New theoretical perspectives on black holes

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with

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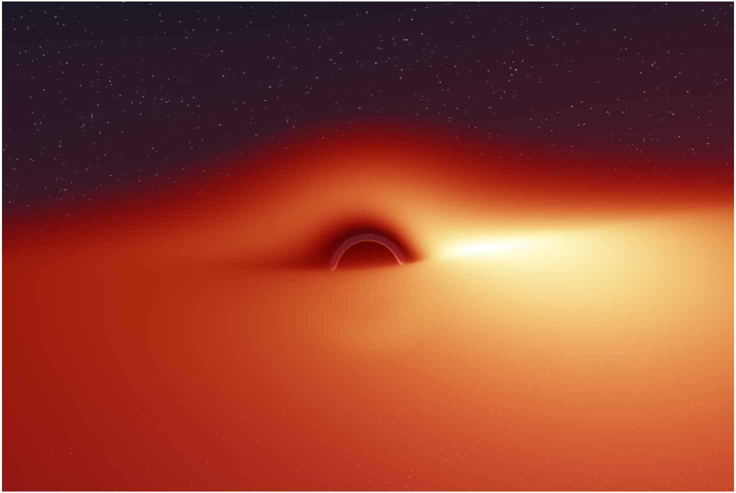
Journées LISA-France
Meudon, 15-16 May 2006

black holes = primary target of LISA

What is a black hole ?

... for the astrophysicist: a very deep gravitational potential well



[J.A. Marck, CQG **13**, 393 (1996)]

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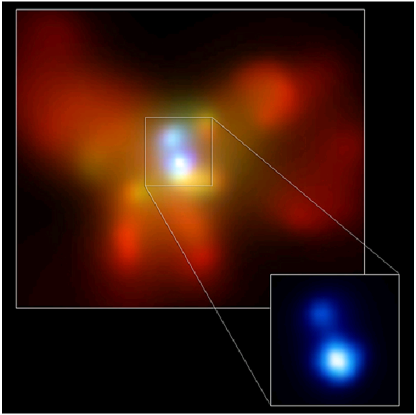
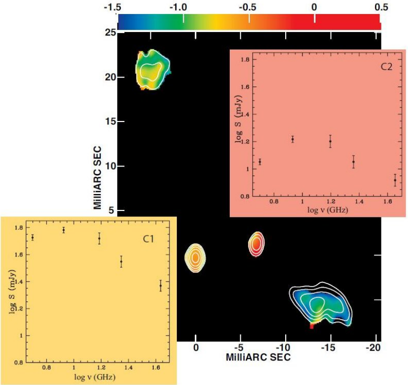


Binary BH in galaxy NGC 6240
$d = 1.4$ kpc
[Komossa et al., ApJ **582**, L15 (2003)]



Binary BH in radio galaxy 0402+379
$d = 7.3$ pc
[Rodriguez et al., ApJ in press, astro-ph/0604042]

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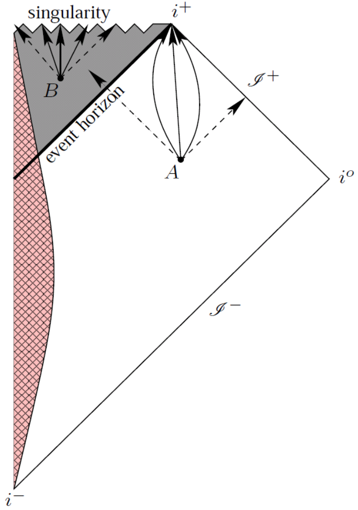

[Booth, Can. J. Phys. **83**, 1073 (2005)]

... for the mathematical physicist:

$$\mathcal{B} := \mathscr{M} - J^-(\mathscr{I}^+)$$

i.e. the region of spacetime where light rays cannot escape to infinity

- $\mathscr{M}$ = asymptotically flat manifold
- $\mathscr{I}^+$ = future null infinity
- $J^-(\mathscr{I}^+)$ = causal past of $\mathscr{I}^+$

event horizon: $\mathcal{H} := \dot{J}^-(\mathscr{I}^+)$
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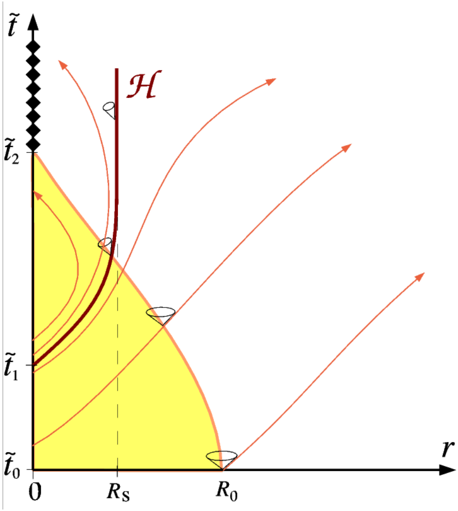

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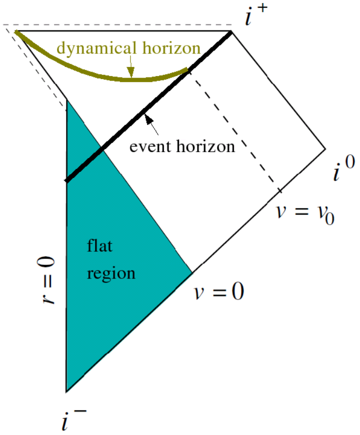
The determination of the boundary of $J^-(\mathscr{I}^+)$ requires the knowledge of the entire future null infinity. Moreover this is not locally linked with the notion of strong gravitational field:



[Ashtekar & Krishnan, LRR **7**, 10 (2004)]

Example of event horizon in a **flat** region of spacetime:

Vaidya metric, describing incoming radiation from infinity:

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dv\,dr + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

with
$m(v) = 0$ for $v < 0$
$dm/dv > 0$ for $0 \leq v \leq v_0$
$m(v) = M_0$ for $v > v_0$

$\Rightarrow$ no local physical experiment whatsoever can locate the event horizon

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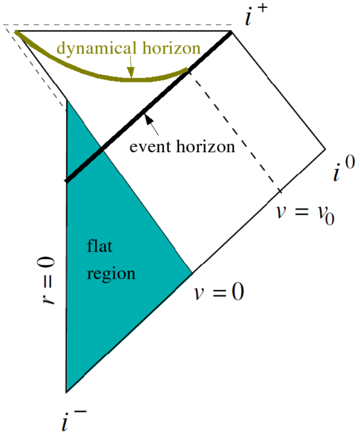

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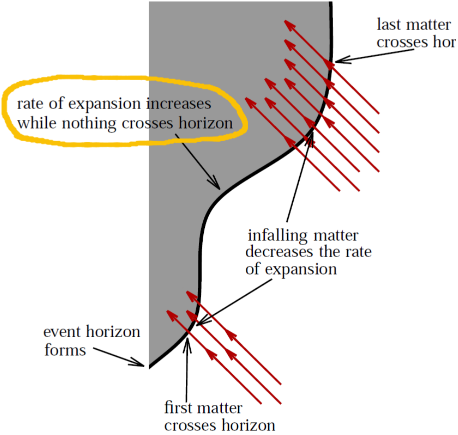
Another non-local feature: teleological nature of event horizons



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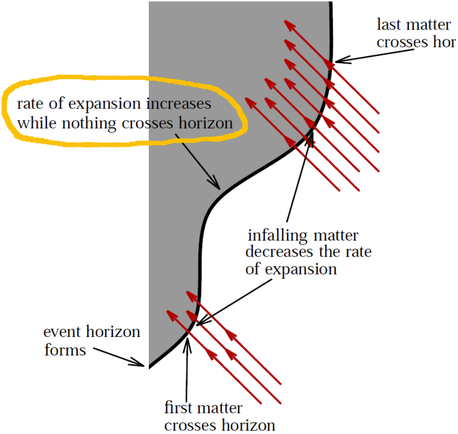

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Local characterizations of black holes

Recently a **new paradigm** appeared in the theoretical approach of black holes: instead of event horizon, black holes are described by

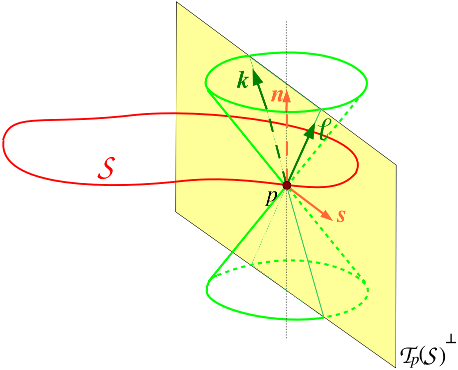
- trapping horizons (Hayward 1994)
- isolated horizons (Ashtekar et al. 1999)
- dynamical horizons (Ashtekar and Krishnan 2002)

All these concepts are **local** and are based on the notion of trapped surfaces

Motivations: quantum gravity, numerical relativity

Trapped surfaces

$\mathcal{S}$: closed (i.e. compact without boundary) spacelike 2-dimensional surface embedded in spacetime $(\mathscr{M}, \boldsymbol{g})$



$\exists$ two future-directed null directions (light rays) orthogonal to $\mathcal{S}$:
$\boldsymbol{\ell}$ = outgoing, expansion $\theta^{(\boldsymbol{\ell})}$
$\boldsymbol{k}$ = ingoing, expansion $\theta^{(\boldsymbol{k})}$

In flat space, $\theta^{(\boldsymbol{k})} < 0$ and $\theta^{(\boldsymbol{\ell})} > 0$

- $\mathcal{S}$ is trapped $\iff$ $\theta^{(\boldsymbol{k})} \leq 0$ and $\theta^{(\boldsymbol{\ell})} \leq 0$
- $\mathcal{S}$ is marginally trapped $\iff$ $\theta^{(\boldsymbol{k})} \leq 0$ and $\theta^{(\boldsymbol{\ell})} = 0$

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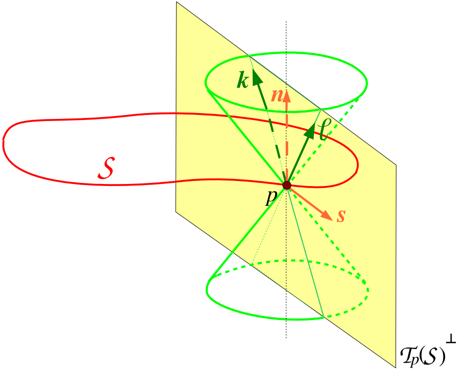

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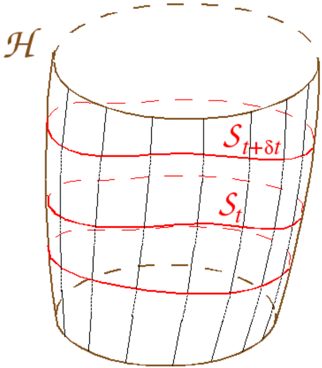
Connection with singularities and black holes

Proposition [Penrose (1965)]: provided that the weak energy condition holds, $\exists$ a trapped surface $\mathcal{S} \Longrightarrow \exists$ a singularity in $(\mathscr{M}, \boldsymbol{g})$ (in the form of a future inextendible null geodesic)

Proposition [Hawking & Ellis (1973)]: provided that the cosmic censorship conjecture holds, $\exists$ a trapped surface $\mathcal{S} \Longrightarrow \exists$ a black hole $\mathcal{B}$ and $\mathcal{S} \subset \mathcal{B}$

A hypersurface $\mathcal{H}$ of $(\mathscr{M}, \boldsymbol{g})$ is said to be



- a **future outer trapping horizon (FOTH)** iff
  (i) $\mathcal{H}$ foliated by marginally trapped 2-surfaces ($\theta^{(\boldsymbol{k})} < 0$ and $\theta^{(\boldsymbol{\ell})} = 0$)
  (ii) $\mathcal{L}_{\boldsymbol{k}}\, \theta^{(\boldsymbol{\ell})} < 0$
  [Hayward, PRD **49**, 6467 (1994)]
- a **dynamical horizon** iff
  (i) $\mathcal{H}$ is foliated by marginally trapped 2-surfaces
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  [Ashtekar & Krishnan, PRL **89** 261101 (2002)]
- a **non-expanding horizon** iff
  (i) $\mathcal{H}$ is null (null normal $\boldsymbol{\ell}$)
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- an **isolated horizon** iff
  (i) $\mathcal{H}$ is a non-expanding horizon
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Local definitions of "black holes"

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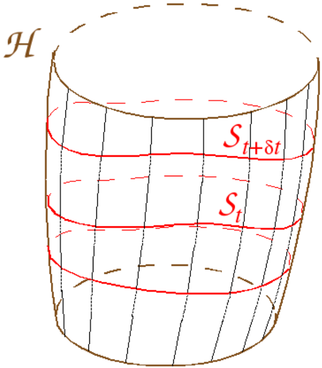

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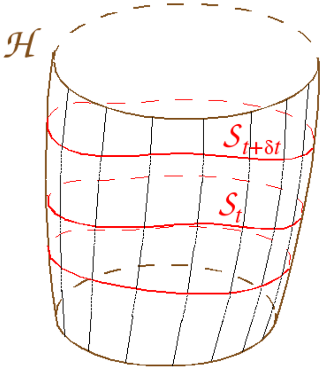

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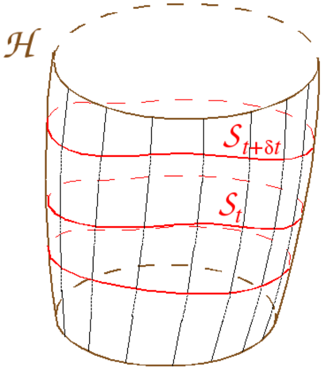

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Dynamics of these new horizons

The *dynamical horizons* and *trapping horizons* have their **own dynamics**, ruled by the Einstein equation.
In particular, one can establish for them

- first and second laws of black hole mechanics
  [Ashtekar & Krishnan, PRD **68**, 104030 (2003)], [Hayward, PRD **70**, 104027 (2004)]
- a Navier-Stokes like equation $\Rightarrow$ viscous membrane behavior as for the event horizon ("membrane paradigm")
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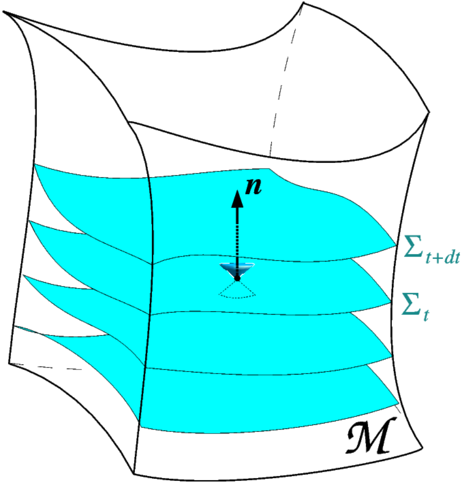
3+1 numerical relativity



3+1 formalism: slicing of the spacetime manifold $\mathscr{M}$ by a family of spacelike hypersurfaces $(\Sigma_t)_{t\in\mathbb{R}}$

t = coordinate time

Σ_t = "the 3-dimensional space" at instant t

$\Rightarrow$ **resolution of Einstein equation = Cauchy problem**

i.e. time evolution from initial data given on some hypersurface Σ_0

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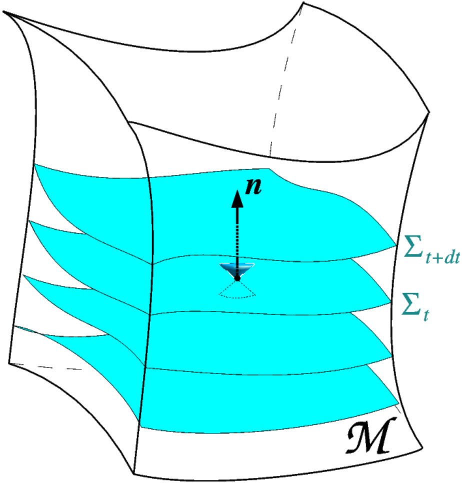

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3+1 numerical relativity and black holes

black hole $\Rightarrow \exists$ singularity in spacetime
$\Rightarrow$ divergent quantities in the 3+1 formalism

However, there is no need to numerically evolve the region around the singularity since it is hidden behind the event horizon and causally disconnected from the exterior.

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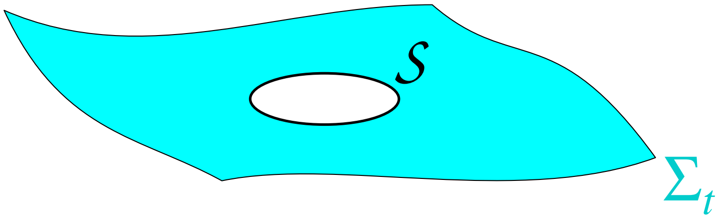

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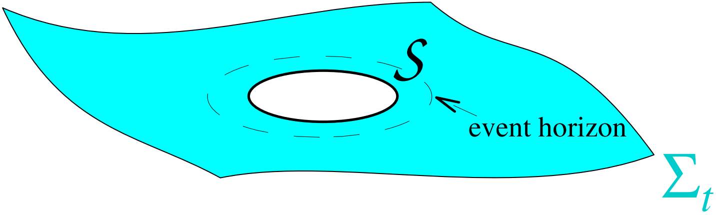
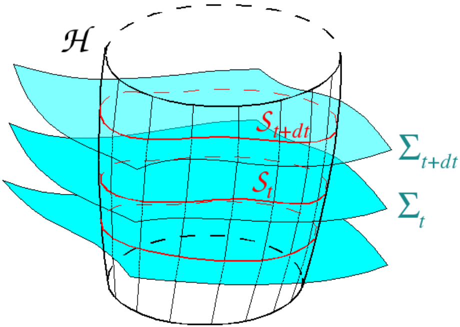


Provided that the excised region is located within the even horizon, the choice of it does not affect the exterior spacetime

Our project

Choose the excision boundary $\mathcal{S}_t$ to be a **marginally trapped surface** for each time t



The tube $\mathcal{H} = \bigcup_{t\in\mathbb{R}} \mathcal{S}_t$

is then a **trapping horizon**

- geometrically well defined excision boundary
- ensures $\mathcal{S}_t$ is located inside the event horizon ◂ reminder
- easy to implement with spherical coordinates and spectral methods

Status

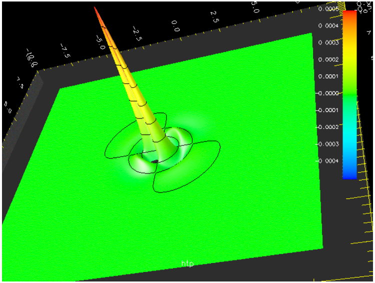
- Equilibrium conditions (isolated horizon) expressed in terms of the quantities of the 3+1 formalism
  - [Jaramillo, Gourgoulhon & Mena Marugán, PRD **70**, 124036 (2004)]
  - [Gourgoulhon & Jaramillo, Phys. Rep. **423**, 159 (2006)]
- Analytical study of the dynamical case completed
- Numerical implementation has started in the framework of the constrained scheme for 3+1 Einstein equations (Dirac gauge)
  - [Bonazzola, Gourgoulhon, Grandclément & Novak, PRD **70**, 104007 (2004)]

Numerical implementation

Numerical code based on the C++ library LORENE (http://www.lorene.obspm.fr) with the following main features:

- multidomain spectral methods based on spherical coordinates (r, θ, φ), with compactified external domain ($\Longrightarrow$ spatial infinity included in the computational domain for elliptic equations)
- very efficient outgoing-wave boundary conditions, ensuring that all modes with spherical harmonics indices $\ell = 0$, $\ell = 1$ and $\ell = 2$ are perfectly outgoing [Novak & Bonazzola, J. Comp. Phys. **197**, 186 (2004)]
  (*recall:* Sommerfeld boundary condition works only for $\ell = 0$, which is too low for gravitational waves)

Results on a pure gravitational wave spacetime



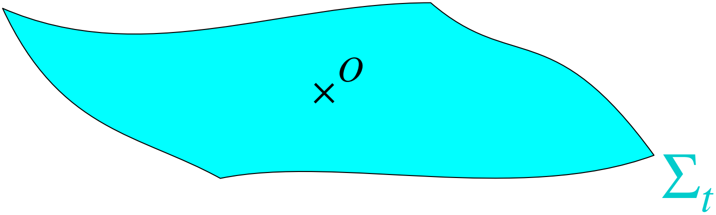
Evolution of $h^{\phi\phi}$ in the plane $\theta = \frac{\pi}{2}$

An alternative approach: the "punctures"

Approach adopted by two American groups:

- *NASA Goddard*
  [Baker, Centrella, Choi, Koppitz & van Meter, PRD **73**, 104002 (2006)], [PRL **96**, 111102 (2006)]
- *Univ. Texas at Brownsville*
  [Campanelli, Lousto, Marronetti & Zlochower, PRL **96**, 111101 (2006)]

Excise only a point O (or two points for a binary system) from the computational domain



O is called a **puncture**. Quantities are diverging at O, but in a finite difference scheme, one may arrange the computational grid in such a way that O never coincide with a grid point...

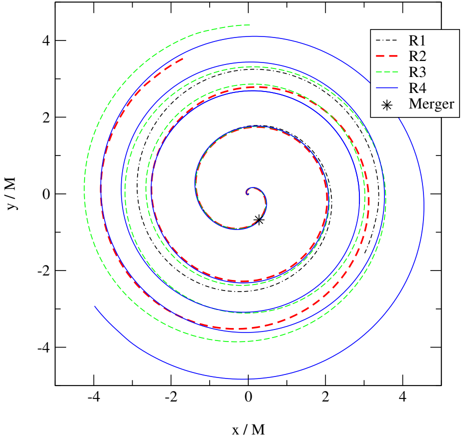
Results from the NASA group

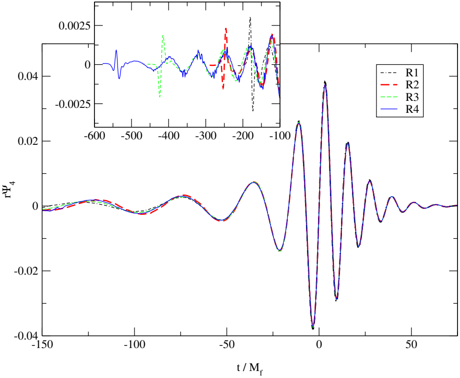
"The largest astrophysical calculations ever performed on a NASA supercomputer" (NASA press release, 18 April 2006)

Columbia supercomputer at NASA's Ames Research Center near Mountain View, California: SGI Altix, 10240 processors Itanium2, Linux, 50 TFlops

Fourth fastest supercomputer in the world (no. 4 in Top500)

Equal mass binary black hole merger





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