Constraints on dense matter equation of state from gravitational wave astrophysics

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Plan

1. A short introduction to gravitational waves
2. Gravitational signal from binary neutron stars
3. Gravitational signal from black hole-neutron star binaries
4. Other types of gravitational radiation from neutron stars

Outline

1. A short introduction to gravitational waves
2. Gravitational signal from binary neutron stars
3. Gravitational signal from black hole-neutron star binaries
4. Other types of gravitational radiation from neutron stars

Spacetime dynamics

- Special relativity : metric tensor $\boldsymbol{g}$ = fixed bilinear form on the vector space $\sim \mathbb{R}^4$ associated with the spacetime affine space
- General relativity : metric tensor $\boldsymbol{g}$ = field of bilinear forms : $\boldsymbol{g} = \boldsymbol{g}(p)$

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Einstein equation : $$\boldsymbol{R} - \frac{1}{2} R\, \boldsymbol{g} = \frac{8\pi G}{c^4} \boldsymbol{T}$$

- $\boldsymbol{R}$ = Ricci tensor = symmetric bilinear form = trace of *curvature tensor* (Riemann tensor) : "$\boldsymbol{R} \sim \boldsymbol{g}\, \partial^2 \boldsymbol{g} + \boldsymbol{g}\, \partial \boldsymbol{g}\, \partial \boldsymbol{g}$"
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  - $E = \boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{u}})$ is the energy density of matter as measured by an observer $\mathcal{O}$ of 4-velocity $\vec{\boldsymbol{u}}$
  - $p_i = -\boldsymbol{T}(\vec{\boldsymbol{u}}, \vec{\boldsymbol{e}}_i)$ component i of the matter momentum density as measured by $\mathcal{O}$ in the direction $\vec{\boldsymbol{e}}_i$
  - $S_{ij} = \boldsymbol{T}(\vec{\boldsymbol{e}}_i, \vec{\boldsymbol{e}}_j)$ component i of the force exerted by matter on the unit surface normal to $\vec{\boldsymbol{e}}_j$

Comparing Newtonian and relativistic gravitation theories

Newtonian gravitation :

fundamental equation : Poisson equation for the gravitational potential Φ :

$$\Delta\Phi = 4\pi G\rho$$

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- only source : mass density ρ

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- non-linear equation
- propagation at finite speed (c)
- source : energy-momentum of matter and electromagnetic field

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Remark : for a weak gravitational field, one of the 10 components of Einstein equation reduces to the Poisson equation (and the other 9 reduced to $0 = 0$).

What is a strong gravitational field ?

Relativity parameter or compacity parameter of a self-gravitating body of mass M and mean radius R :

$$\Xi = \frac{GM}{c^2 R} \sim \frac{|E_{\rm grav}|}{Mc^2} \sim \frac{|\Phi_{\rm surf}|}{c^2} \sim \frac{v_{\rm esc}^2}{c^2}$$

- $E_{\rm grav}$: gravitational potential energy[1]
- $\Phi_{\rm surf}$: gravitational potential at the surface of the body
- $v_{\rm esc}$: escape velocity from the body's surface[2]

	Earth	Sun	white dwarf	neutron star	black hole
Ξ	10^{-10}	10^{-6}	10^{-3}	0.2	1

if $\Xi \gtrsim 0.1$, general relativity must be employed to describe the body (**compact object**)

[1] for a homogeneous ball : $E_{\rm grav} = -\frac{3}{5}\frac{GM^2}{R}$

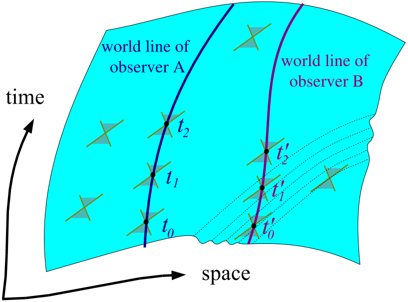
[2] for a spherically symmetric body : $v_{\rm esc} = \sqrt{\frac{2GM}{R}}$

Gravitational waves

Linearization of Einstein equation in weak field : $\boldsymbol{g} = \boldsymbol{\eta} + \boldsymbol{h}$,
$\boldsymbol{\eta}$ = Minkowski metric[3]

$$\Longrightarrow \text{wave equation : } \boxed{\square\, \overline{\boldsymbol{h}} = -\frac{16\pi G}{c^4}\boldsymbol{T}} \qquad \text{(Lorenz gauge)}$$

with $\square = -\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\overline{\boldsymbol{h}} = \boldsymbol{h} - \frac{1}{2} h\, \boldsymbol{\eta}$ and $h = \mathrm{Trace}(\boldsymbol{h})$.



[3] $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$ en Cartesian coordinates

Gravitational wave emission

- For a weakly relativistic source : quadrupole formula :

$$h_{ij}^{\rm TT}(t,\vec{\boldsymbol{x}}) = \frac{2G}{c^4 r}\left[P_i^{\ k}P_j^{\ l} - \frac{1}{2}P_{ij}P^{kl}\right]\ddot{Q}_{ij}\left(t-\frac{r}{c}\right)$$

  - r : distance to the source
  - $P_{ij} = \delta_{ij} - x^i x^j/r^2$: transverse projector
  - $Q_{ij}(t) := \int_{\rm source} \rho(t,\vec{\boldsymbol{x}})\left(x^i x^j - \frac{1}{3}\vec{\boldsymbol{x}}\cdot\vec{\boldsymbol{x}}\,\delta_{ij}\right) d^3\vec{\boldsymbol{x}}$: mass quadrupole

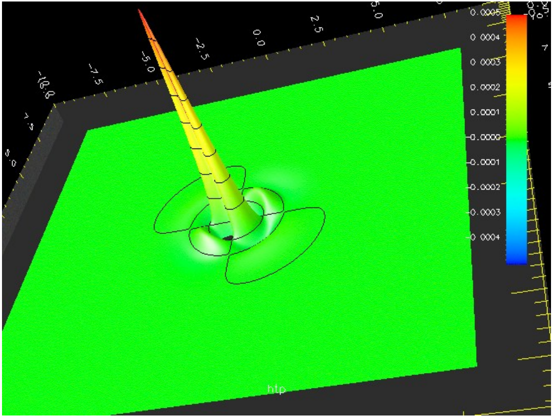
- GW luminosity :

$$L \sim \frac{c^5}{G}\, s^2\, \boldsymbol{\Xi}^2 \left(\frac{v}{c}\right)^6$$

  - s : asymmetry factor ($s=0$ fpr spherical symmetry)
  - $\boldsymbol{\Xi} := GM/(c^2 R)$: compacity parameter
  - v : characteristic velocity of matter in the source

NB : $c^5/G \simeq 4\ 10^{52}$ W !

Numerical integration of Einstein equation



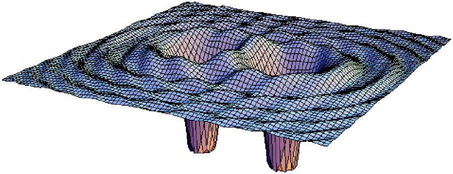
Numerical code constructed on the library

Langage Objet pour la RElativité NumériquE (LORENE)

(`http ://www.lorene.obspm.fr`)

[Bonazzola, Gourgoulhon, Grandclément & Novak, Phys. Rev. D **70**, 104007 (2004)]

Gravitational waves



Bi-dimensional spacelike section of a spacetime generated by a binary system of black holes

gravitational waves = perturbations in spacetime curvature

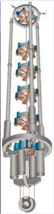
- reveal the **dynamics** of spacetime
- are generated by acceleration of matter
- far from the sources, propagate with the velocity of light
- NB : **electromagnetic waves** (radio waves, IR, optical, UV, X and gamma) are perturbations of the electromagnetic field which propagate *within* spacetime, whereas **gravitational waves** are waves of spacetime *itself*

Detection of gravitational waves

LIGO : USA, Louisiana

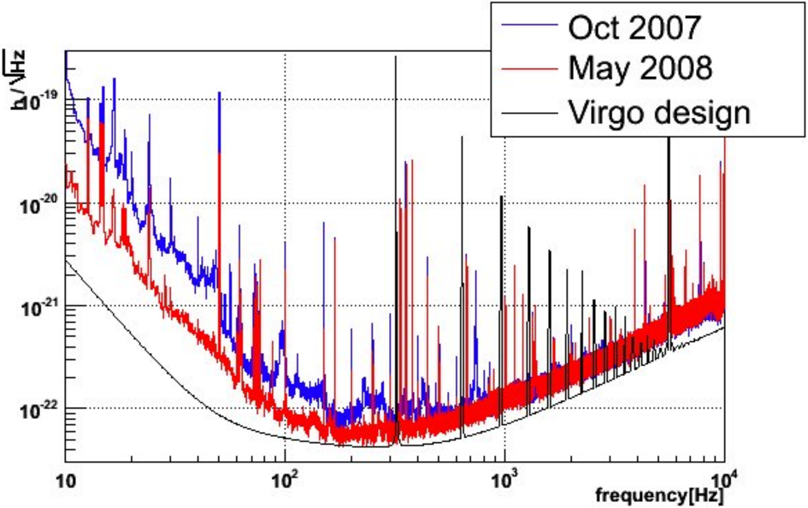
LIGO : USA, Washington

VIRGO : France/Italy (Pisa)



Interferometers VIRGO (3 km) and LIGO (4 km) are currently acquiring data.

VIRGO sensitivity curve



Event rates

Binary coalescences :

		NS-NS	BH-NS	BH-BH
predicted rate$^{(1)}$	$[\mathrm{yr}^{-1} L_{10}^{-1}]$	$5\ 10^{-5}$	$2\ 10^{-6}$	$4\ 10^{-7}$
observed rate$^{(2)}$	$[\mathrm{yr}^{-1} L_{10}^{-1}]$	$< 4\ 10^{-2}$	$< 2\ 10^{-2}$	$< 2\ 10^{-3}$
detection range	LIGO S5$^{(2)}$	30 Mpc	50 Mpc	80 Mpc

$L_{10} = 10^{10} L_\odot$ (blue solar luminosity) ; our galaxy : $\sim 1.7\, L_{10}$

$^{(1)}$ [Kalogera, Belczynski, Kim, O'Shaughnessy & Willems, Phys. Rep. **442**, 75 (2007)]

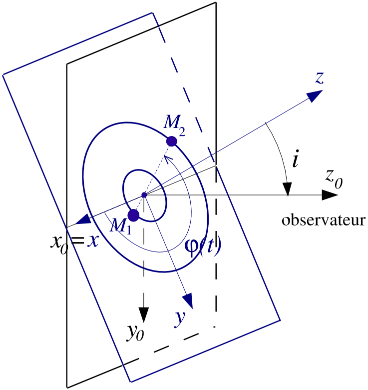
$^{(2)}$ from 1st year of LIGO S5 data, Nov. 2005 - Nov. 2006 [Abbott et al., arXiv :0901.0302 (2009)]

Core collapse supernovae :

rate $\sim 2\ 10^{-2}\ \mathrm{yr}^{-1} L_{10}^{-1}$

Outline

Gravitational radiation from a binary system



Masses : M_1 and M_2

Chirp mass : $\mathcal{M} = \left[\frac{(M_1 M_2)^3}{M_1 + M_2}\right]^{1/5}$

Orbital period : P

Distance to the binary : d

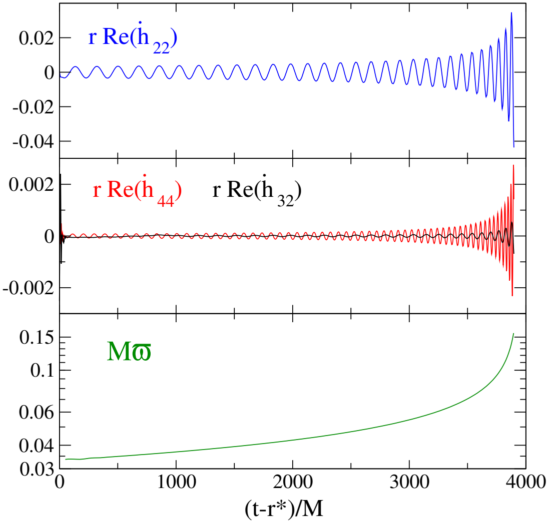
Inclination angle : i

GW for a circular orbit at the 0-PN level (from quadrupole formula) :

$$h_+ = \frac{2}{c^4 d}(G\mathcal{M})^{5/3}\left(\frac{2\pi}{P}\right)^{2/3}(1+\cos^2 i)\ \cos\left(4\pi\frac{t}{P}+\varphi_0\right)$$

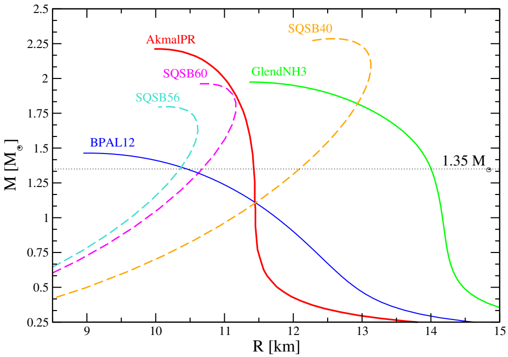
$$h_\times = \frac{4}{c^4 d}(G\mathcal{M})^{5/3}\left(\frac{2\pi}{P}\right)^{2/3}\cos i\ \sin\left(4\pi\frac{t}{P}+\varphi_0\right)$$

Chirp signal



[Boyle et al., PRD **78**, 104020 (2008)]

A panel of different EOS



3 nuclear matter EOS :

- *BPAL12* : phenomenological soft extreme of nucleonic EOS [Bombacci et al. 1995]
- *AkmalPR* : n,p,e,μ with 2-body (Argonne A18) and 3-body (Urbana UIX) nucleon interactions [Akmal, Pandharipande & Ravenhall 1998]
- *GlendNH3* : n,p,e,μ with hyperons for $\rho > 2\rho_{\rm nuc}$ [Glendenning 1985]

3 strange matter EOS :

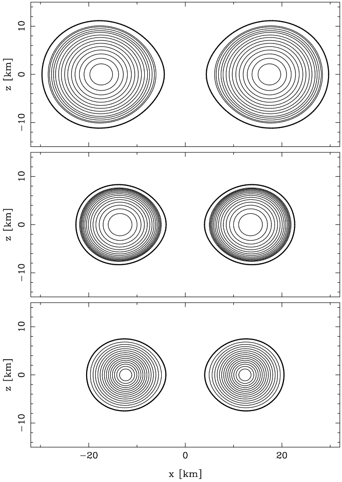
MIT bag model

- *SQSB56* : $m_s c^2 = 200$ MeV, $\alpha = 0.2$, $B = 56$ MeV/fm^3
- *SQSB60* : $m_s c^2 = 0$, $\alpha = 0$, $B = 60$ MeV/fm^3
- *SQSB40* : $m_s c^2 = 100$ MeV, $\alpha = 0.6$, $B = 40$ MeV/fm^3

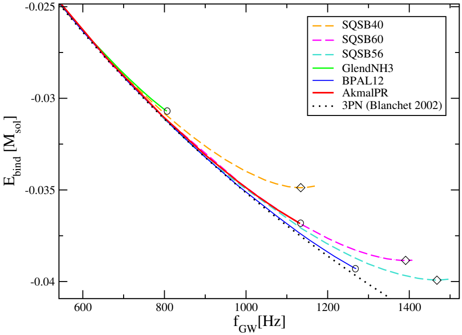
Inspiralling sequences for different EOS

Mass-shedding limit

for $M_1 = M_2 = 1.35\,M_\odot$ and GlendNH3, AkmalPR and BPLA12 EOS :



Binding energy along the sequence :



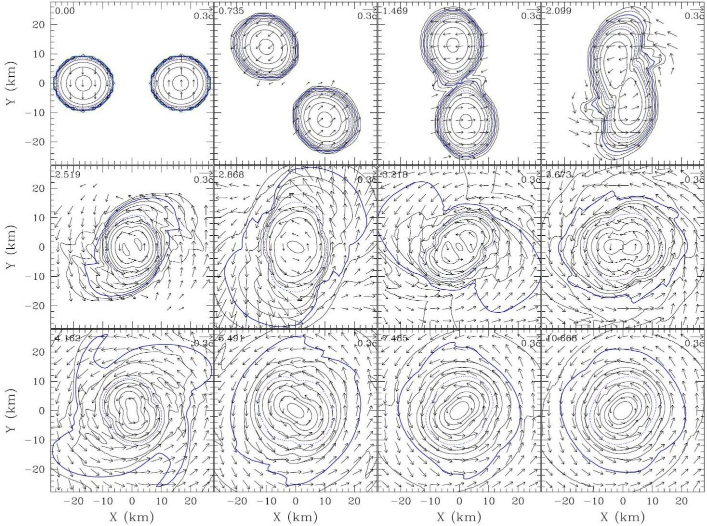
[Bejger, Gondek-Rosińska, Gourgoulhon, Haensel, Taniguchi & Zdunik, A&A **431**, 297 (2005)]

[Limousin, Gondek-Rosińska & Gourgoulhon, PRD **71**, 064012 (2005)]

[Gondek-Rosińska, Bejger, Bulik, Gourgoulhon, Haensel, Limousin, Taniguchi & Zdunik, ASR **39**, 271 (2007)]

The merger

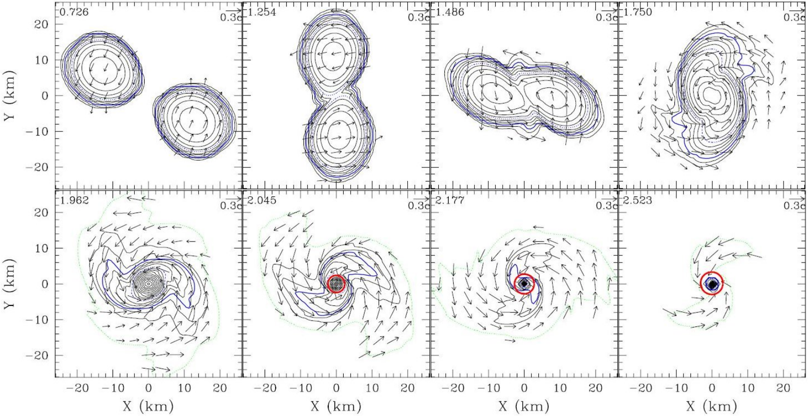
EOS : Akmal, Pandharipande & Ravenhall (1998), $M_1 = M_2 = 1.3\,M_\odot$



[Shibata & Taniguchi, PRD **73**, 064027 (2006)]

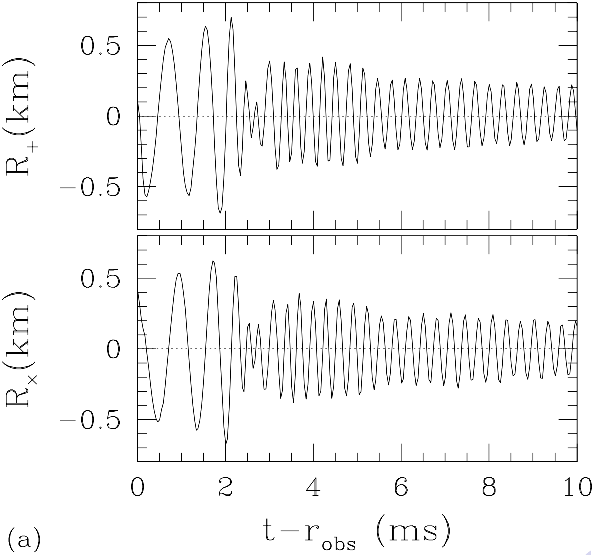
The merger
Larger mass : prompt black hole formation

EOS : Akmal, Pandharipande & Ravenhall (1998), $M_1 = M_2 = 1.5\, M_\odot$



[Shibata & Taniguchi, PRD **73**, 064027 (2006)]

Gravitational wave signal



EOS : APR,

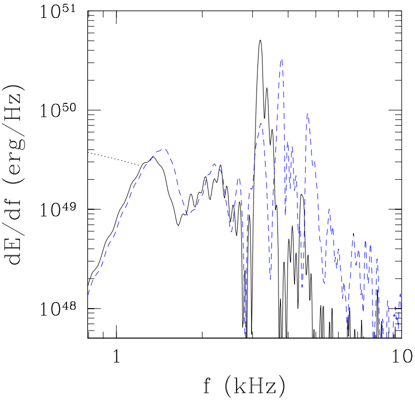
$M_1 = M_2 = 1.3\, M_\odot$

$$h \simeq 10^{-22} R_0 \frac{100 \text{ Mpc}}{d}$$

$$R_0 := \frac{\sqrt{R_+^2 + R_\times^2}}{0.31 \text{ km}}$$

[Shibata & Taniguchi, PRD **73**, 064027 (2006)]

GW Fourier spectrum



← EOS : APR
$M_1 = M_2 = 1.3\, M_\odot$ (solid)
$M_1 = M_2 = 1.4\, M_\odot$ (dashed)
dotted line : 2-PN

$M_{\rm crit}$: total mass for prompt black hole formation
$\exists$ pick at $f \sim 2-3$ kHz $\Rightarrow M_{\rm tot} < M_{\rm crit}$
No pick $\Rightarrow$ prompt BH formation $\Rightarrow$ soft EOS
FPS EOS : $M_{\rm crit} = 2.5\, M_{\rm sol}$
SLy EOS : $M_{\rm crit} = 2.7\, M_{\rm sol}$
APR EOS : $M_{\rm crit} = 2.9\, M_{\rm sol}$
In addition, the frequency of the pick depends on the EOS

[Shibata, PRL **94**, 201101 (2005)]

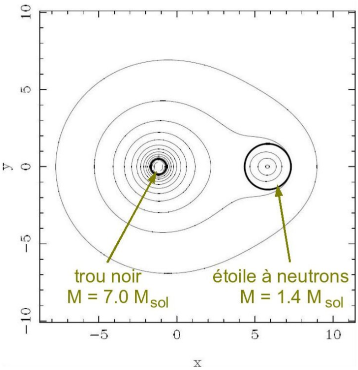
[Shibata & Taniguchi, PRD **73**, 064027 (2006)]

Outline

Black hole-neutron star binaries

The most favorable binary coalescence for VIRGO / LIGO ?

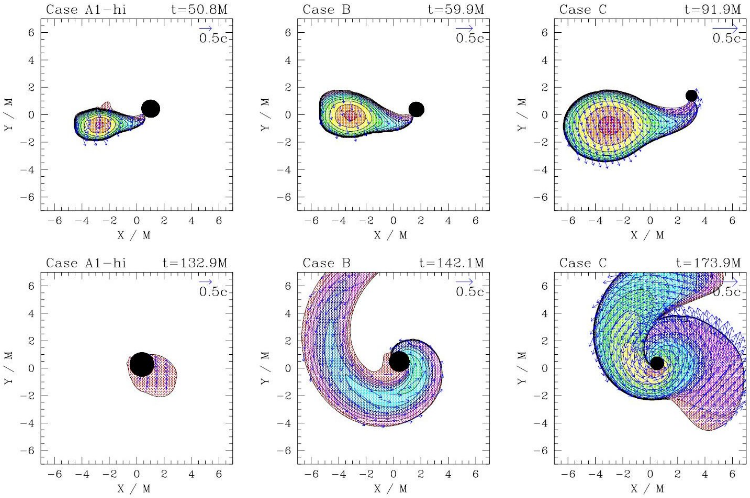
Sources of short gamma-ray burst ?



[Grandclément, PRD **74**, 124002 (2006)]

Black hole-neutron star merger

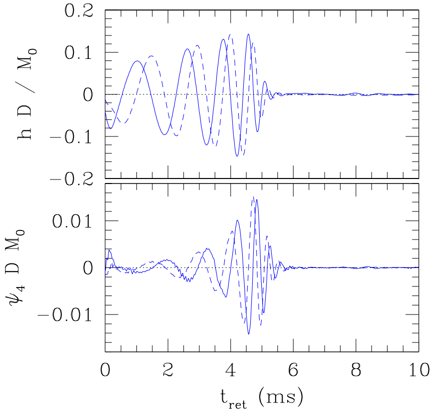
EOS : polyt. $\gamma = 2$, $M/R = 0.145$, mass ratio 3 (A), 2 (B) and 1 (C) :



[Etienne, Faber, Liu, Shapiro, Taniguchi & Baumgarte, PRD **77**, 084002 (2008)]

Gravitational wave signal

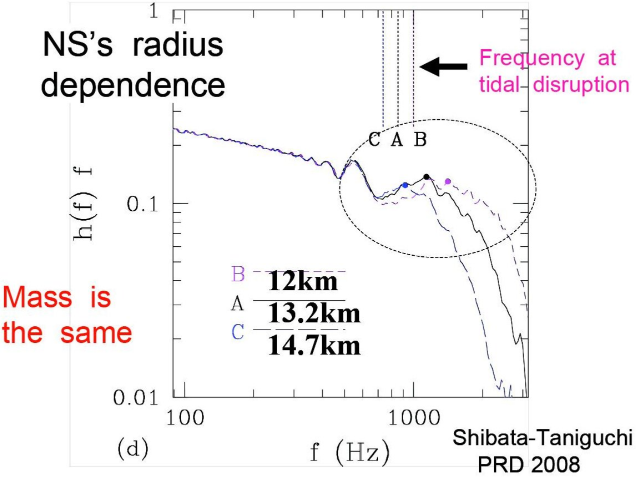
EOS : polyt. $\gamma = 2$, mass ratio 3



[Shibata & Taniguchi, PRD **77**, 084015 (2008)]

GW Fourier spectrum

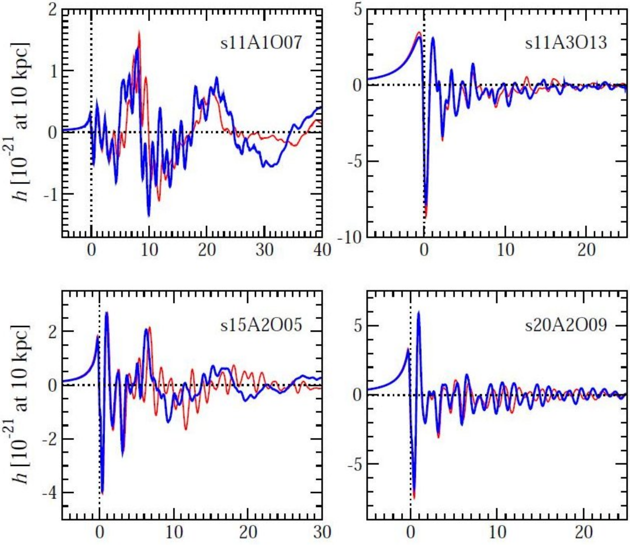
EOS : polyt. $\gamma = 2$, mass ratio 3



[Shibata & Taniguchi, PRD **77**, 084015 (2008)]

Outline

Neutron star formation in core-collapse supernovae



- EOS :
  red : Shen (1998)
  blue : Lattimer & Swesty (1991)
- Progenitor mass :
  s11 $= 11\,M_\odot$,
  s15 $= 15\,M_\odot$,
  s20 $= 20\,M_\odot$,
  s40 $= 40\,M_\odot$
- Rotation profile :
  A1 = uniform,
  A2 = moderately differential, A3 = strongly differential
- $T/|W|$: O1 = small, O15 = large

[Dimmelmeier, Ott, Marek & Janka, PRD **78**, 064056 (2008)]

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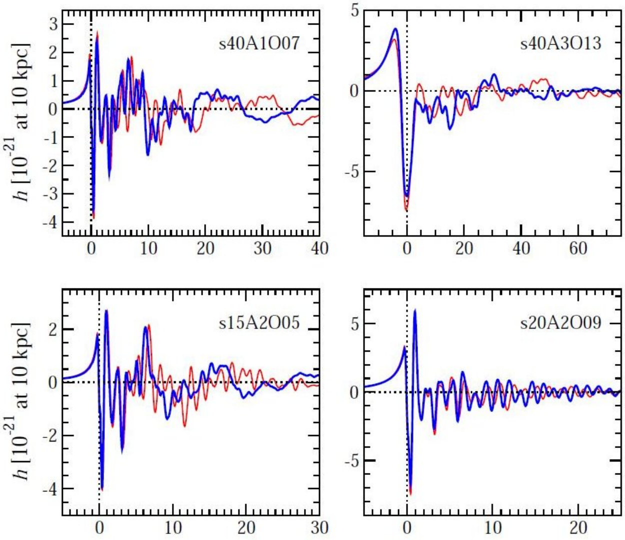

[Dimmelmeier, Ott, Marek & Janka, PRD **78**, 064056 (2008)]

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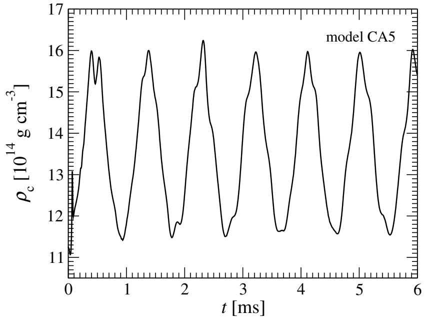
Phase-transition-induced mini-collapse of neutron stars

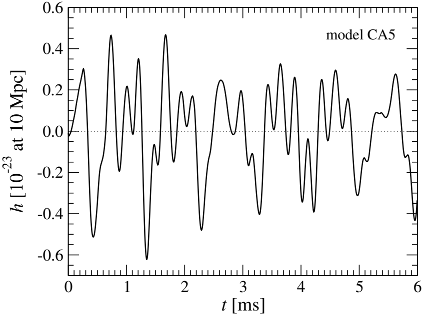


Phase transition from hadronic matter to deconfined quark matter in the core
$\Rightarrow$ compact hybrid quark star

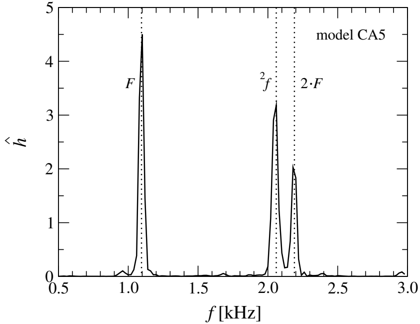
[Abdikamalov, Dimmelmeier, Rezzolla & Miller, arXiv :0806.1700]

GW signal

Waveform



Fourier spectrum



Detectable by VIRGO/LIGO in the Galaxy, needs 3rd generation of detectors at 20 Mpc (Virgo cluster)

[Abdikamalov, Dimmelmeier, Rezzolla & Miller, arXiv :0806.1700]