6

STRING THEORY AND GRAVITY.

String spectrum, Effective actions, Dualities, AdS/CFT, Black Hole entropy, Some phenomenology

Some references:

Textbooks on String Thy: Zwiebach, 'A First Course in String theory', Cambridge U. Press
Polchinski, 'String theory', 2 volumes Cambridge UP
see also IHP semester on string theory Sept-Feb 2001
Green Schwarz Witten, CUP. Becker, Becker, Schwarz 2006
Kiritsis 'Introduction to Superstring Theory' Leuven UP + hepth 9709062

D-branes: Johnson 'D-branes' Cambr. UP; Bachas hepth 9806199

AdS/CFT: Klebanov TASI 2000: hep-th/0009139; Maldacena TASI 2003 hepth/0309246

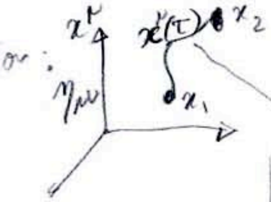
T-duality: Giveon, Porrati, Rabinovici Phys. Rep 244, 77 (1994) hepth 9401139

Black Hole entropy: Damour 'The entropy of black holes: a primer' Poincaré Seminar 2003 hep-th/0401160
BPS black holes in string thy: Strominger Vafa PLB 379, 99 (1996); Callan Maldacena NPB 472 591 (1996)
Schwarzschild BH in string thy: Susskind hep-th 9309145; Horowitz Polchinski Phys. Rev. D 57, 2557 (1998); Damour Veneziano NPB 568, 93 (2000)

Phenomenology:
short-range modifs: Taylor Veneziano PLB 213 450 (1988) Dimopoulos Giudice PLB 379 105 (1996)
Antoniadis, Dimopoulos, Dvali NPB 516, 70 (1998)
long-range modifs: Damour Polyakov NPB 423, 532 (1994); Damour Piazza Veneziano PRD 66, 046007 (2002)
Brane worlds: Rubakov Phys. Usp. 44, 871 (2001); Maartens Living Review
Cosmic superstrings: Witten PLB 153, 243 (1985); Kachru et al. hep-th 0308055
Copeland Myers Polchinski hepth 0312067; Damour Vilenkin PRD 64, 064008 ('01); PRD 71, 063150 ('05)

$\boxed{6.1}$ Quantum Relativistic Particle in Minkowski

Usual action:

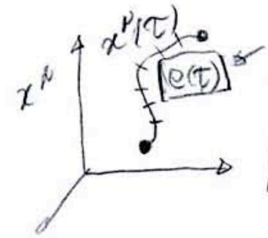


$$S_0[x^\mu(\tau)] = -m\int_1^2 ds$$

$$= -m\int_1^2 \sqrt{-\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}\, d\tau$$

$\boxed{\text{arbitrary parameter}}$

$\sqrt{}$ bad for quantizing



inner metric

$\boxed{d\tilde{s} = e(\tau)\, d\tau}$

$$S[x^\mu(\tau), e(\tau)] = \frac{1}{2}\int_1^2 d\tau \left[\frac{1}{e(\tau)}\eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} - e(\tau)m^2\right]$$

limit $m^2 \to 0$; OK

$\boxed{\text{Quantization}}$:

Feynman: $$\mathcal{A}(x_2, x_1) = \int_1^2 \frac{\mathcal{D}x^\mu \mathcal{D}e}{\text{diff}}\, e^{iS[x,e]} \propto \int_0^\infty dT \int \frac{d^Dp}{(2\pi)^D} e^{i\left[p\cdot(x_2 - x_1) - \frac{p^2+m^2}{2}T\right]}$$

$$\propto \int \frac{d^Dp}{(2\pi)^D} \frac{e^{ip_\mu(x_2^\mu - x_1^\mu)}}{i(p^2+m^2)} = G_F(x_2 - x_1; m)$$

Heisenberg

$$(x^\mu, P_\nu) \to \hat{x}^\mu, \hat{P}_\nu$$

EQS OF MOTION IN GAUGE $e(\tau)=1$: $\quad \frac{d^2x^\mu}{d\tau^2} = 0 \quad \to$

SOLUTION: $x^\mu = x_0^\mu + p^\mu\tau$

CONSTRAINT: $\delta/\delta e(\tau) \to \quad \eta_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = -m^2$

$$\boxed{\begin{gathered}[\hat{x}^\mu, \hat{P}_\nu] = i\hbar\,\delta^\mu_\nu \\ \text{SINGLE PARTICLE STATES } |p_\mu; m\rangle \\ \hat{P}_\mu |p_\mu; m\rangle = p_\mu |p_\mu; m\rangle \\ (\eta^{\mu\nu}\hat{P}_\mu\hat{P}_\nu + m^2)\,|p_\mu; m\rangle = 0\end{gathered}}$$

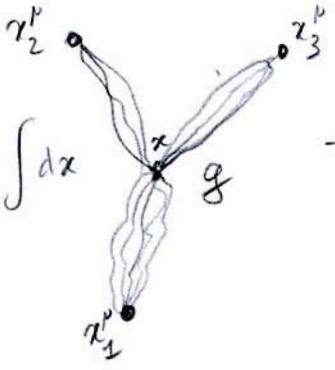
in covariant formalism:
constraint imposed on <u>states</u>

'Second quantization' i.e. multiparticle states

not only  $|p\rangle$ single particle Hilbert space

but  $|p_1, p_2\rangle$ multiparticle Hilbert space

Interactions : line splitting

$\int_{\text{path}} = \int dx$ 

$$\rightarrow \quad A(x_2 x_3; x_1) = g \int d^D x \; G_F(x - x_1)\, G_F(x_2 - x)\, G_F(x_3 - x)$$

$=$ 

Equivalent to quantized scalar field $\hat{\varphi}(x)$

$$\boxed{S[\hat{\varphi}] = \int d^D x \Big[\underbrace{-\tfrac{1}{2}\eta^{\mu\nu}\partial_\mu \hat{\varphi}\,\partial_\nu \hat{\varphi} - \tfrac{1}{2} m^2 \hat{\varphi}^2}_{\text{PROPAGATOR} \sim \frac{1}{p^2+m^2}} \underbrace{- \tfrac{1}{3!} g\, \hat{\varphi}^3_{(x)}}_{\text{INTERACTION: CUBIC VERTEX}} \Big]}$$

(e.g. analytically continued to complex, on-shell p's)

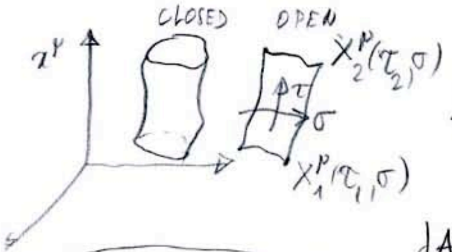
Then: $${}_{\text{out}}\langle p_2, p_3 | p_1 \rangle_{\text{in}} = \int d^D x_1\, d^D x_2\, d^D x_3\, e^{i(p_1\cdot x_1 + p_2 \cdot x_2 + p_3\cdot x_3)} (\Box_1 + m^2)(\Box_2 + m^2)(\Box_3 + m^2)\, A(x_2 x_3; x_1) = g\, \delta^D(p_1 + p_2 + p_3)$$

$\boxed{6.2}$ First quantized relativistic string

AGR 6.3

$\boxed{(\sigma^a)^{a=0,1} = (\tau, \sigma)}$

Nambu-Goto action



STRING TENSION

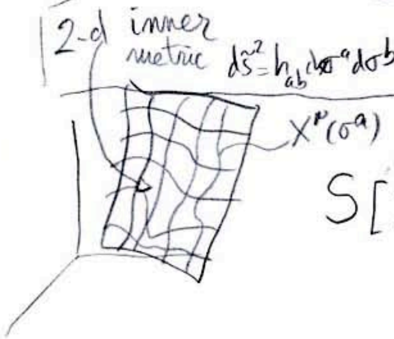
$$S_0[X^\mu(\tau,\sigma)] = -T \iint dA$$

$$dA = \sqrt{-\det \gamma_{ab}}\, d\tau d\sigma \qquad \text{induced metric: } ds^2 = \gamma_{ab}\, d\sigma^a d\sigma^b = (dX^\mu)^2$$

$$= \sqrt{(\eta_{\mu\nu}\dot{X}^\mu X'^\nu)^2 - (\eta_{\mu\nu}\dot{X}^\mu \dot{X}^\nu)(\eta_{\mu\nu}X'^\mu X'^\nu)}$$

2-d inner metric $d\tilde{s}^2 = h_{ab}\, d\sigma^a d\sigma^b$

(Polyakov) action



$$S[X^\mu(\sigma^a), \underbrace{\sqrt{h}\, h^{ab}(\sigma^a)}_{\text{conformal structure}}] = -\frac{T}{2}\iint d\tau d\sigma\, \underbrace{\sqrt{h}\, h^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}}_{\text{analog of}}$$

$$-\frac{1}{2}\int d\tau \frac{1}{e(\tau)} \frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\tau}\eta_{\mu\nu}$$

inner metric $\qquad \boxed{m^2 = 0}$

$\hbar = c = 1$:

$$\boxed{\begin{array}{ll} \text{String tension} & T \equiv \dfrac{1}{2\pi\alpha'} \equiv \dfrac{1}{2\pi\, l_s^2} \equiv \dfrac{m_s^2}{2\pi} \\ \text{Regge slope} & \alpha' \equiv l_s^2 \equiv \dfrac{1}{m_s^2} \end{array}}$$

string length scale $\qquad$ string mass scale

EQS OF MOTION:

in gauge $\sqrt{h}h^{ab} = \eta^{ab}$: $\boxed{0 = \eta^{ab}\partial_{ab}X^\mu = -\partial_\tau^2 X^\mu + \partial_\sigma^2 X^\mu} \longleftrightarrow \partial_\tau^2 x^\mu = 0$

PARTICLE ANALOG

in gauge $e(\tau) = 1$

CONSTRAINTS $\dfrac{\delta}{\delta h^{ab}} = 0 \rightarrow \boxed{0 = \partial_a X^\mu \partial_b X_\mu - \frac{1}{2}\eta_{ab}\eta^{cd}\partial_c X^\mu \partial_d X_\mu} \longleftrightarrow \partial_\tau x^\mu \partial_\tau x_\mu = -m^2$

Heisenberg: $X^\mu(\tau,\sigma)$, $\mathcal{P}_\mu(\tau,\sigma) = \dfrac{\partial \mathcal{L}}{\partial \partial_\tau X^\mu} = +T\,\eta_{\mu\nu}\,\partial_\tau X^\nu$ in gauge $\sqrt{h}\,h^{ab} = \eta^{ab}$

$$\left[\hat{X}^\mu(\tau,\sigma_1),\ \hat{\mathcal{P}}_\nu(\tau,\sigma_2)\right] = i\hbar\,\delta^\mu_\nu\ \delta(\sigma_1-\sigma_2)$$

EQS OF MOTION: $\left(\partial_\tau^2 - \partial_\sigma^2\right)\hat{X}^\mu(\tau,\sigma) = 0 \Rightarrow \hat{X}^\mu(\tau,\sigma) = \hat{X}^\mu_L(\tau+\sigma) + \hat{X}^\mu_R(\tau-\sigma)$

CONSTRAINTS $\quad \left(\partial_\tau \hat{X}^\mu_L\right)^2 \approx 0 \approx \left(\partial_\tau \hat{X}^\mu_R\right)^2$

OPEN STRING WITH FREE ENDS

$\partial_\sigma X^\mu\Big]^{\text{ends}} = 0$

$0 \leq \sigma < \pi$

$$\hat{X}^\mu(\tau,\sigma) = \hat{x}^\mu + 2l_s^2\,\hat{p}^\mu\tau + i\sqrt{2}\,l_s \sum_{n\neq 0} \frac{\hat{\alpha}^\mu_n}{n} e^{-in\tau} \cos n\sigma$$

Center of Mass $\quad [\hat{x}^\mu, \hat{p}^\nu] = i\hbar\eta^{\mu\nu}$

Oscillators $\quad [\hat{\alpha}^\mu_m, \hat{\alpha}^\nu_n] = m\,\delta^0_{m+n}\,\hbar\,\eta^{\mu\nu}$

Usual harmonic oscillators

$n>0 \quad a^\mu_n = \dfrac{\alpha^\mu_n}{\sqrt{n}}\ ;\ (a^\mu_n)^\dagger = \dfrac{\alpha^\mu_{-n}}{\sqrt{n}} \quad [a^\mu_m, a^{\dagger\nu}_n] = \eta^{\mu\nu}\delta_{mn}\hbar$

SPATIAL i

OPEN STRING WITH FIXED SPATIAL ENDS

$\partial_\tau X^i\Big]^{\text{ends}} = 0$

$$\hat{X}^i(\tau,\sigma) = x^i_1 + (x^i_2 - x^i_1)\frac{\sigma}{\pi} + \sqrt{2}\,l_s \sum_{n\neq 0} \frac{\hat{\alpha}^i_n}{n} e^{-in\tau} \sin n\sigma$$

no momentum

CLOSED STRING

$0 \leq \sigma < 2\pi$

$$\hat{X}^\mu(\tau,\sigma) = \hat{x}^\mu + l_s^2\,\hat{p}^\mu\sigma + \frac{i l_s}{\sqrt{2}} \sum_{n\neq 0} \frac{e^{-in\tau}}{n}\left(\hat{\alpha}^\mu_n e^{in\sigma} + \hat{\tilde{\alpha}}^\mu_n e^{-in\sigma}\right)$$

Center of Mass $\quad [\hat{x}^\mu, \hat{p}^\nu] = i\hbar\eta^{\mu\nu}$

LEFT-MOVING + RIGHT-MOVING OSCILLATORS

$[\alpha^\mu_m, \alpha^\nu_n] = m\,\delta^0_{m+n}\,\hbar\,\eta^{\mu\nu} \ldots$

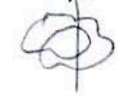
6.3 String spectrum

$a^\mu_n|0\rangle = 0 = \tilde{a}^\mu_n|0\rangle$ AGR 6.5

FREE SINGLE STRING STATES : OSCILLATOR GROUND STATE $|0;p^\mu\rangle$

center of mass p^μ

classically $p=0$



+ TOWER OF EXCITED STATES $a_1^{\mu\dagger} a_1^{\nu\dagger} a_2^{\lambda\dagger} \ldots \tilde{a}_4^{\rho\dagger} \ldots |0;p\rangle$

CONSTRAINTS $\left.\left(\dot{X}^\mu_L\right)^2\right]^{\omega>0} |\Psi\rangle = 0 = \left.\left(\dot{X}^\mu_R\right)^2\right]^{\omega>0} |\Psi\rangle$

à la Gupta-Bleuler positive freq. part

$$L_m |\Psi\rangle = 0 = \tilde{L}_m |\Psi\rangle$$

$$L_n = \frac{1}{2} \sum_m \eta_{\mu\nu} \alpha^\mu_m \alpha^\nu_{n-m} \qquad (\alpha^\mu_0 \propto p^\mu)$$

+ ordering constant for $n=0$

$$L_0 \sim p^2 + \sum \alpha^\dagger \alpha \text{ oscillators}$$

MASS-SHELL CONDITION : $\boxed{\hat{N}_m \equiv \left(\hat{a}^\mu_m\right)^\dagger \hat{a}^\nu_m \eta_{\mu\nu}}$ OCCUPATION NUMBER '$a^\dagger a$' OF n^{th} harmonic $\sim e^{in\sigma}$

Open string free ends : $\boxed{M^2 = m_s^2 \left[\sum_{n=1}^{\infty} n \hat{N}_n - 1 \right]}$

QUANTUM ZERO-POINT ENERGY OF GROUND-STATE

Open string fixed ends : $M^2 = T^2 (x_2^i - x_1^i)^2 + m_s^2 \left[\sum n \hat{N}_n - 1\right]$ TENSION

$(D-2)$ times $\sum_{n=1}^{\infty} \frac{1}{2} \times n = \frac{1}{2}\zeta(-1) = -\frac{1}{24}$

Closed string $\boxed{M^2 = 4 m_s^2 \left[-1 + \frac{1}{2} \sum_1^\infty n N_n + \frac{1}{2} \sum_1^\infty n \tilde{N}_n \right]}$ EQUAL

OPEN STRING (FREE ENDS)

GROUND STATE $|0;p\rangle$; constraints $\Rightarrow p^2 = -M^2 = +m_s^2$ SCALAR TACHYON

FIRST EXCITED STATES $\zeta_\mu (a_1^\mu)^\dagger |0;p\rangle$ constraints $\nearrow p^2 = 0$, $\searrow \zeta_\mu p^\mu = 0$ MASSLESS VECTOR 'PHOTON'

+ equivalence class $\zeta'_\mu = \zeta_\mu + \alpha p_\mu$

CLOSED STRING

GROUND STATE $|0,\tilde{0};p\rangle$; constraints $p^2 = -M^2 = +4m_s^2$ SCALAR TACHYON

FIRST EXCITED STATES $\zeta_{\mu\nu}(a_1^\mu)^\dagger(\tilde{a}_1^\nu)^\dagger|0,\tilde{0};p\rangle$

constraints $\nearrow p^2 = 0$

$\searrow p^\mu \zeta_{\mu\nu} = 0 = \zeta_{\mu\nu} p^\nu$

+ equivalence class $\zeta'_{\mu\nu} = \zeta_{\mu\nu} + a_\mu p_\nu + p_\mu b_\nu$

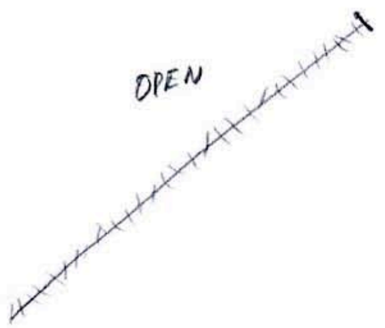
MASSLESS

$\zeta^{TT}_{(\mu\nu)} \leftrightarrow h_{(\mu\nu)}$: EINSTEIN

$\zeta^{T}_{[\mu\nu]} \leftrightarrow B_{[\mu\nu]}$: B-FIELD (Kalb Ramond)

$\zeta^{TS}_{(\mu\nu)} \leftrightarrow \phi$: DILATON

OPEN



$\leftrightarrow$ PHOTON

CLOSED

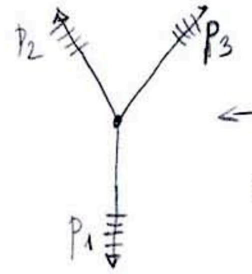
$\leftrightarrow$ GRAVITON

+ B-ON

+ DILATON

6.4 STRING INTERACTIONS

Particle



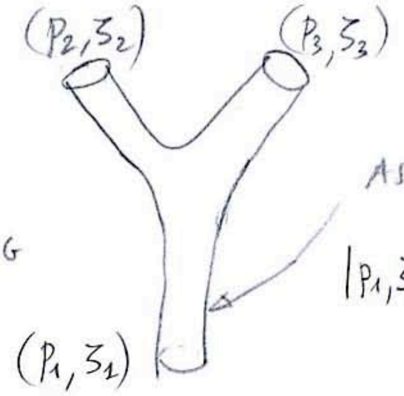
PARTICLE TRANSITION AMPLITUDE

$$\longleftrightarrow \mathcal{A}(p_1 p_2 p_3) = g\ \delta(p_1+p_2+p_3)$$

in FOURIER SPACE

String

SPLITTING



ASYMPTOTIC STRING STATE

$$|p_1, \zeta_1\rangle \equiv \zeta^1_{\mu\nu} (a^\mu_1)^\dagger (\tilde{a}^\nu_1)^\dagger |0, \tilde{0}; p_1^\lambda\rangle$$

STRING TRANSITION AMPLITUDE

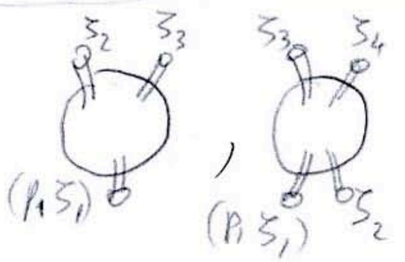
$$\mathcal{A}(p_1\zeta_1, p_2\zeta_2, p_3\zeta_3) \propto g_s \left\langle \hat{V}(p_1\zeta_1)\ \hat{V}(p_2\zeta_2)\ \hat{V}(p_3\zeta_3) \right\rangle$$

'Vertex' operator creating $|p_1, \zeta_1\rangle$ state

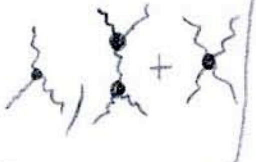
e.g. $$\hat{V}(p_1, \zeta_1) = \int d^2\sigma : e^{i p_1 \cdot X(\sigma)}\ \zeta^1_{\mu\nu}\ \dot{X}^\mu_L(\sigma)\ \dot{X}^\nu_R(\sigma) :$$

Remarkable result: When $p_1, p_2, p_3 \ll m_s$

STRING GRAVITON SCATTERING AMPLITUDES

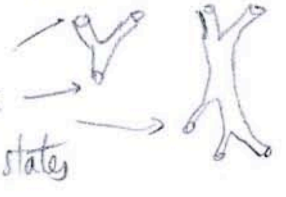


ARE <u>EQUIVALENT</u> TO GR TREE AMPLITUDES



$$\mathcal{L}_{GR} = (\partial h)^2 + h\partial h \partial h + h^2 \partial h \partial h + \cdots$$

Low-energy + tree-level ($E_i \ll m_s$) $\longleftrightarrow$ All-order expansion of

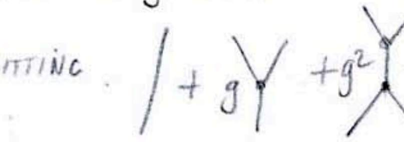


with graviton asymptotic states

$\mathcal{L}_{GR} = \sqrt{g}\, R(g) = (\partial h)^2 + h(\partial h)^2 + \cdots$

String theory 'CONTAINS' CLASSICAL GR in the same way that

QUANTUM SPLITTING PARTICLE



$\longleftrightarrow$ TREE AMPLITUDES FROM

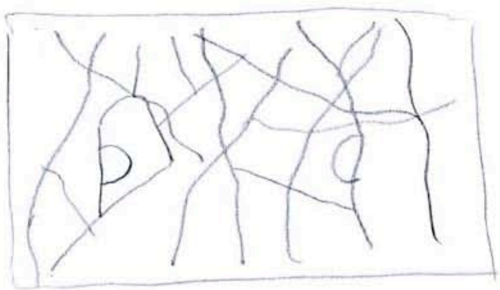
$$-\frac{1}{2}(\partial\hat{\varphi})^2 - \frac{m^2}{2}\hat{\varphi}^2 - g\frac{\hat{\varphi}^3}{3!}$$

$\updownarrow$ 'CONTAINS' CLASSICAL

$$-\frac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2}\varphi^2 - \frac{g}{3!}\varphi^3$$

DIFFERENT STATES IN SAME Theory:

- few particle states: $a^\dagger_{p_1} a^\dagger_{p_2} |0\rangle$
- quasi-classical 'coherent states'



$$\hat{\varphi}(x) = \underbrace{\varphi_{class}(x)}_{\text{large classical background}} + \underbrace{\hat{\phi}(x)}_{\text{small quantum fluctuation}}$$

Basic problem of String thy:

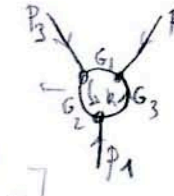
$\nexists$ NO ANALOG OF $(\partial\hat{\varphi})^2 + \hat{\varphi}^2 + \hat{\varphi}^3$ describing the possibility of having many-string arbitrary states (including quasi-classical objects, or backgrounds + quantum excitations)

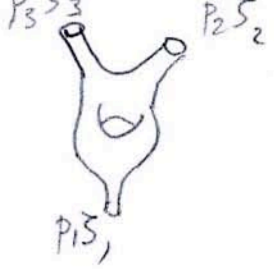
6.5 A GLIMPSE AT THE RICHNESS OF STRING THY

Particle | String

LOOP AMPLITUDES

integral $\int d^D k \left[G_2(k) \times G_3(p_1+k) \times G_1(k-p_3) \right]$



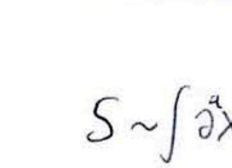


Consistency of string theory (e.g. unitarity at 1-loop) : $\Rightarrow$ $\boxed{D = \text{FIXED}}$ / $D=26$ FOR BOSONIC STRING

$D=10$ FOR SUPERSTRING

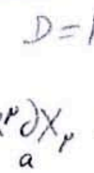
$$S \sim \int \partial^a X^\mu \partial_a X_\mu + i \underbrace{\bar{\psi}^\mu \gamma^a \partial_a \psi^\nu}_{\text{FERMIONIC OSCILLATORS}}$$

Contrary to quantum perturbative GR



quantum perturbative string theory is finite

e.g.



4 gravitons well defined

basically thanks to $\boxed{\alpha' = \ell_s^2}$ FUNDAMENTAL LENGTH $\rightarrow$ UV REGULATOR

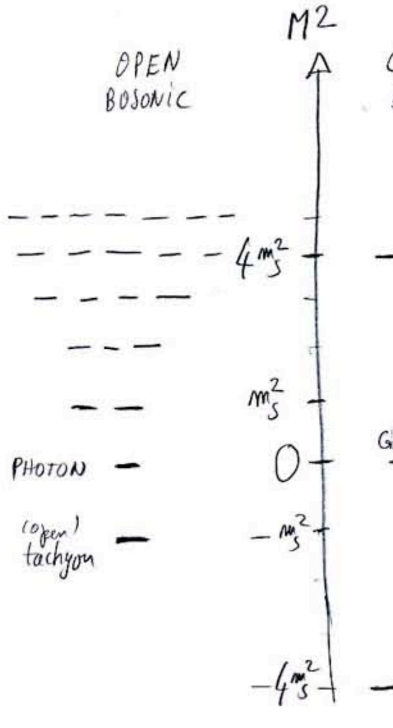
$\nexists$ NO DIMENSIONLESS arbitrary coupling constants

Notably

$$g_s = e^{\Phi_0} \quad \text{DILATON VEV}$$

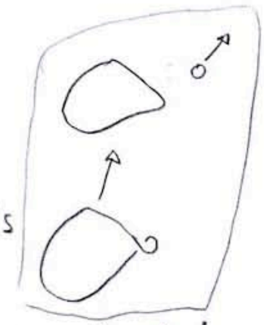
DILATON FIELD $\Phi(x) = \Phi_0 + \hat{\phi}(x)$

$$G_D \sim g_s^2\, \ell_s^{D-2}$$



e.g. $\zeta_{\mu\nu\rho\sigma}\, a_1^{\dagger\mu} a_1^{\dagger\nu} \tilde{a}_1^{\dagger\rho}\tilde{a}_1^{\dagger\sigma}|0\rangle \leftrightarrow$ 'spin 4' field

$A_{\mu\nu\rho\sigma}(x)$: mass $2m_s$



Effective Action

$$S=\int\frac{d^Dx}{16\pi G_N}\sqrt{G}\,e^{-2\Phi}\Big[\overbrace{R(G)-\frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}+4\,\partial_\mu\Phi\,\partial^\mu\Phi}^{\text{massless backgrounds}}\;-\frac{1}{2}(\partial T)^2+(2m_s)^2T^2+\dots$$

tachyon instability

$$-\left(\nabla_\lambda A_{\mu\nu\rho\sigma}\cdots\right)^2-(2m_s)^2A^2_{\mu\nu\rho\sigma}+\cdots$$

$$\sum_n -\left(\partial\varphi_n\right)^2-(2n\,m_s)^2\varphi_n^2\Big]$$

$\rightarrow \alpha'[R^2+\dots]$ at large distances

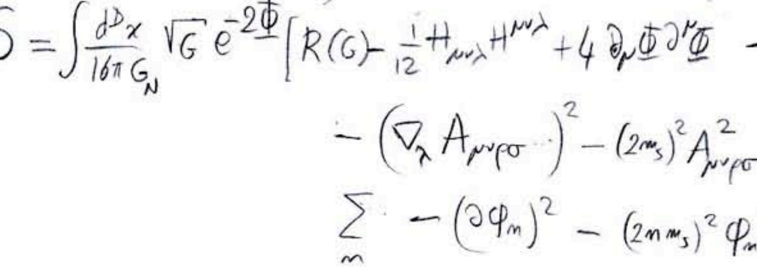
small strings

$$-\sum T\int d^2\sigma\;\partial_a X^\mu\partial_b X^\nu\left[\sqrt{h}\,h^{ab}G_{\mu\nu}(X(\sigma))+\varepsilon^{ab}B_{\mu\nu}(X(\sigma))\right]$$

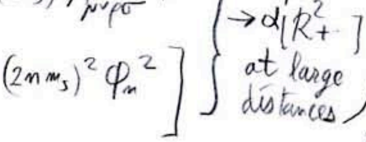
large strings

$$-\sum_p T_p\int d^{p+1}\sigma\;\mathrm{Tr}\Big\{e^{-\Phi}\sqrt{-\det\left(G_{ab}+B_{ab}+2\pi\alpha' F_{ab}\right)}+\mathcal{O}([X,X]^2)\Big\}$$

D-branes



$G\ B\ \Phi$ background
$+\ C_{\mu_1\dots\mu_n}$ = soup of massless strings
$+\dots$
in superstrings



SUPERSTRING THEORIES

$$S_{string} = -\frac{1}{2}T \int d^2\sigma \Big[\underbrace{\sqrt{h}\, h^{ab} \partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}}_{\text{BOSONIC OSCILLATORS}} + \underbrace{i\, \bar{\psi}^\mu \gamma^a \nabla_b \psi^\nu \eta_{\mu\nu}}_{\text{FERMIONIC OSCILLATORS}} \Big]$$

$$X^\mu = x^\mu + p^\mu \tau + \sum_m \frac{\alpha_m^\mu}{m} e^{-im(\tau-\sigma)} + \tilde{\alpha}_m^\mu() \quad \text{eg} \quad \psi_\mu = \begin{pmatrix} \sum_m d_m^\mu e^{-im(\tau-\sigma)} \\ \sum_m \tilde{d}_m^\mu e^{-im(\tau+\sigma)} \end{pmatrix}$$

After (GSO) projection:

NO TACHYON ; critical dimension $D_c = 10$; $\exists$ SPACETIME FERMIONS

OPEN + CLOSED type I $\quad S_{massless} = \int d^{10}x \sqrt{G} \Big[e^{-2\Phi} \big(R + 4(\partial\Phi)^2\big) - \frac{1}{2}\big(\partial_\lambda C_{\mu\nu]} + \big)^2 + e^{-\Phi} \mathrm{Tr}\, F_{\mu\nu}^{A\,2} \Big]$

$G_{\mu\nu}, \Phi ; C_{[\mu\nu]}, A_\mu^A$ + susy partners $\qquad SO(32)$ YM

CLOSED IIA $\quad e^{-2\Phi}\Big[R + 4(\partial\Phi)^2 - \frac{1}{2}\big(\partial_\lambda B_{\mu\nu]} + \big)^2\Big] + \big(\partial_{[\lambda} C_{\mu]}\big)^2 + \big(\partial_\lambda C_{\mu_1\mu_2\mu_3]} + \cdots\big)^2$

$G_{\mu\nu}, \Phi, B_{\mu\nu} ; C_\mu , C_{[\mu_1\mu_2\mu_3]}$

CLOSED IIB $\quad e^{-2\Phi}\Big[R + 4(\partial\Phi)^2 - \frac{1}{2}\big(\partial_{[\lambda} B_{\mu\nu]}\big)^2\Big] + \big(\partial_\lambda C\big)^2 + \big(\partial_{[\lambda} C_{\mu\nu]} + \cdots\big)^2 + \big(\partial_\lambda C_{\mu_1 \cdots \mu_4]}\big)^2$

$G_{\mu\nu}, \Phi, B_{\mu\nu} ; C , C_{[\mu\nu]} , C_{[\mu_1\mu_2\mu_3\mu_4]}$

HETEROTIC

$G_{\mu\nu}, \Phi, B_{\mu\nu} ; A_\mu^A \quad e^{-2\Phi}\Big[R + 4(\partial\Phi)^2 - \frac{1}{2}\big(\partial_{[\lambda} B_{\mu\nu]} + \big)^2 + \mathrm{Tr}\, \big(F_{\mu\nu}^A\big)^2\Big]$

$SO(32)$ or $E_8 \times E_8$

RR FIELDS : $\left.\begin{array}{l} \text{IIA} \quad C_\mu , C_{[\mu_1\mu_2\mu_3]} \\ \text{IIB} \quad C , C_{[\mu\nu]} , C_{[\mu_1 \cdots \mu_4]} \end{array}\right\}$ $\boxed{\text{GENERALIZATIONS OF MAXWELL: } A_\mu + \partial_\mu \Lambda \leftrightarrow \int A_\mu dx^\mu}$

do not couple to fundamental string,

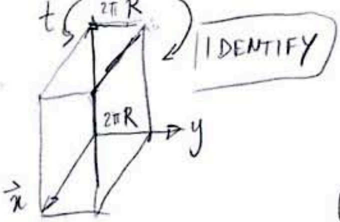
couple to D-branes eg IIA D-particle $\int C_\mu dx^\mu$; D-2 (membrane) $\int C_{\mu\nu\lambda}\, dx^\mu \wedge dx^\nu \wedge dx^\lambda$

IIB D-instanton , D-string , D-3 brane

6.6 DUALITY SYMMETRIES

COMPACTIFYING ON A SMALL RADIUS R : T-DUALITY

Usual field theory: Kaluza-Klein mechanism



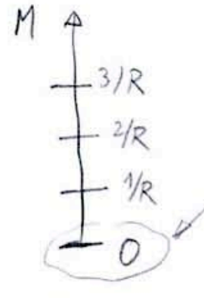
Field: $\varphi(t,\vec{x},y) = \sum_{n\in\mathbb{Z}} \varphi_n(t,\vec{x})\, e^{in\frac{y}{R}}$

$$0 = \left(-\Box_{xy} + m_0^2\right)\varphi = \sum_n \left(-\Box_x - \frac{\partial^2}{\partial y^2} + m_0^2\right)\varphi_n =$$
$$= \sum_n \left(-\Box_x + \frac{n^2}{R^2} + m_0^2\right)\varphi_n$$

MASS SPECTRUM seen from uncompactified dim. $t,\vec{x}$: φ_n : $\boxed{M_n^2 = \frac{n^2}{R^2} + m_0^2}$ in D-1

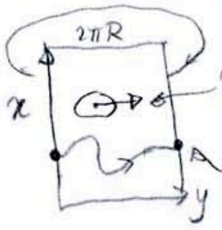
massless in D $\rightarrow$ $\boxed{M_n = \frac{|n|}{R} \text{ in } D-1}$

KK tower of states



As $R\to 0$ only massless field $\varphi_0(t,\vec{x})$ can be excited

New mechanism in string theory : CLOSED STRINGS WITH COMPACT y



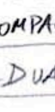
momentum $p_y = n/R$

winding: $w=1$

$$\boxed{M^2_{string} = \frac{n^2}{R^2} + \frac{w^2R^2}{\alpha'^2} + \overbrace{\frac{2}{\alpha'}\left(N+\tilde{N}-2\right)}^{\text{"}m_0^2\text{"}}}$$

KK momentum, winding $w\in\mathbb{Z}$, with $0 = nw + N - \tilde{N}$

As $R\to 0$, appears a new tower of light winding states



$M \sim |w|\frac{R}{\ell_s^2} = \frac{|w|}{\tilde{R}}$

$\rightarrow$ equivalent to Kaluza-Klein tower $\boxed{\tilde{R} = \frac{\ell_s^2}{R}}$: 'DECOMPACTIFY' AS $\begin{matrix} R\to 0 \\ \tilde{R}\to\infty \end{matrix}$!

T-DUALITY

Strong-coupling limit of $\mathrm{II}A_{10}$

$\mathrm{II}A$: closed fundamental strings + RR fields C_μ $\quad C_{[\mu_1\mu_2\mu_3]}$

coupled to D-0 particles

D-brane tension $\boxed{T_{D_p} = \frac{1}{(2\pi)^p} \frac{m_s^{p+1}}{g_s}}$ $\quad$ $\boxed{\text{D-particle mass} \quad m_{D0} = \frac{m_s}{g_s}}$

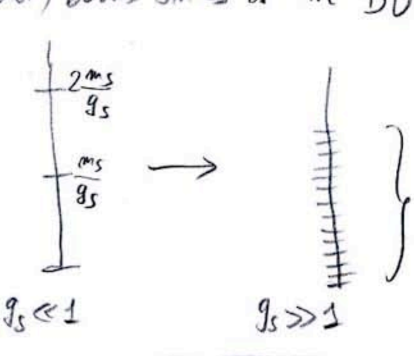
String theory defined only as $g_s \to 0$: $m_{D0} \to \infty$

but $G_{10}\, m_{D0} \sim g_s^2 \ell_s^8 \frac{1}{g_s \ell_s} \propto g_s^{+1} \to 0$

SUSY (BPS object) $\longrightarrow$ $\boxed{m_{D0}(g_s) = \frac{m_s}{g_s}}$ for any value of g_s

+ $\exists$ (threshold) BOUND STATES OF n D0  $\boxed{m_{nD0} = n \frac{m_s}{g_s}}$

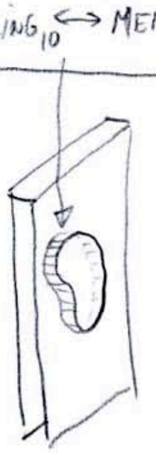
As $g_s \nearrow$:



like KK tower $\boxed{n \frac{m_s}{g_s} = \frac{n}{R}}$

$\boxed{\text{STRING}_{10} \longleftrightarrow \text{MEMBRANE}_{11}}$

$\Rightarrow$ $\boxed{R = g_s\, \ell_s}$ $\quad$ $\boxed{\text{DECOMPACTIFY as } g_s \to \infty \Rightarrow R \to \infty}$



$\mathrm{II}A$ in $D=10$ $\quad \xrightarrow{\quad g_s \nearrow \quad}$ 'M-theory' in $D=11$

$g_s \ll 1$ $\qquad\qquad$ $g_s \gg 1$

Can identify $\lim_{g_s \to \infty}$ D-0 = 'GRAVITONS' + ⋯ in $D=11$ SUPERGRAVITY

$R + (\partial_\mu A_{\mu_1\mu_2\mu_3]})^2 + \cdots$

$\boxed{m_{D0} = \frac{m_s}{g_s} \Leftrightarrow P_{11} = \frac{1}{R}}$

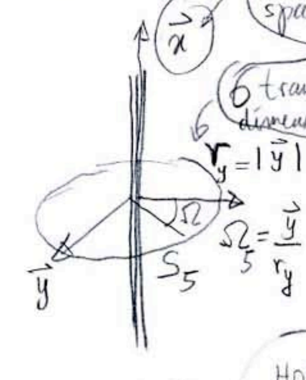
AdS/CFT

N D-3 branes $C_{\mu_1\mu_2\mu_3\mu_4}$ in IIB

3 internal space dimensions

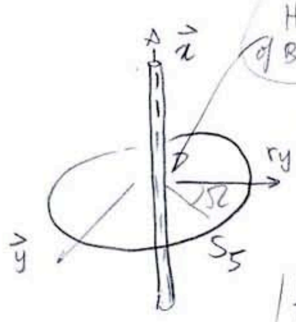
when $g_s \ll 1$

$G T_3 \sim g_s^2 \frac{N}{g_s} \sim g_s N \to 0$



$G_{\mu\nu} = \eta_{\mu\nu} + \underbrace{O\left(\frac{g_s N \ell_s^4}{r_y^4}\right)}_{\ll 1}$

when $g_s N \gg 1$



$ds^2 = H_{(r)}^{-1/2}(-dt^2 + d\vec{x}^2) + H_{(r)}^{1/2}(dr_y^2 + r_y^2 d\Omega_5^2)$

$e^{2\Phi} = g_s^2$

$C_4 = (H^{-1} - 1) g_s^{-1} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$

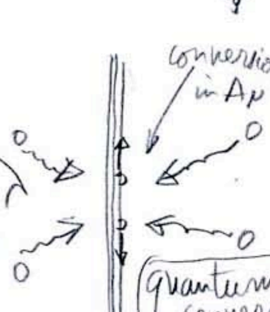
$$\boxed{H(r) = 1 + \frac{4\pi g_s N \ell_s^4}{r_y^4} \equiv 1 + \frac{R^4}{r_y^4}}$$

$\boxed{\text{Near-horizon limit}}$ $r_y \to 0$, $H \sim \frac{R^4}{r_y^4}$

$$ds^2_{10} \simeq \underbrace{\frac{r_y^2}{R^2}(-dt^2 + d\vec{x}^2) + R^2 \frac{dr_y^2}{r_y^2}}_{AdS_5} + \underbrace{R^2 d\Omega_5^2}_{\text{throat: } S_5}$$

Klebanov

scattering of long closed-string modes $h_{\mu\nu}, \phi, B_{\mu\nu} \cdots$

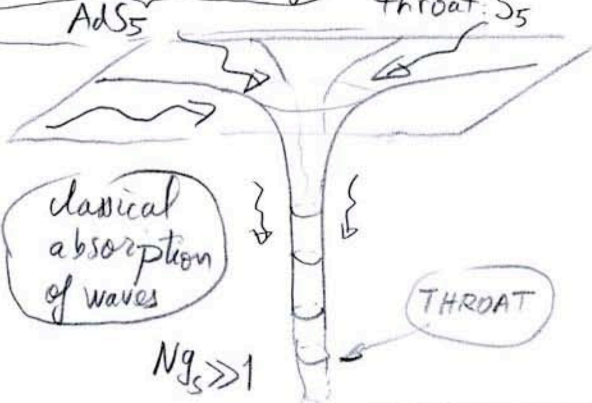


quantum conversion $h_{\mu\nu} \to A\,A$

$N g_s \ll 1$

=

classical absorption of waves



$N g_s \gg 1$

Maldacena

N D3 $A_\mu^{ij}(t,\vec{x})$, $X^{I=1,\dots,6}_{(t,\vec{x})}$, Fermions

Low-energy excitations $=$ Super YM$_{SU(N)}$ in 3+1 dim g_{YM} $\equiv$ 'THROAT' $\equiv$ $ds^2_{10} = AdS_5 \times S_5$

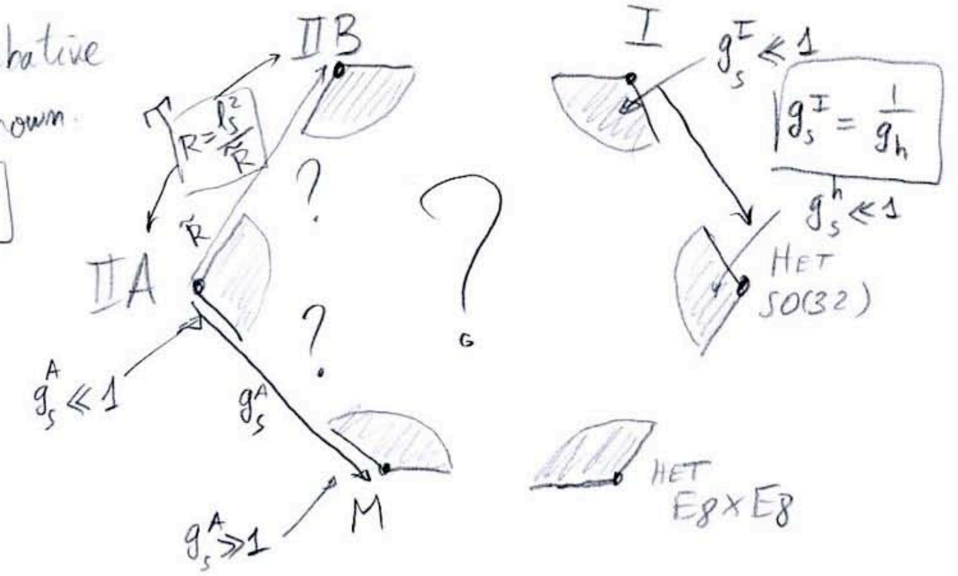
$\boxed{g_{YM}^2 = 2\pi g_s}$

IIB Strings on $ds^2_{10} = AdS_5 \times S_5$, radius $R = \ell_s (4\pi g_s N)^{1/4}$

Web of dualities

only perturbative expansions known

$\boxed{g_s^{\text{theory}} \to 0}$



Like 'multi-chart approach' but WITHOUT OVERLAP in g_s-space

For some results
SUSY allows for

$\boxed{\text{Analytic Continuation of some Perturbative Expansions}}$

analog : $x \to 0$ Taylor expansion $\text{Tayl}[f(x)] = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$

$y \to 0$ Taylor $\text{Tayl}[g(y)] = -\ln y + y - \frac{y^2}{2} + \frac{y^3}{3} \ldots$

RESUMMATION :

$$f(x) = \ln(1+x)$$
$$g(y) = \ln\left(\frac{1+y}{y}\right) = \ln\left(1+\frac{1}{y}\right)$$

$f(x) = g(y)$ with $y = \frac{1}{x}$

$\exists$ NO (USABLE) GLOBAL FORMULATION of the full string + Brane theory

$\boxed{6.7}$ Some lessons from String theory

- Spacetime geometry $\simeq$ soup of more fundamental objects:

  e.g. strings $\left(\overset{m^2=0}{\not{/}} = \not{/}\, h_{\mu\nu} \right)$ or YM A_μ? or Membranes?

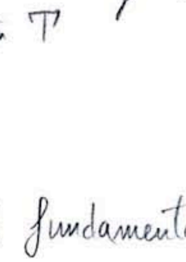
Elastic strings $\rightarrow$ Elasticity of spacetime

tension T Einstein $\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{deformation}} = \underset{\text{elasticity constant (Hooke)}}{\kappa} \underbrace{T_{\mu\nu}}_{\text{tension}}$

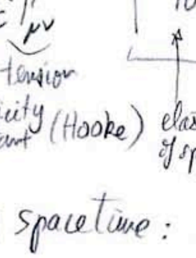
$$\boxed{\underset{\text{elasticity of spacetime}}{\kappa_{10}} \sim g_s^2 \frac{1}{\underset{\text{string tension}}{T'^4}}}$$

- $\exists$ fundamental length in probing spacetime: l_s or $l_{P_{11}} \sim g_s^{1/3} l_s$
- Space dimensions can appear ($g_s \to \infty$, $N g_s \to \infty$), disappear, be regenerated ($R \to 0$)
- gravity may be an emergent phenomenon

  eg AdS/CFT: $A_\mu^{YM} + X^{1\dots6} + \not{4}$ in $\eta_{\mu\nu}$ $D=4$ $\xrightarrow{g_{YM}^2 N \gg 1}$ DYNAMICAL GRAVITY G_{MN} in $D = 5+5$
- OPEN (A_μ) $\leftrightarrow$ CLOSED ($G_{\mu\nu} \dots$) duality
- Suggest new possibilities for describing our 'D=4 world'



KK-type compactification of 6 dimensions $\vec{y}$



Gauge interactions on brane world + gravity in bulk

confined by KK or by warping $e^{2A(y)} g_{\mu\nu}(t\vec{x})$

- $\boxed{g_s = e^{\langle \Phi(x) \rangle}}$ suggest

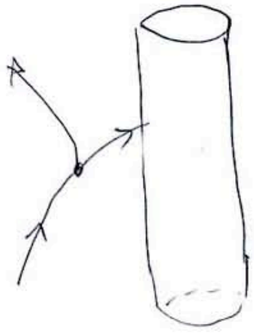
  geometry $\sim$ gravitation $\sim$ gauge couplings $\sim$ gravitational coupling

Less and Less absolute elements $\quad g_{\mu\nu}(x) \sim \quad g(x) \sim \quad G(x)$

suggest possible violations of Equivalence Principle $\frac{\Delta a}{a} \neq 0$, $\dot{\alpha} \neq 0$

6.8 Black Hole Entropy in GR

Irreversibility in BH physics



Energetics of BH

Christodoulou–Ruffini ('71) Mass Formula

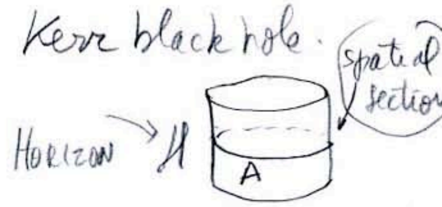
$$\boxed{M^2 = \left(M_{irr} + \frac{Q^2}{4M_{irr}}\right)^2 + \frac{\vec{J}^{\,2}}{4M_{irr}^2}}$$

extractable ('free') energy

non extractable energy

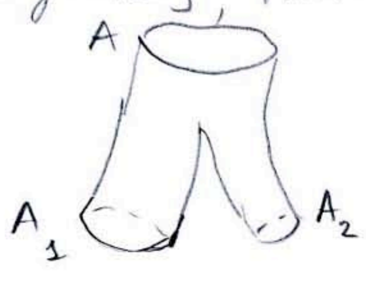
irreducible mass $\boxed{\delta M_{irr} \geqslant 0}$

Kerr black hole.



Area $\boxed{A = 4\pi(r_+^2 + a^2) = 16\pi M_{irr}^2}$

More generally, Hawking ('72) $\boxed{\delta\left(\sum_a A_a\right) \geqslant 0}$



Analogy Black Holes $\longleftrightarrow$ Thermodynamics

AGR 6.18

Area $A \nearrow$ Entropy $S \nearrow$

suggest BH Entropy $\boxed{S_{BH} \equiv \alpha A}$

BH surface gravity $\ell^\nu \nabla_{\ell_\nu} \ell^\mu = \kappa \ell^\mu$

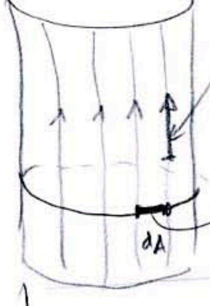
$$dM(M_{irr} \text{ or } A, J, Q) = \underset{\text{angular velocity}}{\Omega} dJ + \underset{\text{electric potential}}{V} dQ + \underbrace{\boxed{\frac{\kappa}{8\pi} dA}}_{T\, dS}$$

$$\boxed{\kappa_{Kerr} = \frac{\sqrt{M^2 - a^2 - Q^2}}{r_+^2 + a^2}}$$

suggests $\boxed{T_{BH} \equiv \frac{1}{\alpha}\left(\frac{\partial M}{\partial A}\right)_{J,Q} = \frac{\kappa}{8\pi\alpha}}$

- Equilibrium black holes: Theorem: κ = uniform on HORIZON
  OK "0th law of thermodynamics" T_{BH} uniform

- Non-equilibrium version of 'second law of BH thermodynamics'
  (Damour '78 '82)



$$\boxed{\frac{d}{dt}(dS_{BH}) - \tau \frac{d^2}{dt^2}(dS_{BH}) = \frac{dA}{T_{BH}}\left[2\eta\, \sigma_{AB}\sigma^{AB} + \zeta\theta^2 + \rho\left(\vec{J}_{BH} - \sigma_{BH}\vec{v}\right)^2\right]}$$

$\tau = \kappa^{-1}$

local increase of BH entropy; BH shear viscosity $\eta = \frac{1}{16\pi}$; shear; expansion; Joule effect, BH surface resistivity $\rho = 4\pi = 377$ OHM

Bekenstein-Hawking entropy

Bekenstein ('72)

Limits to Reversible Transformations $\underbrace{\delta M - \Omega \delta J - V \delta Q}_{\propto \delta M_{irr} \propto \delta A} = \frac{r_+^2 + a^2 \cos^2\theta}{r_+^2 + a^2} |p^r|$

Christodoulou-Ruffini: Reversibility reached when $\boxed{p^r = 0}$ at $\boxed{r = r_+}$ HORIZON

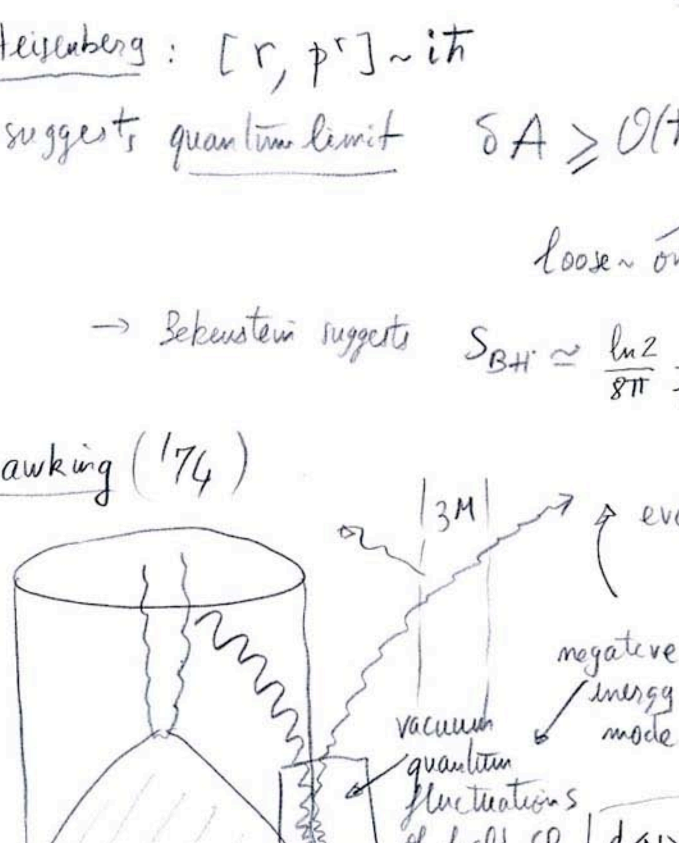
Heisenberg: $[r, p^r] \sim i\hbar$

suggests quantum limit $\delta A \geq \mathcal{O}(\hbar)$ when absorbing one particle

loose ~ one bit of information

→ Bekenstein suggests $S_{BH} \simeq \frac{\ln 2}{8\pi} \frac{c^3}{\hbar G} A$

Hawking ('74)



Hawking: Particle Creation

$$\boxed{\frac{d}{dt}\langle N \rangle = \sum_{l,m} \int \frac{d\omega}{2\pi} \frac{\Gamma_l(\omega)}{e^{\frac{\omega - m\Omega - eV}{T_{BH}}} - 1}}$$

Planck distribution + Grey-Body filter $\Gamma_l(\omega)$

$$\boxed{T_{BH} = \frac{\hbar}{c} \frac{\kappa}{2\pi}} \Rightarrow \boxed{\alpha = \frac{1}{4} \frac{c^3}{\hbar G}} : \boxed{S_{BH} = \frac{1}{4} \frac{c^3}{\hbar G} A \equiv \frac{1}{4} \frac{A}{l_P^2}}$$

Open issues

- Usually, Boltzmann: $S = \ln \left(\#\ \text{microscopic states with given macroscopic charact.} \right)$ E, Q, Vol.

  ? meaning of S_{BH} ?

- Prove generalized second law: $\delta \left(S_{ext} + \frac{1}{4} \frac{A}{\ell_P^2} \right) \geq 0$?

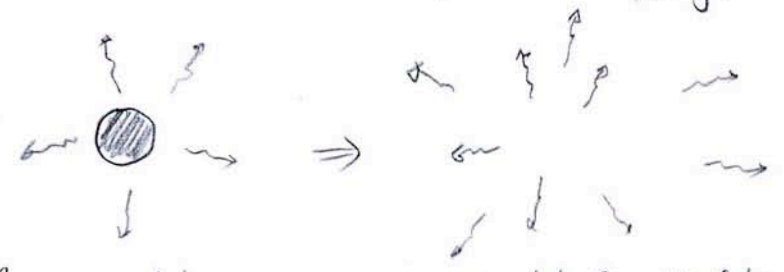
- Final state of evaporation process ?

$$\frac{dM}{dt} \sim -A\, T_{BH}^4 \sim -(M)^2 \left(\frac{1}{M}\right)^4 \sim -M^{-2}$$

$$\Rightarrow \quad M^3 \sim M_0^3 - (t - t_0)$$

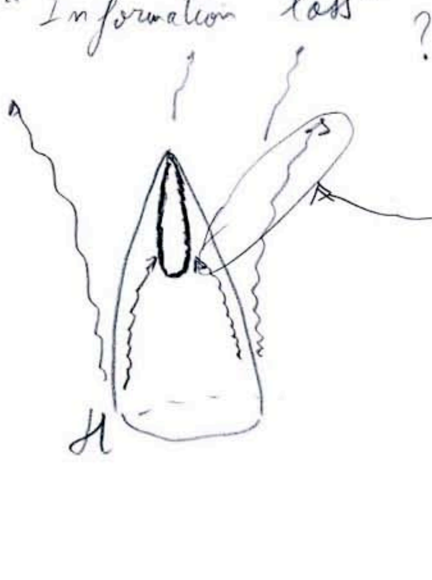
$$\Rightarrow \quad t_{evaporation} \sim t_P \left(\frac{M}{M_P}\right)^3 \sim 10^{-44}\,s \left(\frac{M}{10^{-5}g}\right)^3 \sim 10^{10}\,yr \left(\frac{M}{10^{14}g}\right)^3$$

If



? pure state $\longrightarrow$ thermal state ? Violation of unitarity ?

- "Information loss" ? What happened to the correlations between particle - antiparticle in created pairs



$\boxed{6.9}$ BPS Extremal BH in String Theory

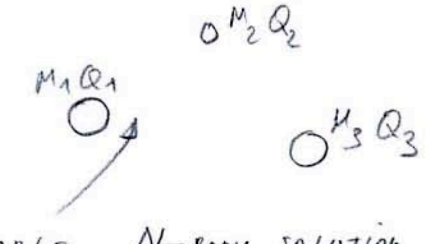
Extremal BH $:\Leftrightarrow$ $\boxed{\kappa = 0}$ $\leftrightarrow$ $\boxed{T_{BH} = 0}$

no evaporation : stable objects

Papapetrou - Majumdar

N extreme RN BH's

$GM^2 = Q^2$



"NO FORCE" : STABLE N-BODY SOLUTION

$GM_1M_2 = Q_1Q_2$

true beyond Newtonian forces

Some of the extremal BH solution correspond to 'Bogomolnyi-Prasad-Sommerfield' (BPS) states i.e. quantum states that preserve a fraction of the (32) global supersymmetry generators present in D=10 vacuum

$\rightarrow$ Strong restriction on the mass of those states

Saturate a general inequality $M \geq f(Q, g)$

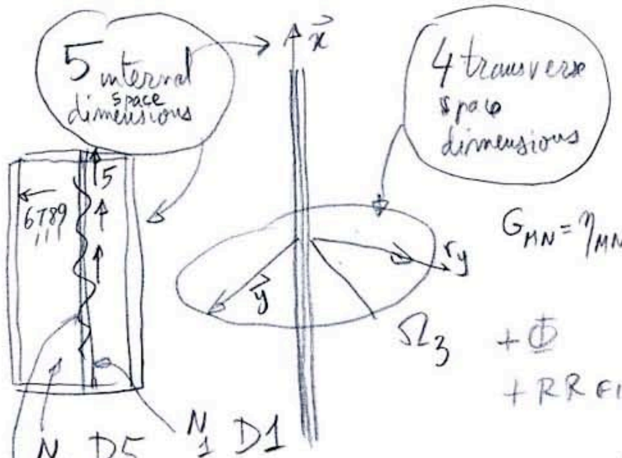
$$\Rightarrow \boxed{M = f(Q, g_s) \quad \forall g_s}$$

can consider analytic continuation in g_s

An example: D1-D5 brane system IIB

Strominger Vafa '96, Callan Maldacena '96

$N g_s \ll 1$ $\qquad\qquad$ $N g_s \gg 1$

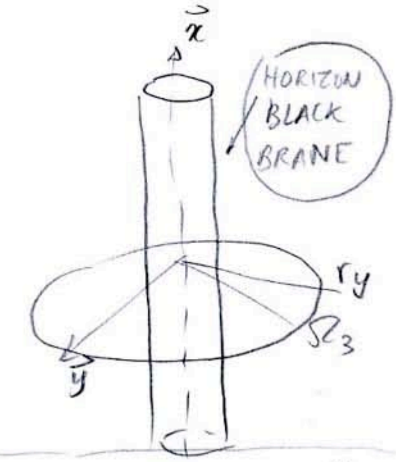


$$G_{MN} = \eta_{MN} + \mathcal{O}\left(\frac{N g_s}{r_y^2}\right)$$

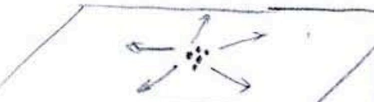
$+ \Phi$

$+$ RR FIELDS

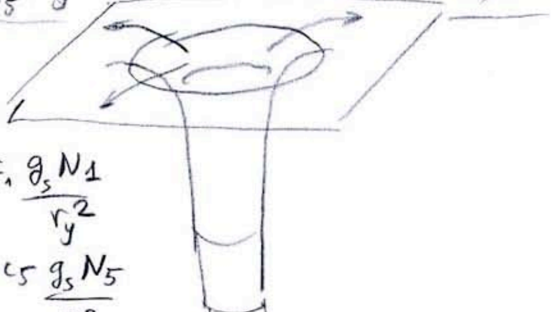
Momentum $P_5 = N/R_5$ in direction x_5



$$\boxed{\begin{aligned} ds^2_{\text{string}} &= H_1^{-1/2} H_5^{-1/2}\left[-dt^2 + dx_5^2 + H_n (dt+dx_5)^2\right] \\ &+ H_1^{1/2} H_5^{1/2}\, d\vec{y}^{\,2} + H_1^{1/2} H_5^{-1/2}\left(dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2\right)\end{aligned}}$$



$$\boxed{e^{-2\Phi} = \frac{H_5}{H_1}}$$



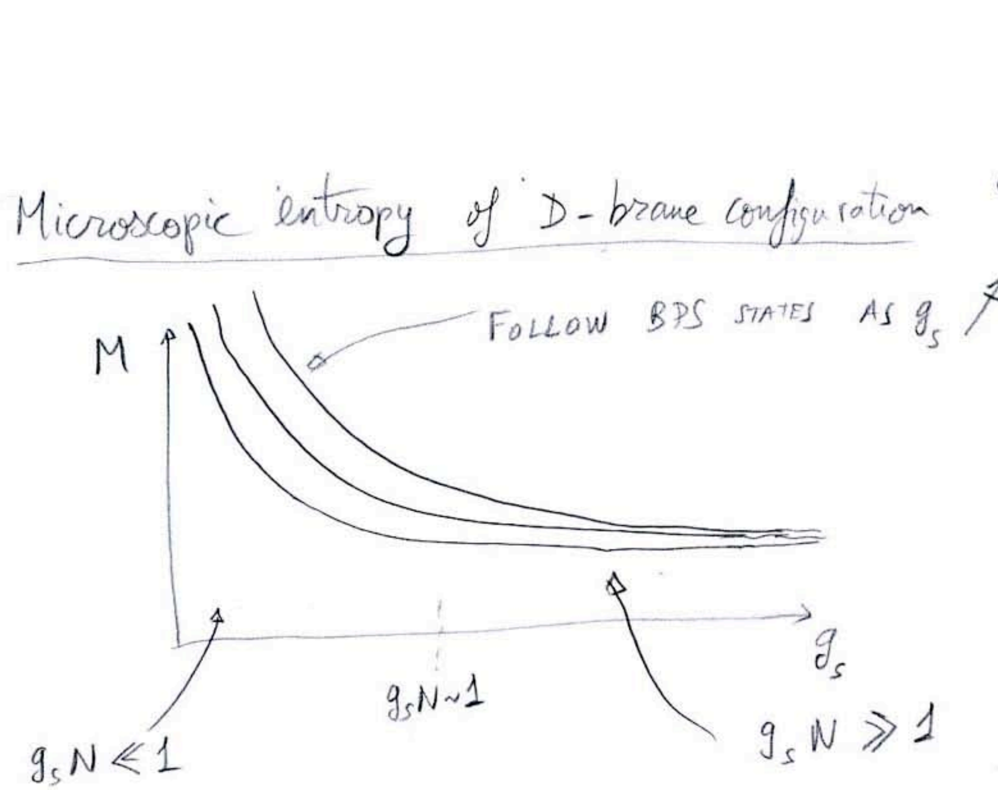
$$\left\{\begin{aligned} H_1 &= 1 + c_1 \frac{g_s N_1}{r_y^2} \\ H_5 &= 1 + c_5 \frac{g_s N_5}{r_y^2} \\ H_n &= c_n \frac{g_s^2 N}{r_y^2}\end{aligned}\right.$$

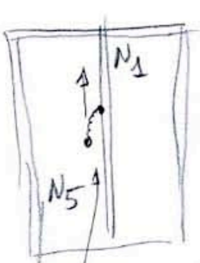
Mass of brane configuration

$$\boxed{M = \underbrace{\frac{N_1 R_5}{g_s}}_{} + \underbrace{\frac{N_5 R_5 V_4}{g_s}}_{} + \frac{N}{R_5}}$$

valid $\forall g_s$ because BPS configuration

Microscopic entropy of D-brane configuration





distribute total momentum $P = \frac{N}{R_5}$

among $4N_1N_5$ bosonic excitations $\begin{pmatrix}1-5\\5-1\end{pmatrix}$ open strings
$+\,4N_1N_5$ fermionic ones

Counting how many ways BOSONIC OSCILL. FERMIONIC OSCILL.

$$N = \sum_{i=1}^{4N_1N_5} \sum_{n} \left\{ n (a^i_n)^\dagger a^i_n + n (b^i_n)^\dagger b^i_n \right\}$$

$\boxed{\text{DEGENERACY}}$

# of moving modes   Bosons   Fermions.

$$\mathcal{D}(N) \simeq \exp\left[2\pi \sqrt{4N_1N_5\left(1+\tfrac{1}{2}\right)\frac{N}{6}}\,\right]$$

$$\boxed{\log \mathcal{D}(N) \simeq 2\pi\sqrt{N_1N_5N}}$$

'Area' of black brane $r_y \to 0$

3-sphere in $\vec{y}$ × 5-6-7-8-9

Einstein-frame metric

$$ds_E^2 = e^{-\frac{\Phi}{2}} ds^2$$
$$= H_1^{-\frac{3}{4}} H_5^{-1/4}\left[-dt^2 + dx_5^2 + H_m (dt+dx_5)^2\right]$$
$$+ H_1^{1/4} H_5^{3/4}\, d\vec{y}^{\,2}$$
$$+ H_1^{1/4} H_5^{-1/4}\left[dx_6^2 + dx_7^2 + dx_8^2 + dx_9^2\right]$$

↓

'Area' of horizon

$$A = 2^6 \pi^7 g_s^2 \ell_s^8 (N_1 N_5 N)^{1/2}$$

using

$$G = 8\pi^6 g_s^2 \ell_s^8$$

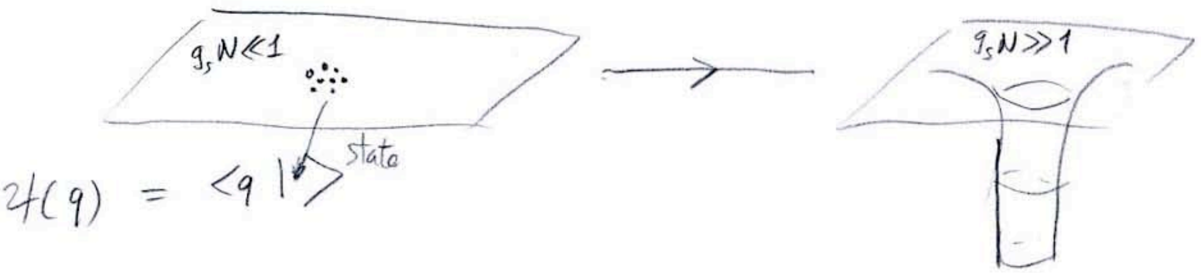
$$\boxed{\frac{A}{4G} = 2\pi\sqrt{N_1N_5N}}$$

EXACT AGREEMENT

This agreement is a deep confirmation of the ability of string theory to describe quantum gravitational effects : remember $S_{BH} \propto \frac{1}{\hbar G}$

However:

- No understanding on how

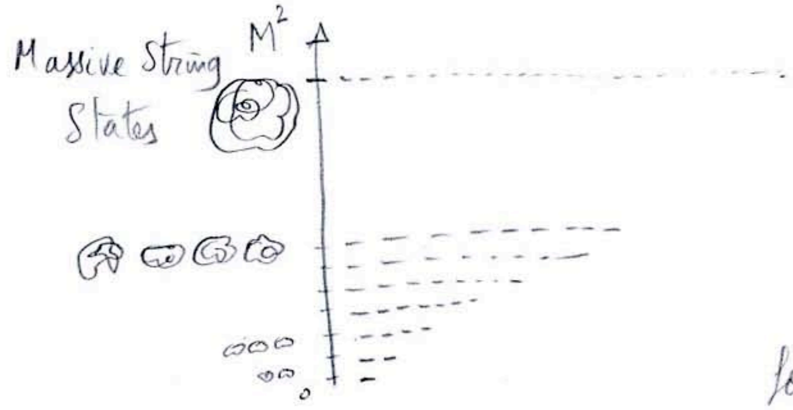


- No understanding of microscopic dof when ↗ in BH state

In addition, $\exists$ many results for BPS or near BPS configurations, but much less understanding of states such as Schwarzschild BHs.

$\boxed{6.10}$ Massive string states and Schwarzschild B Holes

(Susskind '93; Horowitz Polchinski '97 '98; Damour Veneziano '00)



?

do they become black holes as $M \nearrow$?

for which mass M ?

Degeneracy of mass level M

e.g. Open bosonic string (in light-cone gauge: $X^+ \propto \tau$, X^-, X^i) — $i = D-2$ transverse dimensions

$$\boxed{\alpha' M^2 = N - 1} \quad \boxed{N = \sum_{n=1}^{\infty} \sum_{i=1}^{D-2} n \, (a_n^i)^\dagger a_n^i} \quad [a_n^i, (a_m^j)^\dagger] = \delta^{ij} \delta_{nm}$$

oscillators

eigenvalues of each $a^\dagger a = 0, 1, 2, 3, \cdots$

Counting problem: how many partitions

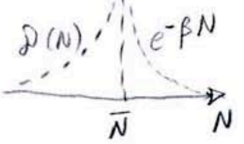
$$\text{if } D-2 = 1 \; : \; N = \underbrace{1+1}_{N_1} + \underbrace{2+2+2}_{N_2} \underset{N_3}{+3} + \cdots + \underbrace{p+p+p+\cdots+p}_{N_p} + \cdots$$

Generating function 'partition function'

$$Z(\beta) = \mathrm{Tr}\, e^{-\beta \hat{N}} = \sum_N \underset{\text{degeneracy}}{\mathcal{D}(N)}\, e^{-\beta N}$$

$$= \prod_i \prod_n \mathrm{Tr}\, e^{-\beta n\, a^{\dagger i}_n a^i_n} = \prod_n \left(\mathrm{Tr}\, e^{-\beta n\, a^\dagger_n a_n}\right)^{D-2}$$

$$= \prod_n \left(1 + e^{-\beta n} + e^{-2\beta n} + e^{-3\beta n} + \dots\right)^{D-2} = \left(\prod_n \frac{1}{1-e^{-\beta n}}\right)^{D-2}$$

but $Z(\beta) = \sum_N$  $\simeq \mathcal{D}(\bar{N})\, e^{-\beta \bar{N}} = e^{S(\bar{N}) - \beta \bar{N}}$

$$\bar{N} = \sum_N N\, \mathcal{D}(N) e^{-\beta N} = -\frac{\partial}{\partial \beta} \ln Z(\beta)$$

Hence $S(\bar{N}) - \beta \bar{N} \simeq \ln Z(\beta) = (D-2) \sum_n \ln\left[\left(1-e^{-\beta n}\right)^{-1}\right]$

Continuous approximation: $x = \beta n$

$$\sum_n -\ln\left(1-e^{-\beta n}\right) \simeq -\frac{1}{\beta} \int_0^\infty dx\, \ln\left(1-e^{-x}\right) = \frac{\pi^2}{6\beta}$$

$$\Rightarrow \quad \boxed{S_{\text{string}}(N) \equiv \ln \mathcal{D}(N) \simeq 2\pi \sqrt{\frac{D-2}{6} N}}$$

in terms of M

$\frac{M^2}{m_s^2} = N-1$

$$\boxed{S_{\text{string}}(M) = \ln \mathcal{D}(M) \simeq 2\pi \sqrt{\frac{D-2}{6}}\, \frac{M}{m_s}}$$

linear in M i.e. $\boxed{\mathcal{D}(M) \sim e^{a_0 M}}$

Black Hole entropy in uncompactified space dimension d

spacetime $d+1$

$$S_{BH}(M) \sim \frac{A}{G} \sim \frac{R_{BH}^{d-1}}{G}$$

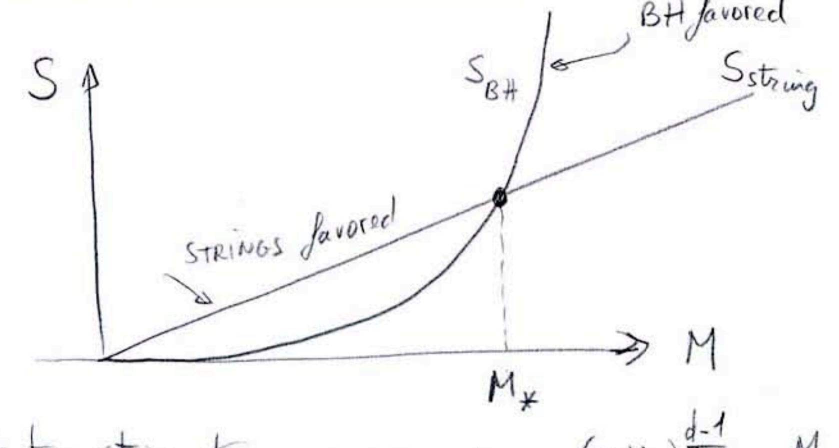
Radius of black hole:

$$-g_{00} = 1 - c\,\frac{GM}{r^{d-2}}$$

$$R_{BH} \sim (GM)^{\frac{1}{d-2}}$$

$$\boxed{S_{BH}(M) \sim \frac{(G_d M)^{\frac{d-1}{d-2}}}{G_d} \sim GM^2 \text{ in } d=3}$$

Hence



expect transition string $\rightleftarrows$ BH when $\frac{(GM_*)^{\frac{d-1}{d-2}}}{G} \sim \frac{M}{m_s}$

i.e. $\boxed{(GM_*)^{\frac{1}{d-2}} \sim l_s}$

or $\boxed{R^*_{BH} \sim l_s}$

with $\boxed{G \sim g_s^2\, l_s^{d-1}}$ $\quad$ $\boxed{g_s^2 M_* \sim m_s}$

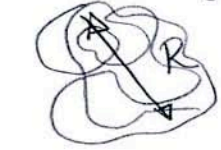
M_* larger than Planck mass

$$M_p \sim \frac{1}{G^{\frac{1}{d-1}}} \sim \frac{m_s}{g_s^{\frac{2}{d-1}}}$$

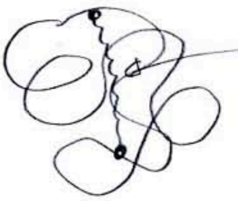
Self-gravity of string and a dynamical study of the transition string → black hole

Consider typical size of string state:

$$\boxed{R^2 \equiv \frac{1}{d}\left\langle \left(\tilde{X}^{\mu}_{\perp\, p'_\mu}(\tau\sigma)\right)^2\right\rangle = \frac{2}{d}\, l_s^2 \sum_{n=1}^{\infty} \frac{a_n^{i\dagger} a_n^i}{n}} \qquad i=1,\dots,d$$



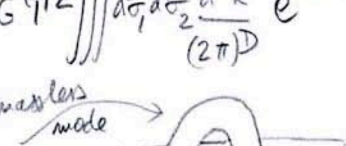
Effect of size on the mass of the string state



exchange of $G_{\mu\nu}$, Φ, $B_{\mu\nu}$

$$\int d\tau\, \delta M^2 = 64\pi G T^2 \iiiint d^2\sigma_1 d^2\sigma_2 \frac{d^D k}{(2\pi)^D}\, e^{ik\cdot(X_1 - X_2)} (\partial_+ X_1^\mu \cdot \partial_+ X_{2\mu}) \times (\partial_- X_1^\nu \cdot \partial_- X_{2\nu})$$

1-LOOP



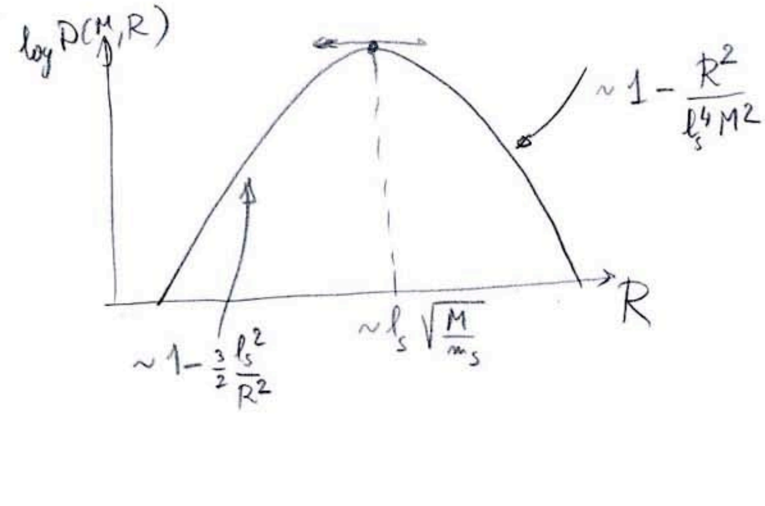
$$\Rightarrow \quad \boxed{M = M_0 - c_d \frac{GM^2}{R^{d-2}}}$$

mass shift depends on size of state

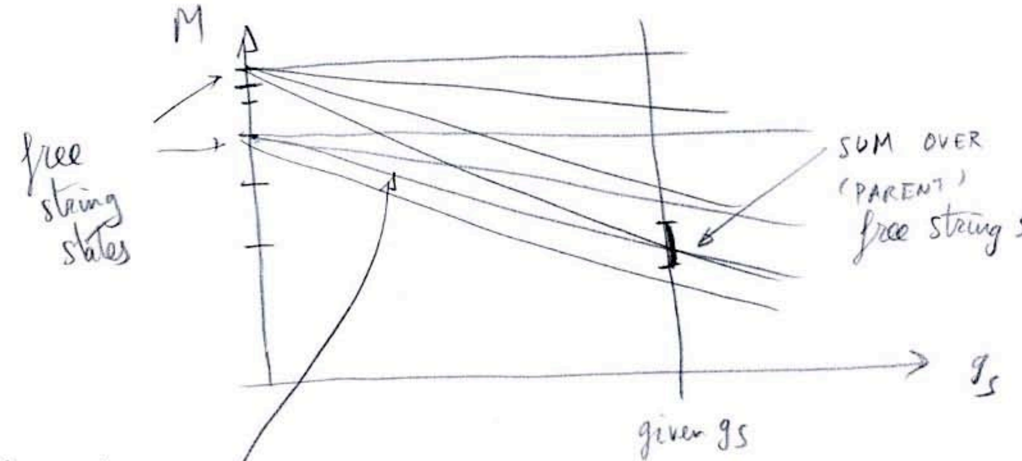
M_0: 'bare mass' when one adiabatically let $g_s \to 0$

Distribution of free string states by size: modified counting by imposing constraint R^2 = given

$\Rightarrow Z(\beta,\gamma) = \mathrm{Tr}\, e^{-\beta \hat{N} - \gamma \hat{R}^2}$



Entropy of self-gravitating strings



self-gravity lifts free degeneracy

Degeneracy of self-gravitating string states

$$\mathcal{D}(M,R) \simeq e^{S(M,R)}$$

$$S(M,R) \simeq a_0 M \left(1-\frac{1}{R^2}\right)\left(1-\frac{R^2}{M^2}\right)\left(1+\frac{g_s^2 M}{R^{d-2}}\right)$$

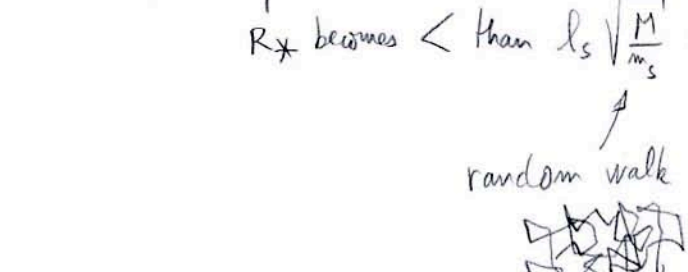
units $l_s \sim 1$

usual string entropy

less states if R small because of constraint on size

more states if R small because $M_0 > M$

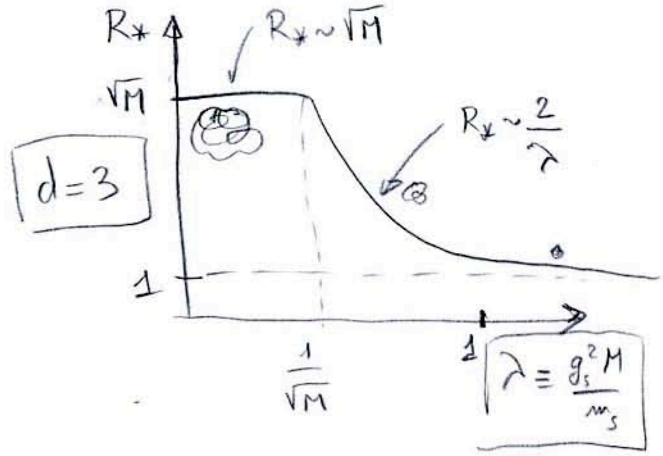
Most probable size of self-gravitating string

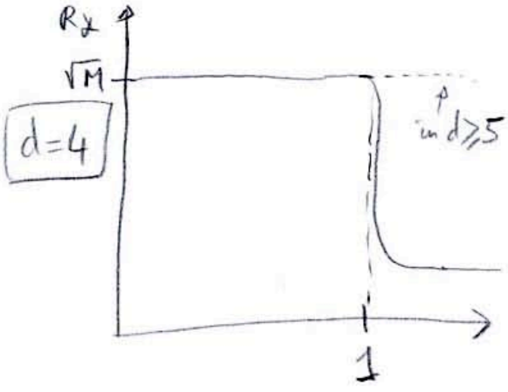


R_* becomes < than $l_s \sqrt{\frac{M}{m_s}}$ of free strings

random walk

Shrinking of string size with self-gravity

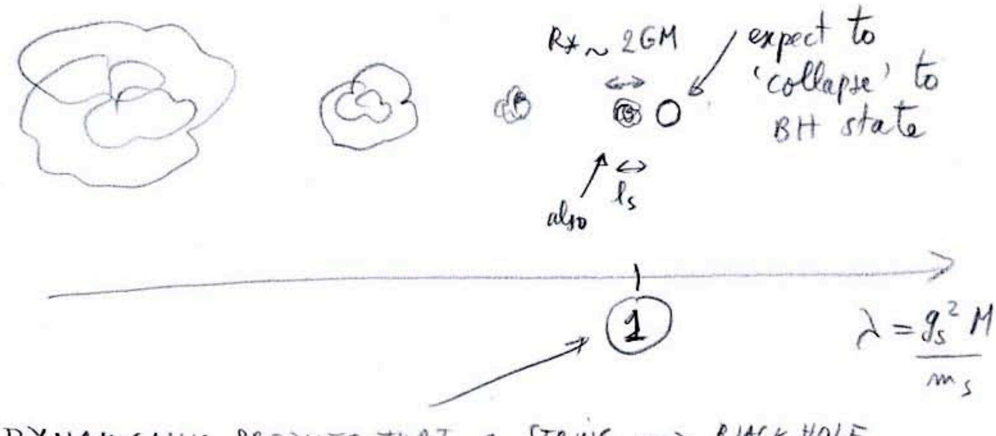




Importance of self-gravity : in $\boxed{d=3}$ 'compactness' $\equiv \frac{2GM}{R_*} \simeq \frac{2\lambda}{\frac{2}{\lambda}} \sim \lambda^2$

when $\boxed{\lambda \to 1}$: $R_* \sim 2GM \Rightarrow$ expect string to become a BH

in other words

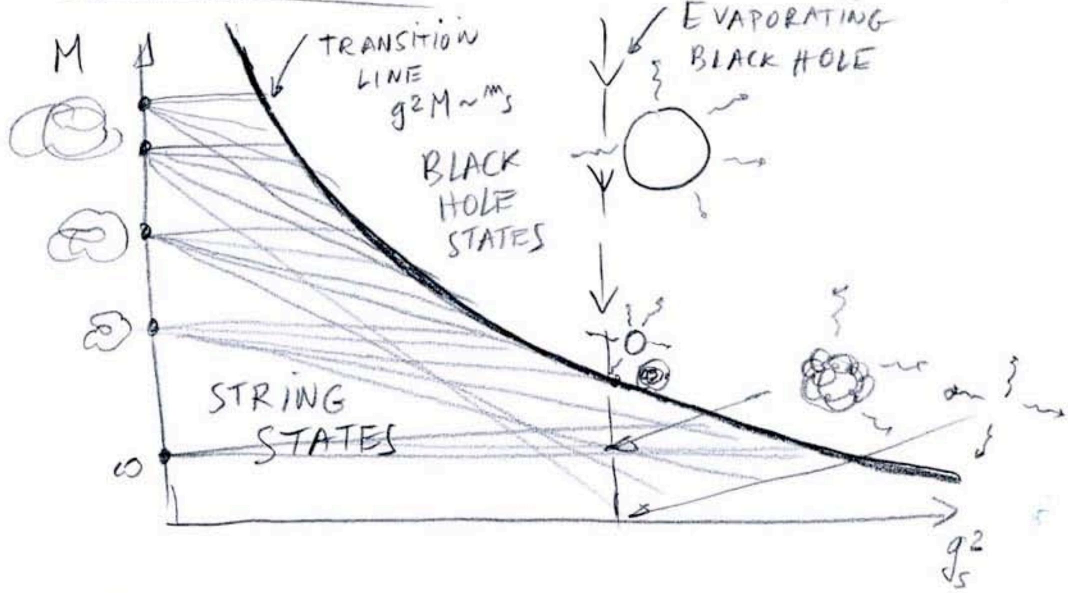


DYNAMICALLY PREDICTS THAT a STRING $\longrightarrow$ BLACK HOLE

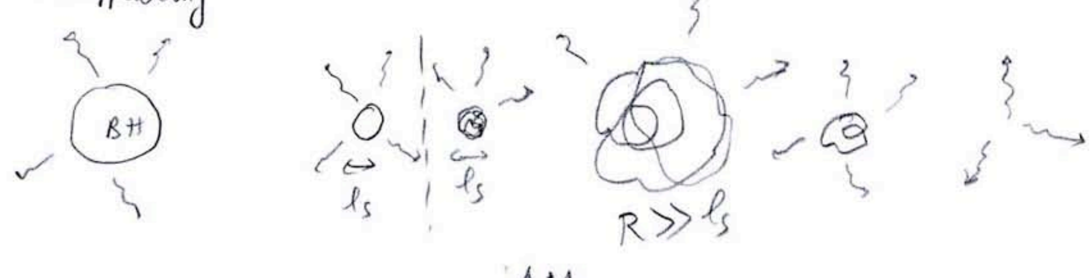
WHEN $\boxed{g_s^2 M = m_s}$

i.e. precisely when $\boxed{\underbrace{S_s}_{\text{STRING entropy}}(M) \sim \underbrace{S_{BH}}_{\text{BH entropy}}(M)}$

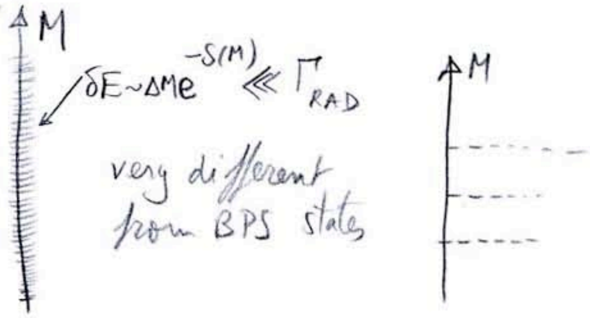
Physical picture



- confirms statistical meaning of $S_{BH}(M)$ as $S_{STRING}(M)$ when $g_s^2 M = M_s$, but does not help to explain statistical meaning when $g^2 M > M_s$
- suggests that an evaporating BH transforms into a very tight string state when $M = \frac{m_s}{g_s^2} \gg M_{Planck}$
  then it expands again to size $R \gg l_s$, before shrinking and disappearing



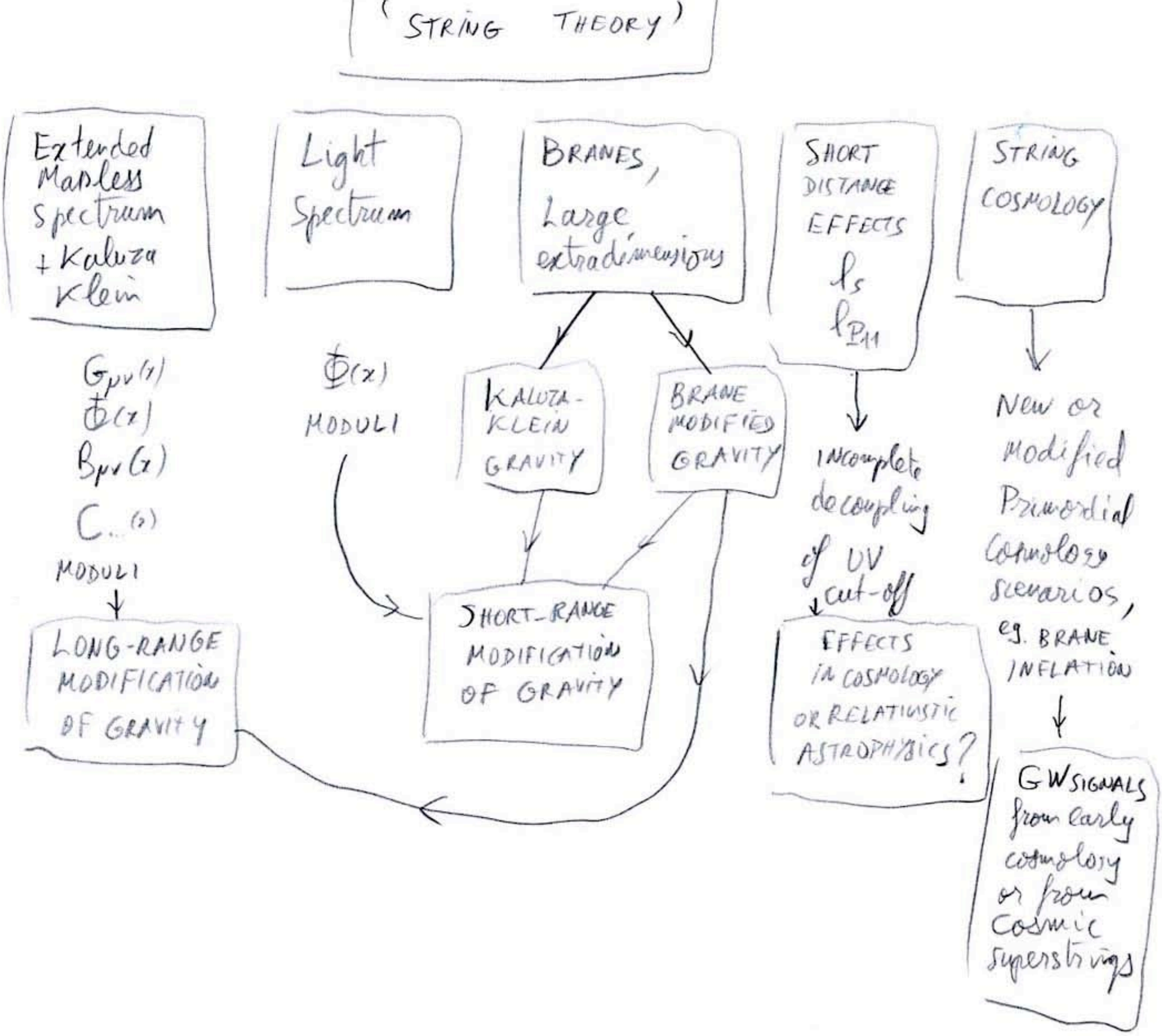
- blurred discreteness of BH quantum states



$\boxed{6.11}$ Some possible phenomenological consequences of string theory

- No clear understanding of link String thy $\longleftrightarrow$ Real World

$\rightarrow$ discuss phenomenological possibilities $\rightarrow$ new experimental opportunities



LONG-RANGE MODIFICATION OF GRAVITY

↓

If long range additional scalar $\varphi(x)$ with gravitational strength expect observable effects in
- Solar-System tests
- Binary PSR tests
- EP violation $\Delta a/a \neq 0$, $\dot{\alpha} \neq 0$

Usually require m_Φ large

However :

$\exists$ Least-Coupling Attractor Mechanism Damour Polyakov '94, Damour Piazza Veneziano '00

→ Naturally small, but possibly observable effects ($n=2$: χ^2 inflation)

$\alpha_0^2 \sim 10 \left(\frac{\delta\rho}{\rho}\right)^{\frac{8}{n+2}} \sim 2.5 \times 10^{-8}$

→ $\gamma^{PPN} - 1 \sim -5 \times 10^{-8}$

$\Delta a/a \sim 5 \times 10^{-5} \alpha_0^2 \sim 10^{-12}$

$\dot{\alpha}_{em}/\alpha_{em} \sim \sqrt{10^{12} \frac{\Delta a}{a}}\ 10^{-16}\,yr^{-1}$

Or Combining Attractor mech + Quintessence-like $V(\varphi)$

→ Chameleon mechanism Khoury, Weltman '04; Brax...04

Possible signals in

MICROSCOPE (STEP) $\frac{\Delta a}{a} \sim 10^{-15} (10^{-18})$

MORE, GAIA ($\gamma^{PPN}_{-1} \sim 10^{-7}$), LATOR ($\gamma^{PPN}_{-1} \sim 10^{-9}$)

SHORT-RANGE MODIFICATION OF GRAVITY

↓

If moduli φ has mass m_φ linked to SUSY breaking

$V(\varphi) \sim m_{SUSY}^4\, \mathcal{V}\!\left(\frac{\varphi}{\tilde{m}_P}\right)$

↓

$$\boxed{m_\varphi \sim \frac{m_{SUSY}^2}{\tilde{m}_P}}$$

$? \sim \frac{(TeV)^2}{2.4 \times 10^{18}\,GeV} \sim 10^{-3}\,eV$

↓

Possibly observable in Cavendish-type expts $\boxed{\text{below } 0.1\,mm}$

Taylor Veneziano '88

Ferrara et al '94

Dimopoulos Giudice '96

Antoniadis Dimopoulos Dvali '98

↑

BRANES AND GRAVITY

Various possibilities and consequences

Antoniadis Arkani-Hamed, Dimopoulos, Dvali

Randall Sundrum

Dvali Gabadadze Porrati

Kogan, Mouslopoulos, Papazoglou, Ross

CUT-OFF RELATED PHYSICS

↓

? effect on propagation of UHECR?

? effect on $\frac{\delta\rho}{\rho}$ from inflation

↑

microscope of ultra UV fluctuations

GW SIGNALS FROM COSMOLOGY

↓

Various possible stochastic GW backgrounds

also : possibility of $\boxed{\text{COSMIC SUPERSTRINGS}}$

Witten '85

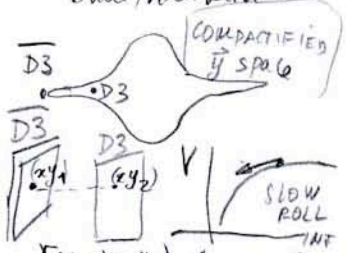
Dvali, Tye

Tye...

Kachru Kallosh Linde Maldacena McAllister Trivedi

Copeland Myers Polchinski

Dvali, Vilenkin



COMPLEX TACHYON

↓

STRINGS @ end inflation

$10^{-11} \lesssim G T \lesssim 10^{-6}$

↓

GW bursts

Potentially observable in LIGO/VIRGO, LISA, PULSAR TIMING Damour, Vilenkin

CONCLUSIÒN

THE NEW GRAVITY FRONTIER

- String theory puts gravity in a central place, and suggest many new ways in which gravity could be richer than ordinary GR

- Observational discoveries suggest gravity might need modifications

  DARK MATTER

  'DARK ENERGY' ≡ ACCELERATED EXPANSION

  ? PIONEER 10, 11 anomalous acceleration $a \simeq 9 \times 10^{-8}$ cm/s^2 $\simeq c H_0$?

- Observational opportunities are ahead:

CMB, Dark Matter, Dark Energy, EP tests, Variation of Constants, Short-range tests, GW observations, Pulsar timing, LHC, ...