Black holes and gravitational waves: general relativity at work

Eric Gourgoulhon

Laboratoire Univers et Théories (LUTH)
CNRS / Observatoire de Paris / Université Paris Diderot
F-92190 Meudon, France

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eric.gourgoulhon@obspm.fr

http://www.luth.obspm.fr/~luthier/gourgoulhon/

Faculty of Science, University of the Ryukyus
Nishihara, Okinawa, Japan
11 August 2008

Plan

1. Introduction
2. Relativistic spacetime
3. Generation and detection of gravitational waves
4. Black holes and gravitational waves
5. Conclusions

Outline

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Road map of Pisa neighbourhood (Italy)...

VIRGO: a giant interferometer...

Gravitational wave detector VIRGO in Cascina, near Pisa (Italy) [CNRS/INFN]

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Gravitational waves: spacetime vibrations

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- front / behind,
- left / right,
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$\Longrightarrow$ 3 numbers (x, y, z) *(coordinates)* to specify the position of a point in space

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$\Longrightarrow$ 1 single number t *(date)* to localise an event in time

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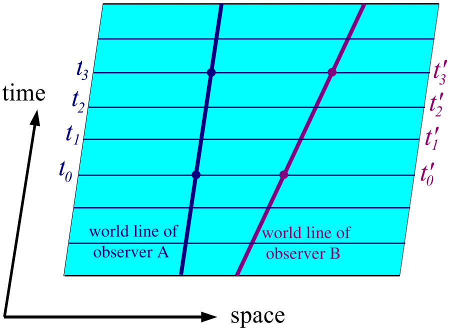
Space and time can be unified in a mathematical continuum with *four dimensions*: spacetime

Concept developed by Henri Poincaré, Herman Minkowski and Albert Einstein at the beginning of the 20th Century

Spacetime



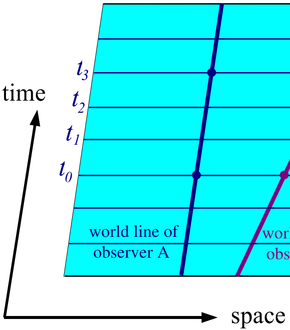
Newtonian spacetime:

math. description: affine space $\mathbb{R}^4$

absolute structure: universal time

All observers measure the same time

Spacetime



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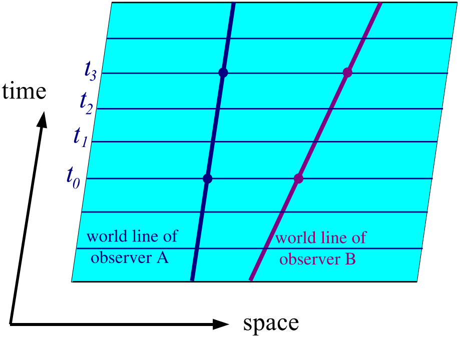

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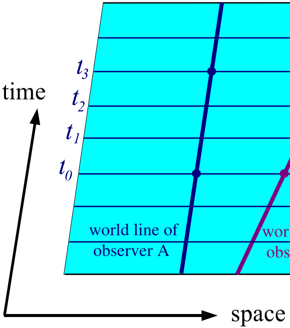

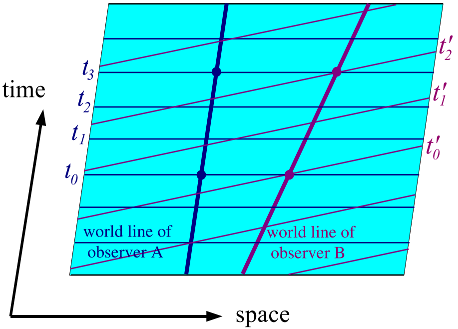

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$\Longrightarrow$ simultaneity is relative
$\Longrightarrow$ *time dilation* phenomenon

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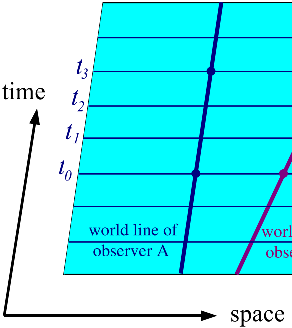

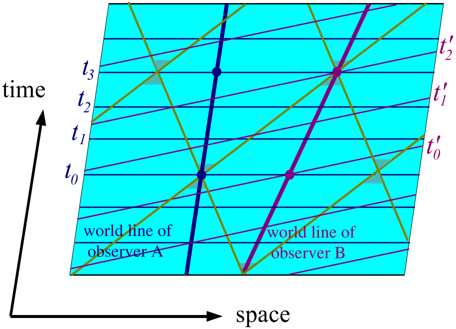

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Special relativity spacetime:

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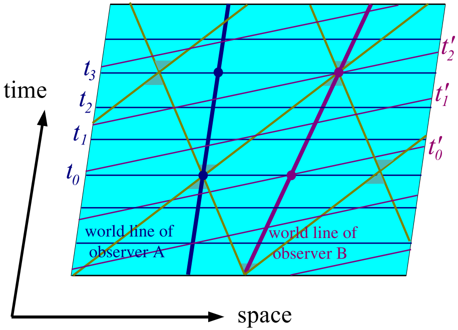
absolute structure: light cones

$\Longrightarrow$ simultaneity is relative

$\Longrightarrow$ *time dilation* phenomenon

Relativistic spacetime



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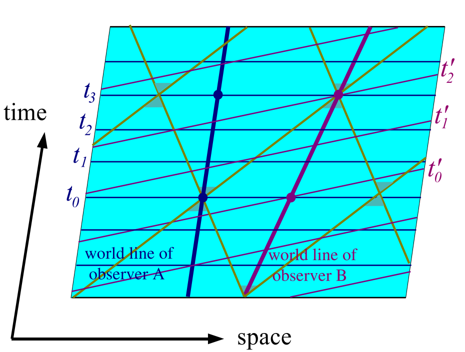

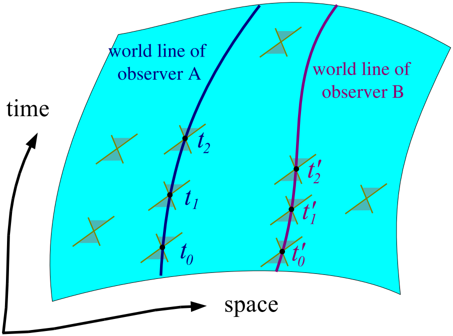

Special relativity spacetime:
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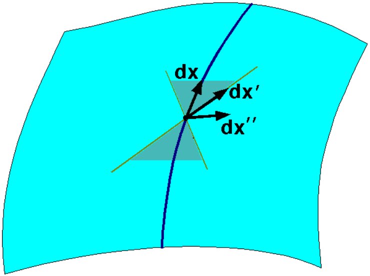
- not satisfactory for **gravitation**

General relativity spacetime:
math. description: 4-dimensional curved space (manifold)

- OK for **electromagnetism**
- OK for **gravitation**

The metric tensor

Algebraic translation of the absolute structure provided by the light cones:



metric tensor $\boldsymbol{g}$

$\boldsymbol{g}$ = symmetric bilinear form of signature $(-,+,+,+)$ such that:

- proper time $d\tau$ for a displacement $d\vec{\boldsymbol{x}}$: $c^2 d\tau^2 = -\boldsymbol{g}(d\vec{\boldsymbol{x}}, d\vec{\boldsymbol{x}}) = -d\vec{\boldsymbol{x}} \cdot d\vec{\boldsymbol{x}}$
- along a light cone : $\boldsymbol{g}(d\vec{\boldsymbol{x}'}, d\vec{\boldsymbol{x}'}) = 0$
- proper distance along a displacement $d\vec{\boldsymbol{x}''}$: $dl^2 = \boldsymbol{g}(d\vec{\boldsymbol{x}''}, d\vec{\boldsymbol{x}''})$

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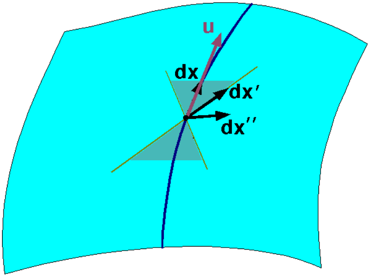

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- along a light cone : $\boldsymbol{g}(d\vec{x}', d\vec{x}') = 0$
- proper distance along a displacement $d\vec{x}''$:
  $dl^2 = \boldsymbol{g}(d\vec{x}'', d\vec{x}'')$

4-velocity along a world line: $\vec{u} = \dfrac{d\vec{x}}{d\tau}$

NB: $\boldsymbol{g}(\vec{u}, \vec{u}) = -c^2$

Spacetime dynamics

- Special relativity: metric tensor $\boldsymbol{g}$ = fixed bilinear form on the vector space $\sim \mathbb{R}^4$ associated with the spacetime affine space
- General relativity: metric tensor $\boldsymbol{g}$ = field of bilinear forms: $\boldsymbol{g} = \boldsymbol{g}(p)$

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Einstein equation : $\boxed{\boldsymbol{R} - \frac{1}{2} R\, \boldsymbol{g} = \frac{8\pi G}{c^4} \boldsymbol{T}}$

- $\boldsymbol{R}$ = Ricci tensor = symmetric bilinear form = trace of *curvature tensor* (Riemann tensor) : "$\boldsymbol{R} \sim \boldsymbol{g}\, \partial^2 \boldsymbol{g} + \boldsymbol{g}\, \partial \boldsymbol{g}\, \partial \boldsymbol{g}$"
- $R = \mathrm{Trace}(\boldsymbol{R})$
- $\boldsymbol{T}$ = *energy-momentum tensor* of matter = symmetric bilinear form such that

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- $\boldsymbol{T}$ = *energy-momentum tensor* of matter = symmetric bilinear form such that
  - $E = \boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{u}})$ is the energy density of matter as measured by an observer $\mathcal{O}$ of 4-velocity $\vec{\boldsymbol{u}}$
  - $p_i = -\boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{e}}_i)$ component i of the matter momentum density as measured by $\mathcal{O}$ in the direction $\vec{\boldsymbol{e}}_i$
  - $S_{ij} = \boldsymbol{T}(\vec{\boldsymbol{e}}_i, \vec{\boldsymbol{e}}_j)$ component i of the force exerted by matter on the unit surface normal to $\vec{\boldsymbol{e}}_j$

Comparing Newtonian and relativistic gravitation theories

Newtonian gravitation:

fundamental equation: Poisson equation for the gravitational potential Φ:

$$\Delta\Phi = 4\pi G\rho$$

- scalar equation
- linear equation
- elliptic equation
  ($\Rightarrow$ instantaneous propagation)
- only source: mass density ρ

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Relativistic gravitation:

fundamental equation: Einstein equation for the metric tensor $\boldsymbol{g}$:

$$\boldsymbol{R}(\boldsymbol{g}) - \frac{1}{2}R(\boldsymbol{g})\,\boldsymbol{g} = \frac{8\pi G}{c^4}\,\boldsymbol{T}$$

- tensorial equation (10 scalar equations)
- non-linear equation
- propagation at finite speed (c)
- source: energy-momentum of matter and electromagnetic field

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Remark: for a weak gravitational field, one of the 10 components of Einstein equation reduces to the Poisson equation (and the other 9 reduced to $0 = 0$).

What is a strong gravitational field ?

Relativity parameter or compacity parameter of a self-gravitating body of mass M and mean radius R:

$$\Xi = \frac{GM}{c^2 R} \sim \frac{|E_{\rm grav}|}{Mc^2} \sim \frac{|\Phi_{\rm surf}|}{c^2} \sim \frac{v_{\rm esc}^2}{c^2}$$

- $E_{\rm grav}$: gravitational potential energy[1]
- $\Phi_{\rm surf}$: gravitational potential at the surface of the body
- $v_{\rm esc}$: escape velocity from the body's surface[2]

	Earth	Sun	white dwarf	neutron star	black hole
Ξ	10^{-10}	10^{-6}	10^{-3}	0.2	1

if $\Xi \gtrsim 0.1$, general relativity must be employed to describe the body (**compact object**)

[1] for a homogeneous ball: $E_{\rm grav} = -\frac{3}{5}\frac{GM^2}{R}$

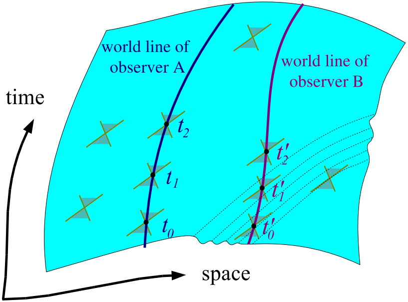
[2] for a spherically symmetric body: $v_{\rm esc} = \sqrt{\frac{2GM}{R}}$

Gravitational waves

Linearization of Einstein equation in weak field: $\boldsymbol{g} = \boldsymbol{\eta} + \boldsymbol{h}$,
$\boldsymbol{\eta}$ = Minkowski metric[3]

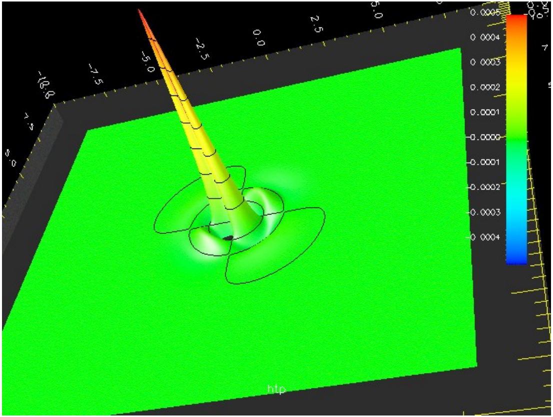
$$\Longrightarrow \text{wave equation: } \boxed{\Box \bar{\boldsymbol{h}} = -\frac{16\pi G}{c^4}\boldsymbol{T}} \quad \text{(Lorenz gauge)}$$

with $\Box = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\bar{\boldsymbol{h}} = \boldsymbol{h} - \frac{1}{2} h\, \boldsymbol{\eta}$ and $h = \mathrm{Trace}(\boldsymbol{h})$.



[3] $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ en Cartesian coordinates

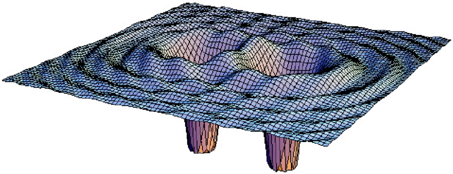
Numerical integration of Einstein equation



Numerical code constructed on the library
Langage Objet pour la RElativité NumériquE (LORENE)
(http://www.lorene.obspm.fr)

[Bonazzola, Gourgoulhon, Grandclément & Novak, Phys. Rev. D **70**, 104007 (2004)]

Gravitational waves



Bi-dimensional spacelike section of a spacetime generated by a binary system of black holes

gravitational waves = perturbations in spacetime curvature

- reveal the **dynamics** of spacetime
- are generated by acceleration of matter
- far from the sources, propagate with the velocity of light
- NB: **electromagnetic waves** (radio waves, IR, optical, UV, X and gamma) are perturbations of the electromagnetic field which propagate *within* spacetime, whereas **gravitational waves** are waves of spacetime *itself*

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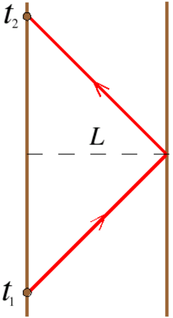
Measurable effects of a gravitational wave passage

Measure of the distance L between two free masses by a "radar" method:

$$L = \frac{1}{2}\, c(t_2 - t_1)$$



Variation of length L when a gravitational wave passes by:

$$\boxed{\delta L \simeq h\, L}$$

h = amplitude of the gravitational wave

In practice, h is so small that our senses are not sensitive to it:
for the most important **astrophysical sources:** $h \sim 10^{-21}$!!!

Generation of gravitational waves in the lab

In the 19th Century, Hertz has demonstrated the existence of electromagnetic waves by producing them in his laboratory.
Is the same experiment possible for gravitational waves ?

- electromagnetic waves: produced by the acceleration of *electric charges*
- gravitational waves: produced by the acceleration of *masses*

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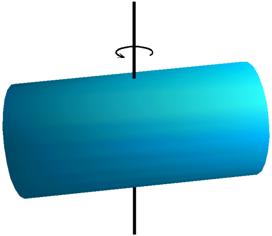
A simple mean of providing a constant acceleration to some material: make it *rotate*



Steel cylinder: diameter = 1 m, length = 20 m, mass = 490 t, rotating at 28 rad/s (break-up limit)
$\Longrightarrow$ emitted energy in gravitational waves per unit of time: 2×10^{-29} W !

$\Longrightarrow$ No hope of detection !

Generation of gravitational waves by astrophysical sources

Emitted energy per unit of time in the form of gravitational waves:

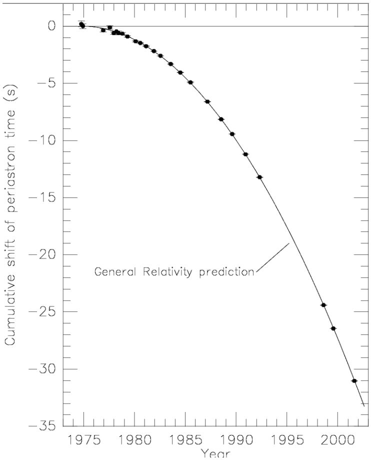
$$\boxed{\mathcal{L} \sim \frac{c^5}{G} s^2 \left(\frac{v}{c}\right)^6 \boldsymbol{\Xi}^2}$$

gravitational luminosity

- G : Newton's constant $\rightarrow$ *gravitation*
- c : velocity of light $\rightarrow$ *relativity*
- s : asymmetry factor: $s = 0$ if spherical symmetry
- v : characteristic speed of motions inside the source
- $\boldsymbol{\Xi}$: *compacity parameter* reminder or *relativity parameter*

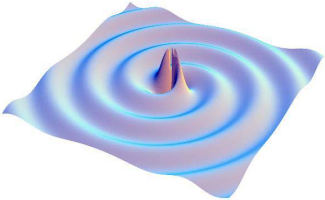
Only compact objects are good emitters of gravitational waves

Gravitational waves do exist !



[Weisber & Taylor (2002)]

Emission of gravitational waves by the neutron star binary system PSR B1913+16 (*binary pulsar*)



← Observed decay of the orbital period $P = 7\,\mathrm{h}\,45\,\mathrm{min}$ of the binary pulsar PSR B1913+16 produced by the *reaction to gravitational radiation*

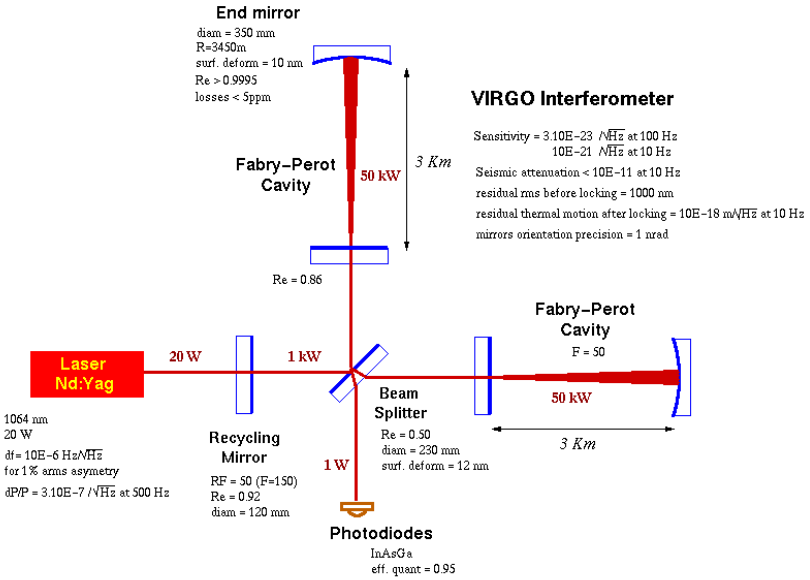
$\Longrightarrow$ coalescence in 140 millions year.

Nobel Prize in Physics to R. Hulse & J. Taylor (1993)

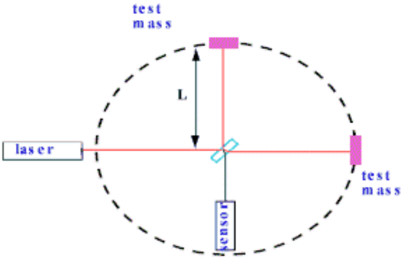
Detection of gravitational waves on Earth

Interferometer detector VIRGO in Cascina, near Pisa (Italy) [CNRS/INFN]

Optical scheme of the VIRGO interferometer



Response of the interferometer to a gravitational wave



Super-attenuators

Mirror suspension device: chain of 7 pendula (horizontal seismic insulation), the elements of which are constituted by oscillating triangular blades (vertical seismic insulation)

[CNRS/INFN]

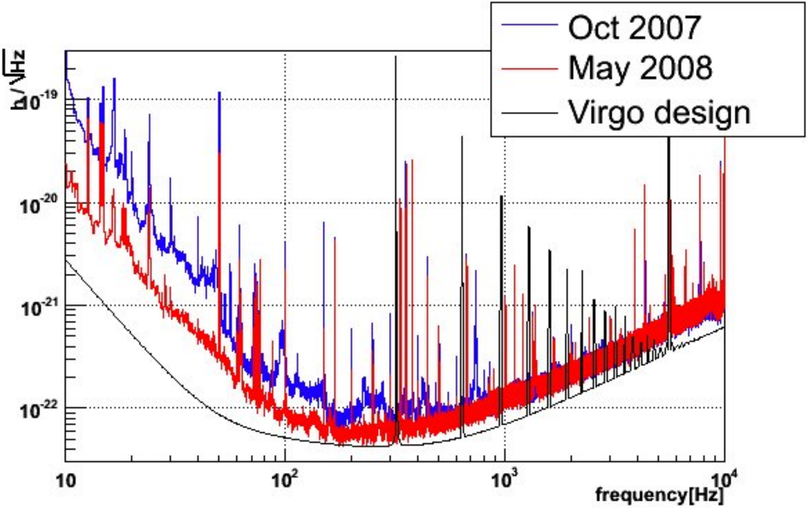
Working principle

The detector is tuned on the dark fringe; a gravitational wave (represented by its two modes of polarization $(h_+(t), h_\times(t))$) induces the following variation of the arm lengths:

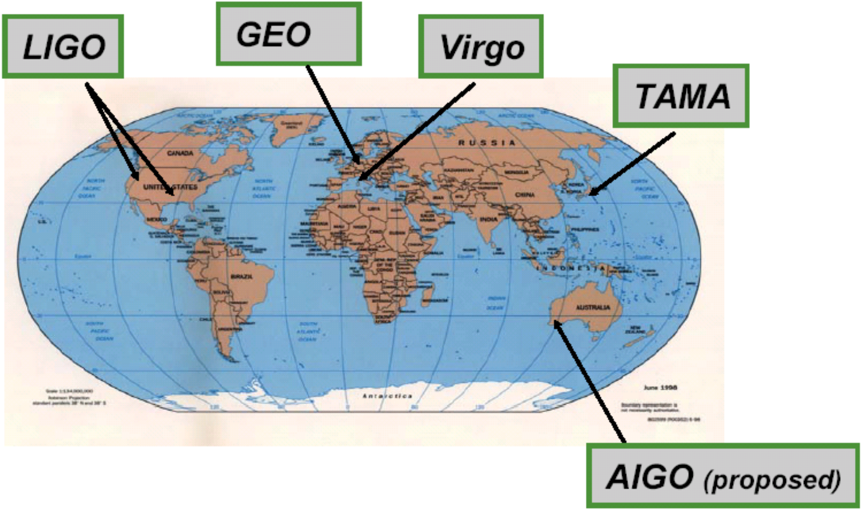
$$\frac{\delta L}{L} =: h(t) = F_+(\theta, \phi, \psi)\, h_+(t) + F_\times(\theta, \phi, \psi)\, h_\times(t). \tag{1}$$

This δL generates a change in the interferometric pattern and therefore some luminous signal in the photodiode which observes the dark fringe.
In Equation (1), F_+ and $F_\times$ are the two response functions of the detector; they depend on the arrival direction (θ, ϕ) of the wave with respect to the detector's arms and of the polarization angle ψ of the wave with respect to the detector orientation

VIRGO sensitivity curve



Other interferometric detectors operating in the world

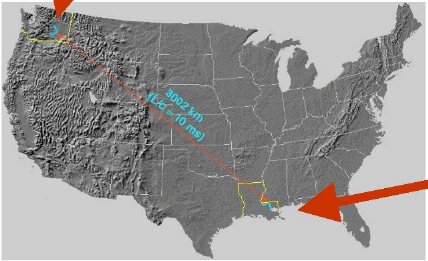


TAMA interferometer

TAMA-300 on the Mitaka campus of National Astronomical Observatory in Tokyo

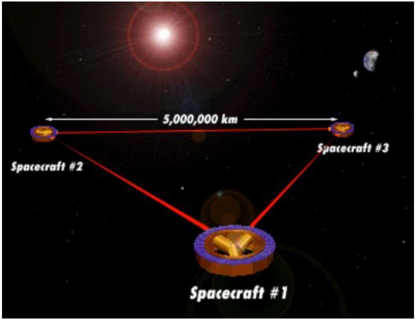
LIGO interferometers

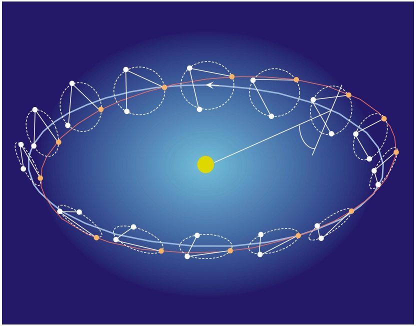
Hanford (H1=4km, H2=2km)



Livingston (L1=4km)

Space project LISA (ESA/NASA)





Frequency bandwith: 0.1 mHz $\rightarrow$ 0.1 Hz

LISA Pathfinder: end of 2009

LISA launch $\sim$ 2018

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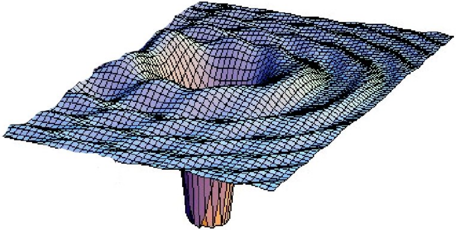
Black holes and gravitational waves



Link between black holes and gravitational waves:

Black holes and gravitational waves are both spacetime distortions:

- extreme distortions (black holes)
- small distortions (gravitational waves)

In particular, black holes and gravitational waves are both vacuum solutions of general relativity equations (Einstein equations)

Black hole concept

A black hole is a region of spacetime *causally disconnected* from the rest of the Universe, i.e. no lightlike geodesic (photon trajectory) can escape from it. The (immaterial) boundary which separates the black from the rest of the universe is called the event horizon.

The gravitational field, particularly intense, is responsible for this behavior. Black holes are thus the more compact among the compact objects. General relativity is mandatory to describe them. reminder

← Computer generated image of a black surrounded by a disk of matter (accretion disk) [Jean-Alain Marck]

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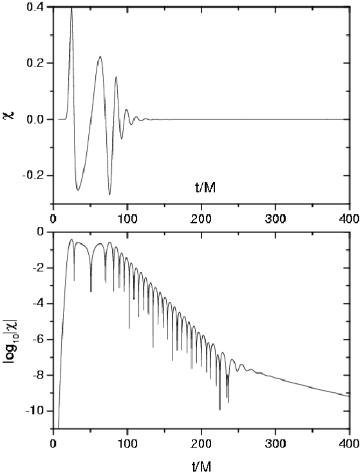
Uniqueness theorem (B. Carter, S. Hawking, W. Israel): A black hole *in equilibrium* is entirely described by only two parameters: its mass M and its angular momentum J $\Longrightarrow$ "a black hole has no hair"

A black hole out of equilibrium "loses its hair" via the emission of gravitational waves

Black hole oscillations



[Kokkotas & Schmidt, Liv. Rev. Relat. **2**, 2 (1999)]

Black holes out of equilibrium:

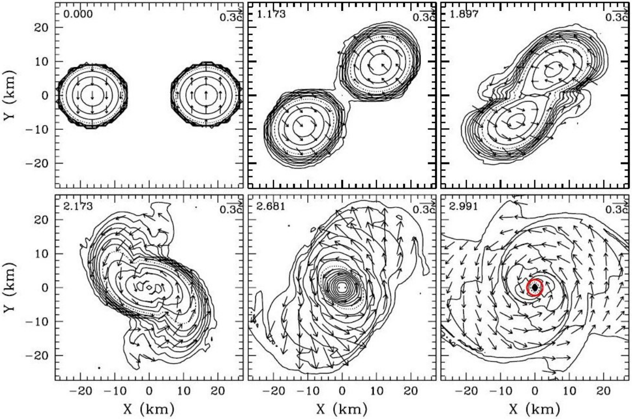
- newly born black hole: *supernova core, coalescing binary compact objects*
- black hole excited by some *fall of matter* (star or plasma accretion *(microquasars)*)

De-excitation by emission of gravitational waves under the form of quasi-normal modes.
The detection of these gravitational waves would permit the direct measure of the black hole's mass M and angular momentum J

$$M = 10\,M_\odot \Rightarrow \left\{ \begin{array}{lcl} f & = & 1.2\ \mathrm{kHz} \\ \tau & = & 0.55\ \mathrm{ms} \end{array} \right. \quad \text{(VIRGO)}$$

$$M = 10^6\,M_\odot \Rightarrow \left\{ \begin{array}{lcl} f & = & 12\ \mathrm{mHz} \\ \tau & = & 55\ \mathrm{s} \end{array} \right. \quad \text{(LISA)}$$

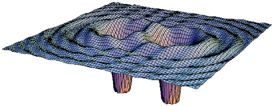
Formation of a black hole by the merger of two neutron stars



[Shibata, Taniguchi & Uryu, Phys. Rev. D **71**, 084021 (2005)]

$$M_1 = M_2 = 1.4\ M_{\odot},\ \text{EOS: SLy}$$

Coalescence of a black hole binary system



Detection: the expected main source for LIGO and VIRGO

Theory:

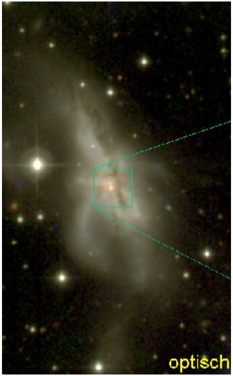
- binary black hole: the two-body problem par excellence of general relativity
- test of general relativity in a dynamical regime and in strong field

Astrophysics:

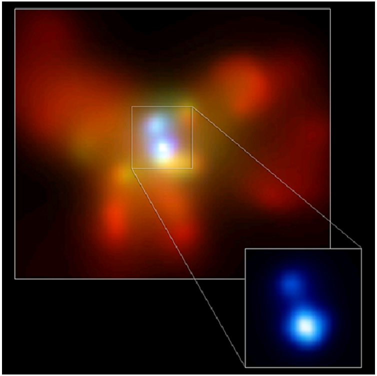
- coalescence rate $\Longrightarrow$ hints on massive star evolution
- gravitational signal from the inspiralling phase $\Longrightarrow$ measure of the Hubble constant H_0
- observations of supermassive black holes at high redshift $\Longrightarrow$ hints on the formation of galaxies in the primordial universe

Binary black hole systems in the Universe

Double nucleus of galaxy NGC 6240



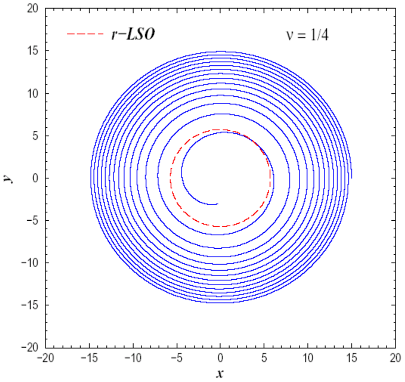
optical



X (Chandra satellite)

[Komossa et al., ApJ **582**, L15 (2003)]

Evolution of a black hole binary



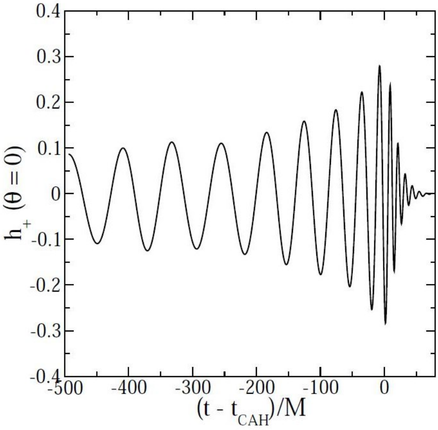
Inspiralling orbit
[Buonanno & Damour, Phys. Rev. D **62**, 064015 (2000)]]

Contrary to Newtonian gravity, there does not exist any stationary solution for the two-body problem in general relativity: the loss of energy and angular momentum by gravitational emission leads to the shrinking of the orbits

reminder

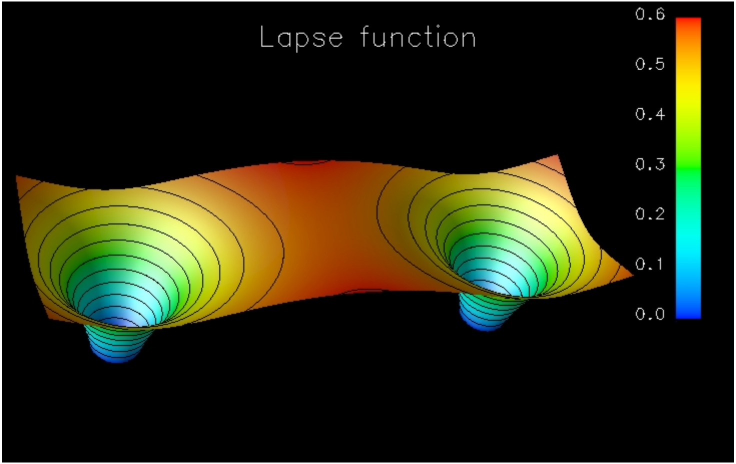
Another effect of the emission of gravitational waves:
circularization of the orbits: $e \to 0$

Emitted gravitational waves



[Pretorius, arXiv:0710.1338 (2007)]

Binary black hole system in the last orbits

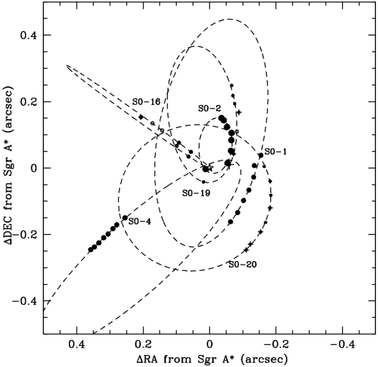


[Grandclément, Gourgoulhon, Bonazzola, Phys. Rev. D **65**, 044021 (2002)]

Detection rates of coalescing binary black holes

- Stellar black holes ($2 \times 10\, M_\odot$):
  - First generation of detectors (LIGO, VIRGO):
    - detectability distance: $d_{\rm max} \simeq 100$ Mpc
    - detection rate: ~ 1 per year
  - Second generation:
    - detectability distance: $d_{\rm max} \simeq 1$ Gpc
    - detection rate: ~ 1 per day
- Supermassive black holes ($2 \times 10^6\, M_\odot$):
  - LISA:
    - detectability distance: $d_{\rm max} >$ Hubble radius
    - detection rate: ~ 1 to 1000 per year

Capture of a star by a supermassive black hole



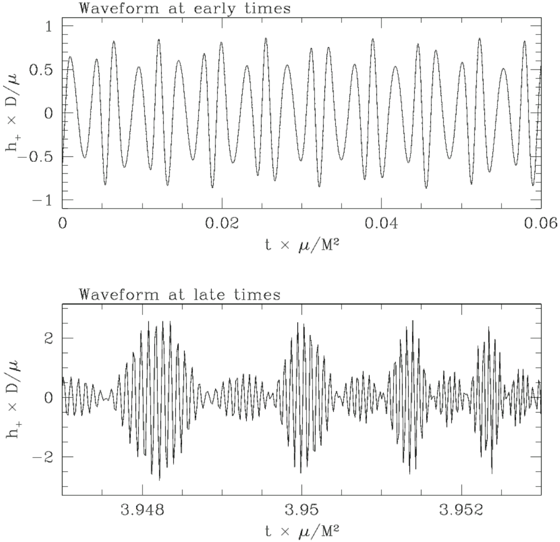
Orbits of stars around the black hole Sgr A* in the center of our galaxy [Ghez (2004)]

Capture of a compact object of stellar mass (neutron star or black hole) by a supermassive black hole in the nucleus of a galaxy

The orbit shrinks (reaction to the emission of gravitational waves) until the last stable orbit is reached $\Longrightarrow$ plunge and absorption by the central black hole

The emitted gravitational waves are in the bandwidth of LISA

Emitted gravitational waves



h_+ mode of gravitational waves emitted during the inspiral of a compact object around a rapidly rotating supermassive black hole ($J = 0.998\, M^2$)

[Hughes, Phys. Rev. D **64**, 064004 (2001)]

Final proof of existence of black holes

Inspiral of a stellar black hole $m = 5\,M_\odot$ into a rapidly rotating black hole $M = 10^6\,M_\odot$:

- Elapsed time between the orbit of radius $r = 8M$ and the last stable orbit: ~ 1 year
- Number of gravitational wave cycles: 10^5
- Frequency bandwidth swept by the gravitational signal: 3 mHz $\leq f \leq$ 30 mHz
- Detectability distance by **LISA** (signal-to-noise ratio $>$ 10): ~ 1 Gpc

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Measure of a large number of cycles $\Rightarrow$ Detailed cartography of spacetime around the central object
Comparison with the spacetime generated by a rotating black hole (Kerr solution) $\Rightarrow$ definite proof of the existence of black holes in our Universe
Expected detection rate by LISA: 1 to 10 per year until 1 Gpc.

Outline

1 Introduction

2 Relativistic spacetime

3 Generation and detection of gravitational waves

4 Black holes and gravitational waves

5 Conclusions

Conclusions

- Although they propagate at the same speed (c), gravitational waves are very different from electromagnetic waves (photons); they are waves of the spacetime structure, generated by the mass-energy motions.
- They are emitted mostly by the most relativistic objects in the Universe (compact objects): black holes, neutron stars.
- Contrary to electromagnetic waves, they interact very weakly with matter:
  - *advantage:* they can emerge from very dense regions, opaque to photons and neutrinos, like the cores of supernovæ, and cross the interstellar medium without any absorption;
  - *drawback:* they are difficult to detect !
- The proof of their existence has been obtained (binary pulsar), but they have not been detected on Earth yet.
- Large interferometric detectors (GEO600, LIGO, TAMA and VIRGO) are actually acquiring data; they should provide the first detection soon, opening the era of **gravitational astronomy**.