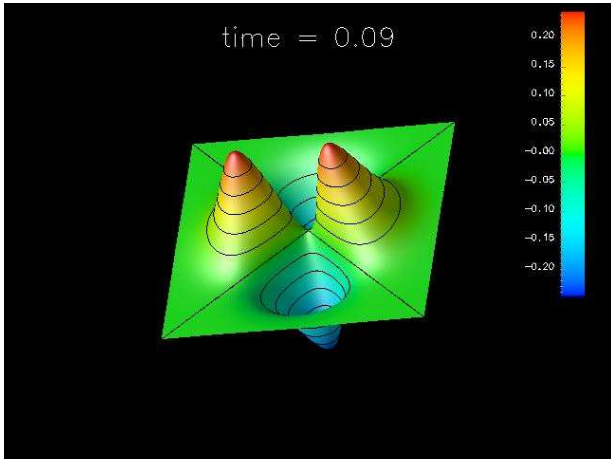
[Faber, Grandclément, Rasio & Taniguchi, PRL **89**, 231102 (2002)]

Dynamics of the gravitational field

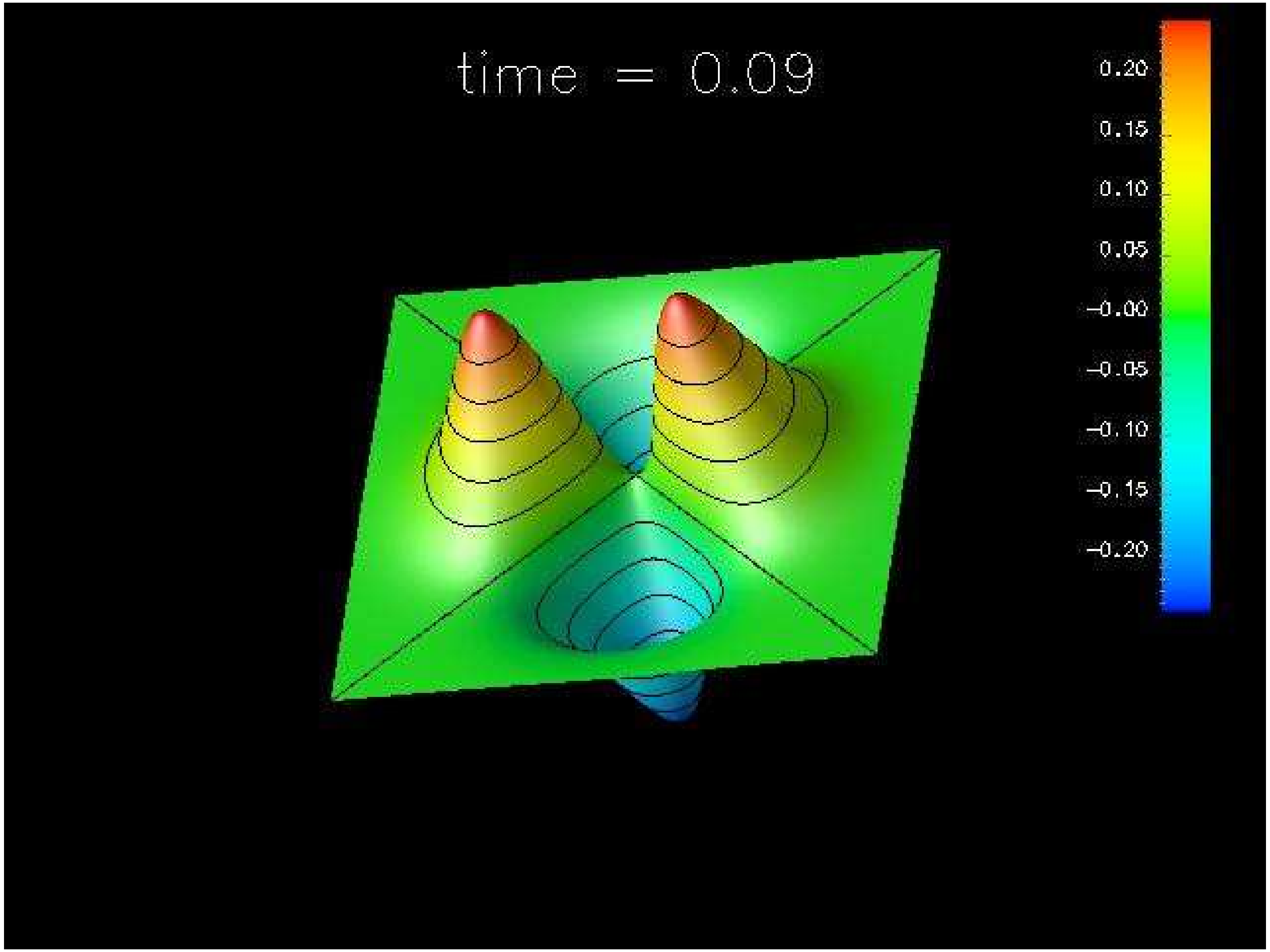
Recent analytical work: new formulation of the 3+1 Einstein equations adapted to numerical time evolution [Bonazzola, Gourgoulhon, Grandclément & Novak, gr-qc/0307082]

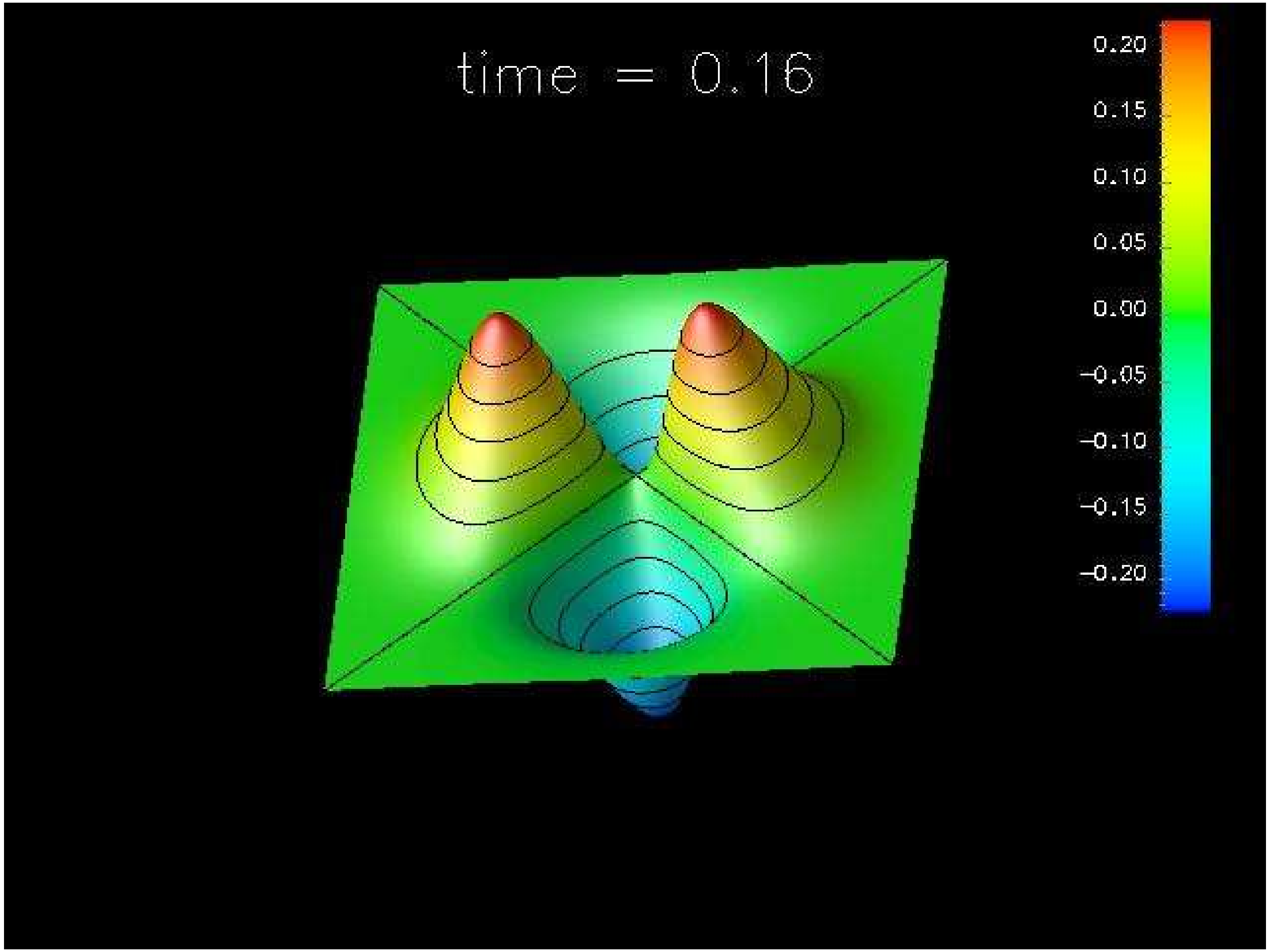
Recent technical progress: **Non-reflecting** boundary conditions for d'Alembert equation, adapted from the Bayliss & Turkel (1980) [Novak & Bonazzola, gr-qc/0203102]

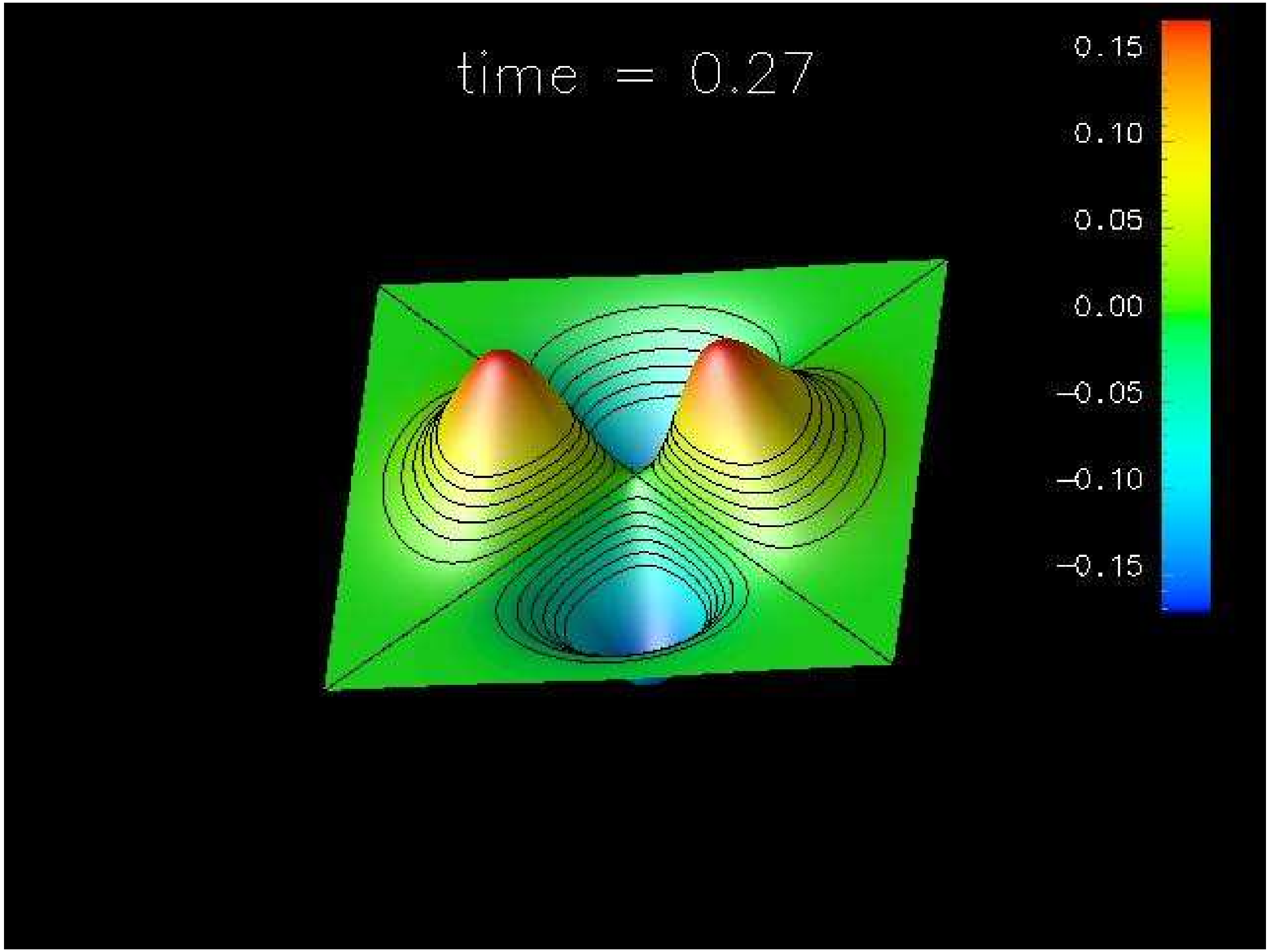
$\Rightarrow$ ensures that spherical harmonics with $\ell = 0$, $\ell = 1$ and $\ell = 2$ are perfectly outgoing. This is important for gravitational waves.

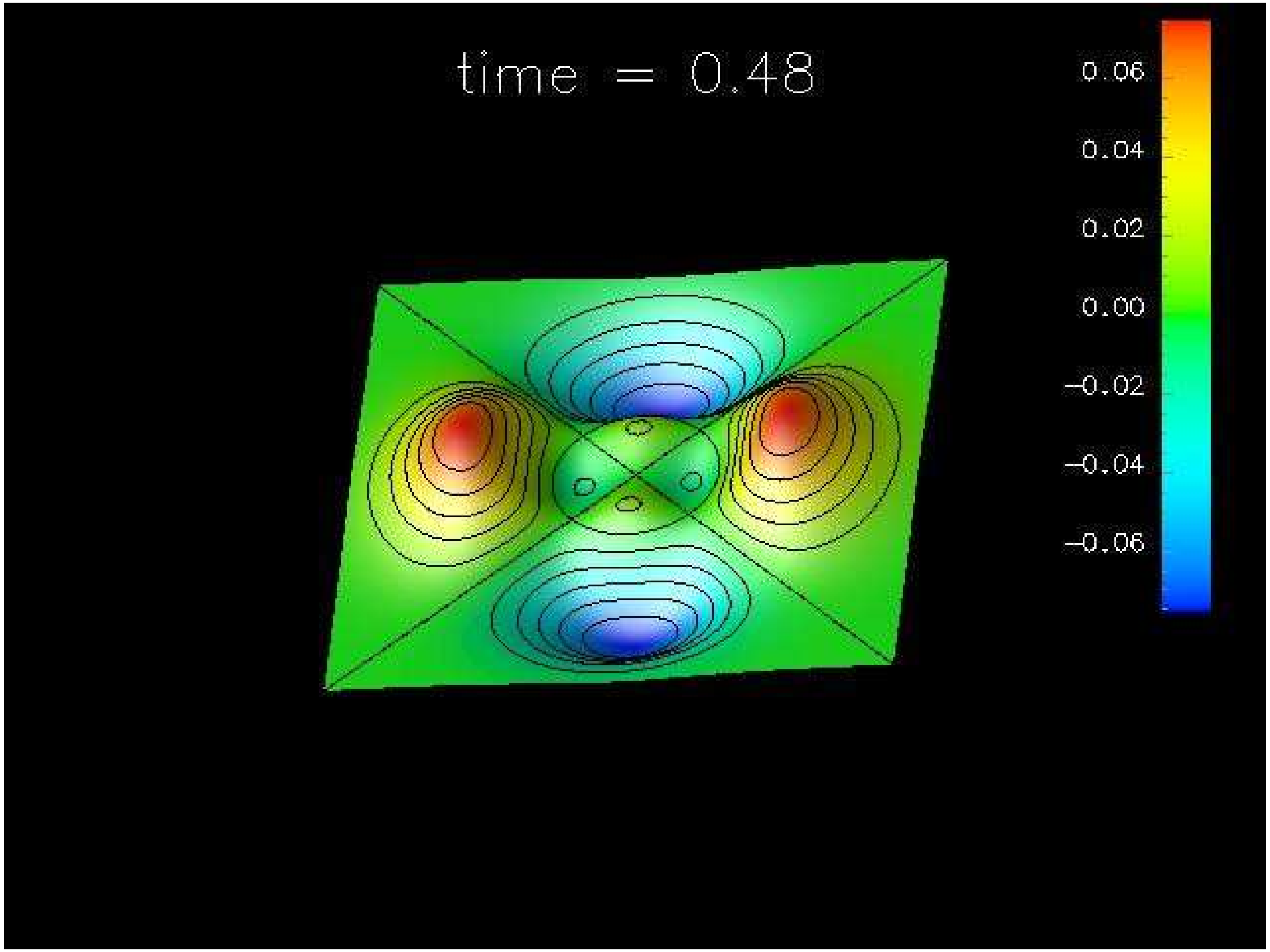


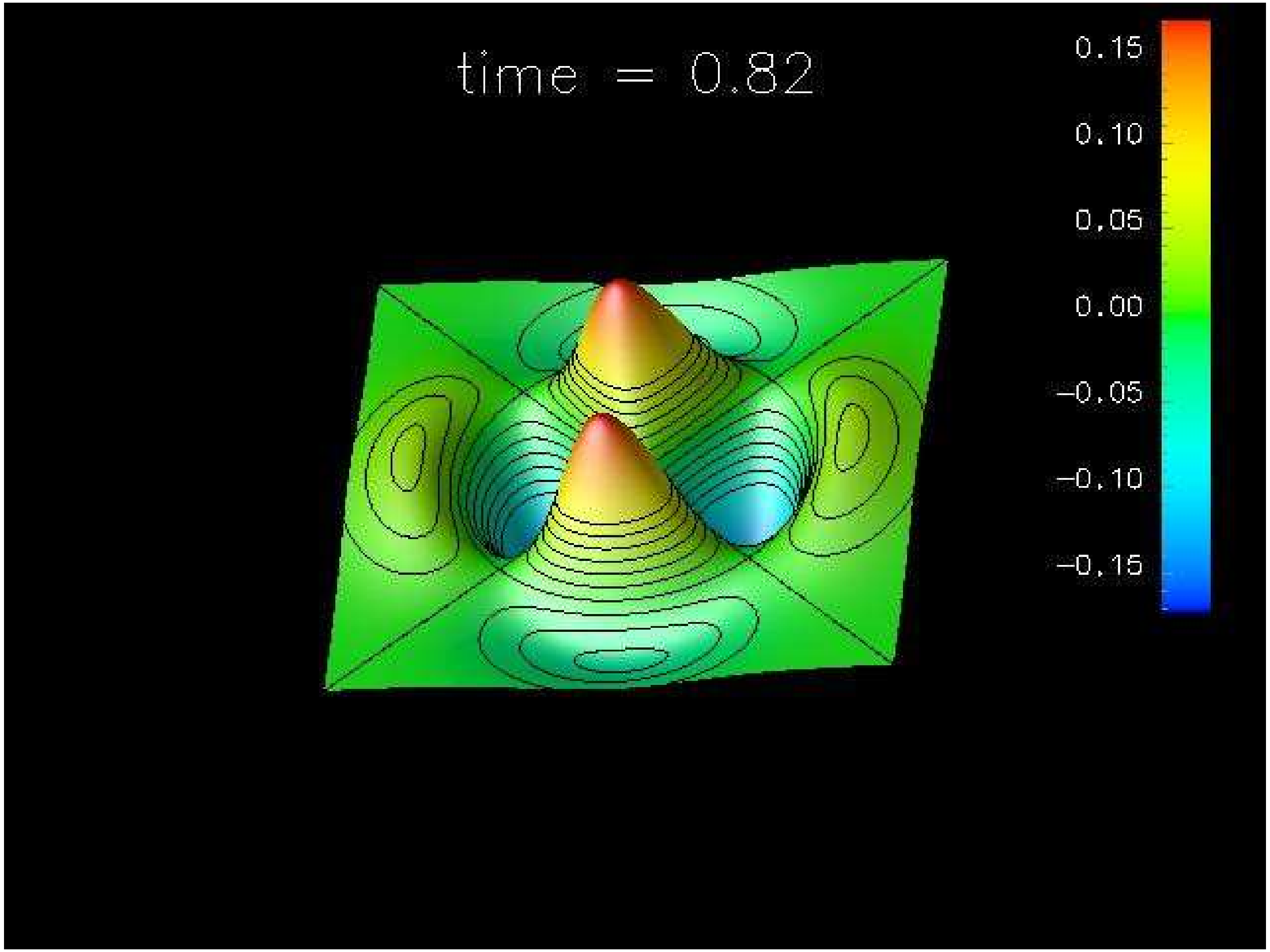
$\ell = 2, m = 2$ scalar wave

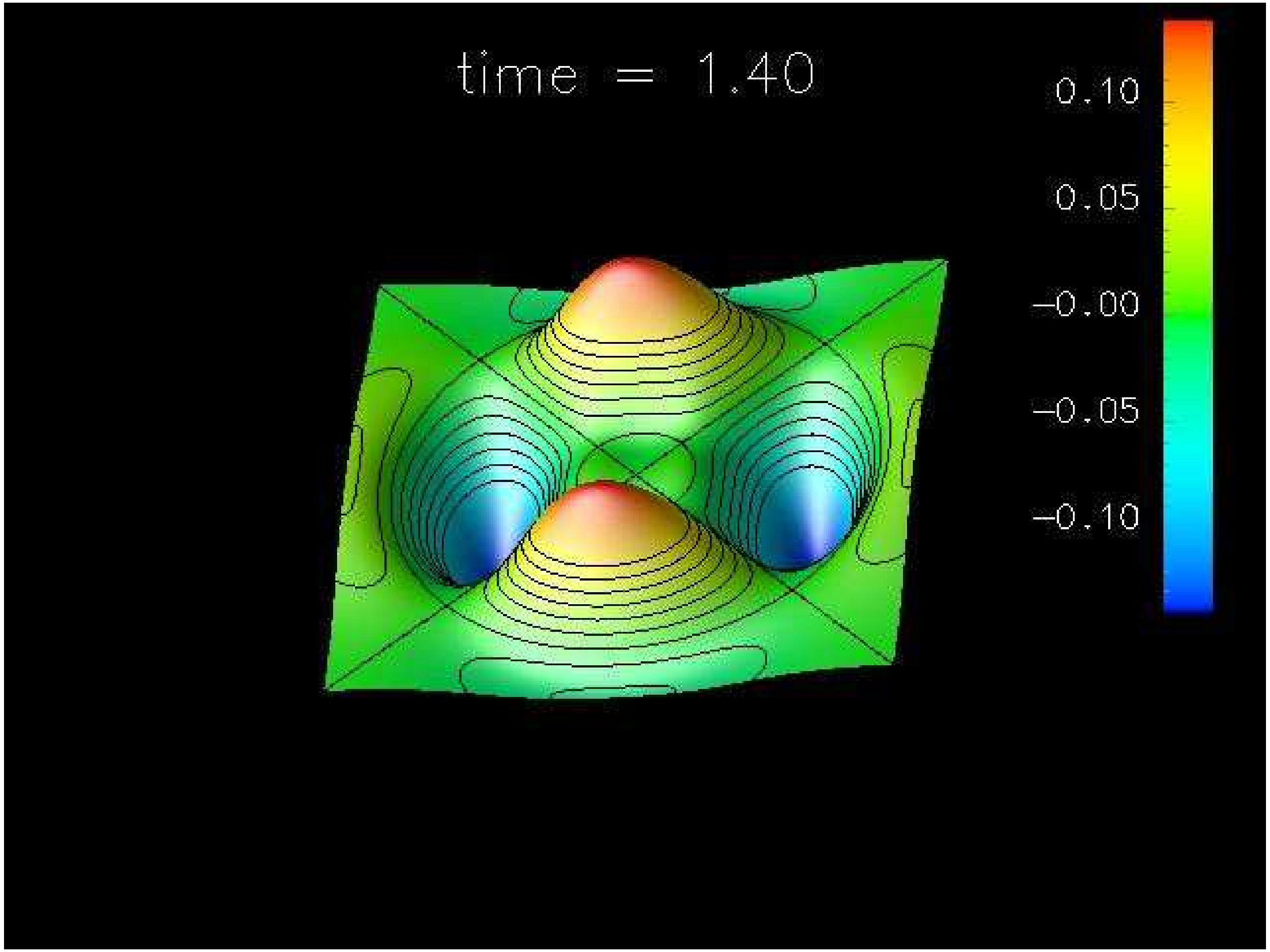

time = 0.09
0.20
0.15
0.10
0.05
−0.00
−0.05
−0.10
−0.15
−0.20

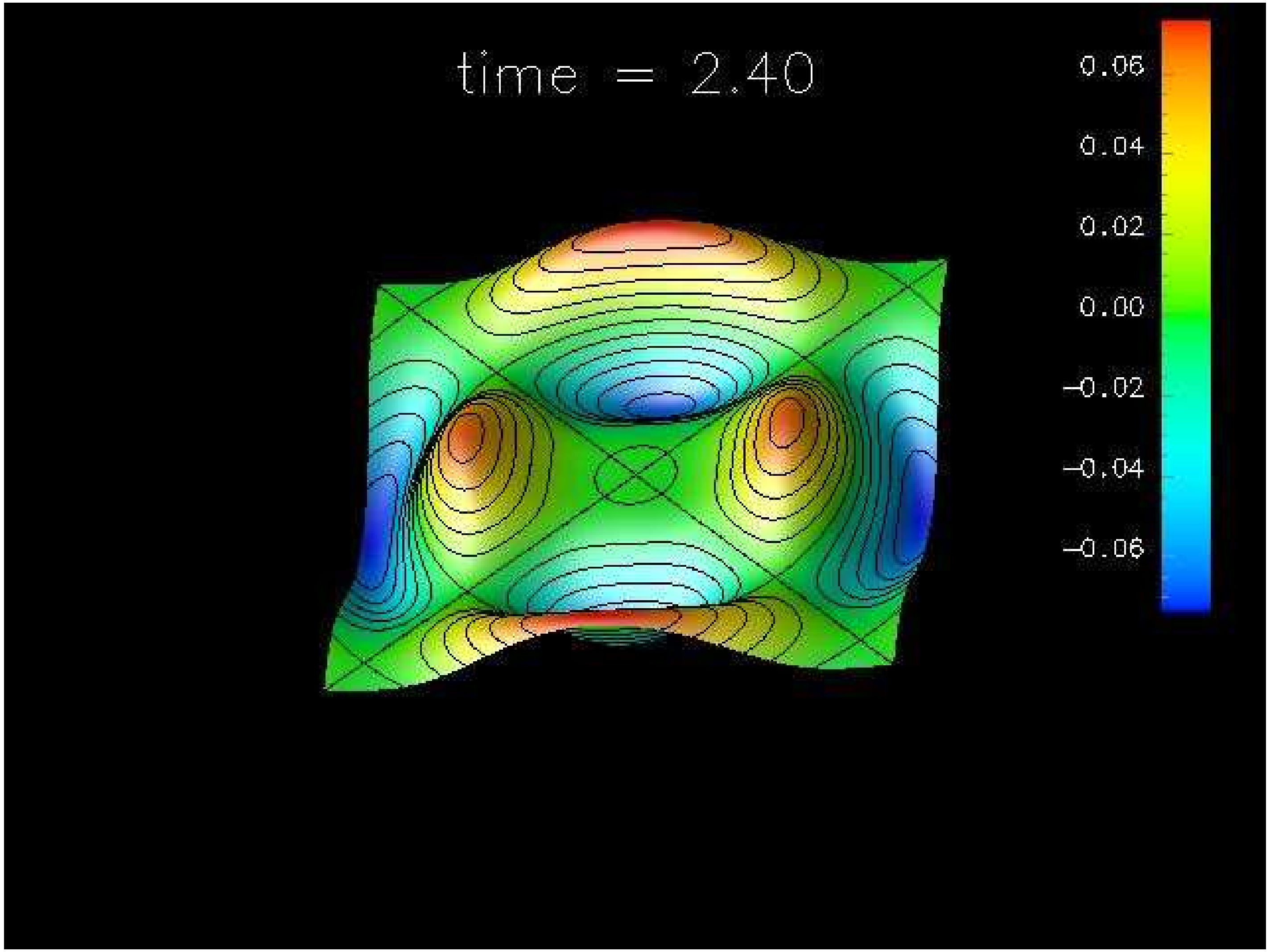

time = 0.16
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20

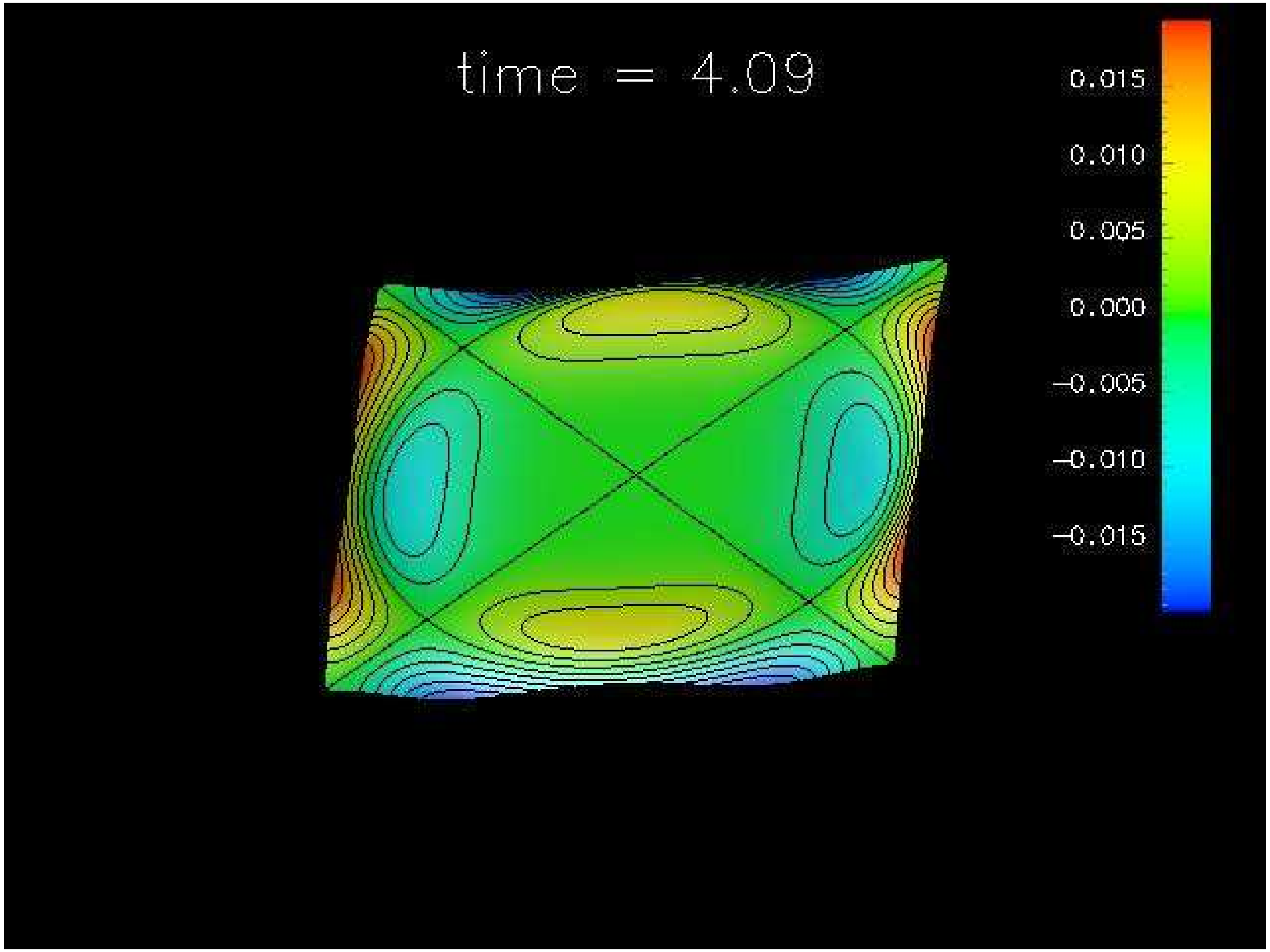

time = 0.27
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15

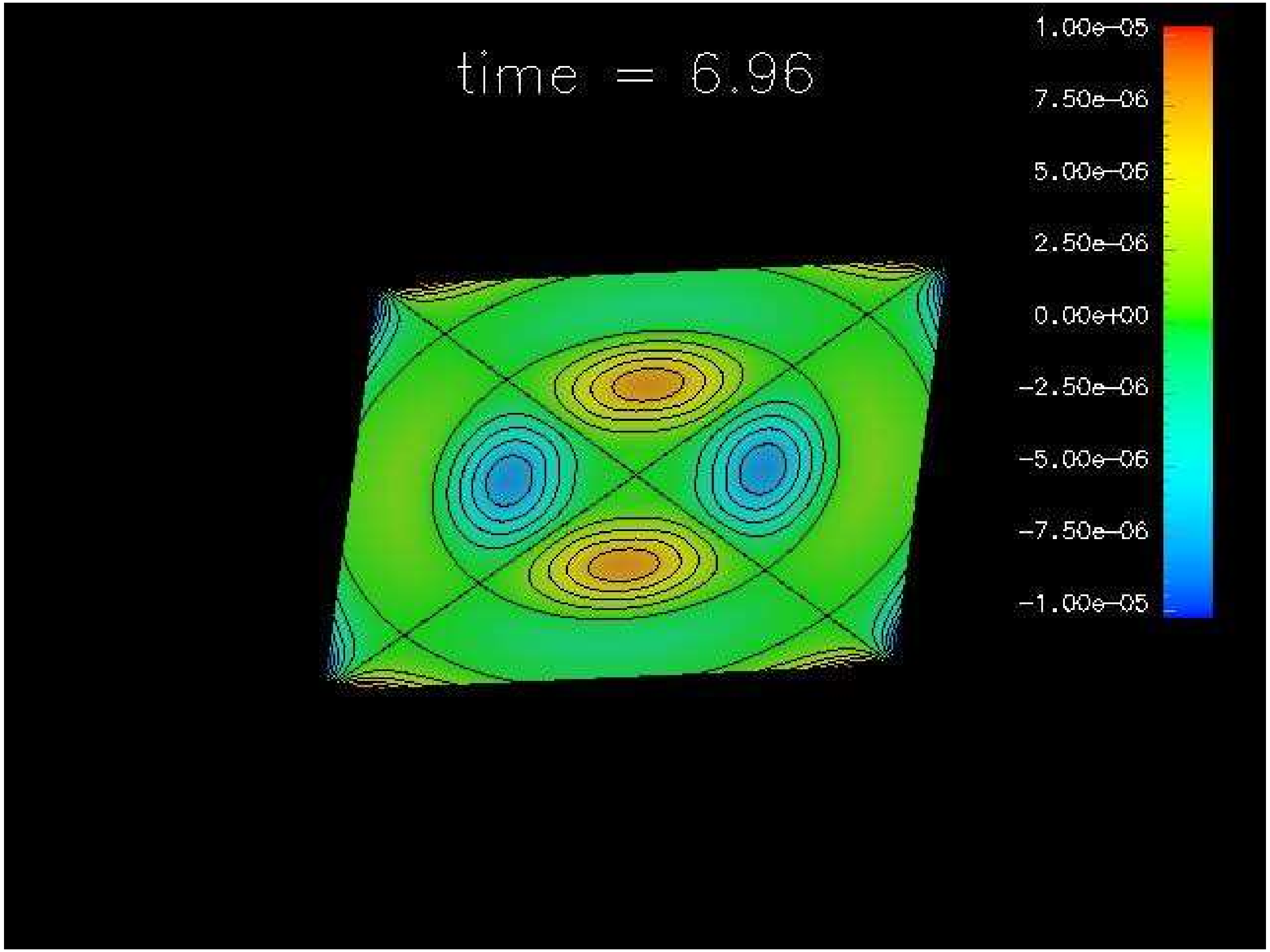

time = 0.48
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06


time = 0.82
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15


time = 1.40
0.10
0.05
−0.00
−0.05
−0.10

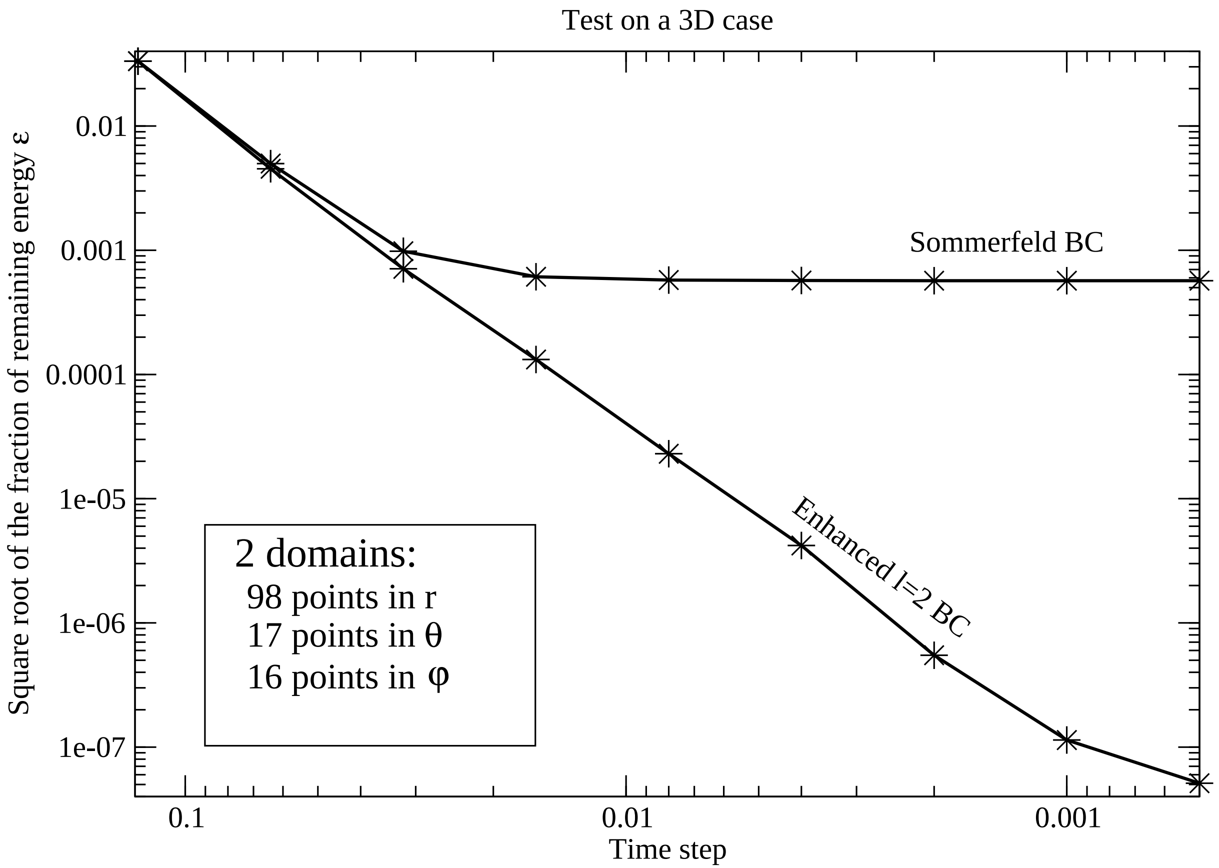

time = 2.40
0.06
0.04
0.02
0.00
−0.02
−0.04
−0.06


time = 4.09
0.015
0.010
0.005
0.000
−0.005
−0.010
−0.015


time = 6.96
1.00e-05
7.50e-06
5.00e-06
2.50e-06
0.00e+00
-2.50e-06
-5.00e-06
-7.50e-06
-1.00e-05

Comparison with Sommerfeld boundary condition

Sommerfeld boundary condition: used by other numerical relativity groups



[Novak & Bonazzola, gr-qc/0203102]