4

BINARY PULSARS:

Motion, Timing and Use as Probes of Relativistic Gravity

Some relevant references:

2.5PN dynamics: Damour Deruelle PLA 87, 81 (1981); CRAS II, 293 537 (1981)
293 877 (1981)
Damour CRAS II 294, 1355 (1982).
in Gravitational Radiation ed. N. Deruelle T. Piran, North Holland (1983), p 59
PRL 51, 1019 (1983)
see also Blanchet Faye, Ponsot PRD 58, 124002 (1998)
and for ADM approach: Schäfer, Ann Phys 161, 81 (1985); GRG 18, 255 (1986)

Solving 1PN and 2PN Motion: Damour PRL 51, 1019 (1983); Damour Deruelle AIHP 43, 107 (1985)
Damour Schäfer Nuovo Cim. 101B, 127 (1988)

Variation of constants: PRL 51, 1019 and Damour Gopakumar Iyer PRD 70, 064028 (2004)

Timing Formula: Damour Deruelle AIHP 44, 263 (1986)

Parametrized Post Keplerian Formalism: Damour Taylor PRD 45, 1840 (1992)

Binary Pulsar Tests: Damour Schäfer PRL 66, 2549 (1991)
Taylor et al. Nature 355, 132 (1992). Weisberg Taylor astro-ph/0407149
see also Stairs' Living Review → Stairs et al. ApJ 581, 501 (2002); Kramer et al. Science (2006)
Damour Esposito-Farèse. PRL 70, 2220 (1993); CQG 9 2093 (1992);
Phys Rev D 54, 1474 (1996); 58, 042001 (1998); ... (2006)

Short Review of Exp. Tests: see chapter 18 of Particle Data Group Review: http://pdg.lbl.gov/

$\boxed{4.1}$ Binary pulsars and relativistic gravity

EXPERIMENT	THEORY
Discovery of PSR 1913+16 Hulse, Taylor '74	
	Spin-orbit effects Damour, Ruffini '74
	Energy loss in gravit. waves Wagoner '75
	Dipole radiation as test of gravity Eardley '75, Will, Eardley '77
Observation of secular acceleration of the orbital motion, $\dot{P}_b$ Taylor, Fowler, McCullough '79 Taylor, Weisberg '82	Relativistic "Timing Formula" Blandford, Teukolsky '76 ... Damour Deruelle '86
	Relativistic dynamics of compact objects $a = \frac{Gm}{r^2}\left(1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} + \frac{v^5}{c^5}\right)$ Damour Deruelle '81; Damour '82 '83
	Strong-field tests of gravity and Parametrized Post-Keplerian formalism Damour '87; Damour Taylor '92
	Spin-orbit and NS inertia moments Damour Schäfer '88
Discovery of PSR 1534+12 Wolszczan '91	Tests of Strong Equivalence Principle Damour Schäfer '91
Experimental constraints on strong-field relativistic gravity Taylor, Wolszczan, Damour, Weisberg '92	Non-perturbative strong-field effects in tensor-scalar gravity Damour Esposito-Farèse '93
Observation of spin-orbit effects Kramer '98; Weisberg, Taylor '02	
Discovery of Double Binary Psr 0737-3039AB Burgay et al '03; Lyne et al '04	

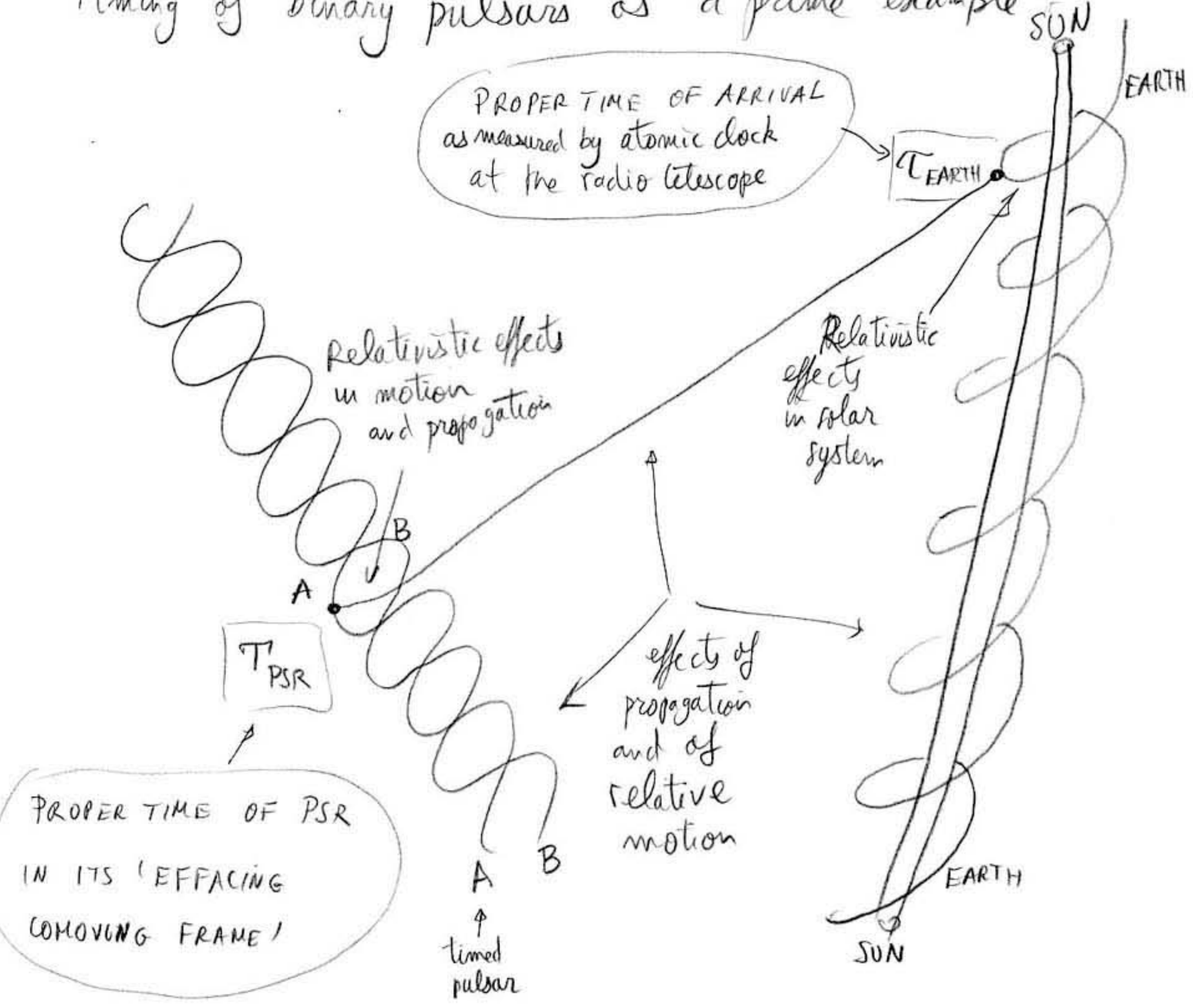
$\boxed{4.2}$ Binary pulsars and the physical meaning of General Relativity

? Physical meaning of GR with its 'general covariance', 'coordinates', ...

- Einstein confused for several years
- 'Problem of motion' marred by 'coordinate ambiguities' for years

$\boxed{\text{GR defines its own interpretation}}$

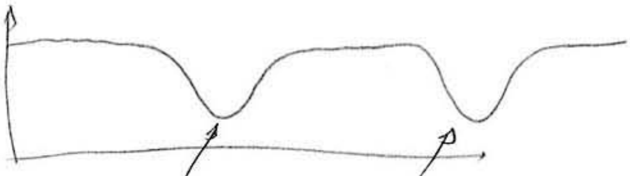
Timing of binary pulsars as a prime example



4.3 Motion of binary pulsars: strong fields, grav. waves

Binary Pulsar

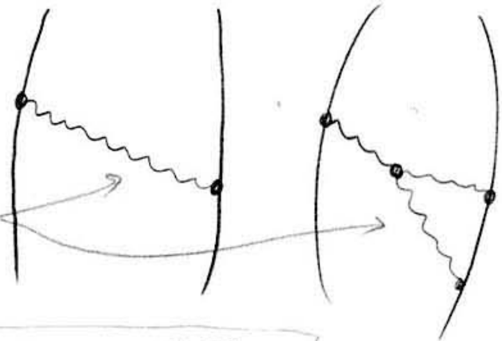
- strongly self-gravitating



need multi-chart approach

- high timing accuracy

need to carefully take into account the



PROPAGATION OF GRAV. INTERACTION AT FINITE VELOCITY: C

- Text book 'quadrupole formula' approach is unsatisfactory
- Often said that binary pulsars are only an indirect proof of $\exists$ of G Waves.

Actually, when done more correctly (, Eqs of Motion) binary pulsars are a direct proof of the propagation of the gravitational interaction between A and B at velocity c and with spin 2 propagator.

+ confirmations of the fully non linear structure (strong-field effects)

$\boxed{4.4}$ Relativistic dynamics of binary systems

Matched Multi-chart (+ Analytically Continued Skeletonized) approach

$\Rightarrow$ Eqs of Motion (say in harmonic coordinates : Damour Deruelle '81 ; Damour '82)

$$\boxed{2.5\ \text{PN}} \quad \frac{d^2 z_A^i}{dt^2} = \underbrace{A_0^{Ai}(\vec{z}_A - \vec{z}_B)}_{\text{Newton}} + \frac{1}{c^2} A_2^{Ai}(\vec{z}_A - \vec{z}_B, \vec{v}_A, \vec{v}_B) + \frac{1}{c^4} A_4^{Ai}(z_A - z_B, v_A, v_B) + (\text{spin-orbit terms}) + \underbrace{\frac{1}{c^5} A_5^i(z_A - z_B, v_A, v_B)}_{\text{needed to explain PSR observations}} + \cdots$$

? How to solve this complicated dynamics

(1) Consider the time-symmetric $\boxed{2\,\text{PN}}$ dynamics

$$\frac{d^2 \vec{z}_A}{dt^2} = \vec{A}_0^A + \frac{1}{c^2} \vec{A}_2^A + \frac{1}{c^4} \vec{A}_4^A$$

(2) Exploit the conserved quantities at 2 PN to solve 2 PN dynamics

(3) Use Lagrange method of Variation of Constants of 2PN dynamics to compute the effect of $\vec{A}_5$, and spin-orbit terms.

$\boxed{4.5}$ Action, Symmetries and Conserved Quantities at 2 PN

From EoM $\rightarrow$ brute force construction of $\overbrace{\text{Lagrangian}}^{2\,PN}$ (Damour, Deruelle '81)

$$\boxed{L^{2PN}(\vec{z}_A, \vec{z}_B, \vec{v}_A, \vec{v}_B, \vec{a}_A, \vec{a}_B) = L_0(z, v) + \frac{1}{c^2} L_2(z, v) + \frac{1}{c^4} L_4(z, v, a)}$$

velocities

accelerations $\vec{a}_A = \frac{d^2 \vec{z}_A}{dt^2}$

Newton

$$\sum \frac{1}{2} m_A \vec{v}_A^{\,2} + \frac{1}{2} \sum_{A \neq B} \frac{G m_A m_B}{|\vec{z}_A - \vec{z}_B|}$$

Lorentz Droste '17
Fichtenholz '50

$$\boxed{L_4 \sim v^6 + v^4 \frac{G}{r} + v^2 \frac{G^2}{r^2} + \frac{G^3}{r^3} + G a v^2}$$

numerically similar if $a \sim \frac{G}{r^2}$

but not allowed to replace EoM in L

($\Rightarrow$ change in coordinate system
Schäfer '84, Damour Schäfer '85 ...)

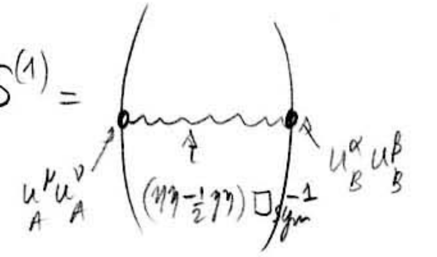
however $+ \, Q(a - A(z, v))$ do not contribute

$\geq$ quadratic ('double zero')

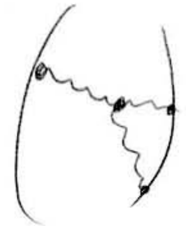
and can be used to reduce a-dependence to be linear in accelerations

Symmetries

Thinking of $S^{(1)} =$



$+ \; S^{(2)}$

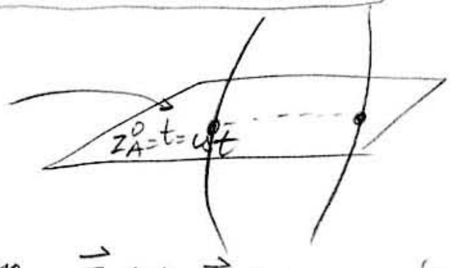


$+ \cdots$

manifestly Poincaré invariant

→ manifest invariance under $\boxed{z^{\mu\,\prime}_A = \Lambda^\mu{}_\nu z^\nu_A + b^\mu}$

Z doest not respect



→ more complicated action on $\vec{z}_A(t), \vec{z}_B(t)$ 'same instant'

N.B.: Poincaré symmetry = Residual symmetry of harmonic coords gauge-fixed Einstein action

$$S = \underbrace{\int R - \sum m_A \int ds_A}_{\text{locally diff-invariant } x' = f(x)} + k \frac{g_{\mu\nu}}{\sqrt{g}} \partial_\alpha(\sqrt{g} g^{\mu\alpha}) \partial_\beta(\sqrt{g} g^{\nu\beta})$$

gauge-fixing term: only invariant under $x'^\mu = A^\mu{}_\nu x^\nu + B^\mu$

+ boundary condition: $\boxed{\text{asymptotic flatness}}$

$g_{\mu\nu}(x) \longrightarrow \eta_{\mu\nu}$

⟶ restricts $A^\mu_\nu = \Lambda^\mu_\nu$

> In other gauges, eg. ADM, asymptotic flatness → Poincaré gp acts as global symmetry group. However, not realized as $z' = \Lambda z + b$ on worldlines

<u>Noether</u>

$$\overbrace{J_{ij} \qquad J_{0i}}^{J_{\mu\nu}} \qquad P_\mu = (P_0, P_i)$$

Poincaré group: 3 rotations + 3 boosts + 4 translations

$\rightarrow$ leaves $L(z, v, \underline{\underline{a}})$ invariant <u>modulo</u> some $\frac{d}{dt}(\cdots)$

$\rightarrow$ 10 Noetherian conserved quantities (see Damour Deruelle '81)

$$\boxed{\begin{aligned}
&\text{(total energy)} && E^{2PN}(z, v, a) = \sum_A \left(p_{Ai}\, v_A^i + q_{Ai}\, a_A^i \right) - L_{2PN}(z, v, a) \\
&\text{(lin. momentum)} && P_i^{2PN}(z, v, a) = \sum_A p_{Ai} \\
&\text{(angul. momentum)} && J_{ij}^{2PN}(z, v, a) = \sum_A \left(z_A^i\, p_{Aj} - z_A^j\, p_{Ai} + v_A^i\, q_{Aj} - v_A^j\, q_{Ai} \right) \\
&\text{(center of mass conserved quantity)} && K^i_{2PN}(z, v, a; t) = G^i_{2PN}(z, v) - t \,.\, P^i
\end{aligned}}$$

$$G^i_{2PN} = \sum_A \left(m_A + \frac{1}{c^2}\left(m_A \vec{v}_A^{\,2} - \frac{1}{2}\frac{G m_A m_B}{r_{AB}} \right) + \frac{1}{c^4}\left(v^4 + \frac{v^2 G}{r} + \frac{G^2}{r^2} \right) \right) z_A^i - \frac{7}{4c^4} G m_A m_B \left(\vec{n}_{AB} \cdot (\vec{v}_A + \vec{v}_B) \right) (v_A^i - v_B^i)$$

$\boxed{4.6}$ Variation of 2PN conserved quantities under 2.5PN dynamics

after reduction

$$\frac{d}{dt} C^{2PN}(z, v) = \sum_A \frac{\partial C^{2PN}}{\partial z_A^i} v_A^i + \frac{\partial C^{2PN}}{\partial v_A^i} \left(A_0^{Ai} + \frac{1}{c^2} A_2^{Ai} + \frac{1}{c^4} A_4^{Ai} + \frac{1}{c^5} A_5^{Ai} \right)$$

$$= \sum_A \frac{\partial C^{2PN}}{\partial v_A^i} \frac{1}{c^5} A_5^{Ai} + O\left(\frac{1}{c^6}\right)$$

$$= \sum_A \frac{\partial C^{Newt}}{\partial v_A^i} \frac{1}{c^5} A_5^{Ai} + O\left(\frac{1}{c^6}\right)$$

Insert explicit expression of

$$\boxed{A_5^{Ai} = \frac{4}{5} \frac{G^2 m_A m_B}{r_{AB}^3} \left[v_{AB}^i \left(-v_{AB}^2 + \frac{2Gm_A}{r_{AB}} - \frac{8Gm_B}{r_{AB}} \right) + n_{AB}^i (\vec{n}_{AB} \cdot \vec{v}_{AB}) \underbrace{[\ \ ]}_{\left[3 v_{AB}^2 - 6\frac{Gm_A}{r_{AB}} + \frac{52}{3} \frac{Gm_B}{r_{AB}} \right]} \right]}$$

after reduction

$$\frac{d}{dt} P_i^{2PN}(z,v) = -\frac{1}{c^5} \frac{d}{dt} P_i^5(z,v) + 0 + O(c^{-6})$$

$$\frac{d}{dt}\left(G_i^{2PN} - t P_i^{2PN} \right) = -\frac{1}{c^5} \frac{d}{dt}\left(G_i^5(z,v) - t P_i^5(z,v) \right) + 0 + O(c^{-6})$$

→ Allows one to define 6 2.5 PN conserved quantities

$$P_i^{2.5PN} \equiv P_i^{2PN} + \frac{P_i^5}{c^5} \; ; \quad K_i^{2.5PN} \equiv \left(G_i^{2PN} + \frac{G_i^5}{c^5} \right) - t P_i^{2.5PN}$$

→ Definition of <u>2.5 PN accurate center of mass frame</u>

$$\boxed{0 = P_i^{2.5PN} = K_i^{2.5PN} = G_i^{2.5PN}}$$

Explicit calculations with A^i_5 harmonic $\rightarrow$

$$\boxed{\begin{aligned}\frac{d}{dt}E^{2PN}(z,v) &= -\frac{1}{c^5}\frac{d}{dt}E^5(z,v) - \frac{G}{5c^5}I^{(3)}_{ij}I^{(3)}_{ij} + O\left(\frac{1}{c^6}\right)\\ \frac{d}{dt}J^{2PN}_{ij}(z,v) &= -\frac{1}{c^5}\frac{d}{dt}J^5_{ij}(z,v) - \frac{2G}{5c^5}\left(I^{(2)}_{is}I^{(3)}_{js} - I^{(2)}_{js}I^{(3)}_{is}\right) + O\left(\frac{1}{c^6}\right)\end{aligned}}$$

where $\boxed{I_{\langle ij \rangle} = \sum_A m_A z_A^{\langle i} z_A^{j \rangle}}$

Schwarzschild masses of A, B

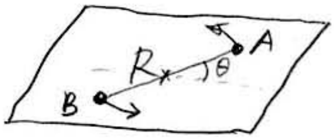
coincides with expected 'quadrupolar' losses of energy and ang. mom. in GWaves at ∞

Actually, this agreement is comforting for showing that A^i_5 drains out of the binary system the expected losses (so that A^i_5 = Radiation Damping = Radiation Reaction) but is useless as far as computing the observable effects in the binary dynamics.

4.7 Lagrange method of 'variation of constants'

In 2.5PN center of mass frame

$\frac{d}{dt} J_{ij}^{2PN} \propto J_{ij}^{2PN} \Rightarrow$ Fixed Plane of Motion



In fixed plane → Polar coordinates for relative motion

$$\vec{Z} \equiv \vec{z}_A - \vec{z}_B = (R\cos\theta,\ R\sin\theta,\ 0)$$

Consider first 2PN relative motion

→ conserved quantities

$$\boxed{\begin{aligned} c_1 &\equiv \left[E^{2PN}\right]_{com} \\ c_2 &\equiv \left[\vec{J}^{2PN}\right]_{com} \end{aligned}}$$

$$\boxed{\begin{aligned} \left(\frac{dR}{dt}\right)^2 &= A + \frac{2B}{R} + \frac{C}{R^2} + \frac{D_1}{R^3} + \frac{D_2}{R^4} + \frac{D_3}{R^5} \equiv \mathcal{R}(R, c_1, c_2) \\ \frac{d\theta}{dt} &= \frac{H}{R^2} + \frac{I_1}{R^3} + \frac{I_2}{R^4} + \frac{I_3}{R^5} \equiv \mathcal{G}(R, c_1, c_2) \end{aligned}}$$

where, e.g.,

$$\begin{cases} A = 2E^{2PN}\left(1 + \frac{3}{2}(3\nu - 1)\frac{E}{c^2} + \frac{\cdots}{c^4}\right) = 2c_1\left(1 + \frac{3}{2}(3\nu-1)\frac{c_1}{c^2} + \frac{f(c_1,c_2)}{c^4}\right) \\ \vdots \\ H = J^{2PN}\left(1 + (3\nu-1)\frac{E^{2PN}}{c^2} + \frac{\cdots}{c^4}\right) = c_2\left(1 + (3\nu-1)\frac{c_1}{c^2} + \frac{g(c_1,c_2)}{c^4}\right) \end{cases}$$

with

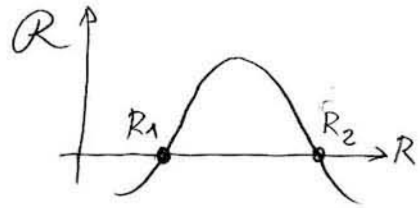
$$\boxed{\nu \equiv \frac{m_A m_B}{(m_A + m_B)^2} = \frac{\mu}{M} \qquad 0 \leqslant \nu \leqslant \frac{1}{4}}$$

Structure of 2PN solution

Introduce TWO ANGULAR VARIABLES $\ell, m \in [0, 2\pi)$

$$\boxed{\begin{aligned} R &= S(\ell; c_1, c_2) \\ \theta &= m + W(\ell; c_1, c_2) \\ \ell &= 2\pi \frac{t}{P(c_1,c_2)} + c_0 \\ m &= 2\pi \frac{t}{Q(c_1,c_2)} + c_3 \end{aligned}}$$

hyperelliptic functions of ℓ; periodic on $[0, 2\pi]$



$$P(c_1,c_2) = 2\int_{R_1}^{R_2} \frac{dR}{\sqrt{\mathcal{R}(R, c_1, c_2)}} \quad ; \quad \frac{P(c_1,c_2)}{Q(c_1,c_2)} = \frac{1}{\pi}\int_{R_1}^{R_2} dR \, \frac{\mathcal{G}(R, c_1, c_2)}{\sqrt{\mathcal{R}(R, c_1, c_2)}}$$

Structure of 2.5PN solution (Damour '83; Damour, Gopakumar, Iyer '04)

$$\boxed{\begin{aligned} R &= S(\ell; c_1(t), c_2(t)) \\ \theta &= m + W(\ell; c_1(t), c_2(t)) \\ \ell &= 2\pi \int_0^t \frac{dt'}{P(c_1(t'), c_2(t'))} + c_0(t) \\ m &= 2\pi \int_0^t \frac{dt'}{Q(c_1(t'), c_2(t'))} + c_3(t) \end{aligned}}$$

Same functions S, W, P, Q as before, but now

($c_\alpha \equiv (c_1, c_2, c_0, c_3)$
$\alpha = 1, 2, 0, 3$

vary with time accordingly with)

$$\boxed{\frac{dc_\alpha(t)}{dt} = \frac{1}{c^5} F_\alpha(\ell; c_\beta)}$$

$F_\alpha(\ell)$ — periodic in ℓ; $\propto \vec{A}_5$

4.8 Explicit form of 'first post-Keplerian' solution

1PN Lagrangian

(Damour Deruelle '85)

$$L = \sum_A \left(\frac{1}{2} m_A \vec{v}_A^{\,2} + \frac{1}{8} m_A \frac{\vec{v}_A^{\,4}}{c^2} \right) + \frac{G m_A m_B}{r_{AB}}$$

$$+ \frac{G m_A m_B}{2 c^2 r_{AB}} \left[3 \vec{v}_A^{\,2} + 3 \vec{v}_B^{\,2} - 7 (\vec{v}_A \cdot \vec{v}_B) - (\vec{n}_{AB} \cdot \vec{v}_A)(\vec{n}_{AB} \cdot \vec{v}_B) - \frac{G(m_A + m_B)}{r_{AB}} \right]$$

→ invariant under Poincaré group → 10 Noetherian quantities

$E^{1PN}, \vec{P}, \vec{J}, \vec{K}^{1PN}$ → 1PN center of mass frame

→ 1PN Lagrangian for relative motion in center of mass frame

$$\boxed{\hat{L}(\vec{R}, \vec{V}) = \frac{1}{2} \vec{V}^2 + \frac{GM}{R} + \frac{1}{8}(1-3\nu) \frac{\vec{V}^4}{c^2} + \frac{GM}{2Rc^2} \left[(3+\nu) \vec{V}^2 + \nu (\vec{N} \cdot \vec{V})^2 - \frac{GM}{R} \right]}$$

$\hat{L}$ divided by $\mu \equiv \frac{m_A m_B}{M}$

$\vec{R} \equiv \vec{z}_A - \vec{z}_B$

$\vec{V} \equiv \vec{v}_A - \vec{v}_B$

$M \equiv m_A + m_B$

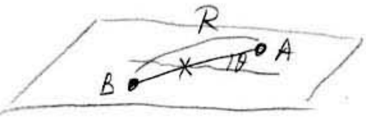
$\nu \equiv \frac{m_A m_B}{M^2} \equiv \mu / M$

conserved quantities

$$\begin{cases} \hat{E} = \vec{V} \cdot \dfrac{\partial \hat{L}}{\partial \vec{V}} - \hat{L} \\[2ex] \hat{\vec{J}} = \vec{R} \times \dfrac{\partial \hat{L}}{\partial \vec{V}} \end{cases}$$

AGR 4.13

In polar coordinates in plane of motion



$$\vec{R} = (R\cos\theta,\ R\sin\theta,\ 0)$$

$$\boxed{\left(\frac{dR}{dt}\right)^2 = \underbrace{A + \frac{2B}{R} + \frac{C}{R^2}}_{} + \frac{D}{R^3}} \ ; \quad \boxed{\frac{d\theta}{dt} = \underbrace{\frac{H}{R^2}}_{} + \frac{I}{R^3}}$$

as in Newtonian approx

but $A = 2\hat{E}\left(1 + \frac{\cdots}{c^2}\right)$

etc.

$\boxed{D = \mathcal{O}\left(\frac{1}{c^2}\right)}$

purely 1PN

as Newton

$H = \hat{J}\left(1 + \frac{\cdots}{c^2}\right)$

$\boxed{I = \mathcal{O}\left(\frac{1}{c^2}\right)}$

purely 1PN

NB: Contrary to usual Newtonian solution, one is not interested in eliminating t to get the orbit $\left(\frac{dR}{d\theta}\right)^2 = f(R)$.

One needs the motion: $(R(t),\ \theta(t))$

→ Conchoidal transformation trick

absorb $D = \mathcal{O}(c^{-2})$

into a constant shift δR: $\quad \delta R \frac{\partial}{\partial R}\left(A + \frac{2B}{R} + \frac{C}{R^2}\right) = -2\delta R\left(\frac{B}{R^2} + \frac{C}{R^3}\right)$

$$\boxed{R = \bar{R} + \frac{D}{2C}}$$

neglect $\delta R^2 = \mathcal{O}(c^{-4})$

modifies C 　 absorbs D

$$\boxed{\left(\frac{d\bar{R}}{dt}\right)^2 = A + 2\frac{B}{\bar{R}} + \frac{\bar{C}}{\bar{R}^2}} \qquad \bar{C} \equiv C - \frac{BD}{C}$$

as in Newtonian theory

→ can be solved by Kepler's parametrization

- Finally (after using another conchoidal trick for solving $d\theta/dt = \dots$) one can write explicit 1PN solution in '<u>QUASI-KEPLERIAN FORM</u>'

$$\boxed{n \equiv \frac{2\pi}{P_b}} \quad n(t - t_0) = u - e_t \sin u$$

angle $u \in [0, 2\pi]$ 'relativistic eccentric anomaly'

$$R \equiv r_{AB} = a_R (1 - e_R \cos u)$$

$$\theta - \theta_0 = (1+k) A_{e_\theta}(u)$$

$$A_e(u) \equiv 2 \arctan\left[\sqrt{\frac{1+e}{1-e}} \tan\frac{u}{2}\right]$$

- Differs from Newton only in

  - eccentricity $e \left\langle \begin{matrix} e_t \\ e_R \\ e_\theta \end{matrix} \right.$ $\Big\}$ explicitly computed in terms of $\hat{E}, \hat{J}$

  - θ motion multiplied by $1 + k$

<u>Periastron advance</u>

$u : 0 \to 2\pi \Rightarrow R_{min} \to R_{min} = a_R(1 - e_R)$ "periastron to periastron"

$t \to t + \frac{2\pi}{n} \Leftrightarrow \boxed{n = \frac{2\pi}{P_b}}$ periastron-to-periastron (binary) period

$\theta \to \theta + 2\pi(1+k) \Rightarrow \boxed{k = \left[\frac{\Delta\theta}{2\pi}\right]_{\text{orbital period}}}$

$$\boxed{k = \frac{3}{c^2}\left(\frac{GM}{\hat{J}}\right)^2 + O\left(\frac{1}{c^4}\right)}$$

For binary pulsars : need higher accuracy on k

2PN + spin-orbit effects : need <u>hyperelliptic (complete) integrals</u>
(evaluated à la Sommerfeld by contour integrals : Damour Schäfer '88)

$$\boxed{k = \frac{3}{c^2 j^2}\left[1+\left(\frac{5}{2}-\nu\right)\frac{\hat{E}}{c^2} + \left(\frac{35}{4}-\frac{5}{2}\nu\right)\frac{1}{c^2 j^2}\right] + \sum_A \frac{B_A}{\sin^2 i}\left[(1-3\sin^2 i)(\vec{k}\cdot\vec{s}_A) - \cos i\, \vec{K}_0\cdot\vec{s}_A\right]}$$

$j \equiv \frac{\hat{J}}{GM} = \frac{J}{GM\mu}$ 2PN correction spin-orbit contributions

- Need also the separate motions of A and B :

conchoidal transformation again

$$\boxed{\begin{aligned} |\vec{z}_A-\vec{z}_B| &= R = a_R\,(1-e_R\cos u) \\ |\vec{z}_A-\vec{z}_{cm}| &= r_A = a_r\,(1-e_r\cos u) \\ |\vec{z}_B-\vec{z}_{cm}| &= r_B = a_{r'}\,(1-e_{r'}\cos u) \end{aligned}}$$

again like in Keplerian solution,

but with

$$\underline{\text{eccentricity } e} \begin{cases} e_t \\ e_\theta \\ e_R \\ e_r \\ e_{r'} \end{cases} \text{all explicitly computed in terms of } \hat{E}, \hat{J}$$

4.9 Radial motion at 2.5PN approximation

At 2PN, $R = r_{AB}$ is a periodic function of the 'true anomaly' ℓ

$$\boxed{R = S(\ell; c_1, c_2)} \quad \text{with} \quad \boxed{\ell^{2PN} = 2\pi \frac{t}{P(c_1, c_2)} + c_0}$$

defined by — constant at 2PN

$$\boxed{\begin{aligned} R &= a_R(c_1,c_2)\left(1 - e_R(c_1,c_2)\cos u\right) \\ \ell &= u - e_t(c_1,c_2)\sin u + \frac{f(c_1,c_2)}{c^4}\sin A_{e_\theta}(u) + \frac{g(c_1,c_2)}{c^4}\left(A_{e_\theta}(u) - u\right) \end{aligned}}$$

2PN corrections — Damour Schäfer '88, Schäfer Wex '93

Lagrange variation of constants: $\boxed{\dfrac{dc_\alpha}{dt} = \dfrac{1}{c^5} F_\alpha(\ell; c_1, c_2)}$

Radial motion at 2.5PN:

$$\boxed{\begin{aligned} R &= a_R(c_1(t), c_2(t))\left(1 - e_R(c_1(t), c_2(t))\cos u\right) \\ \ell &= u - e_t(c_1,c_2)\sin u + \frac{1}{c^4} \cdots c_1(t) \cdots c_2(t) \end{aligned}}$$

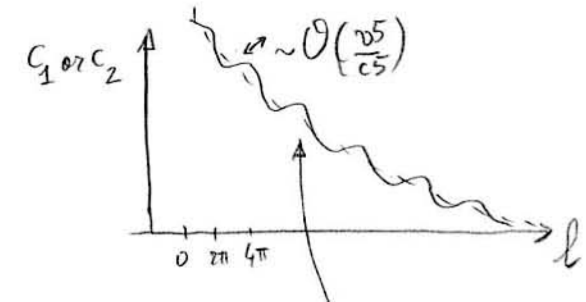
but with $\boxed{\ell^{2.5PN} = 2\pi \displaystyle\int_0^t \frac{dt'}{P(c_1(t'), c_2(t'))} + c_0(t)}$

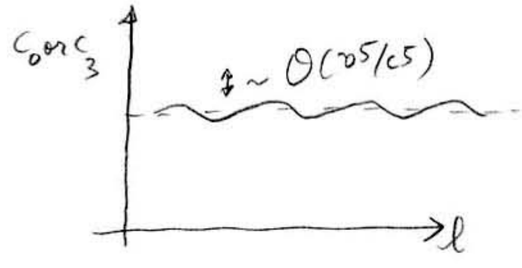
CRUCIAL nontrivial time dependence, because periastrons are reached for $\ell = 0, 2\pi, 2\times 2\pi, \ldots, N \times 2\pi, \ldots$

$\boxed{4.10}$ Evolution of the $c_\alpha(t)$ under $\vec{A}_5$

$$\frac{dc_\alpha}{d\ell} = \frac{1}{c^5}\widetilde{F}_\alpha(\ell; c_1, c_2) \propto A_5^i$$

- ℓ-even for $\alpha = 1, 2$
- ℓ-odd for $\alpha = 0, 3$





'secular' evolution
given by the <u>averaged system</u>

$$\frac{d}{d\ell}\bar{c}_\alpha = \frac{1}{c^5}\left\langle \widetilde{F}_\alpha(\ell; c_1, c_2)\right\rangle_\ell^{\text{with } c_1, c_2 \text{ fixed}}$$

$$c_\alpha = \bar{c}_\alpha + \tilde{c}_\alpha \quad \mathcal{O}\left(\frac{v^5}{c^5}\right)$$

<u>negligible in pulsar observables</u>
(but possibly measurable in GW phasing of eccentric binaries, Damour Gopakumar Iyer '04)

Over times $\ll$ time-scale of radiation reaction A_5, i.e. $\frac{P_b}{(v/c)^5}$

$$\bar{c}_{\alpha=1,2} = \bar{c}_\alpha^0 + \dot{c}_\alpha^0\, t$$

$$\Rightarrow \ell \overset{2.5PN}{\simeq} 2\pi \int_0^t \frac{dt'}{P(c_1^0 + \dot{c}_1^0 t', c_2^0 + \dot{c}_2^0 t')} + \bar{c}_0^0 + \cancel{\dot{c}_0^0 t} + \frac{\text{periodic}}{c^5}$$

Simplification

orbital frequency "ω"

$$\frac{2\pi}{P(c_1,c_2)} \equiv n(c_1,c_2) = \frac{\left(-2\hat{E}\right)^{3/2}}{GM}\left[1 + \frac{15-\nu}{4}\frac{\hat{E}}{c^2} + \frac{1}{c^4} f(\hat{E},\hat{J})\right]$$

At N and 1PN only $c_1 = \hat{E}$ enters

c_1 AND c_2 only at 2PN

$$\rightarrow \quad P(\bar{c}_1^0 + \dot{c}_1^0 t') \simeq P_0 + \dot{P}_0 t' \quad \text{with } \dot{P}_0 = \frac{\partial P_0}{\partial c_1^0}\dot{c}_1^0$$

$$= \frac{\partial P}{\partial E}\left\langle \frac{dE}{dt}\right\rangle_{\text{orbital period}}$$

$$\boxed{\dot{P}_0 = \frac{\partial P}{\partial E}\left\langle \vec{v}_A \cdot \frac{\vec{A}_5^A}{c^5} + \vec{v}_B \cdot \frac{\vec{A}_5^B}{c^5}\right\rangle}$$

direct consequence of 

$+ \cdots$

direct calculation from A_5

$$\boxed{\dot{P}_0 = -\frac{192\pi}{5c^5}\frac{1 + \frac{73}{24}e^2 + \frac{37}{96}e^4}{(1-e^2)^{\frac{7}{2}}}(GMn)^{\frac{5}{3}}\nu}$$

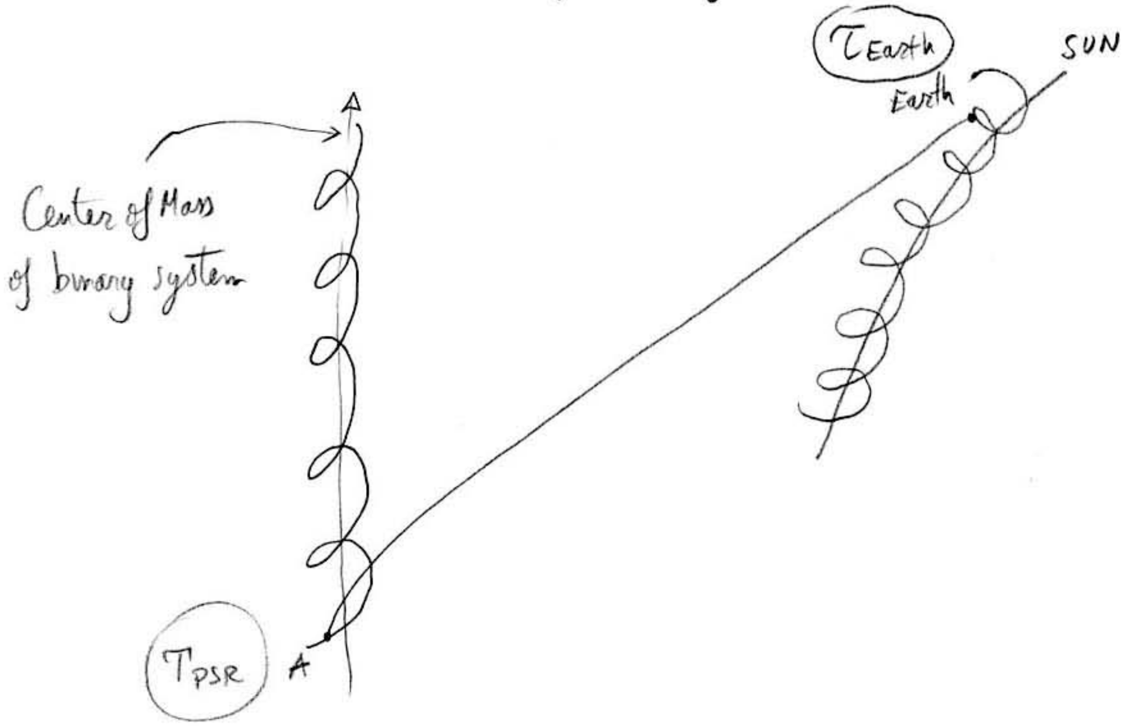
$$\boxed{l \simeq 2\pi\int_0^t \frac{dt'}{P_0 + \dot{P}_0 t'} + \bar{c}_0^0 \simeq 2\pi\left[\frac{t}{P_0} - \frac{1}{2}\dot{P}_0\left(\frac{t}{P_0}\right)^2\right] + \bar{c}_0^0}$$

$\rightarrow$ date t_N of N^{th} periastron passage : $\quad l_N = 2\pi N$

$$\boxed{t_N \simeq t_0 + P_0 N + \frac{1}{2}P_0\dot{P}_0 N^2}$$

4.11 Relativistic timing of binary pulsars

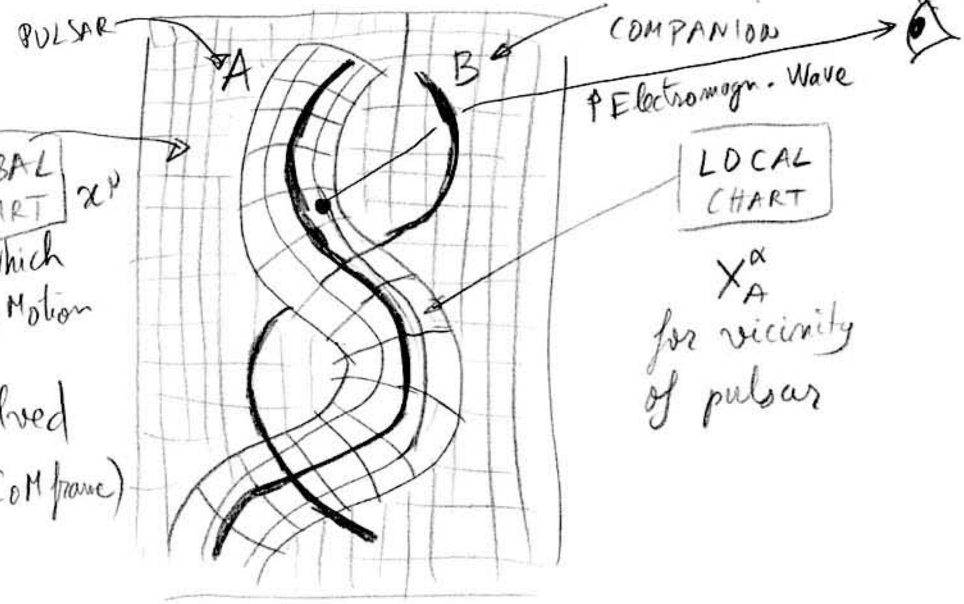
Blandford Teukolsky '76 ... Damour Deruelle '86



CHAIN OF TRANSFORMATIONS: $T_{PSR} \longrightarrow \tau_{EARTH}$

- Within binary system — focus on these
- Between binary system and solar system
  - Dispersion of radio waves $\sim D/f^2$
  - Variation of Doppler factor between CoM worldline and Barycenter of solar system
- Within solar system
  - travel time in solar system
  - gravitational delay (Shapiro)
  - variable redshift (Einstein) $d\tau_{Earth} / d\tau_{Barycenter}$

$\boxed{4.12}$ Various contributions to TIMING of PSR



need $$\boxed{x^\mu = z^\mu(T) + e^\mu_a(T)X^a + \tfrac{1}{2} f^\mu_{ab}(T) X^a X^b + \dots}$$

('PROPER TIME' of PULSAR FRAME) ('ASYMPTOTIC FRAME')

$$ds^2 = -\left(1-\frac{2GM}{c^2R}\right)c^2dT^2 + \left(1+\frac{2GM}{c^2R}+\dots\right)\delta_{ij}\,dX^i dX^j$$

<u>Matching</u> inner and outer problem $\rightarrow$

- $$\boxed{T = \frac{1}{c}\int\sqrt{-\bar{g}_{\mu\nu}(z^\lambda_A)\,dz^\mu_A\,dz^\nu_A}} \simeq \int dt\sqrt{1-\frac{2Gm_B}{c^2r_{AB}}-\frac{\vec{v}_A^{\,2}}{c^2}} \simeq \int dt\left(1-\frac{Gm_B}{c^2r_{AB}}-\frac{\vec{v}_A^{\,2}}{2c^2}\right)$$

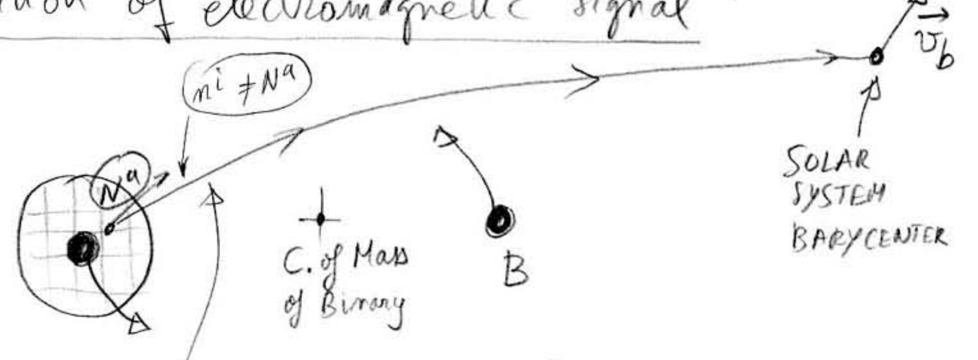
'A-EXTERNAL METRIC' (see end of lecture 2)

- $e^0_a = \frac{v_a}{c}\left[1+\frac{1}{2}\frac{\vec{v}^2}{c^2}+\frac{3Gm_B}{c^2r_{AB}}+\dots\right] \longrightarrow$ ANGLE between $N^a\big|^{\text{emission}}_{X^\alpha}$ and $n^i\big|_{x^\mu}$

direction emitted radio wave

ABERRATION $$\boxed{\left[\delta^a_i + O\left(\frac{v^2}{c^2}\right)\right] n^i \simeq N^a(T) + \frac{v^a_A}{c} - N^a\left(\frac{\vec{N}\cdot\vec{v}_A}{c}\right)}$$

Propagation of electromagnetic signal



NULL GEODESIC : $$0 = ds^2 = \overbrace{g_{\mu\nu}(x^\lambda)\,dx^\mu dx^\nu}^{\text{after}} = \overbrace{G^A_{\alpha\beta}(X_A)\,dX^\alpha_A dX^\beta_A}^{\text{beginning}}$$

$$-\left(1-\frac{2U}{c^2}\right)c^2 dt^2 + \left(1+\frac{2U}{c^2}\right)d\vec{x}^2 = 0$$

$$dt \simeq \frac{1}{c}\left(1+\frac{2U}{c^2}\right)|d\vec{x}|$$

$$\int_{t_{match}}^{t_{arrival}} dt \simeq \frac{1}{c}\int_{t_m}^{t_a}|d\vec{x}| + \frac{2}{c^3}\int_{t_m}^{t_a}\left(\frac{Gm_A}{|\vec{x}-\vec{z}_A|} + \frac{Gm_B}{|\vec{x}-\vec{z}_B|}\right)|d\vec{x}|$$

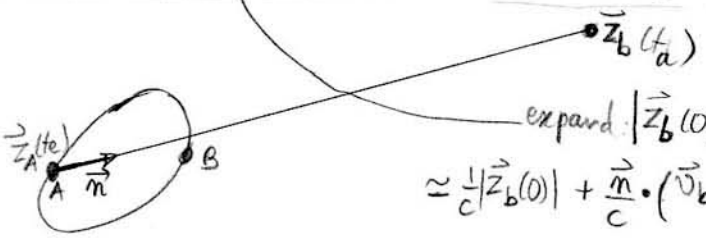
along straight line

expanded in tidal series, $G(T) \sim \frac{Gm_B}{r_{AB}}$ absorbed in $x^\mu = f^\mu(X_A)$

+ similar contributions in local frame with $\frac{Gm_A}{|\vec{X}_A|}$

Matching

$$\boxed{t_{arriv} = t_{emiss} + \frac{1}{c}|\vec{z}_b(t_a) - \vec{z}_A(t_e)| + \frac{2}{c^3}\int_{t_e}^{t_a}\frac{Gm_B}{|\vec{x}-\vec{z}_B|}d|\vec{x}|}$$

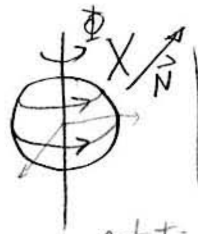


expand $|\vec{z}_b(0) + \vec{v}_b t_a + \text{accel} - \vec{z}_A(t_e)|$

$$\simeq \frac{1}{c}|\vec{z}_b(0)| + \frac{\vec{n}}{c}\cdot\left(\vec{v}_b t_a - \vec{z}_A(t_e)\right) + \mathcal{O}\left(\frac{v^2}{|z_b|} + \text{accel}\right)$$

$\boxed{4.13}$ The DD Timing formula (Damour Deruelle '86)

PULSAR PROPER TIME T



$$\boxed{\frac{\Phi}{2\pi} \equiv N = N_0 + \nu_p T + \frac{1}{2}\dot{\nu}_p T^2 + \frac{1}{6}\ddot{\nu}_p T^3}$$

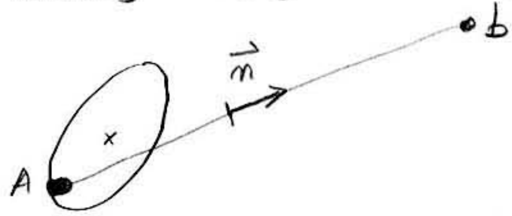
Rotational phase — # of turns — PSR frequency $\nu_p \equiv \dfrac{1}{P_p}$

integer $N \longleftrightarrow T_N =$ proper time of emission of N^{th} pulse, corrected for aberration

then

$$\boxed{D \cdot \tau_a = T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)}$$

Doppler factor — Proper time of arrival at solar barycenter (corrected for dispersion + solar-system analogs of $\Delta_R, \Delta_E, \Delta_S$) — ROEMER time delay — EINSTEIN time delay — SHAPIRO time delay — ABERRATION time delay



$$D = \frac{1 - \vec{n}\cdot\vec{v}_b/c}{\sqrt{1 - \vec{v}_b^{\,2}/c^2}}$$

$$\Delta_R = -\frac{1}{c}\,\vec{n}\cdot\vec{z}_A$$

Time of flight across PSR orbit

$$\Delta_E = +\frac{1}{c^2}\int dt \left(\frac{Gm_B}{r_{AB}} + \frac{1}{2}\vec{v}_A^{\,2} - \left\langle \frac{Gm_B}{r_{AB}} + \frac{1}{2}\vec{v}_A^{\,2} \right\rangle \right)$$

after rescaling T by mean GR + SR effects in binary $\rightarrow$ modifies D

$$\Delta_S = +\frac{1}{c^3}\int_{t_e}^{t_a} \frac{2\,Gm_B\; d|\vec{x}|}{|\vec{x} - \vec{z}_B|}$$

$$\Delta_A = \frac{1}{2\pi\nu_p}\,\frac{\vec{v}_A\cdot(\vec{n}\times\vec{e}_3)}{c\,(\vec{n}\times\vec{e}_3)^2}$$

AGR 4.23

Orders of magnitude of timing effects

$$D = 1 + \mathcal{O}\left(\frac{v_b^{\text{solar}} - v_{\text{CoM}}^{\text{binary}}}{c}\right) \simeq 1 + \mathcal{O}(10^{-3})$$

can be exactly absorbed in some rescalings, except for its time-dependent piece $<$ accel. $\frac{v^2}{d}$

orbital effect

$$\frac{GM}{a} \sim v^2$$

$$v \sim n a \sim \frac{2\pi}{P_b} a$$

typically $\boxed{\frac{v}{c} \sim (\text{few}) \times 10^{-3}}$ $\boxed{\frac{a}{c} \sim \text{second}}$

$$\Delta_R \sim \frac{a}{c}\left[1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} + \cdots\right] \sim \frac{v}{c}\frac{P_b}{2\pi}\left[1 + \frac{v^2}{c^2} + \frac{v^4}{c^4} + \cdots\right]$$

$$\sim \frac{a}{c} + \frac{GM}{c^3} + \frac{v^2}{c^2}\frac{GM}{c^3}$$

$$\boxed{\frac{GM_\odot}{c^3} = 4.92549\ldots\ \mu s}$$

$$\sim \underline{\underline{s}} + \text{few } \underline{\underline{\mu s}} + 10^{-12}\,s$$

$$\Delta_E \sim \frac{v^2}{c^2}\frac{P_b}{2\pi} \sim \frac{v}{c}\Delta_R\left[1 + \frac{v^2}{c^2} + \cdots\right] \sim \underline{\underline{ms}} + 10^{-9}s$$

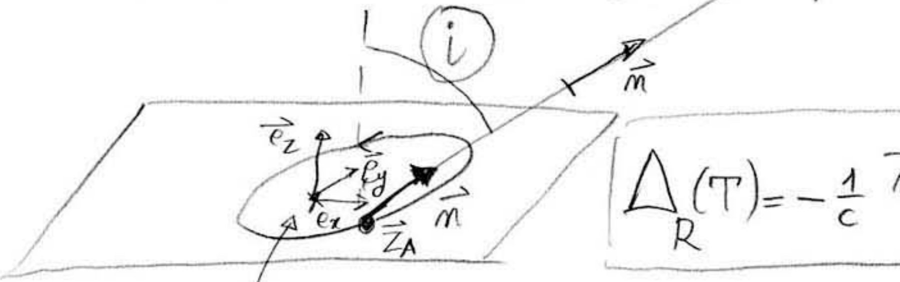
$$\Delta_S \sim \frac{v^3}{c^3}\frac{P_b}{2\pi} \sim \frac{v^2}{c^2}\Delta_R \sim \frac{GM}{c^3} \sim \underline{\underline{\mu s}} + 10^{-9}s$$ from $\mathcal{O}\left(\frac{v}{c}\right)$ corrections

$$\Delta_A \sim \frac{v}{c}\frac{P_b}{2\pi} \sim 10^{-3}\frac{P_p}{2\pi}$$

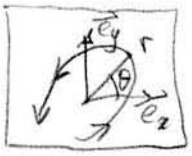
$\Rightarrow$ $\boxed{\text{1PK TIMING FORMULA ACCURATE ENOUGH}}$

Roemer time delay

AGR 4.24



orbital frame



t_{emission} in GLOBAL x^μ

$$\boxed{\Delta_R(T) = -\frac{1}{c}\,\vec{n}\cdot\vec{Z}_A(t)}$$

need to treat $\vec{Z}_A$ to $\frac{v^2}{c^2}$ accuracy

$$\vec{n} = -\sin i\,\vec{e}_y - \cos i\,\vec{e}_z$$

inclination $\boxed{0 \leq i < \pi}$

$$\vec{Z}_A = r_A \cos\theta\,\vec{e}_x + r_A \sin\theta\,\vec{e}_y$$

$$\boxed{\Delta_R = -\frac{1}{c}\vec{n}.\vec{Z}_A(t) = +\frac{1}{c}\sin i\; r_A\; \sin\theta}$$

$$r_A = a_r(1 - e_r \cos u) \qquad \theta = \omega_0 + (1+k)\,\underbrace{2\arctan\left[\sqrt{\frac{1+e_\theta}{1-e_\theta}}\tan\frac{u}{2}\right]}_{A_{e_\theta}(u)}$$

$$a_r = \left(\frac{m_B}{M} + \frac{O}{c^2}\right)a_R$$

where $\quad n(t - t_0) = u - e_t \sin u$

but $\quad T = \int^{t_e} dt\left(1 - \frac{Gm_B}{c^2 r_{AB}} - \frac{1}{2}\frac{\vec{v}_A^{\,2}}{c^2} + \langle\,\rangle\right) = t - \Delta_E$

$$\boxed{\Delta_E = \frac{Gm_B}{c^2}\left(1 + \frac{m_B}{M}\right)\int\frac{dt}{R} - \langle\,\rangle^{dt} = \frac{e}{n}\,\frac{Gm_B(1 + m_B/M)}{c^2 a_R}\,\sin u}$$

i.e. $\quad \boxed{t = T + \Delta_E = T + \gamma \sin u}$

At this stage

$$\boxed{t \to T} \Rightarrow \quad n(T - T_0) = u - \overset{\circ}{e}_T \sin u \qquad \boxed{x_B \equiv \frac{m_B}{M}}$$

$$\overset{\circ}{e}_T = e_t \left(1 + \frac{G m_B (1 + x_B)}{c^2 a_R} \right)$$

Can further simplify unpleasant ratios $\frac{1 - e_r \cos u}{1 - e_\theta \cos u}$ by redefining the relativistic eccentric anomaly u

$$\boxed{u \longrightarrow u^{new}} \; : \; n(T - T_0) = u^{new} - e_T \sin u^{new} \qquad e_T \equiv \overset{\circ}{e}_T + e_\theta - e_r$$

Final DD Timing Formula

$$N = N_0 + \nu_p T + \frac{1}{2} \dot{\nu}_p T^2 + \frac{1}{6} \ddot{\nu}_p T^3$$

$$D \, \tau_{\substack{\text{arrival} \\ \text{barycenter}}} = T + \Delta_R(T) + \Delta_E(T) + \Delta_S(T) + \Delta_A(T)$$

$$\Delta_R(T) = x \sin\omega \left[\cos u - e(1 + \delta_r) \right] + x \sqrt{1 - e^2 (1 + \delta_\theta)^2} \cos\omega \sin u$$

$$\Delta_E(T) = \gamma \sin u$$

$$\Delta_S(T) = -2 r \ln\left\{ 1 - e\cos u - s\left[\sin\omega(\cos u - e) + \sqrt{1-e^2} \cos\omega \sin u \right] \right\}$$

$$\Delta_A(T) = A\left\{ \sin\left[\omega + A_e(u)\right] + e\sin\omega \right\} + B\left\{ \cos\left[\omega + A_e(u)\right] + e\cos\omega \right\}$$

with

$$x = x_0 + \dot{x}(T - T_0), \quad e = e_0 + \dot{e}(T - T_0)$$

$$\omega = \omega_0 + k \, A_e(u)$$

$$\boxed{A_e(u) \equiv 2 \arctan\left[\sqrt{\frac{1+e}{1-e}} \tan\frac{u}{2} \right]}$$

$$\boxed{u - e \sin u = 2\pi \left[\frac{T - T_0}{P_b} - \frac{1}{2} \dot{P}_b \left(\frac{T - T_0}{P_b} \right)^2 \right]}$$

$\boxed{4.14}$ PARAMETRIZED POST-KEPLERIAN FORMALISM

(Damour Taylor '92)

- Fact: The mathematical structure of the DD timing formula

$$\tau_a = F[T_N;\ \underbrace{P_b, T_0, e_0, \omega_0, x_0}_{\substack{\text{Keplerian parameters} \\ p_i^K}};\ \underbrace{k, \gamma, \dot{P}_b, r, s, \delta_\theta, \dot{e}, \dot{x}}_{\substack{\text{separately measurable} \\ \text{post-Keplerian param.} \\ p_i^{PK}}};\ \underbrace{\delta_r, A, B, D}_{\substack{\text{not separately} \\ \text{measurable} \\ \text{post-Kepler. para.} \\ q_i^{PK}}}]$$

Set to FIDUCIAL VALUES

holds in a large class of alternative relativistic theories of gravity: tensor-scalar

- the PK param. p_i^{PK} reflect various aspects of relativistic gravity, notably strong-field and/or radiative effects

  k : periastron advance : depend on α_A spin α_B

  γ : Einstein delay : Einstein + SR redshift, depend on $A \overset{G_{AB}}{\cdots} B$ ...

  $\dot{P}_b$: radiation damping, ... spin + non-linearities

  r, s : range and shape of Shapiro delay, depend A light

  δ_θ : depend on many terms in 1PK Lagrangian

  $\dot{e}, \dot{x}$: depend on radiation damping and spin-orbit effects notably after absorbing A, B

- the relations $p_i^{PK} = f(p^K_i; m_A, m_B)$ depend on the theory of gravity and will be strongly modified wrt GR, if effacement is violated

PHENOMENOLOGICAL ANALYSIS OF BINARY PULSAR DATA

('Parametrized Post Keplerian Formalism')

RAW PULSAR DATA
t_N^{OBS}
OR FLUX $\{\mathcal{F}(t)\}$

5 Kepler, 8 PK

$t_N^{PPK} = F[N, p_i^K, p_i^{PK}]$

OR

$\mathcal{F}^{PPK}_{(t)} = F'[t, p_i^K, \tilde{p}_i^{PK}]$

11 $\tilde{p}^{PK}$

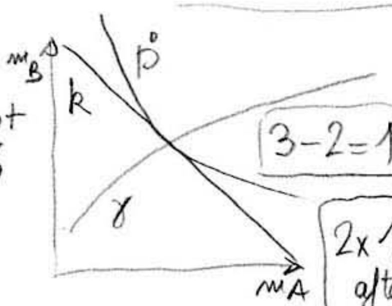
Least-squares FIT

$p_i^{K\,OBS} = \cdots \pm \cdots$

$p_i^{PK\,OBS} = \cdots \pm \cdots$

THEN $\boxed{\text{COMPARE}}$ results to GR predictions

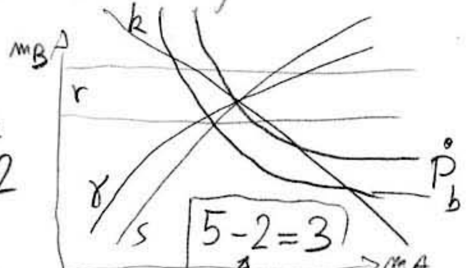
1913+16



$\boxed{3-2=1}$ test

2x10^{-3} level after subtracting $\dot{D}$

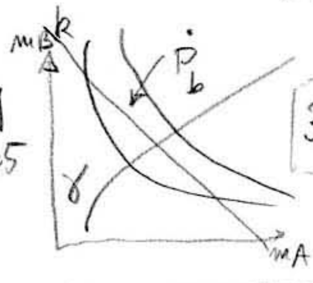
1534+12



$\boxed{5-2=3}$

2 first pure strong-field tests

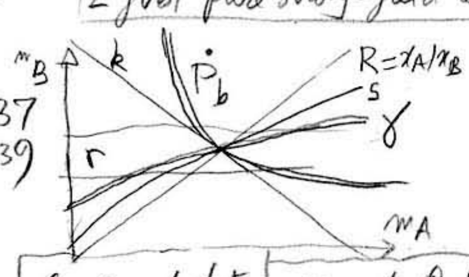
1141-6545



$\boxed{3-2=1}$ test

?+ $s_{scintill}$?

0737-3039



$\boxed{6-2=4 \text{ tests}}$ s: 5x10^{-4} level

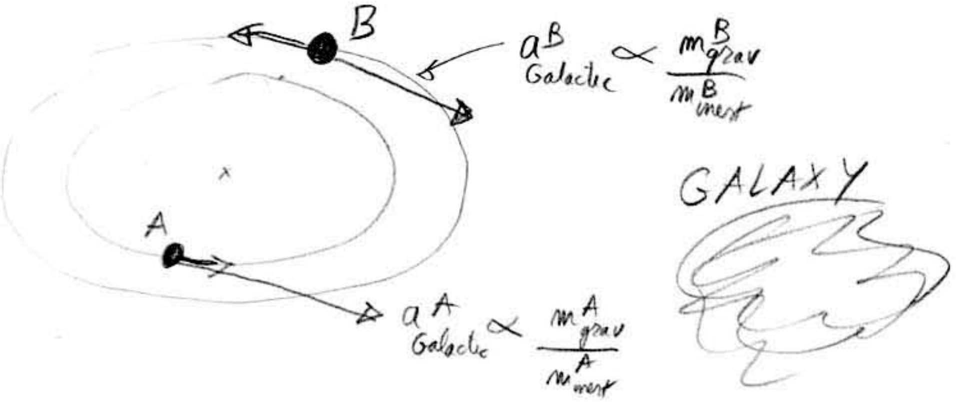
GR OK WITH ALL TESTS

Other phenomenological tests:

Binary pulsar tests of violation of Strong Equivalence Principle

(Damour Schäfer '91)

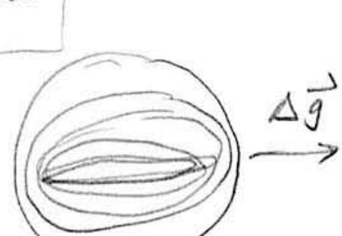
Certain quasi-circular neutron-star white-dwarf systems



if $\Delta = \frac{m^{A}_{\text{grav}}}{m^{A}_{\text{inert}}} - \frac{m^{B}_{\text{grav}}}{m^{B}_{\text{inert}}} \neq 0 \Rightarrow$ 'gravitational Stark effect'

$$\boxed{\frac{d^2\vec{r}}{dt^2} + \underset{+\, v^2/c^2 \text{ terms}}{G_{AB} M \frac{\vec{r}}{r^3}} = \Delta\, \vec{g}_{\text{galactic}}}$$ like $e\vec{E}_{\text{external}}$

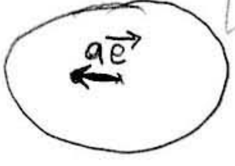
Secular instability of Keplerian ellipse:



$\rightarrow$ POLARIZATION after including v^2/c^2 terms

eccentricity vector

$$\boxed{\vec{e}(t) = \underset{\text{Precessing}}{\vec{e}_{\text{Relativistic}}(t)} + \vec{e}_{\Delta}}$$



latest data Stairs et al 05

$$\boxed{e_{\Delta} = \frac{3}{2} \frac{\Delta g_{\perp}}{\dot{\omega}_R\, n a}}$$

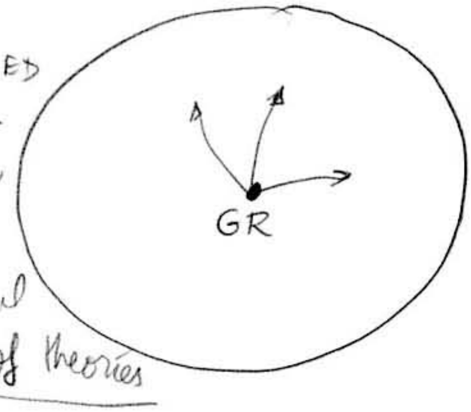
$\Delta \lesssim 5 \times 10^{-3}$

4.15 Theory-space approach to testing relativistic gravity with binary pulsar data

Instead of

YES GR or NO ~~GR~~

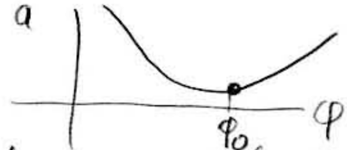
EMBED GR WITHIN A multi-dimensional Space of theories



Need to go beyond the weak-field PPN parametrization

"post-Einstein" para.: $\bar{\gamma} \equiv \gamma^{PPN} - 1$; $\bar{\beta} \equiv \beta^{PPN} - 1$; ...

E.g. TENSOR-SCALAR GRAVITY:



$\boxed{\text{Arbitrary coupling function } \; a(\varphi)}$ if only one scalar φ

tensor-multi-scalar: $\varphi^a = (\varphi^1, \dots, \varphi^n)$ (Damour Esposito-Farèse '92)

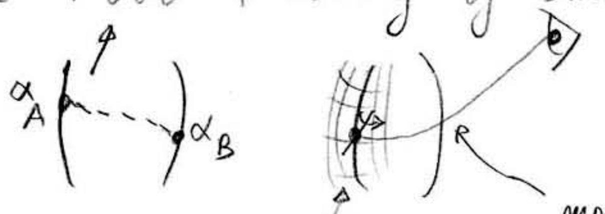
$$\underbrace{\gamma_{ab}(\varphi^c)}_{\text{METRIC in } \varphi\text{-space}} \nabla_\mu \varphi^a \nabla^\mu \varphi^b \; + \; \underbrace{a(\varphi^a)}_{\text{COUPLING FUNCTION}}$$

Various possibilities:

- Beyond PPN: $\overbrace{\bar{\gamma}, \bar{\beta}}^{1PN}$, $\overbrace{\epsilon, \zeta}^{2PN \text{ level}}$ + expansion in $\frac{G m_A}{c^2 R_A}$
- Special classes of tensor-bi-scalar $T_2(\beta', \beta'')$
- One scalar and 2-parameter $T_1(\alpha_0, \beta_0)$: $\boxed{a_{\alpha_0, \beta_0}(\varphi) = \alpha_0 (\varphi - \varphi_0) + \frac{1}{2} \beta_0 (\varphi - \varphi_0)^2}$
  strong-field generalization of PPN

Modifications in $p_i^{PK} = f(p^K; m_A, m_B)$ AGR 4.30

Need to rederive Motion + Timing of binary pulsar



modified $x^\mu \leftrightarrow X_A^\alpha$ modified propagation

ROTATING NS

has $J_A = I(\varphi_{aA})\,\Omega_A$

adiabatic invariant; varies along orbit

E.g. $\boxed{M = m_A + m_B}$ $\boxed{x_A \equiv \frac{m_A}{M}}$, $\boxed{x_B \equiv \frac{m_B}{M}}$

$$k^{GR}(m_A, m_B) = \frac{3}{1-e^2}\left(\frac{GMn}{c^3}\right)^{2/3} \rightarrow k^{TS}(m_A, m_B) = \frac{3}{1-e^2}\left(\frac{G_{AB}Mn}{c^3}\right)^{2/3} \times \left[\frac{1-\frac{1}{3}\alpha_A\alpha_B}{1+\alpha_A\alpha_B} - \frac{x_A\beta_B\alpha_A^2 + x_B\beta_A\alpha_B^2}{6(1+\alpha_A\alpha_B)^2}\right]$$

$$\gamma^{GR}(m_A, m_B) = \frac{e}{n}\left(\frac{GMn}{c^3}\right)^{\frac{2}{3}} x_B(x_B+1) \rightarrow \gamma^{TS} = \frac{e}{n}\left(\frac{G_{AB}Mn}{c^3}\right)^{2/3} \frac{x_B\left[x_B(1+\alpha_A\alpha_B) + 1 + k_A\alpha_B\right]}{1+\alpha_A\alpha_B}$$

$-\partial \ln I_A(\varphi_0)/\partial\varphi_0$

$$\dot{P}_b^{GR}(m_A, m_B) = -\frac{192\pi}{5}\left(\frac{GMn}{c^3}\right)^{5/3} \frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}}\, x_A x_B \rightarrow \dot{P}_b^{TS} = \dot{P}_b^{\text{quadrupole}} + \underbrace{\dot{P}_b^{\text{dipole}}}_{\sim v^3/c^3\,!} + \cdots$$

$\dot{P}_b^{\text{quadrupole}} \sim v^5/c^5$

$\dot{P}_b^{\text{dipole}} \propto (\alpha_A - \alpha_B)^2$

Non-perturbative strong-field effects

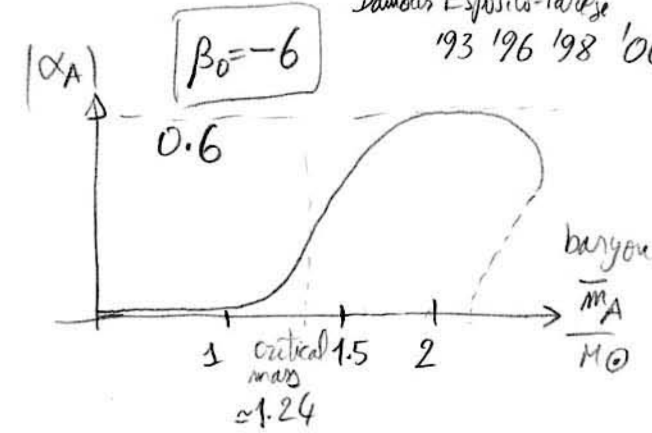
Damour Esposito-Farèse '93 '96 '98 '06

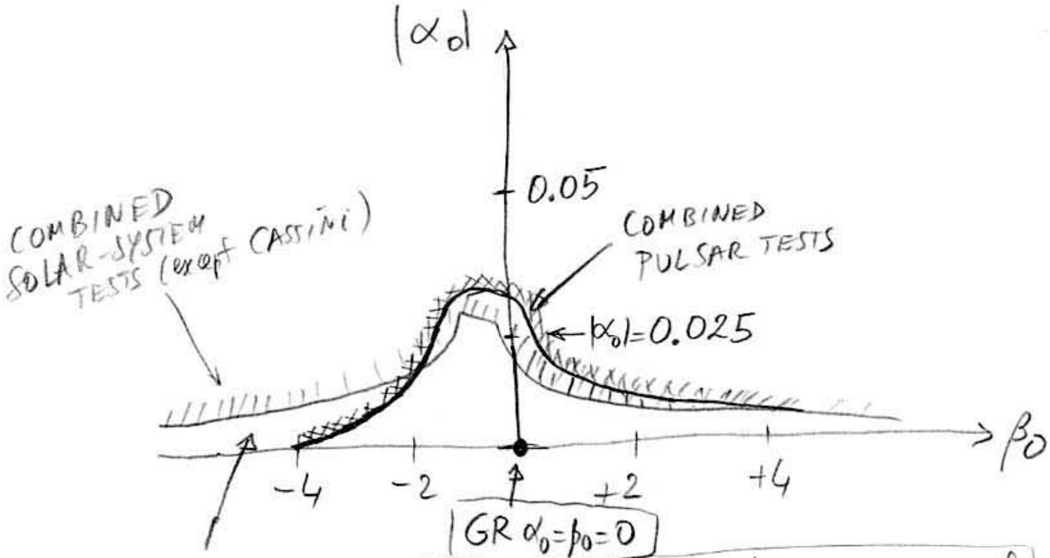
Even if $\alpha_0 \ll 1$

$\Downarrow$

$$\bar{\gamma} = -\frac{2\alpha_0^2}{1+\alpha_0^2} \ll 1$$

$$\bar{\beta} = \frac{1}{2}\,\frac{\beta_0 \alpha_0^2}{(1+\alpha_0^2)^2} \ll 1$$





BINARY PULSARS exclude a whole domain of theory space ($\beta_0 \lesssim -4$) which is allowed by all solar-system tests

(see course of Esposito-Farèse)
and Bourbaphy 28 october: Kramer

This shows that binary pulsar tests do go beyond stationary-weak-field tests and probe strong-field + radiative regime of relativistic gravity → <u>deep</u> confirmations of GR in many of its essential aspects (non-linear, wave propagation, ...)
[spin-orbit effects have also been confirmed]