Gravitational waves from binary black holes

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Plan

1. Introduction

2. The inspiral *(the most understood phase)*

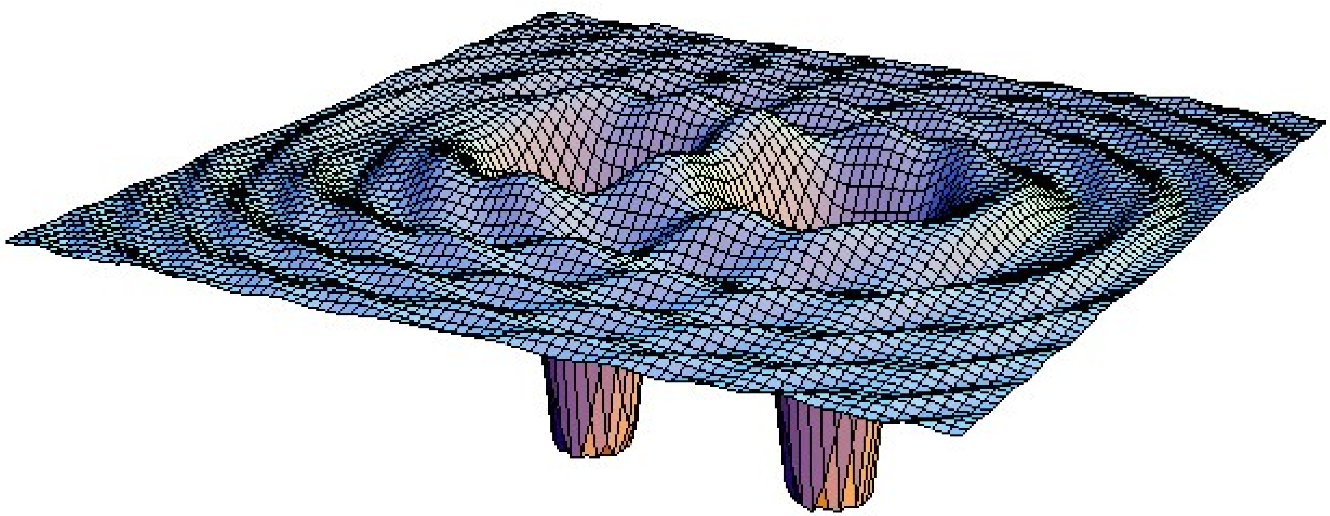
3. The ISCO problem *(when and how does the inspiral terminate ?)*

4. The final merger *(progress report)*

1. Introduction

Gravitational waves: the only detectable radiation which comes directly from a black hole.

(Hawking radiation negligible)



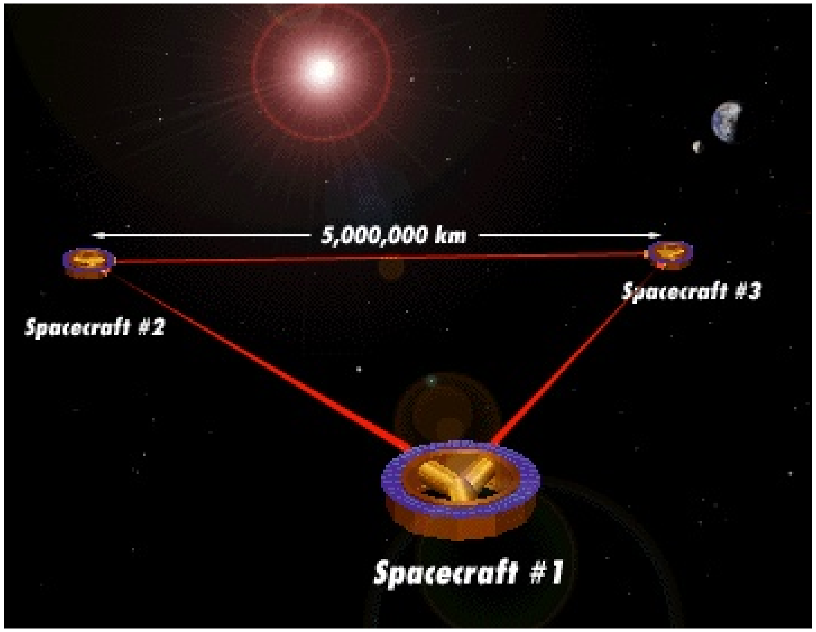
Gravitational wave detectors are coming on line...

VIRGO, Cascina, Italy

$10 \text{ Hz} < f < 10^3 \text{ Hz}$

also LIGO, GEO600, TAMA

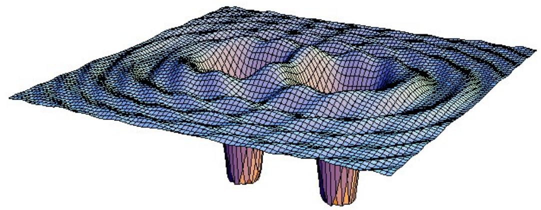
...or will be launched in the not too far future (2011)



LISA (ESA/NASA)

$10^{-4} \text{ Hz} < f < 10^{-1} \text{ Hz}$

Binary black holes



From the GW detection point of view: the most promising source

From the theoretical point of view:

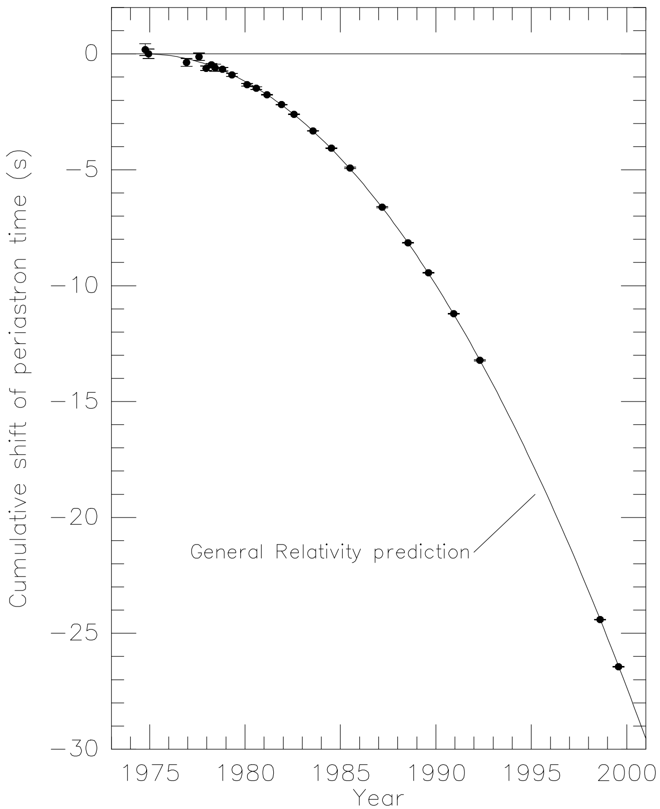
- Binary BH = the two body problem in General Relativity
- Extreme GR $\Longrightarrow$ probes the limit of GR (as weak field limit of string theory)

From the astrophysical point of view:

- Rate of binary black hole coalescence $\Longrightarrow$ massive star evolution
- Inspiral GW signal $\Longrightarrow$ precise measure of Hubble constant H_0
- GW observations of supermassive BH at high z $\Longrightarrow$ large structure formation

Evolution of binary black holes

Contrary to Newtonian 2-body problem, no stationary solution for 2 bodies in GR : *Energy and angular momentum loss due to gravitational radiation* $\Longrightarrow$ *shrink of the orbits*

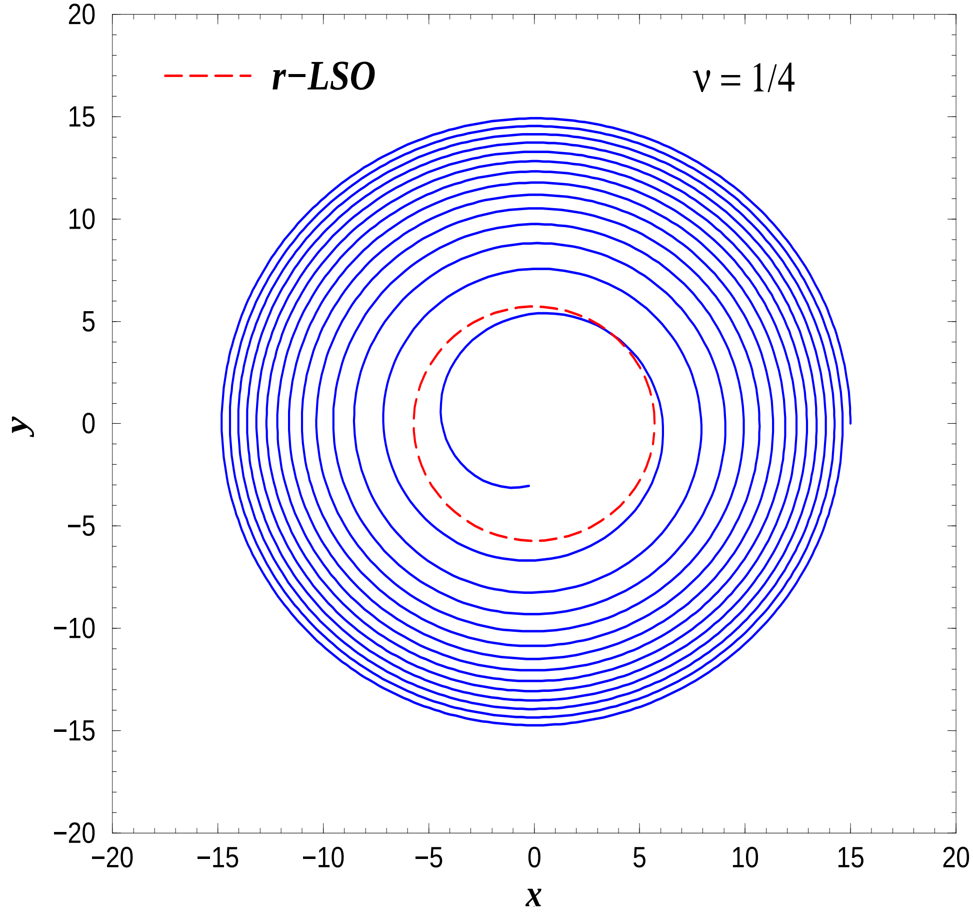


[from Lorimer (2001)]

$\longleftarrow$ Observed decay of the orbital period $P = 7\,\mathrm{h}\,45\,\mathrm{min}$) of the binary pulsar PSR B1913+16 due to gravitational radiation reaction $\Longrightarrow$ merger in 140 Myr.

Another effect of gravitational wave emission: *circularisation of the orbits:* $e \longrightarrow 0$

Inspiraling motion



2-PN Effective One Body computation

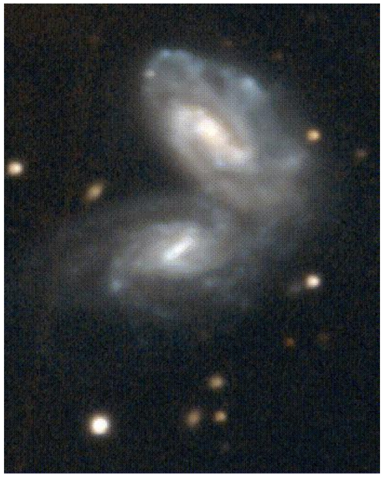
[Buonanno & Damour, PRD 62, 064015 (2000)]

Two types of binary BH coalescence

(1) Coalescence of stellar BH: from massive star evolution

event rate:

- up to $\sim 20/\mathrm{Myr}$ per galaxy (Belczynski, Kalogera, Bulik (2002), astro-ph/0111452)
- $1.6 \times 10^{-7}\ \mathrm{yr}^{-1}\mathrm{Mpc}^{-3}$ from binary BH formation in globular clusters (Portegies Zwart & McMillan, ApJ 528, L17 (2000))

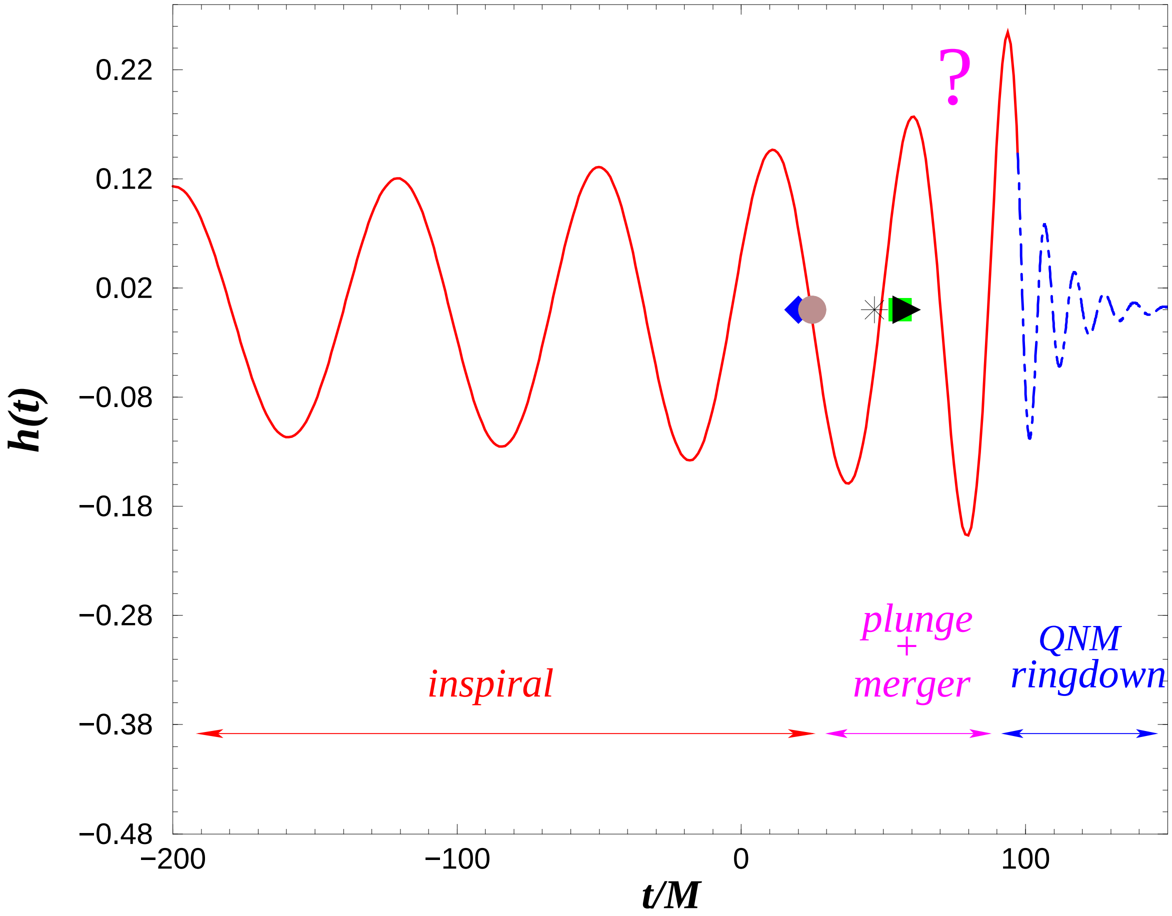


(2) Coalescence of supermassive BH: from galaxy encounters

event rate : possibly large (cf. K. Menou's talk)

NB: Same physics (scaling with M)

Gravitational waveform

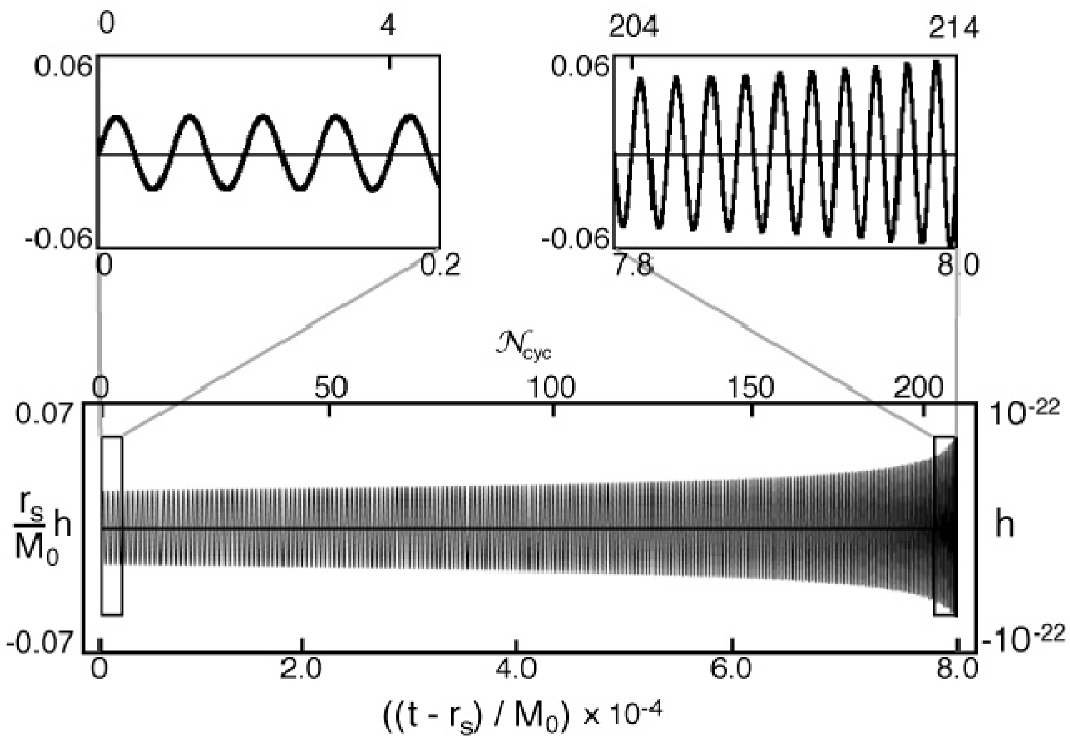


[from Buonanno & Damour, PRD 62, 064015 (2000)]

2. The inspiral

(the most understood phase)

Inspiral waveform



[Duez, Baumgarte & Shapiro, PRD 63, 084030 (2001)]

Chirp signal:

$$h_+ \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \cos(2\pi f t)$$

$$h_\times \propto \frac{\mathcal{M}^{5/3}}{r} f^{2/3} \sin(2\pi f t)$$

$$f = K_0\, \mathcal{M}^{-5/8} (t_{\rm coal} - t)^{-3/8}$$

with the "chirp mass":

$$\mathcal{M} = (M_1 M_2)^{3/5} (M_1 + M_2)^{-1/5}$$

and the constant:

$$K_0 = \frac{5^{3/8}}{8\pi} \left(\frac{c^3}{G} \right)^{5/8}$$

More precise formulae:

- More harmonics in $h_+(t)$ and $h_\times(t)$ (up to 6 at the 2.5PN level)
- Orbital phase ($\Longrightarrow$ number of cycles) at the 3.5PN level:

$$\phi(t) = -\frac{1}{\nu}\Big\{\tau^{5/8} + \left(\frac{3715}{8064} + \frac{55}{96}\nu\right)\tau^{3/8} - \frac{3}{4}\pi\tau^{1/4}$$

$$+ \quad \left(\frac{9275495}{14450688} + \frac{284875}{258048}\nu + \frac{1855}{2048}\nu^2\right)\tau^{1/8} + \left(-\frac{38645}{172032} - \frac{15}{2048}\nu\right)\pi\ln\left(\frac{\tau}{\tau_0}\right)$$

$$+\left(\frac{831032450749357}{57682522275840} - \frac{53}{40}\pi^2 - \frac{107}{56}C + \frac{107}{448}\ln\left(\frac{\tau}{256}\right)\right.$$

$$+\left[-\frac{123292747421}{4161798144} + \frac{2255}{2048}\pi^2 + \frac{385}{48}\lambda - \frac{55}{16}\theta\right]\nu + \frac{154565}{1835008}\nu^2$$

$$\left.-\frac{1179625}{1769472}\nu^3\right)\tau^{-1/8} + \left(\frac{188516689}{173408256} + \frac{140495}{114688}\nu - \frac{122659}{516096}\nu^2\right)\pi\tau^{-1/4}\Big\}$$

Blanchet, Faye, Iyer & Joguet, PRD 65, 061501(R) (2002)

Chirp time

Characteristic evolution time at the frequency f:

$$\tau := \frac{f}{\dot{f}} = \frac{8}{3}(t_{\rm coal} - t) = \frac{5}{96\pi^{8/3}} \frac{c^5}{G^{5/3}} \mathcal{M}^{-5/3} f^{-8/3}$$

- for stellar black holes ($M_1 = M_2 = 10\, M_\odot \Rightarrow \mathcal{M} = 8.7\, M_\odot$):

$$\tau = 100 \text{ s} \left(\frac{10\,\rm Hz}{f}\right)^{8/3} \left(\frac{8.7\, M_\odot}{\mathcal{M}}\right)^{5/3}$$

- for supermassive black holes ($M_1 = M_2 = 10^6\, M_\odot \Rightarrow \mathcal{M} = 8.7 \times 10^5\, M_\odot$):

$$\tau = 116 \text{ d} \left(\frac{10^{-4}\,\rm Hz}{f}\right)^{8/3} \left(\frac{8.7 \times 10^5 M_\odot}{\mathcal{M}}\right)^{5/3}$$

NB: $h\tau f^2 = \dfrac{K}{r}$ with K independent of $\mathcal{M}$ $\Longrightarrow$ standard candle

Signal in an interferometric detector

Gravitational wave strain:

$$h(t) = F_+(\theta, \phi, \psi)\, h_+(t) + F_\times(\theta, \phi, \psi)\, h_\times(t)$$

θ, ϕ : direction of the source with respect to the detector arms

ψ : polarization angle of the wave with respect to the detector orientation

$F_+,\ F_\times$: beam-pattern functions

Detector's output:

$$o(t) = h(t) + n(t)$$

with the noise $n(t)$ in most cases larger than $h(t) \Longrightarrow$ signal filtering necessary

Optimal signal filtering

Characterization of the noise: the r.m.s. noise in a bandwidth $[f, f + df]$ is $\sqrt{\langle n(t)^2 \rangle} =: \sqrt{S(f)\, df}$, where $S(f)$ is the noise power spectral density. A stationary Gaussian noise is fully characterized by $S(f)$.

Signal filtering: $C := \int_{-\infty}^{+\infty} o(t)\, F(t)\, dt$ (F: filter)

Signal-to-noise ratio: $\frac{S}{N} := \frac{\langle C \rangle}{\sqrt{\langle C^2 \rangle_{h=0}}}$

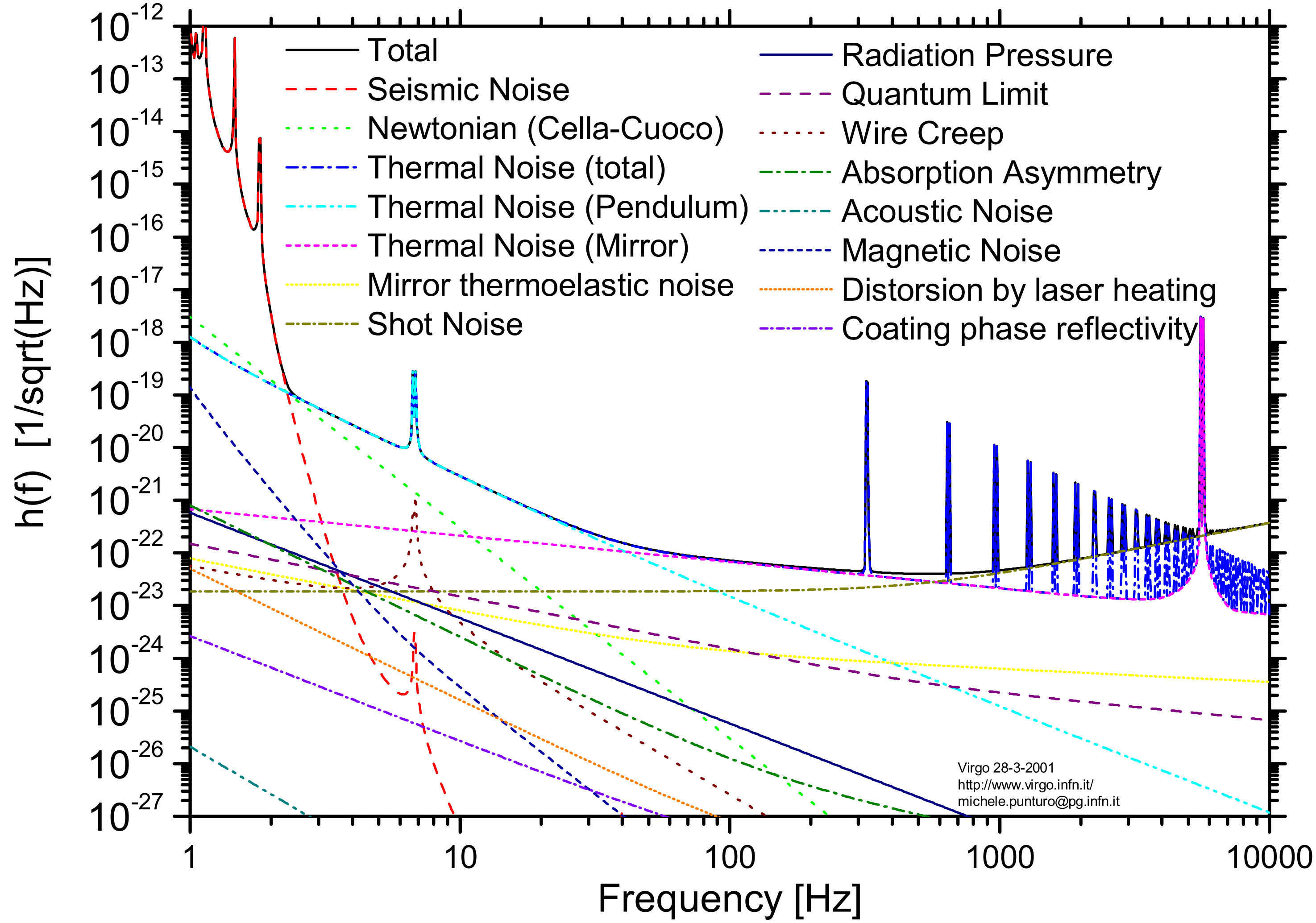
Wiener theorem: SNR maximal $\Leftrightarrow$ $\tilde{F}(f) = \frac{\tilde{h}(f)}{S(f)}$ (*optimal* or *matched* filter)

Then

$$\frac{S}{N} = 2 \left(\int_0^{\infty} \frac{\left| \tilde{h}(f) \right|^2}{S(f)}\, df \right)^{1/2}$$

$\Longrightarrow$ a priori knowledge of $h(t)$ is required

Expected noise density $S(f)^{1/2}$ for the VIRGO detector

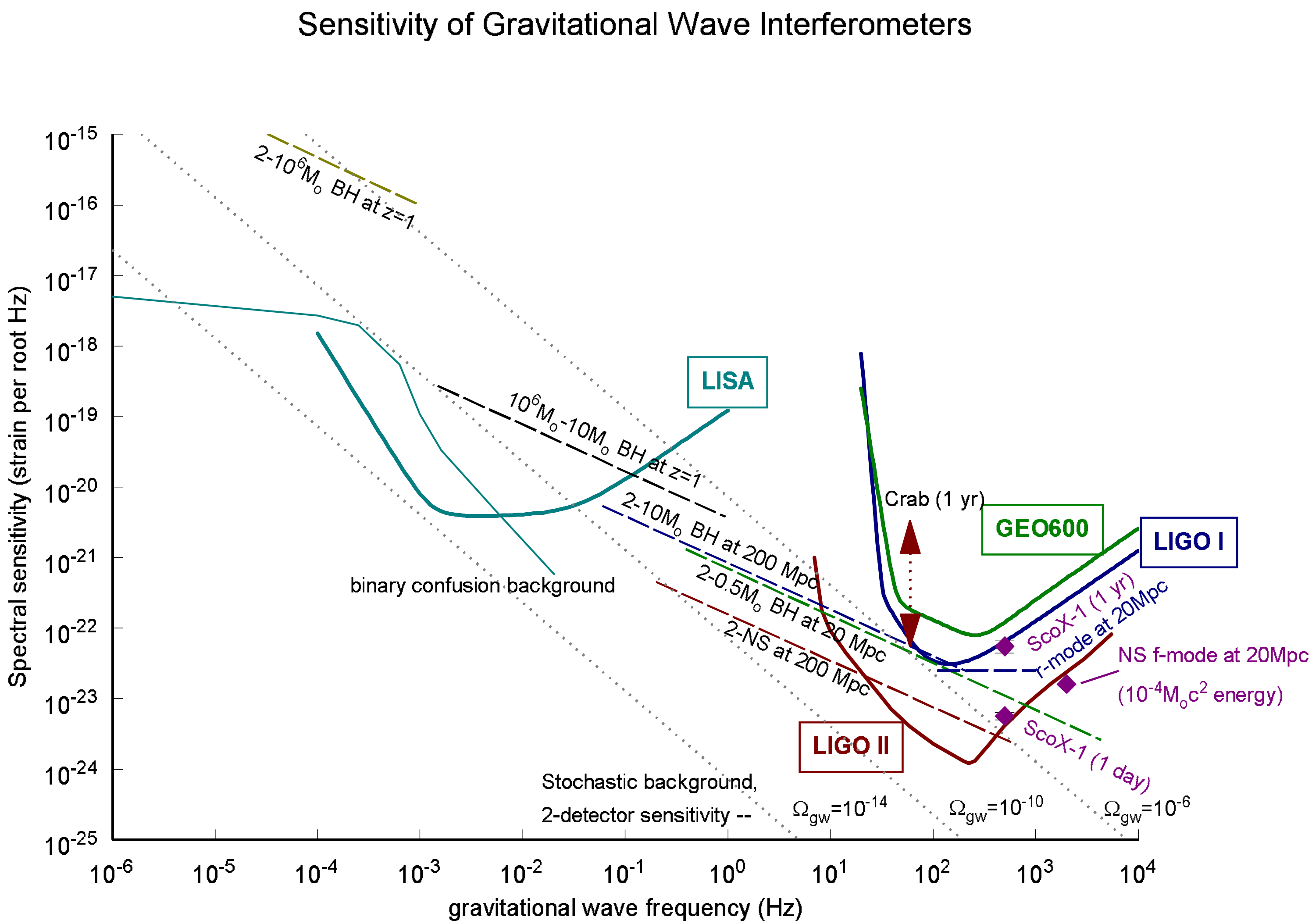


Inspiralling binary SNR

Approximately $\frac{S}{N} \sim \frac{h\sqrt{\mathcal{N}}}{S(f)^{1/2}\sqrt{f}}$, where $\mathcal{N}$ is the number of cycles spent within a bandwidth $\Delta f \sim f$ centered around f: $\mathcal{N} = f^2/\dot{f} = f\tau \propto (\mathcal{M}f)^{-5/3}$.

Hence

$$\frac{S}{N} \propto \frac{\mathcal{M}^{5/6}}{S(f)^{1/2}\, f^{2/3}}$$



[Schutz, CQG 16, A131 (1999)]

Range of detection and expected event rate

Stellar BH ($2 \times 10\, M_\odot$):

Detection range:

- first generation (LIGO-I, VIRGO): $d_{\rm max} \simeq 100$ Mpc
- second generation: $d_{\rm max} \simeq 1$ Gpc

Expected event rate:

- first generation (LIGO-I, VIRGO): ~ 1 per year
- second generation: daily

Supermassive BH ($2 \times 10^6\, M_\odot$):

$d_{\rm max} >$ Hubble radius for LISA $\Longrightarrow$ expected rate: a few per year up to 10^3 per year

3. The ISCO problem

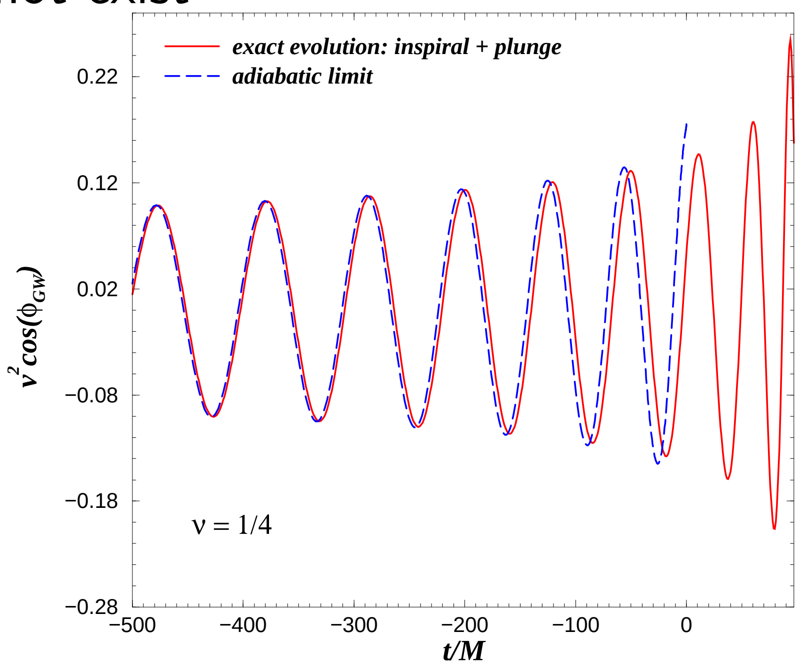
(when and how does the inspiral terminate ?)

The last stable orbit

Very small mass ratio (Schwarzschild spacetime) : there exists an *innermost stable circular orbit (ISCO)* :

$$R_{\rm ISCO}^{\rm Schw} = 6M \qquad\qquad \Omega_{\rm ISCO}^{\rm Schw} = 6^{-3/2} M^{-1} \simeq 0.068\, M^{-1}$$

Equal mass ratio : gravitational radiation dissipation $\Longrightarrow$ strictly circular orbits do not exist



The ISCO is then defined in terms of the conservative part in the equation of motions, which give rise to circular orbits (adiabatic approximation). Consider a sequence of circular orbits of smaller and smaller radius, mimicking the inspiral. The ISCO is defined as the *turning point* in the *binding energy* of this sequence.

← Buonanno & Damour, PRD 62, 064015 (2000)

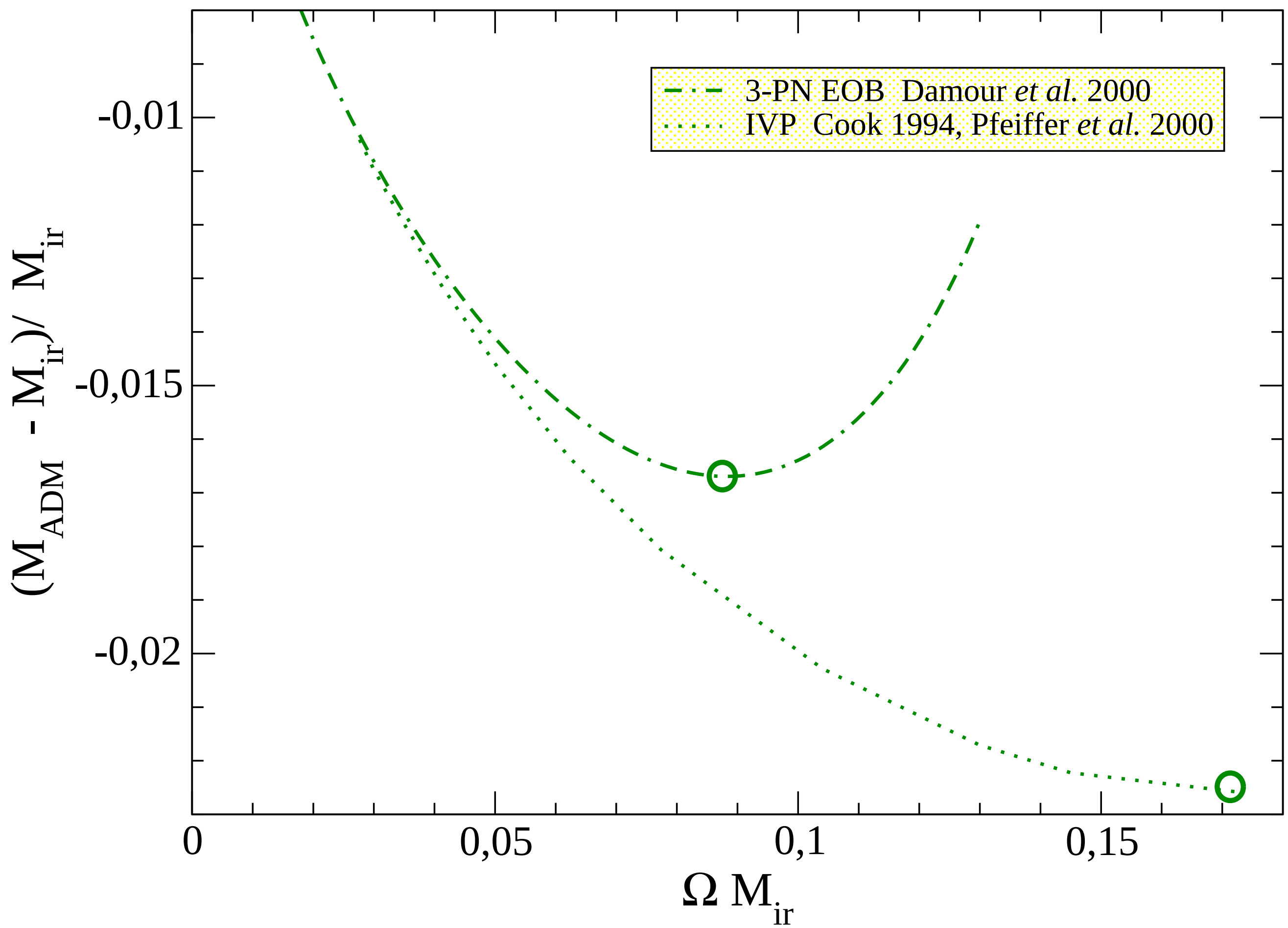
Binary BH ISCO computations

- *Post-Newtonian computations :* at the 3-PN level:
  - Effective One Body approach (EOB) : Damour, Jaranowski & Schäfer, PRD 62, 084011 (2000)
  - point masses approach : Blanchet, PRD in press, gr-qc/0112056 (2002)
- *Numerical computations :* based on the initial value problem (IVP) :
  - Cook, PRD 50, 5025 (1994)
  - Pfeiffer, Teukolsky & Cook, PRD 62, 104018 (2000)
  - Baumgarte, PRD 62, 024018 (2000)

$\longrightarrow$ Big discrepancy between the two types of computations

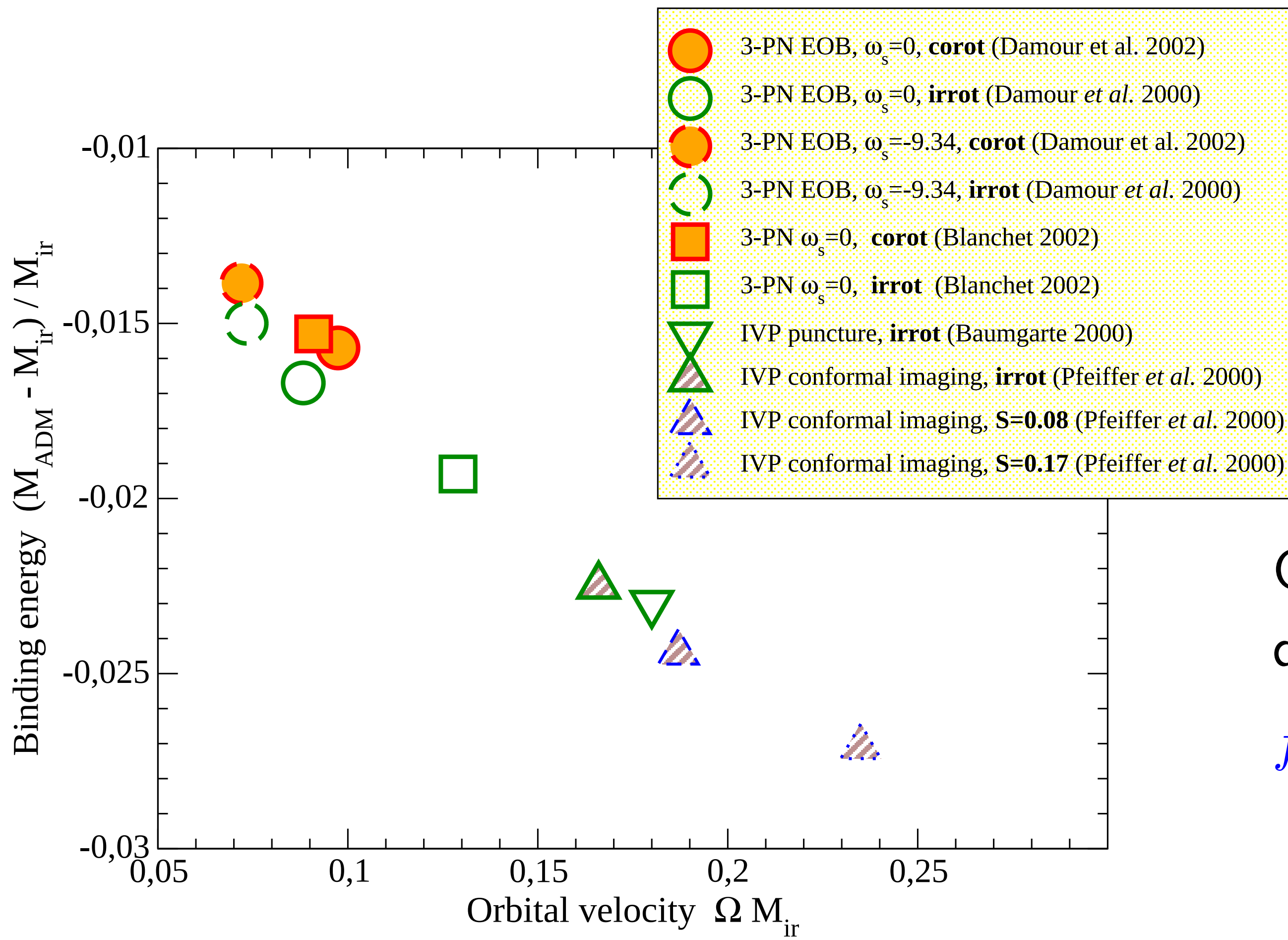
Discrepancy between analytical and numerical methods

Binding energy along an evolutionary sequence of equal-mass binary black holes



Discrepancy between analytical and numerical methods

Location of the ISCO



Gravitational wave frequency:

$$f = 320 \; \frac{\Omega M_{\rm ir}}{0.1} \; \frac{20\, M_\odot}{M_{\rm ir}} \; {\rm Hz}$$

Our new numerical approach

Problem treated:

Binary black holes in the pre-coalescence stage

$\Rightarrow$ the notion of *orbit* has still some meaning

Basic idea:

Construct an approximate, but full spacetime (i.e. *4-dimensional*) representing 2 orbiting black holes

Previous numerical treatments (IVP) : 3-dimensional (initial value problem on a spacelike 3-surface)

4-dimensional approach $\Rightarrow$ rigorous definition of orbital angular velocity

First results:

Gourgoulhon, Grandclément & Bonazzola, PRD 65, 044020 (2002)

Grandclément, Gourgoulhon & Bonazzola, PRD 65, 044021 (2002)

Helical symmetry

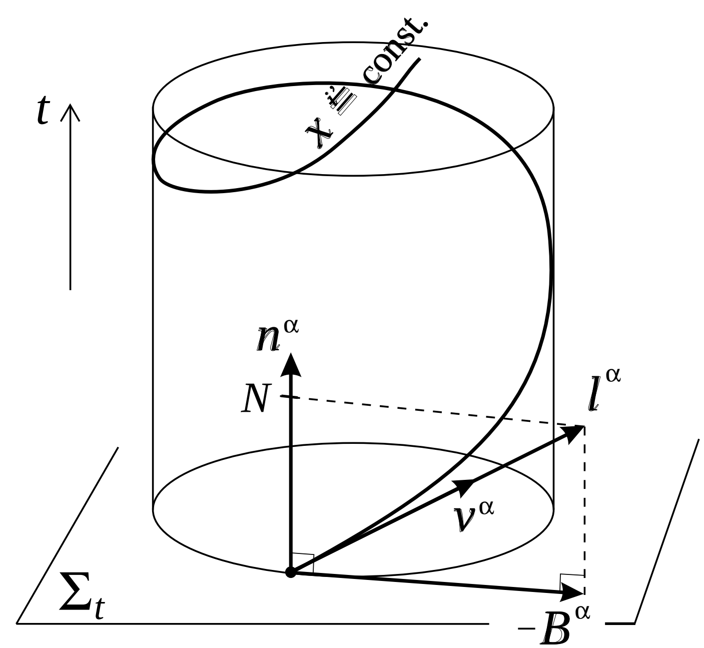
Physical assumption: when the two holes are sufficiently far apart, the radiation reaction can be neglected $\Rightarrow$ closed orbits

Gravitational radiation reaction circularizes the orbits $\Rightarrow$ circular orbits

Geometrical translation: there exists a Killing vector field $\boldsymbol{\ell}$ such that:

far from the system (asymptotically inertial coordinates $(t_0, r_0, \theta_0, \varphi_0)$),

$$\boldsymbol{\ell} \to \frac{\partial}{\partial t_0} + \Omega \frac{\partial}{\partial \varphi_0}$$



Einstein equations

Assumption: Maximal slicing: $K = 0$

Approximation: conformally flat spatial metric: $\boldsymbol{\gamma} = \Psi^4 \mathbf{f}$

Amounts to solve 5 of the 10 Einstein equations (one more than IVP !) :

$$\Delta\Psi = -\frac{\Psi^5}{8}\hat{A}_{ij}\hat{A}^{ij} \qquad \textit{(Hamiltonian constraint)}$$

$$\Delta\beta^i + \frac{1}{3}\bar{D}^i\bar{D}_j\beta^j = 2\hat{A}^{ij}\left(\bar{D}_jN - 6N\bar{D}_j\ln\Psi\right) \qquad \textit{(momentum constraint)}$$

$$\Delta N = N\Psi^4\hat{A}_{ij}\hat{A}^{ij} - 2\bar{D}_j\ln\Psi\,\bar{D}^jN \qquad \textit{(trace of } \frac{\partial K_{ij}}{\partial t} = \cdots\textit{)}$$

with $\hat{A}_{ij} := \Psi^{-4}K_{ij}$ and $\hat{A}^{ij} := \Psi^4 K^{ij}$

Kinematical relation between $\boldsymbol{\gamma}$ and $\mathbf{K}$:

$$\hat{A}^{ij} = \frac{1}{2N}(L\beta)^{ij} \qquad \textit{(traceless part)}$$

$$\bar{D}_i\beta^i = -6\beta^i\bar{D}_i\ln\Psi \qquad \textit{(trace part)}$$

with $(L\beta)^{ij} := \bar{D}^i\beta^j + \bar{D}^j\beta^i - \frac{2}{3}\bar{D}_k\beta^k\,f^{ij}$

Determination of Ω

Virial assumption: $O(r^{-1})$ part of the metric ($r \to \infty$) same as Schwarzschild

[The only quantity "felt" at the $O(r^{-1})$ level by a distant observer is the total mass of the system.]

A priori

$$\Psi \sim 1 + \frac{M_{\rm ADM}}{2r} \qquad \text{and} \qquad N \sim 1 - \frac{M_{\rm K}}{r}$$

Hence

$$\text{(virial assumption)} \iff M_{\rm ADM} = M_{\rm K}$$

Note

$$\text{(virial assumption)} \iff \Psi^2 N \sim 1 + \frac{\alpha}{r^2}$$

Defining an evolutionary sequence

An evolutionary sequence is defined by:

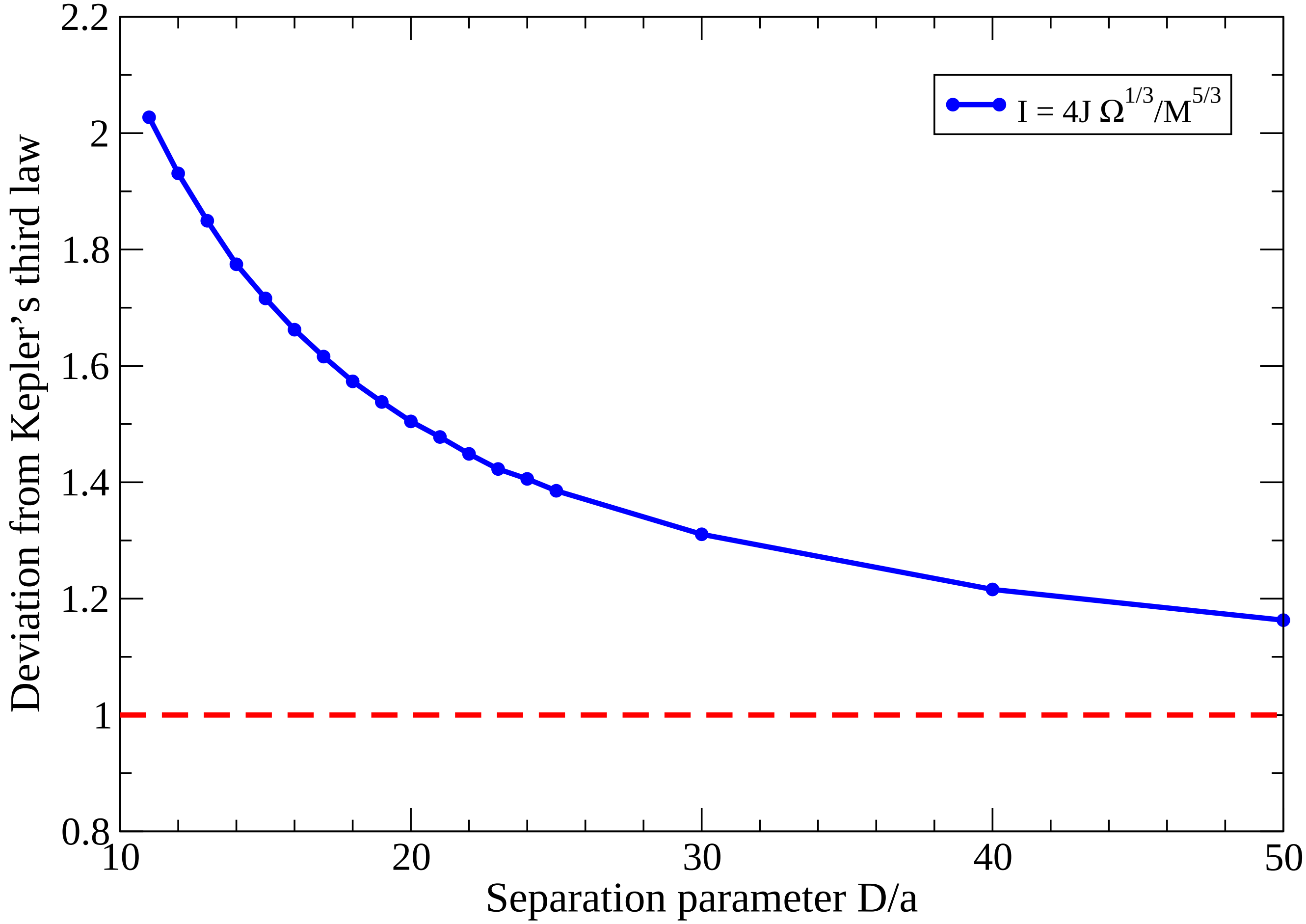
$$\left. \frac{dM_{\rm ADM}}{dJ} \right|_{\rm sequence} = \Omega$$

This is equivalent to requiring the constancy of the horizon area of each black hole, by virtue of the First law of thermodynamics for binary black holes :

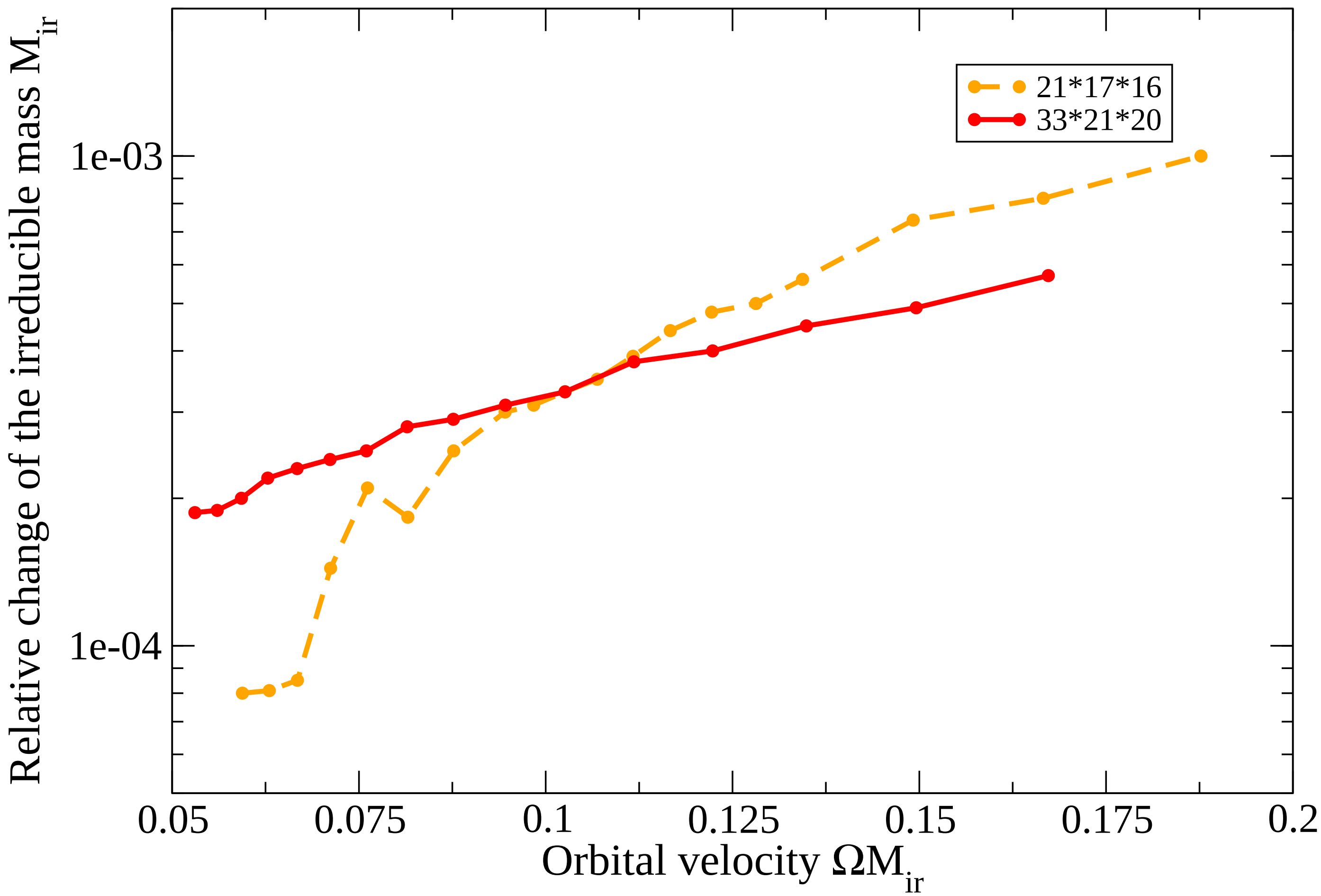
$$dM_{\rm ADM} = \Omega \, dJ + \frac{1}{8\pi} \left(\kappa_1 \, dA_1 + \kappa_2 \, dA_2 \right)$$

recently established by *Friedman, Uryu & Shibata, PRD in press, gr-qc/0108070*.

Test : getting Kepler's third law at large separation



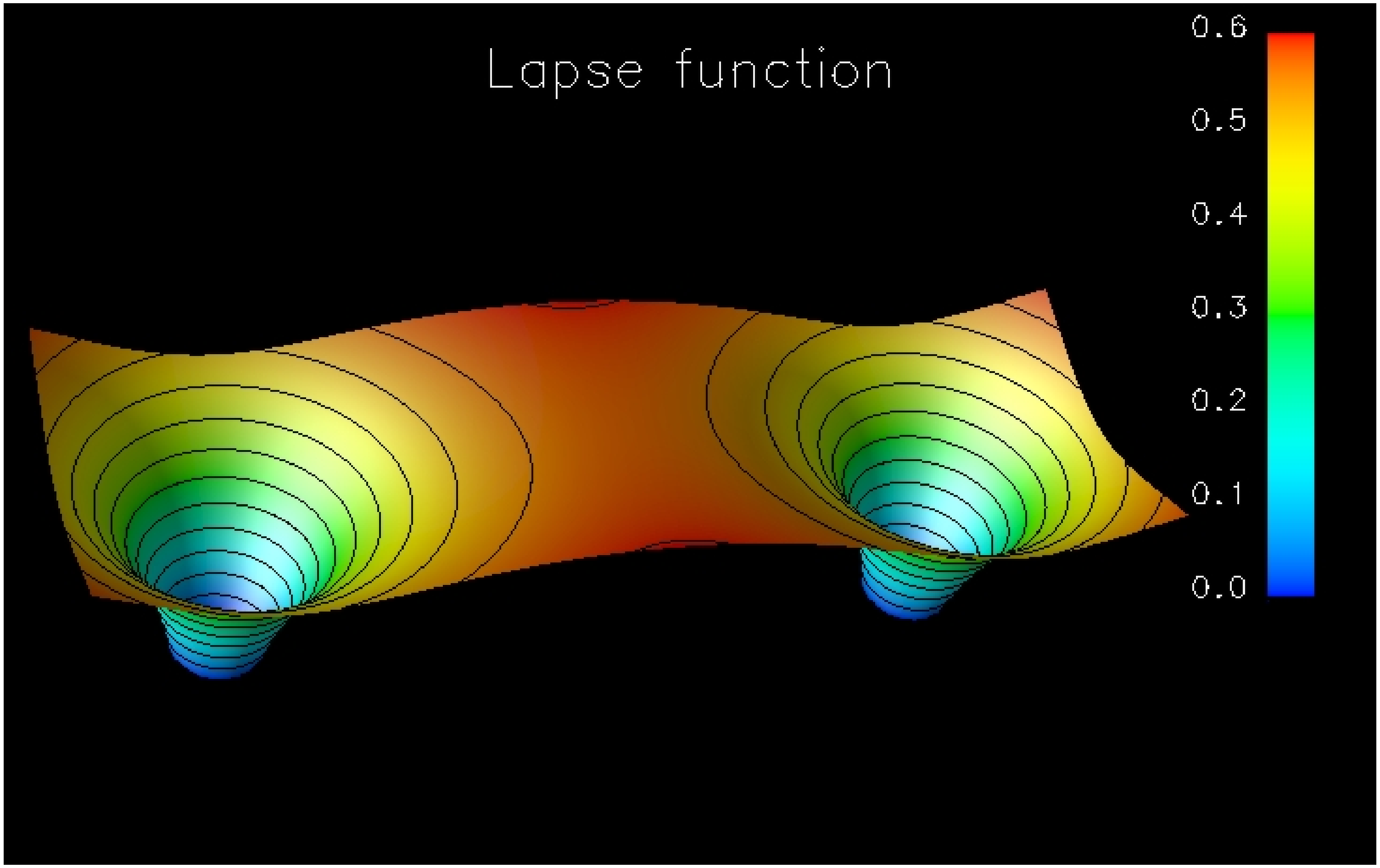
Test : conservation of the horizon area along a sequence



Relative change of the horizon area along an evolutionary sequence

Lapse in the orbital plane

ISCO configuration



Lapse in the orbital plane

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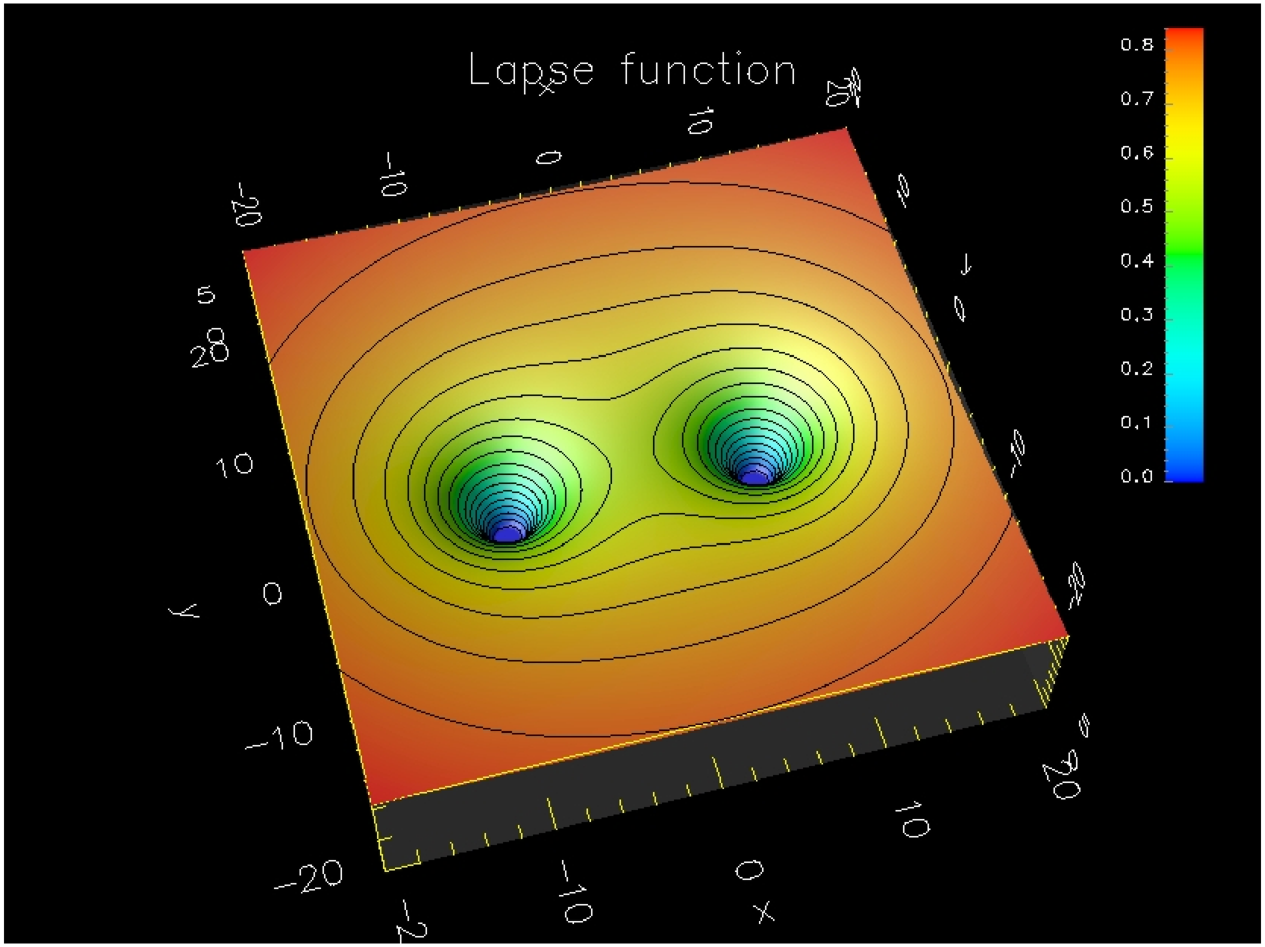
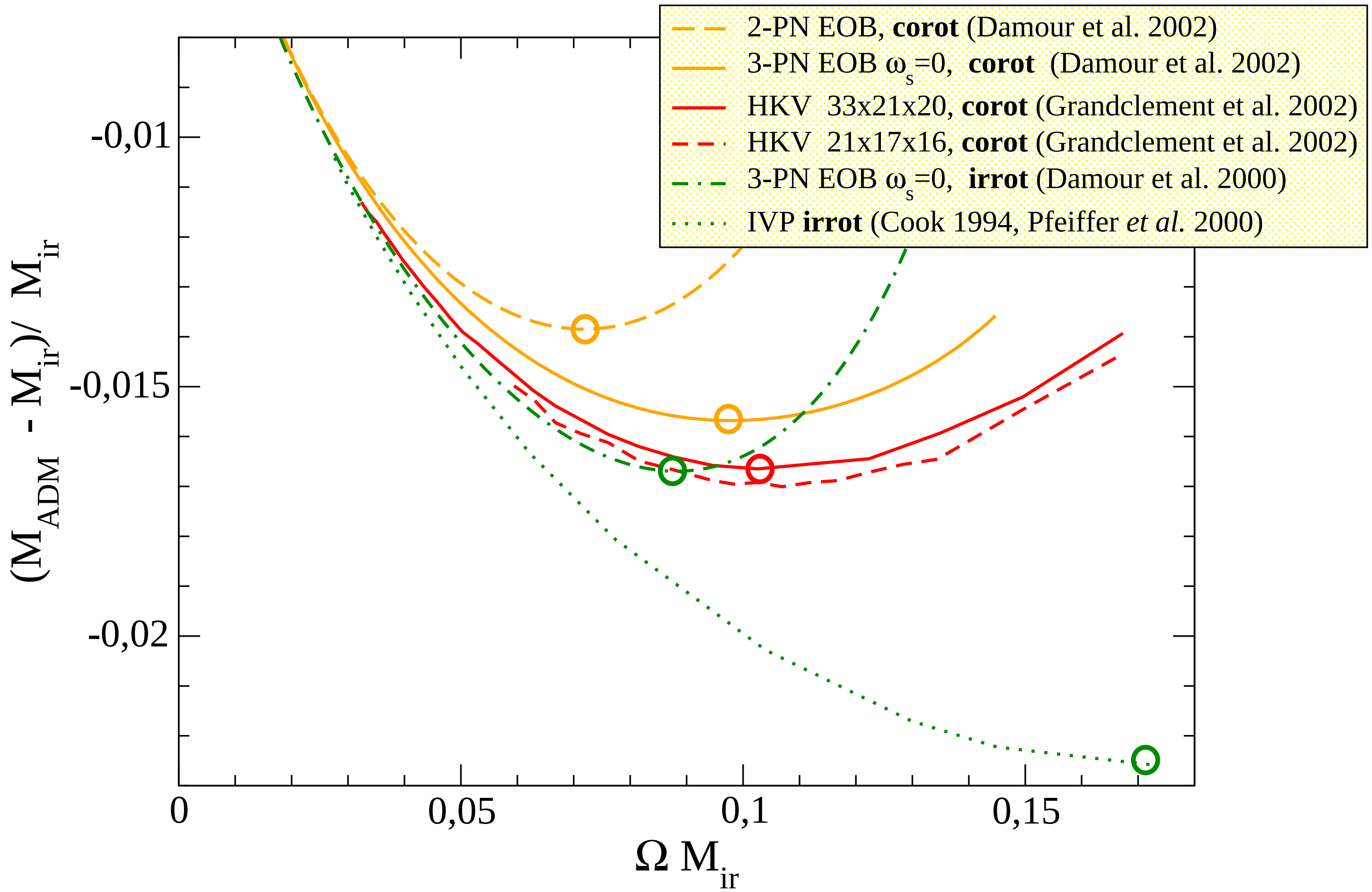


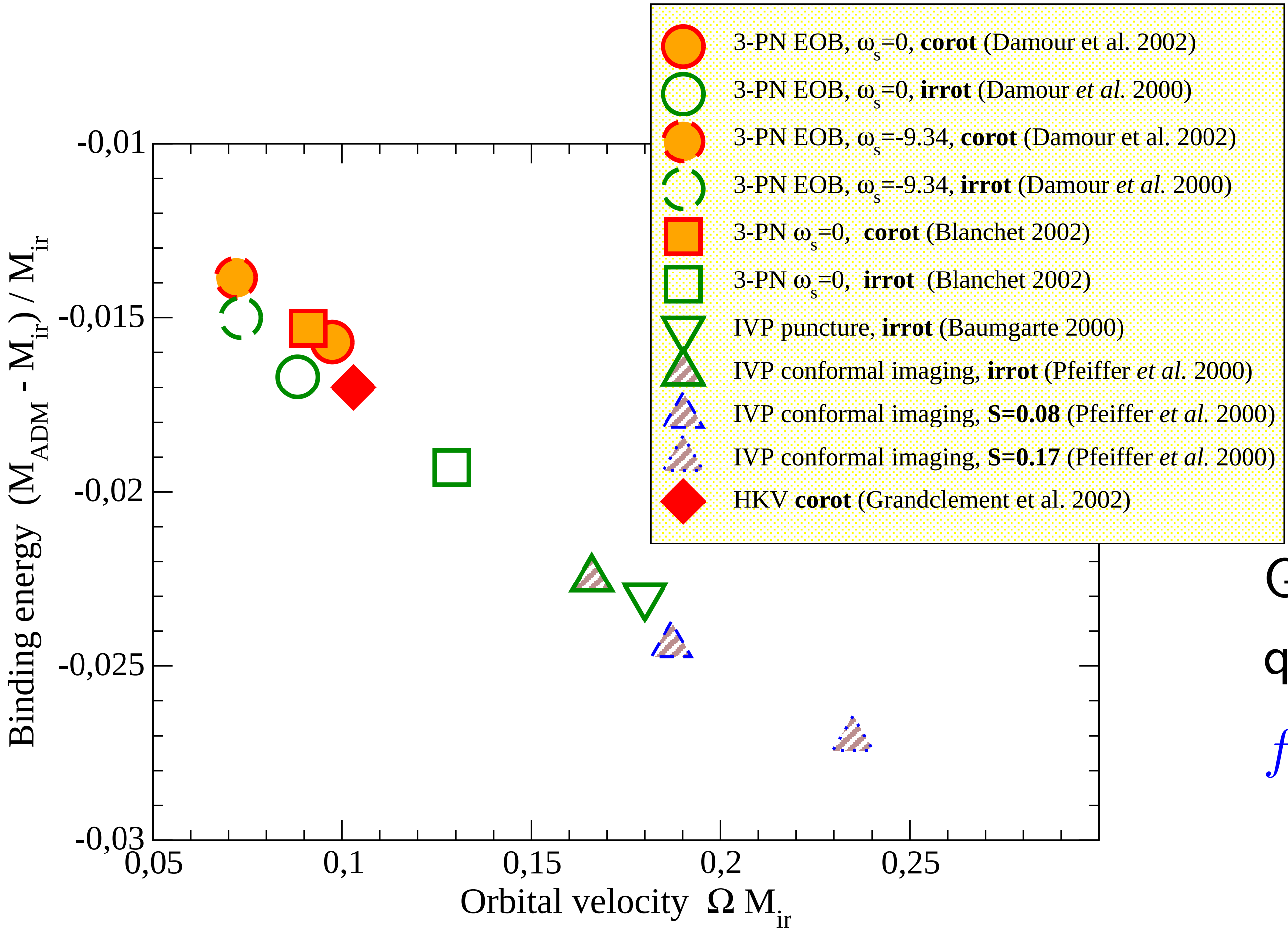
Comparison with Post-Newtonian computations

Binding energy along an evolutionary sequence of equal-mass binary black holes



Comparison with Post-Newtonian computations

Location of the ISCO



Gravitational wave frequency:

$$f = 320 \, \frac{\Omega M_{\rm ir}}{0.1} \, \frac{20 \, M_\odot}{M_{\rm ir}} \; \mathrm{Hz}$$

4. The final merger

(…for the next "three years after" meeting)

Numerical relativity attempts

• *Combining numerical relativity and linearized pertubation theory around the final Kerr BH:*

Baker, Brügmann, Campanelli, Lousto & Takahashi, PRL 87, 121103 (2001)

But

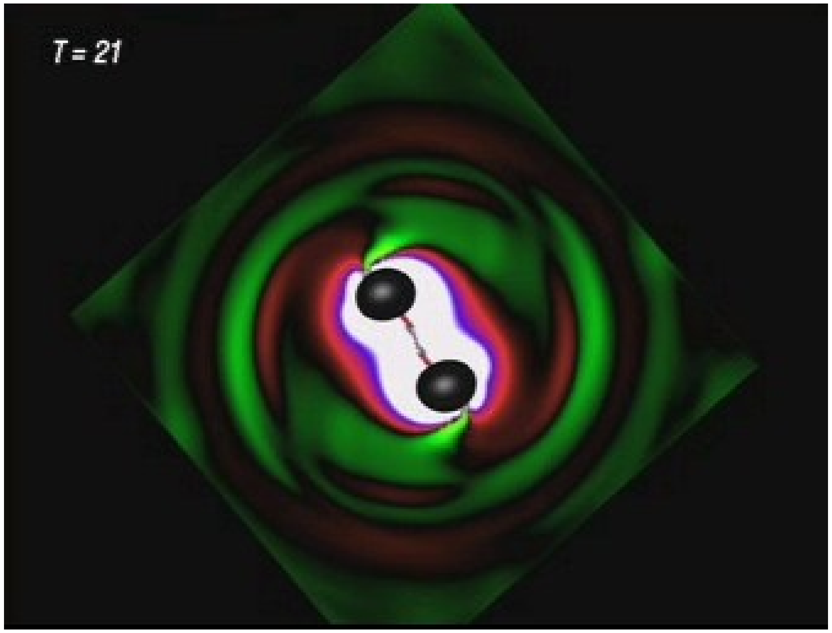
- crude time evolution: old fashioned ADM + zero-shift $\Longrightarrow$ code crashes after only $t = 15\,M$, before a common apparent horizon forms
- bad initial data (Baumgarte ISCO)

• *Full numerical relativity with improved coordinate choice and astrophysical initial data:*

work in progress at Albert Einstein Institute (Seidel et al.), in the framework of the European Union Network "Sources of gravitational waves" (`http://www.eu-network.org/`) :

Meudon ISCO data, computed by means of spectral methods (Lorene), exported on finite-differences grid (Cactus).

Recent merger computation by the AEI group



movie

Corotating coordinates + conformal decomposition of Einstein equations $\Longrightarrow$ formation of a common apparent horizon

But still non-astrophysical initial data (Baumgarte ISCO).

Results with new initial data coming soon...

Energy emitted by gravitational radiation

Absolute upper bounds:

Hawking (1971) : $\frac{E_{\rm rad}}{M} < 0.5$ for merger of maximaly rotating Kerr BH, such that the final BH does not rotate

$\frac{E_{\rm rad}}{M} < 0.29$ for merger of non-rotating BH

Inspiral stage: $\frac{E_{\rm rad}}{M} \simeq 0.017$

Plunge + merger phase: $\frac{E_{\rm rad}}{M} \sim 0.1$?? Flanagan & Hughes, PRD 57, 4535 (1998)

Ringdown phase: $\frac{E_{\rm rad}}{M} \simeq 0.03$?

Brandt & Seidel, PRD 52, 870 (1995), Flanagan & Hughes, PRD 57, 4535 (1998)

Conclusions

- Weakness of expected GW signal $\Longrightarrow$ adapted filters $\Longrightarrow$ theoretical prediction of waveforms necessary to detect the signal
- Inspiral phase: well described by analytical tools (post-Newtonian expansions)
- First agreement between analytical methods and numerical ones about the termination point of the inspiral (ISCO), resulting in a strong reliability of the result
- Advantage of numerical methods about PN ones in this regime: treat the BH as extended objects (horizons) and naturally provide initial data (γ_{ij}, K_{ij}) for subsequent time evolution.
- The full merger, starting from these realistic initial data, seems now feasible within three years...