Testing gravitation theories with the Event Horizon Telescope and GRAVITY

Éric Gourgoulhon

Laboratoire Univers et Théories (LUTH)
CNRS / Observatoire de Paris / Université Paris Diderot
Paris Sciences et Lettres Research University
92190 Meudon, France
http://luth.obspm.fr/~luthier/gourgoulhon/

based on a collaboration with
Philippe Grandclément, Marion Grould, Carlos Herdeiro, Frédéric Lamy, Zakaria Meliani, Jérôme Novak, Thibaut Paumard, Guy Perrin, Eugen Radu, Claire Somé, Odele Straub, Karim Van Aelst and Frédéric H. Vincent

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Outline

1. Observing the black hole at the Galactic center
2. The theoretical framework and the no-hair theorem
3. Examples : boson stars and hairy black holes

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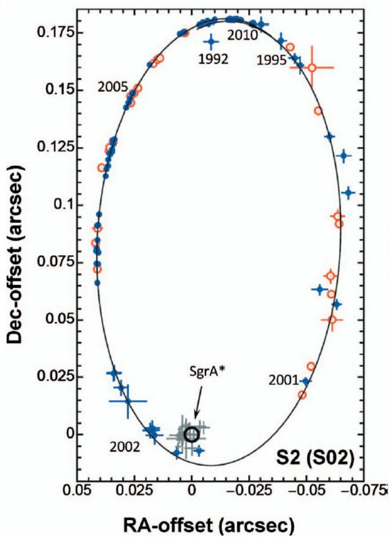
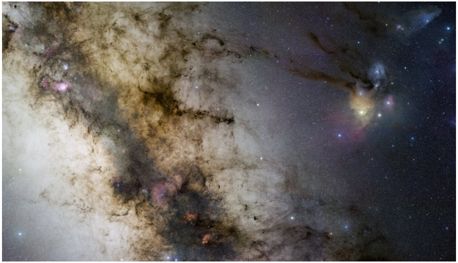
The black hole at the centre of our galaxy : Sgr A*





[ESO (2009)]

Mass of Sgr A* black hole deduced from stellar dynamics :

$$M_{\rm BH} = 4.3 \times 10^6 \, M_\odot$$

← Orbit of the star S2 around Sgr A*
$P = 16\,{\rm yr}$, $r_{\rm per} = 120\,{\rm UA} = 1400\,R_{\rm S}$, $V_{\rm per} = 0.02\,c$

[Genzel, Eisenhauer & Gillessen, RMP **82**, 3121 (2010)]

Next periastron passage : mid 2018

Can we see it from the Earth ?

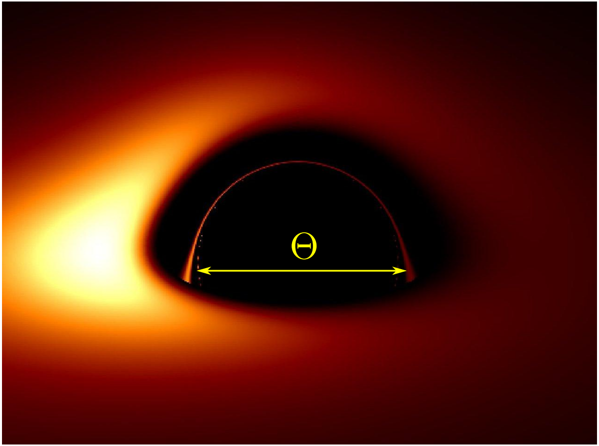


Image of a thin accretion disk around a Schwarzschild BH
[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

Angular diameter of the silhouette of a Schwarzschild BH of mass M seen from a distance d :

$$\boxed{\Theta = 6\sqrt{3}\,\frac{GM}{c^2 d} \simeq 2.60\,\frac{2R_S}{d}}$$

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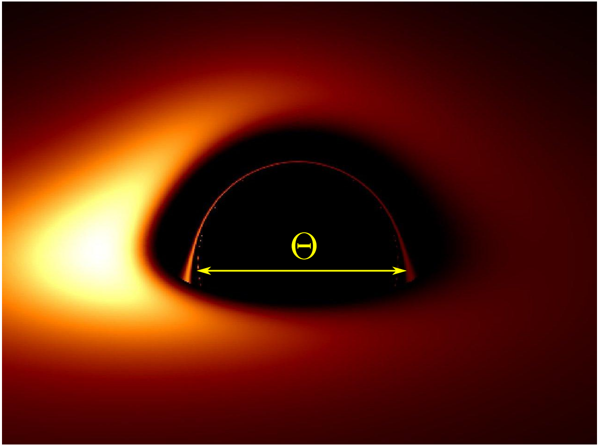

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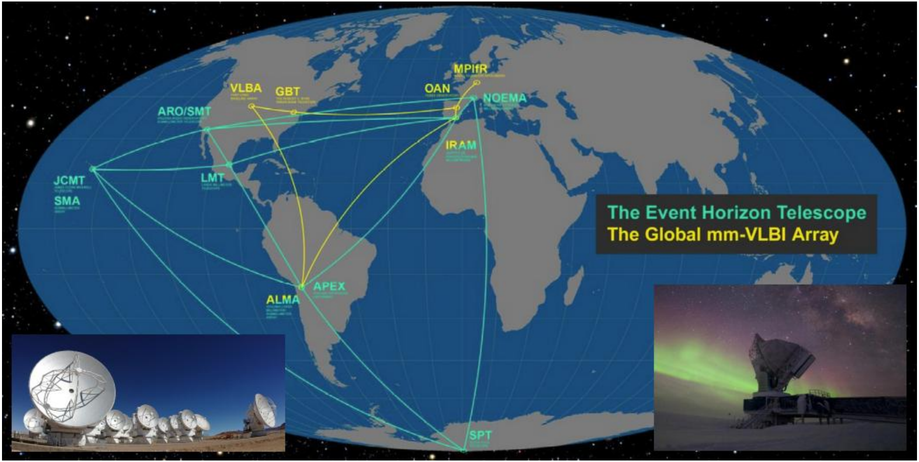
Largest black holes in the Earth's sky :

Sgr A* : $\Theta = 53\ \mu\text{as}$
M87 : $\Theta = 21\ \mu\text{as}$
M31 : $\Theta = 20\ \mu\text{as}$

Remark : black holes in X-ray binaries are $\sim 10^5$ times smaller, for $\Theta \propto M/d$

Reaching μas resolution : the Event Horizon Telescope



http://eventhorizontelescope.org/

Very Large Baseline Interferometry (VLBI) at $\lambda = 1.3$ mm

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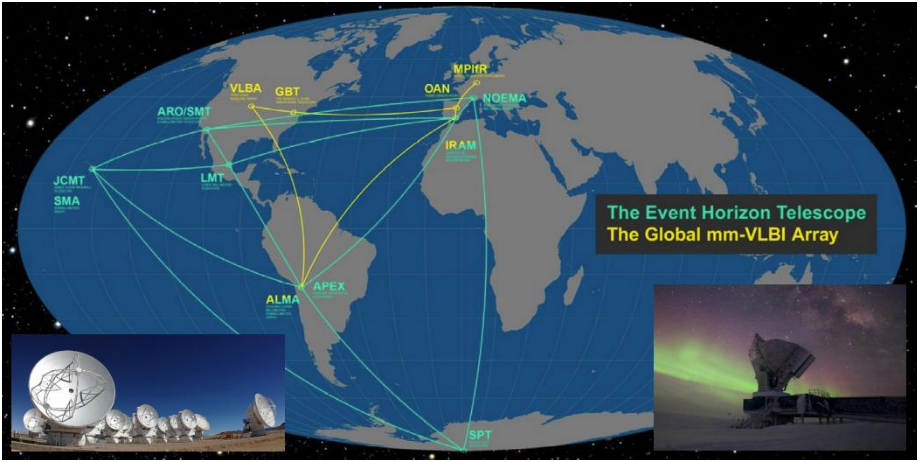

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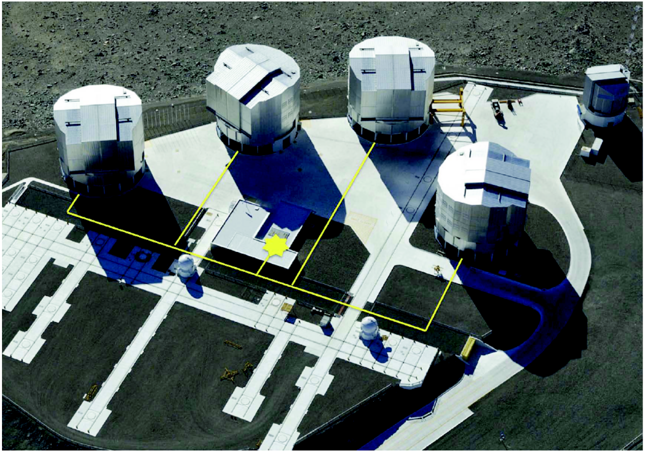
Very Large Baseline Interferometry (VLBI) at $\lambda = 1.3$ mm

April 2017 : large observation campaign $\Longrightarrow$ first image soon ?

Near-infrared optical interferometry : GRAVITY



[Gillessen et al. 2010]

GRAVITY instrument at VLTI (start : 2016)

Beam combiner (the four 8 m telescopes + four auxiliary telescopes)

astrometric precision on orbits : $10\ \mu\mathrm{as}$

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The no-hair theorem

Dorochkevitch, Novikov & Zeldovitch (1965), Israel (1967), Carter (1971), Hawking (1972)

Within 4-dimensional general relativity, a *stationary* black hole in an otherwise empty universe is necessarily a Kerr-Newmann black hole, which is an electro-vacuum solution of Einstein equation described by only 3 numbers :

- the total mass M
- the total specific angular momentum $a = J/(Mc)$
- the total electric charge Q

$\Longrightarrow$ *"a black hole has no hair"* (John A. Wheeler)

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Astrophysical black holes have to be electrically neutral :

- $Q = 0$: Kerr solution (1963)

Other special cases :

- $a = 0$: Reissner-Nordström solution (1916, 1918)
- $a = 0$ and $Q = 0$: Schwarzschild solution (1916)
- $a = 0$, $Q = 0$ and $M = 0$: Minkowski metric (1907)

The Kerr metric is specific to black holes

Spherically symmetric (non-rotating) bodies :

Birkhoff theorem

Within 4-dimensional general relativity, the spacetime outside any spherically symmetric body is described by Schwarzschild metric

$\Longrightarrow$ No possibility to distinguish a non-rotating black hole from a non-rotating dark star by monitoring orbital motion or fitting accretion disk spectra

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Rotating axisymmetric bodies :

No Birkhoff theorem

Moreover, no "reasonable" matter source has ever been found for the Kerr metric (the only known source consists of two counter-rotating thin disks of collisionless particles [Bicak & Ledvinka, PRL **71**, 1669 (1993)])

$\Longrightarrow$ The Kerr metric is specific to rotating black holes
(in 4-dimensional general relativity)

Lowest order no-hair theorem : quadrupole moment

Asymptotic expansion (large r) of the metric in terms of multipole moments $(\mathcal{M}_k, \mathcal{J}_k)_{k\in\mathbb{N}}$ [Geroch (1970), Hansen (1974)] :

- $\mathcal{M}_k$: mass 2^k-pole moment
- $\mathcal{J}_k$: angular momentum 2^k-pole moment

$\Longrightarrow$ For the Kerr metric, all the multipole moments are determined by (M, a) :

- $\mathcal{M}_0 = M$
- $\mathcal{J}_1 = aM = J/c$
- $\boxed{\mathcal{M}_2 = -a^2 M = -\frac{J^2}{c^2 M}}$ (1) $\leftarrow$ mass quadrupole moment
- $\mathcal{J}_3 = -a^3 M$
- $\mathcal{M}_4 = a^4 M$
- $\cdots$

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- $\cdots$

Measuring the three quantities M, J, $\mathcal{M}_2$ provides a compatibility test w.r.t. the Kerr metric, by checking (1)

Theoretical alternatives to the Kerr black hole

Within general relativity

The compact object is not a black hole but

- boson stars
- gravastar
- dark stars
- ...

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Beyond general relativity

The compact object is a black hole but in a theory that differs from 4-dimensional GR :

- Horndeski theories
- Chern-Simons gravity
- Hořava-Lifshitz gravity
- Higher-dimensional GR
- ...

Viable scalar-tensor theories after GW170817

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [42] Brans-Dicke/$f(R)$ [43, 44] Kinetic Gravity Braiding [46]	quartic/quintic Galileons [13, 14] Fab Four [15, 16] de Sitter Horndeski [45] $G_{\mu\nu}\phi^\mu\phi^\nu$ [47], Gauss-Bonnet
beyond H.	Derivative Conformal (20) [18] Disformal Tuning (22) DHOST with $A_1 = 0$	quartic/quintic GLPV [19] DHOST [20, 48] with $A_1 \neq 0$
	Viable after GW170817	Non-viable after GW170817

[Ezquiaga & Zumalacárregui, arXiv:1710.05901]

→ *see talks by Ed Porter and Christos Charmousis*

Testing the Kerr black hole hypothesis

Observational tests

Search for

- stellar orbits deviating from Kerr timelike geodesics (GRAVITY)

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  - EMRI : extreme-mass-ratio binary inspirals (LISA)

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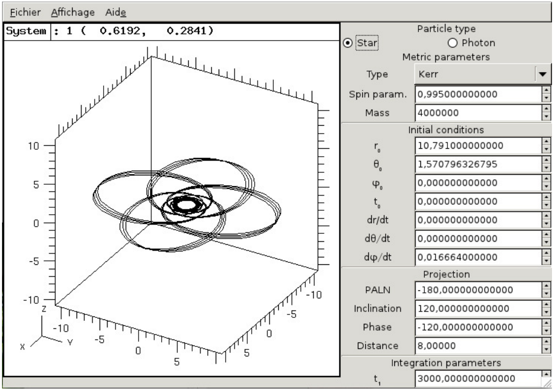
Need for a good and versatile **geodesic integrator**
to compute timelike geodesics (orbits) and null geodesics (ray-tracing) in any kind of metric

Gyoto code

Main developers : T. Paumard & F. Vincent



- Integration of geodesics in Kerr metric
- Integration of geodesics in any numerically computed 3+1 metric
- Radiative transfer included in optically thin media
- Very modular code (C++)
- Yorick and Python interfaces
- Free software (GPL) : http://gyoto.obspm.fr/

[Vincent, Paumard, Gourgoulhon & Perrin, CQG **28**, 225011 (2011)]

[Vincent, Gourgoulhon & Novak, CQG **29**, 245005 (2012)]

Geodesics with the free computer algebra system SageMath

`SageMath` : Python-based open-source mathematical software

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Timelike geodesic in Schwarzschild spacetime

This Jupyter/SageMath worksheet presents the numerical computation of a timelike geodesic in Schwarzschild spacetime, given an initial point and tangent vector. It uses the `integrated_geodesic` functionality introduced by Karim Van Aelst in **SageMath 8.1**, in the framework of the SageManifolds project.

A version of SageMath at least equal to 8.1 is required to run this worksheet:

In [1]:
```
version()
```
Out[1]: 'SageMath version 8.1.rc0, Release Date: 2017-11-08'

In [2]:
```
%display latex  # LaTeX rendering turned on
```

We define first the spacetime manifold M and the standard Schwarzschild-Droste coordinates on it:

In [3]:
```
M = Manifold(4, 'M')
X.<t,r,th,ph> = M.chart(r't r:(0,+oo) th:(0,pi):\theta ph:\phi')
X
```
Out[3]: $(M,(t,r,\theta,\phi))$

http://nbviewer.jupyter.org/github/sagemanifolds/SageManifolds/blob/master/Worksheets/v1.1/SM_simple_geod_Schwarz.ipynb

Geodesics with the free computer algebra system SageMath

In [3]:
```
M = Manifold(4, 'M')
X.<t,r,th,ph> = M.chart(r't r:(0,+oo) th:(0,pi):\theta ph:\phi')
X
```

Out[3]: $(M, (t, r, \theta, \phi))$

For graphical purposes, we introduce $\mathbb{R}^3$ and some coordinate map $M \to \mathbb{R}^3$:

In [4]:
```
R3 = Manifold(3, 'R^3', latex_name=r'\mathbb{R}^3')
X3.<x,y,z> = R3.chart()
to_R3 = M.diff_map(R3, {(X, X3): [r*sin(th)*cos(ph),
                                  r*sin(th)*sin(ph), r*cos(th)]})
to_R3.display()
```

Out[4]:

$$\begin{array}{llcl} & M & \longrightarrow & \mathbb{R}^3 \\ & (t, r, \theta, \phi) & \longmapsto & (x, y, z) = (r\cos(\phi)\sin(\theta), r\sin(\phi)\sin(\theta), r\cos(\theta)) \end{array}$$

Then, we define the Schwarzschild metric:

In [5]:
```
g = M.lorentzian_metric('g')
m = var('m'); assume(m >= 0)
g[0,0], g[1,1] = -(1-2*m/r), 1/(1-2*m/r)
g[2,2], g[3,3] = r^2, (r*sin(th))^2
g.display()
```

Out[5]:

$$g = \left(\frac{2\,m}{r} - 1\right) \mathrm{d}t \otimes \mathrm{d}t + \left(-\frac{1}{\frac{2\,m}{r} - 1}\right) \mathrm{d}r \otimes \mathrm{d}r + r^2 \mathrm{d}\theta \otimes \mathrm{d}\theta + r^2 \sin(\theta)^2 \mathrm{d}\phi \otimes \mathrm{d}\phi$$

We pick an initial point and an initial tangent vector:

In [6]:
```
p0 = M.point((0, 8*m, pi/2, 1e-12), name='p_0')
v0 = M.tangent_space(p0)((1.297513, 0, 0, 0.0640625/m), name='v_0')
v0.display()
```

Out[6]:

$$v_0 = 1.29751300000000 \frac{\partial}{\partial t} + \frac{0.0640625000000000}{m} \frac{\partial}{\partial \phi}$$

Geodesics with the free computer algebra system SageMath

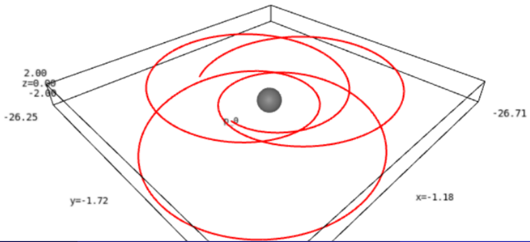
We declare a geodesic with such initial conditions, denoting by s the affine parameter (proper time), with $(s_{\min}, s_{\max}) = (0, 1500\, m)$:

In [7]:
```
s = var('s')
geod = M.integrated_geodesic(g, (s, 0, 1500), v0); geod
```

Out[7]: Integrated geodesic in the 4-dimensional differentiable manifold M

We ask for the numerical integration of the geodesic, providing some numerical value for the parameter m, and then plot it in terms of the Cartesian chart X3 of $\mathbb{R}^3$:

In [8]:
```
sol = geod.solve(parameters_values={m: 1})  # numerical integration
interp = geod.interpolate()                 # interpolation of the solution for the plot
graph = geod.plot_integrated(chart=X3, mapping=to_R3, plot_points=500,
                             thickness=2, label_axes=False)          # the geodesic
graph += p0.plot(chart=X3, mapping=to_R3, size=4, parameters={m: 1}) # the starting point
graph += sphere(size=2, color='grey')                                 # the event horizon
show(graph, viewer='threejs', online=True)
```



Geodesics with the free computer algebra system SageMath

In [11]:
```
g.christoffel_symbols_display()
```

Out[11]:

$$
\begin{array}{lcl}
\Gamma^{t}{}_{t\,r} & = & -\frac{m}{2\,mr-r^2} \\
\Gamma^{r}{}_{t\,t} & = & -\frac{2\,m^2-mr}{r^3} \\
\Gamma^{r}{}_{r\,r} & = & \frac{m}{2\,mr-r^2} \\
\Gamma^{r}{}_{\theta\,\theta} & = & 2\,m-r \\
\Gamma^{r}{}_{\phi\,\phi} & = & (2\,m-r)\sin(\theta)^2 \\
\Gamma^{\theta}{}_{r\,\theta} & = & \frac{1}{r} \\
\Gamma^{\theta}{}_{\phi\,\phi} & = & -\cos(\theta)\sin(\theta) \\
\Gamma^{\phi}{}_{r\,\phi} & = & \frac{1}{r} \\
\Gamma^{\phi}{}_{\theta\,\phi} & = & \frac{\cos(\theta)}{\sin(\theta)}
\end{array}
$$

In [12]:
```
g.riemann().display_comp()
```

Out[12]:

$$
\begin{array}{lcl}
\mathrm{Riem}(g)^{t}{}_{r\,t\,r} & = & -\frac{2\,m}{2\,mr^2-r^3} \\
\mathrm{Riem}(g)^{t}{}_{r\,r\,t} & = & \frac{2\,m}{2\,mr^2-r^3} \\
\mathrm{Riem}(g)^{t}{}_{\theta\,t\,\theta} & = & -\frac{m}{r} \\
\mathrm{Riem}(g)^{t}{}_{\theta\,\theta\,t} & = & \frac{m}{r} \\
\mathrm{Riem}(g)^{t}{}_{\phi\,t\,\phi} & = & -\frac{m\sin(\theta)^2}{r} \\
\mathrm{Riem}(g)^{t}{}_{\phi\,\phi\,t} & = & \frac{m\sin(\theta)^2}{r}
\end{array}
$$

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Boson stars

Boson star = localized configurations of a self-gravitating complex scalar field Φ
≡ "Klein-Gordon geons" [Bonazzola & Pacini (1966), Kaup (1968), Ruffini & Bonazzola (1969)]

- Minimally coupled scalar field : $\mathcal{L} = \frac{1}{16\pi} R - \frac{1}{2}\left[\nabla_\mu \bar{\Phi} \nabla^\mu \Phi + V(|\Phi|^2)\right]$
- Scalar field equation : $\nabla_\mu \nabla^\mu \Phi = V'(|\Phi|^2)\, \Phi$
- Einstein equation : $R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} = 8\pi T_{\alpha\beta}$

  with $T_{\alpha\beta} = \nabla_{(\alpha} \bar{\Phi} \nabla_{\beta)} \Phi - \frac{1}{2}\left[\nabla_\mu \bar{\Phi} \nabla^\mu \Phi + V(|\Phi|^2)\right] g_{\alpha\beta}$

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Examples :

- free field : $V(|\Phi|^2) = \frac{m^2}{\hbar^2} |\Phi|^2$, $\quad m$: boson mass

  $\Longrightarrow$ field equation = Klein-Gordon equation : $\nabla_\mu \nabla^\mu \Phi = \frac{m^2}{\hbar^2} \Phi$
- a standard self-interacting field : $V(|\Phi|^2) = \frac{m^2}{\hbar^2} |\Phi|^2 + \lambda |\Phi|^4$

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Boson stars as black-hole mimickers

Boson stars can be very compact and are the less exotic alternative to black holes : they require only a scalar field and since 2012 we know that at least one fundamental scalar field exists in Nature : the Higgs boson !

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Maximum mass

- Free field : $M_{\rm max} = \alpha \frac{\hbar}{m} = \alpha \frac{m_{\rm P}^2}{m}$, with $\alpha \sim 1$
- Self-interacting field : $M_{\rm max} \sim \left(\frac{\lambda}{4\pi}\right)^{1/2} \frac{m_{\rm P}^2}{m} \times \frac{m_{\rm P}}{m}$

$m_{\rm P} = \sqrt{\hbar} = \sqrt{\hbar c/G} = 2.18\ 10^{-8}\ {\rm kg}$: Planck mass

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m	$M_{\max}$ (free field)	$M_{\max}$ (self-interacting field, $\lambda = 1$)
$125\ \mathrm{GeV}$ (Higgs)	$2\ 10^{9}\ \mathrm{kg}$	$2\ 10^{26}\ \mathrm{kg}$
$1\ \mathrm{GeV}$	$3\ 10^{11}\ \mathrm{kg}$	$2\ M_{\odot}$
$0.5\ \mathrm{MeV}$	$3\ 10^{14}\ \mathrm{kg}$	$5\ 10^{6}\ M_{\odot}$

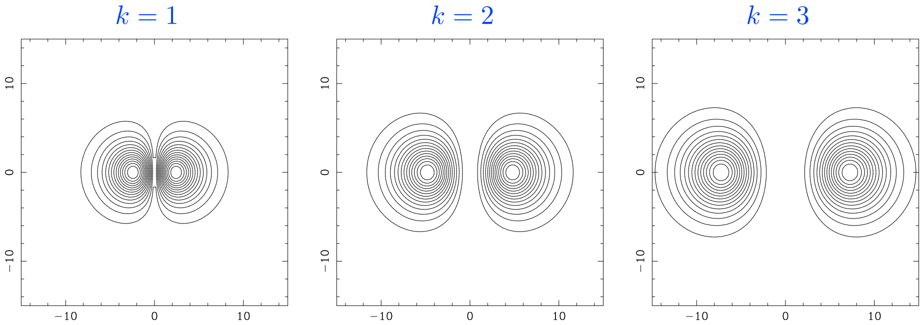
Rotating boson stars

Solutions computed by means of Kadath [Grandclément, JCP **229**, 3334 (2010)]

http://kadath.obspm.fr/

→ *see Philippe Grandclément's talk*

Isocontours of $\Phi_0(r, \theta)$ in the plane $\varphi = 0$ for $\omega = 0.8 \frac{m}{\hbar}$:



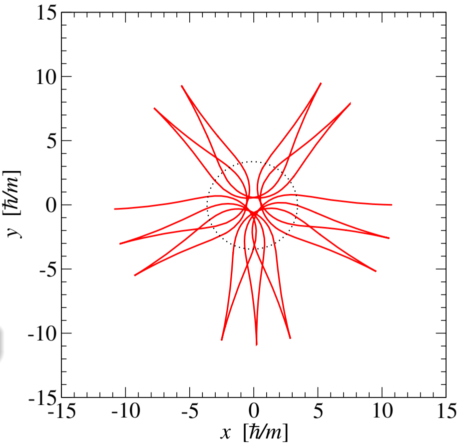
[Granclément, Somé & Gourgoulhon, PRD **90**, 024068 (2014)]

Initially-at-rest orbits around rotating boson stars

Orbit with a rest point around a rotating boson star based on the scalar field $\Phi = \Phi_0(r,\theta)e^{i(\omega t + k\varphi)}$
with $k = 2$ and $\omega = 0.75\, m/\hbar$

Orbit = timelike geodesic computed by means of Gyoto

$\Longrightarrow$ strong Lense-Thirring effect



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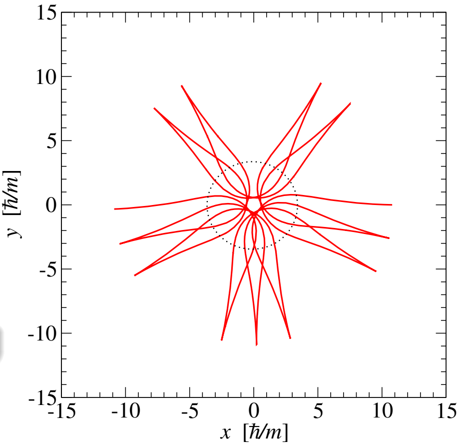
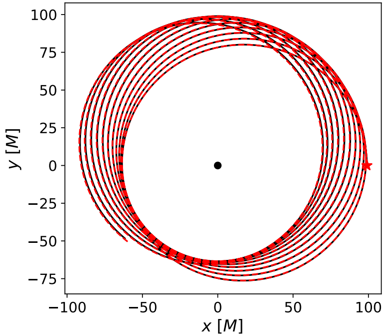


[Grandclément, Somé & Gourgoulhon, PRD **90**, 024068 (2014)]

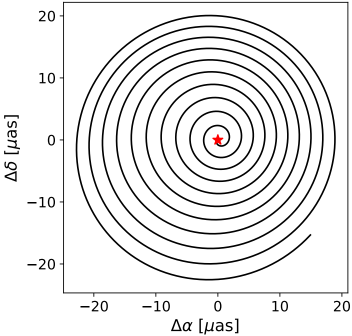
No equivalent in Kerr spacetime

Comparing orbits with a Kerr BH

Same reduced spin for the boson star and the Kerr BH : $a = 0.802\,M$
Boson star (BS) : $k = 1$ and $\omega = 0.8\,m/\hbar$
Orbit with pericenter of $60\,M$ and apocenter of $100\,M$



The two orbits

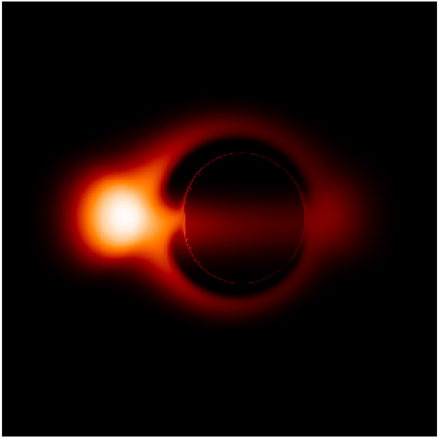


Difference between the BS orbit and the BH one for Sgr A*

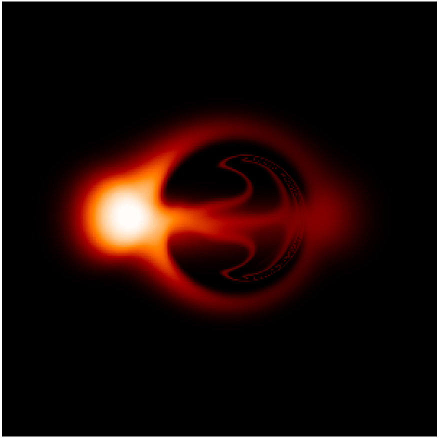
[Grould, Meliani, Vincent, Grandclément & Gourgoulhon, CQG **34**, 215007 (2017)]

Image of an accretion torus : comparing with a Kerr BH

Kerr BH $a/M = 0.9$



Boson star $k = 1,\ \omega = 0.70\, m/\hbar$



[Vincent, Meliani, Grandclément, Gourgoulhon & Straub, CQG **33**, 105015 (2016)]

Hairy black holes

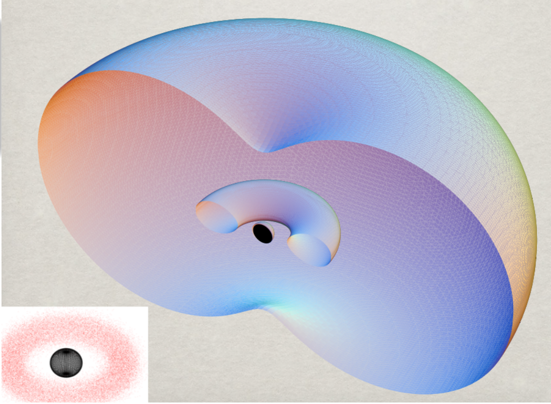
Herdeiro & Radu discovery (2014)

A black hole can have a complex scalar hair

Stationary axisymmetric configuration with a self-gravitating massive complex scalar field Φ and an event horizon

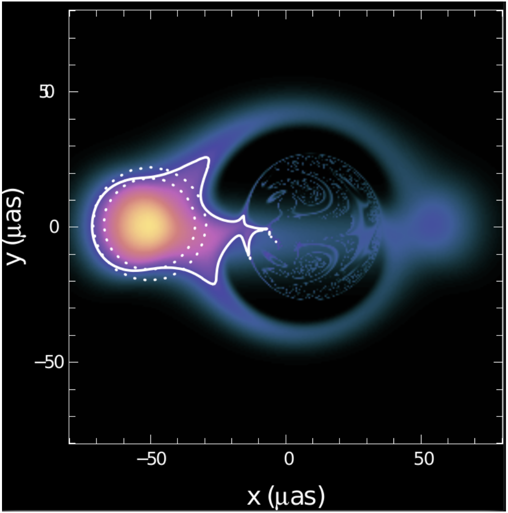
$$\Phi(t, r, \theta, \varphi) = \Phi_0(r, \theta) e^{i(\omega t + k\varphi)}$$

$$\omega = k\Omega_{\rm H}, \quad k \in \mathbb{N}$$



[Herdeiro & Radu, PRL **112**, 221101 (2014)]

Hairy black hole

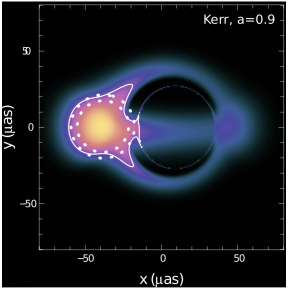


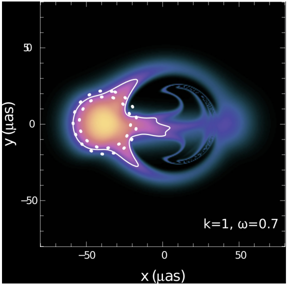
Accretion torus around a scalar-field-hairy rotating black hole

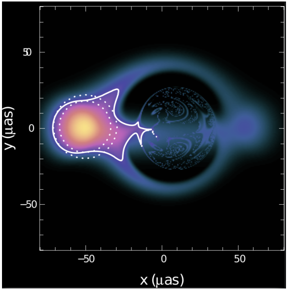
[Vincent, Gourgoulhon, Herdeiro & Radu, Phys. Rev. D **94**, 084045 (2016)]

Alternatives to the Kerr black hole

Kerr black hole	boson star [[1]]	hairy black hole [[2]]
$a/M = 0.9$	$k=1,\ \omega=0.7\, m/\hbar$	$a/M = 0.9$







Boson star:

Kadath → metric

Gyoto → ray-tracing

Hairy black hole:

HR code → metric

(via Lorene)

Gyoto → ray-tracing

[[1] Vincent, Meliani, Grandclément, Gourgoulhon & Straub, Class. Quantum Grav. **33**, 105015 (2016)]

[[2] Vincent, Gourgoulhon, Herdeiro & Radu, Phys. Rev. D **94**, 084045 (2016)]

Conclusions and perspectives

Black hole physics is entering a new observational era, with the advent of high-angular-resolution telescopes and gravitational wave detectors, which provide unique opportunities to test general relativity in the strong field regime, notably by searching for some violation of the *no-hair theorem*.

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To conduct these tests, it is necessary to conduct studies of theoretical alternatives of the Kerr black hole, like *boson stars* and *hairy black holes*.

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To conduct these tests, it is necessary to conduct studies of theoretical alternatives of the Kerr black hole, like *boson stars* and *hairy black holes*.

Work in progress

- rotating *regular* black holes [F. Lamy et al.]
- rotating black holes in cubic Galileon gravity [K. Van Aelst et al.]