Equation of state for compact stars

Lecture 2

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IHP Paris, October 31st, 2006

Neglecting NN interactions $\Longrightarrow$ a model of neutron stars built of ideal Fermi gas of neutrons. This is what Oppenheimer & Volkoff (1939) did (they could be excused for doing that). They got **maximum allowable mass** for neutron stars $M_{\rm max} = 0.71 M_\odot$, half of the Chandrasekhar mass limit for white dwarfs. This was a puzzle! Today, the precisely measured mass of the Hulse-Taylor binary pulsar is $1.44 M_\odot$. **Observations** tell us that

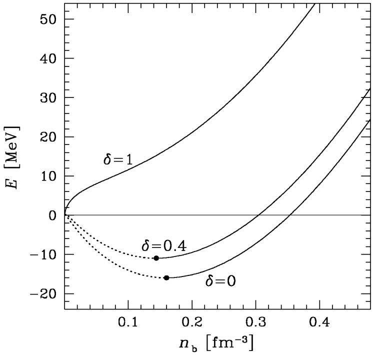
$$M_{\rm max}(\text{with interactions})/M_{\rm max}(\text{no interactions}) > 2$$

Strong interactions are crucial for understanding neutron stars.

Nuclear matter and experiment

Nuclei are droplets of self-bound nuclear matter. Ideal infinite system: $A = N + Z \longrightarrow \infty$, Coulomb forces being switched-off. Uniform medium, with nucleon density $n_{\rm b} = n_n + n_p$ and the asymmetry parameter $\delta = (n_n - n_p)/n_{\rm b}$, so that $n_n = (1+\delta)n_{\rm b}/2$, $n_p = (1-\delta)n_{\rm b}/2$. Charge symmetry of nuclear forces implies that $E(n_{\rm b}, \delta) = E(n_{\rm b}, -\delta)$.

General nuclear physics reference: Bhaduri & Preston (1975)



Energy per nucleon versus baryon number density for symmetric nuclear matter ($\delta = 0$), asymmetric nuclear matter with $\delta = 0.4$ (such an asymmetry corresponds to the neutron-drip point in a neutron star crust and to a central core of a newly born protoneutron star), and pure neutron matter ($\delta = 1$). Minima of the $E(n_{\rm b})$ curves are indicated by filled dots. Dotted segments correspond to negative pressure. Calculations are performed for the SLy4 model of effective nuclear Hamiltonian, which was used to calculate the SLy EOS by Douchin & Haensel (2001). It yields $n_0 = 0.16~{\rm fm}^{-3}$ and $E_0 = -16.0$ MeV.

$B_0 = -E_0 =$ maximum binding energy per nucleon in nuclear matter ("at saturation point" - $n_{\rm b} = n_0$, $\delta = 0$). In the vicinity of saturation point:

$$E(n_{\rm b}, \delta) \simeq E_0 + S_0\ \delta^2 + \frac{K_0}{9}\left(\frac{n_{\rm b} - n_0}{n_0}\right)^2 , \qquad (1)$$

where S_0 and K_0 are, respectively, the nuclear *symmetry energy* and *incompressibility* at the saturation point,

$$S_0 = \frac{1}{2}\left(\frac{\partial^2 E}{\partial \delta^2}\right)_{n_{\rm b}=n_0,\ \delta=0} , \qquad K_0 = 9\left(n_{\rm b}^2\,\frac{\partial^2 E}{\partial n_{\rm b}^2}\right)_{n_{\rm b}=n_0,\ \delta=0} . \qquad (2)$$

Experiment: $n_0 = 0.16 \pm 0.01\ {\rm fm}^{-3}$,
$B_0 = 16.0 \pm 1.0$ MeV, $S_0 = 32 \pm 6$ MeV, $K_0 \approx 230$ MeV

From principles to practice: **Principle**: interaction is given by the quantum chromodynamics (QCD). The electromagnetic interaction is negligible for the EOS, and the weak interaction enters the problem only indirectly by opening some channels for reaching the ground state of the matter. **Practice**: We have to use an effective theory, where quark degrees of freedom are not treated explicitly but are replaced by hadrons – baryons and mesons – in which quarks are confined. Hadronic Hamiltonian cannot be presently derived from the QCD, we have to use phenomenological models of strong (hadronic) interaction, based partly on mesonic theories, where strong interaction between hadrons is modeled by the exchange of mesons. Most refined and complete phenomenological models constructed for the NN interactions. Tested using thousands of experimental data on NN scattering cross sections supplemented with experimental deuteron (^{2}H) properties. Experimental information on the NH and HH interactions for the lowest-mass hyperons Λ and Σ only. Mainly obtained from studies of **hypernuclei**. Generally: the interaction models for NH and HH are incomplete and plagued by uncertainties due to scarcity (or non-existence) of experimental data.

Hadronic interactions and dense matter

Three body interactions. Two-body hadronic interactions yield only a part of the hadronic Hamiltonian of dense matter. At $\rho \sim 10^{15}$ g cm^{-3}, interactions involving three and more hadrons might be important. Our experimental knowledge of three-body interaction is restricted to nucleons. The three-nucleon (NNN) force is **necessary** to reproduce properties of ^{3}H and ^{4}He and to obtain correct parameters of symmetric nuclear matter at saturation.

Minimal model and beyond. In view of such a high degree of our ignorance, it seems reasonable to start with a model which is the simplest, and not obviously wrong. Such a "minimalistic" approach consists in extending the $npe\mu$ model to $\rho \gtrsim 2\rho_0$. The calculated EOS has to be confronted with observations, to see whether it is *sufficient* to explain observational data. After fulfilling this minimal program, we can try richer models, including hyperons and exotic phases of hadronic matter. Whatever model of dense matter we assume, we should calculate its ground state as a function of density.

Phenomenological NN interaction - 1

Introductory reference: Preston & Bhaduri (1975) Since the dawn of nuclear physics the determination of forces which bind atomic nuclei has been a central problem for experimentalists and theoreticians. In its most basic formulation, the problem consists in determining the nucleon-nucleon potential which would explain the nucleon-nucleon scattering data and the properties of ^{2}H. It has turned out to be a very difficult task. Bethe (1953) estimated that during the preceding 25 years more hours of human work had been devoted to this problem than to any other scientific problem.

Present phenomenological NN potentials fit very precisely a few thousand of NN scattering data in the energy range up to 350 MeV (in laboratory reference frame). At higher energies, non-elastic processes of pion production switch on and the potential model represented by a Hermitian operator becomes meaningless. For an ij pair of interacting nucleons these quantities are represented by the following operators: the relative position vector $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$; spins $\boldsymbol{\sigma}_i$ and $\boldsymbol{\sigma}_j$ (in the units of $\hbar/2$); isospins $\boldsymbol{\tau}_i$ and $\boldsymbol{\tau}_j$ (in the units of 1/2); the relative momentum $\hat{\boldsymbol{p}}_{ij} = \hat{\boldsymbol{p}}_i - \hat{\boldsymbol{p}}_j$; the total orbital angular momentum $\hat{\boldsymbol{L}} = \boldsymbol{r}_{ij} \times \hat{\boldsymbol{p}}_{ij}$ and its square $\hat{L}^2$ in the center-of-mass system. Let us introduce also the operators of the total spin $\hat{\boldsymbol{S}} = \frac{1}{2}(\boldsymbol{\sigma}_i + \boldsymbol{\sigma}_j)$ (in units of $\hbar$) and the total isospin $\hat{\boldsymbol{T}} = \frac{1}{2}(\boldsymbol{\tau}_i + \boldsymbol{\tau}_j)$, which act in spin and isospin spaces, respectively.

Phenomenological NN interaction - 2

The tensor coupling enters via the tensor operator

$$\hat{S}_{ij} = 3(\boldsymbol{\sigma}_i \cdot \boldsymbol{n}_{ij})(\boldsymbol{\sigma}_j \cdot \boldsymbol{n}_{ij}) - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \ , \tag{3}$$

where $\boldsymbol{n}_{ij} = \boldsymbol{r}_{ij}/r_{ij}$, and the spin-orbit coupling enters via $\hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{S}}$. Both couplings are necessary for explaining experimental data.

The NN potential acting between a nucleon pair ij is a Hermitian operator $\hat{v}_{ij}$ in coordinate, spin, and isospin spaces. The operator $\hat{v}_{ij}$ commutes with $\hat{\boldsymbol{J}} = \hat{\boldsymbol{L}} + \hat{\boldsymbol{S}}$, $\hat{T}^2$, and $\hat{S}^2$, which leads to vanishing matrix elements of $\hat{v}_{ij}$ between states with different (JST). However, because of the tensor force, the $S = 1$ (spin triplet) states with different $L = J \pm 1$ can mix. The Pauli exclusion principle allows only for two-nucleon states with an odd value of the sum $L + S + T$.

The form of $\hat{v}_{ij}$, which is sufficiently general to reproduce the wealth of NN scattering data, is

$$\hat{v}_{ij} = \sum_{u=1}^{18} v_u(r_{ij}) \hat{O}_{ij}^u \ , \tag{4}$$

where the first fourteen operators are charge-independent, i.e., invariant with respect to rotation in the isospin space.

Phenomenological NN interaction

These operators have the form

$$\hat{\mathcal{O}}_{ij}^{u=1,\ldots,14} = 1, \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \,, \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \,, (\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,, \hat{S}_{ij} \,, \hat{S}_{ij}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,,$$
$$\hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{S}} \,, \hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{S}}(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,, \hat{L}^2 \,, \hat{L}^2(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,, \hat{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j) \,,$$
$$\hat{L}^2(\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,, (\hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{S}})^2 \,, (\hat{\boldsymbol{L}} \cdot \hat{\boldsymbol{S}})^2(\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j) \,. \quad (5)$$

The form of $\hat{v}_{ij}$, Eq. (4), is still quite restrictive, because apart from the angular-momentum dependent terms, the interaction is *local*, i.e., depends only on $\boldsymbol{r}_{ij}$. The terms with $\hat{\mathcal{O}}_{ij}^{u=15,\ldots,18}$ are small and break charge independence; they are not invariant with respect to a rotation in the isospin space. The charge independence corresponds to $v_{np}(T=1) = v_{nn} = v_{pp}$, while the charge symmetry implies only that $v_{nn} = v_{pp}$. Modern fits to very precise nucleon scattering data indicate the existence of charge-independence breaking.

One-pion exchange + the rest. One splits NN interaction

$$\hat{v}_{ij} = \hat{v}_{ij}^{\pi} + \hat{v}_{ij}^{\mathrm{IS}} \,, \quad (6)$$

where $\hat{v}_{ij}^{\pi}$ is a one-pion exchange part and $\hat{v}_{ij}^{\mathrm{IS}}$ is a phenomenological intermediate- and short-range (IS) component.

Three-body (NNN) interaction

Calculations show that the two-body interactions which satisfactorily reproduce NN scattering and ^{2}H properties, give the binding energies of ^{3}H and ^{4}He systematically lower than experimental ones $\Longrightarrow$ necessity of introducing three-body interaction into the nuclear Hamiltonian.

The underbinding of light nuclei can be corrected by introducing three-body forces.

$$\hat{V}_{ijk} = \hat{V}_{ijk}^{2\pi} + \hat{V}_{ijk}^{\rm IS} \ , \qquad (7)$$

$\hat{V}_{ijk}^{2\pi}$ - longest range 3-body, resulting from exchange of two pions. Can be calculated theoretically.
$\hat{V}_{ijk}^{\rm IS}$ - phenomenological Intermediate-Short (IS) range component, its parameters are to be obtained by fitting experimental data on many-nucleon systems.

Four and more-body interactions seems to be not needed.

Meson-exchange nucleon-nucleon interaction - 1

NN interaction results from exchange of virtual mesons. Lightest meson - postulated by Yukawa (1935). Range $\hbar/m_\pi c \simeq 1.4$ fm. Lowest order: one pion exchange (OPE) $\Longrightarrow$ one pion exchange potential (OPEP). Contemporary model - nucleon fields coupled to meson fields: meson exchange model (MEM; modern review in Machleidt (1989).
Mesons involved:

pseudoscalar (ps) mesons π, η $(J^{\mathcal{P}} = 0^-)$

scalar (s) mesons σ, δ $(J^{\mathcal{P}} = 0^+)$

vector (v) mesons ρ, ω $(J^{\mathcal{P}} = 1^+)$

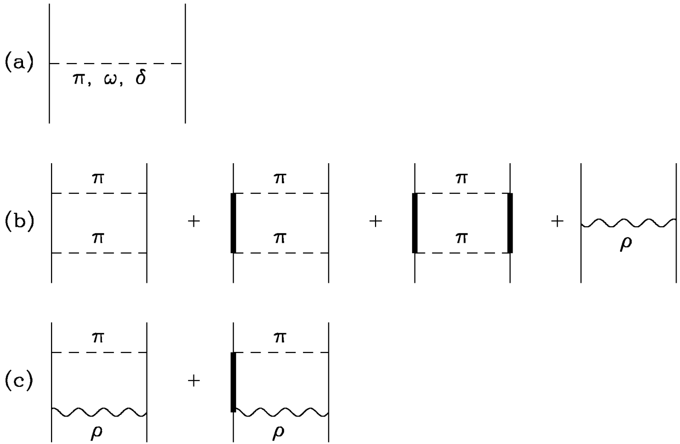
where $J^{\mathcal{P}}$ denotes the meson spin J and parity $\mathcal{P}$.

Meson-exchange nucleon-nucleon interaction - 2

The experimentally measured meson masses are: $m_\pi c^2 = 138$ MeV, $m_\eta c^2 = 548$ MeV, $m_\rho c^2 = 769$ MeV, $m_\omega c^2 = 783$ MeV, and $m_\delta c^2 = 983$ MeV. The scalar σ meson plays a special role: it represents a scalar state of an exchanged pion pair $(\pi\pi)$, and its mass is found from fitting the MEM to NN scattering data (in this way, one gets $m_\sigma c^2 = 550$ MeV).

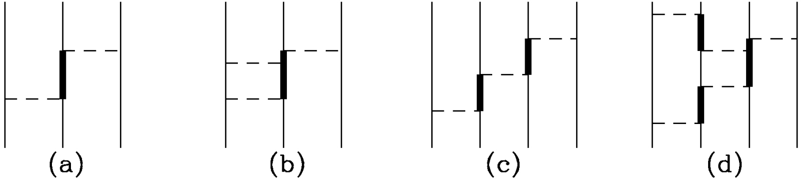
Apart from experimental meson masses, the MEM contains **coupling constants** determined by fitting experimental data. Finally, in order to account for finite sizes of interacting hadrons, one has to introduce **form-factors** at every meson-nucleon vertex. The form-factors describe the effect of shortest-range strong interactions, which depend on the quark structure of baryons and are not calculable within the MEM.

Meson-exchange nucleon-nucleon interaction - 3



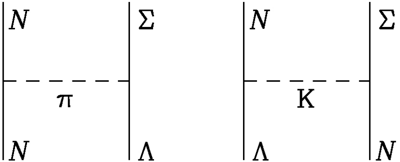
Some Feynman diagrams describing the most important meson-exchange processes which contribute to the NN interaction. Time goes upwards. Thin vertical lines: nucleons. Thick vertical segments: Δ $(=N\pi)$ resonance in an intermediate state.

Many-body interactions arise naturally in the meson-exchange models: they are represented by Feynman diagrams which cannot be reduced to a sequence of NN interactions.



Some meson-exchange Feynman diagrams describing processes contributing to NNN and four-nucleon interactions. Time goes upwards. Thin vertical solid lines: nucleon states. Thick vertical segments: Δ resonance in intermediate states. Dashed horizontal lines: exchanged mesons.

New type of strong interactions - with conversion (new baryon in the final state - not "just an exchange" of baryons in the initial pair)



Two Feynman diagrams describing strong-interaction one-meson-exchange processes accompanied by Λ–Σ conversion. Notice that K meson in the right-hand-side diagram transfers strangeness. Examples

$$\Lambda + p \longrightarrow \Sigma^{+} + n, \qquad \Lambda + p \longrightarrow \Sigma^{0} + p. \tag{8}$$

Solving the many-body problem – an overview - 1

The basic formula for the ground-state energy per baryon of a system of A_b baryons is

$$E_\mathrm{B} = \frac{\left(\Psi_0|\hat{H}_\mathrm{B}|\Psi_0\right)}{A_\mathrm{b}\left(\Psi_0|\Psi_0\right)} \ , \tag{9}$$

where $\hat{H}_\mathrm{B}$ is a baryon (B) Hamiltonian operator and Ψ_0 is a ground-state wave function of the system. In our case E_B should be calculated in the thermodynamic limit ($A_\mathrm{b} \longrightarrow \infty$, volume of the system $\longrightarrow \infty$).

In the simplest ("minimal model") case of nucleon matter (B=N), the calculation yields E_N as a function of n_n and n_p. The knowledge of $E_\mathrm{N}(n_n, n_p)$ is sufficient for calculating the EOS of matter consisting of nucleons and leptons (the so called $npe\mu$ matter). Leptons $(e, \mu) \approx$ free Fermi gases. In a more general case of hyperonic matter, one needs E_B as a function of all baryon densities n_B $(B = n, p, \Sigma^-, \Lambda, \ldots)$.

Perturbative expansions, summed to infinite order.

Systematic and convergent expansions of expression (9) for $E_{\rm B}$. Interaction is strong ! **Brueckner-Bethe-Goldstone theory (BBG)**. Formulated in 1950s-1960s. Monumental achievement. Reviewed in Baldo (1999), Baldo et al. (2001). Starts with NN in vacuum. Suitable to treat strong and complicated interactions in dense baryon matter. Initially: to explain properties of nuclear matter staring from NN interaction. Also - not so popular but with great formal beauty - **Green's Function Theory** (Martin & Schwinger 1958; reviewed in Weber 1999).

Variational calculations.

Based on variational principle of Quantum Mechanics (e.g., Schiff 1968):

$$E_{\rm B}^{\rm (var)} = \frac{\left(\Psi_{\rm var}|\hat{H}_{\rm B}|\Psi_{\rm var}\right)}{A_{\rm b}\left(\Psi_{\rm var}|\Psi_{\rm var}\right)} \geq E_{\rm B}^{\rm (exact)} \ , \tag{10}$$

where $\Psi_{\rm var}$ is a **trial wave function**. This method consists in minimizing the energy functional $E_{\rm B}^{\rm (var)}$ within a set of trial wave functions, which should be sufficiently rich in their structure, reflecting the structure of $\hat{H}_{\rm B}$. Monumental computational achievements (Wiringa et al. 1988; Akmal et al. 1998; Morales et al. 2002).

Effective baryon Hamiltonians or Langrangians

Simple calculations, provided that effective Hamiltonian (or Lagrangian) is known.

Nonrelativistic. Effective Hamiltonian based partly on experimental data and partly on selected many body results for pure neutron matter (see Chabanat et al. 1997, 1998 - SLy model; also Pandharipande & Ravenhall 1989 - FPS model).

$$E_{\rm B}^{\rm (HF)} = \frac{\left(\Psi_{\rm HF}|\hat{H}_{\rm B}^{\rm eff}|\Psi_{\rm HF}\right)}{A_{\rm b}\left(\Psi_{\rm HF}|\Psi_{\rm HF}\right)} \simeq E_{\rm B}^{\rm (exact)} \; . \tag{11}$$

Relativistic mean-field approximation. Correlations neglected. Usually Hartree approximation only. Proud names: Relativistic Mean Field Theory or Relativistic Hadrodynamics. Reviewed in Glendenning (2000). Parameters fixed by nuclear matter saturation experimental data.

The equation of state of the outer core - 1

The theoretical description of the matter at $\rho \lesssim 2\rho_0$ is within the reach of the modern nuclear theory. The nuclear Hamiltonian, albeit very complicated, is known reasonably well. The calculation of the ground state of nucleon matter requires big computing resources but can be carried out with a reasonable accuracy. Consider the matter composed of nucleons, electrons, and possibly muons (if $\mu_e > m_\mu c^2 = 106$ MeV). Nucleons form a strongly interacting Fermi liquid, while electrons and muons constitute nearly ideal Fermi gases. The energy per unit volume is

$$\mathcal{E}(n_n, n_p, n_e, n_\mu) = \mathcal{E}_{\rm N}(n_n, n_p) + \mathcal{E}_e(n_e) + \mathcal{E}_\mu(n_\mu) \ , \tag{12}$$

where $\mathcal{E}_{\rm N}$ is the nucleon contribution. In what follows, we will assume **full thermodynamic equilibrium** (= **cold catalyzed matter**). The pressure and energy-density depend on a single parameter; best choice - baryon density $n_{\rm b}$. The equilibrium at given $n_{\rm b}$ corresponds to the minimum of $\mathcal{E}$ under the condition of electrical neutrality.

We derive the equilibrium equations using a general method of Lagrange multipliers, particularly suitable for calculating the minimum of a function of many variables under additional constrains.

The equation of state of the outer core - 2

The variables are the number densities n_j, $j = n, p, e, \mu$, and the constrains are

$$\text{fixed baryon density:} \quad n_n + n_p - n_\mathrm{b} = 0 \ , \tag{13a}$$

$$\text{electrical neutrality:} \quad n_e + n_\mu - n_p = 0 \ . \tag{13b}$$

Auxiliary function $\widetilde{\mathcal{E}}$, defined by

$$\widetilde{\mathcal{E}} = \mathcal{E} + \lambda_1(n_e + n_\mu - n_p) + \lambda_2(n_n + n_p - n_\mathrm{b}) \ . \tag{14}$$

λ_i are Lagrange multipliers to be determined from the unconstrained minimization of $\widetilde{\mathcal{E}}$ by requiring $\partial\widetilde{\mathcal{E}}/\partial n_j = 0$ for all j:

$$\partial\widetilde{\mathcal{E}}/\partial n_n = \mu_n + \lambda_2 = 0 \ , \tag{15a}$$

$$\partial\widetilde{\mathcal{E}}/\partial n_p = \mu_p - \lambda_1 + \lambda_2 = 0 \ , \tag{15b}$$

$$\partial\widetilde{\mathcal{E}}/\partial n_e = \mu_e + \lambda_1 = 0 \ , \tag{15c}$$

$$\partial\widetilde{\mathcal{E}}/\partial n_\mu = \mu_\mu + \lambda_1 = 0 \ , \tag{15d}$$

with $\partial\mathcal{E}/\partial n_j = \mu_j =$ chemical potential of particles j..

The equation of state of the outer core - 3

Eliminating λ_i from Eqs. (15) one gets the relation between the chemical potentials

$$\mu_n = \mu_p + \mu_e \ , \qquad \mu_\mu = \mu_e \ , \tag{16}$$

which expresses the equilibrium with respect to the weak-interaction processes

$$n \longrightarrow p + e + \overline{\nu}_e \ , \qquad p + e \longrightarrow n + \nu_e \ , \tag{17a}$$
$$n \longrightarrow p + \mu + \overline{\nu}_\mu \ , \qquad p + \mu \longrightarrow n + \nu_\mu \ . \tag{17b}$$

We consider a neutron-star core transparent for neutrinos (which occurs, typically, as soon as $T \lesssim 10^9 - 10^{10}$ K). In this case neutrinos do not affect the matter thermodynamics, and we can put $\mu_{\nu_e} = \mu_{\overline{\nu}_e} = \mu_{\nu_\mu} = \mu_{\overline{\nu}_\mu} = 0$.
Equations (16) supplemented by the constraints (13) form a closed system of equations which determine the equilibrium composition of the $npe\mu$ matter.

$$n_j = p_{\mathrm{F}j}^3/(3\pi^2)$$

Electrons are ultra-relativistic, so that $\mu_e = \hbar c p_{\mathrm{F}e} \approx 122.1\,(n_e/0.05n_0)^{1/3}$ MeV
while muons are mildly relativistic

$$\mu_\mu = m_\mu c^2 \sqrt{1 + (\hbar p_{\mathrm{F}\mu}/m_\mu c)^2} \ . \tag{18}$$

Muons are present only if $\mu_e > m_\mu c^2 = 105.65$ MeV.

Pressure is calculated from the first law of thermodynamics (at $T = 0$):

$$P = n_{\rm b}^2 \, \frac{{\rm d}(\mathcal{E}/n_{\rm b})}{{\rm d}n_{\rm b}} \ . \tag{19}$$

The derivative is taken at the equilibrium composition.

Comment: density-dependent particle composition $\Longrightarrow x_j \equiv n_j/n_{\rm b}$ depending on $n_{\rm b}$ give a non-vanishing contribution to the density derivative of the energy per nucleon. Let us treat $\mathcal{E}$ as a function of $n_{\rm b}$, x_p, x_e, and x_μ. Then

$$P = n_{\rm b}^2 \left(\frac{\partial(\mathcal{E}/n_{\rm b})}{\partial n_{\rm b}} \right)_{\rm eq} + \frac{1}{n_{\rm b}} \sum_{j=p,e,\mu} \left(\frac{\partial \mathcal{E}}{\partial x_j} \right)_{\rm eq} \left(\frac{{\rm d}x_j}{{\rm d}n_{\rm b}} \right)_{\rm eq} \ , \tag{20}$$

where derivatives are taken at equilibrium. However, using Eqs. (13) and (16) one can see that the second term on the right-hand-side of Eq. (20) vanishes in equilibrium, i.e., both formulae for P give the same result.

Symmetry energy and the proton fraction - 1

Many-body calculations of the energy per nucleon in an asymmetric nuclear matter with realistic nucleon-nucleon interactions show that, to a very good approximation, the dependence on the neutron excess $\delta = 1 - 2x_p$ is quadratic (see, e.g., Lagaris & Pandharipande 1981c; Wiringa et al. 1988; Akmal et al. 1998):

$$E_{\rm N}(n_{\rm b}, \delta) \simeq E_0(n_{\rm b}) + S(n_{\rm b})\, \delta^2 \ . \tag{21}$$

Here, $E_0(n_{\rm b})$ refers to the symmetric nuclear matter and $S(n_{\rm b})$ is the symmetry energy. A very high precision of this formula, even for $\delta \simeq 1$, indicates that the higher-order terms of the expansion in δ are small.

In this context it is instructive to consider the free-Fermi gas (FFG) model of the nuclear matter, where the energy per baryon is

$$E^{\rm FFG}(n_{\rm b}, \delta) = \frac{3}{10}\, \epsilon_{\rm F}(n_{\rm b}) \left[(1+\delta)^{5/3} + (1-\delta)^{5/3} \right] \ . \tag{22}$$

Here, $m \equiv (m_n + m_p)c^2/2 = 938.93$ MeV is the mean nucleon mass and $\epsilon_{\rm F}$ is the Fermi energy in the symmetric nuclear matter at a given $n_{\rm b}$,

$$\epsilon_{\rm F} = \frac{\hbar^2}{2m} \left(\frac{3}{2}\, \pi^2 n_{\rm b} \right)^{2/3} \approx 36.8 \left(\frac{n_{\rm b}}{n_0} \right)^{2/3} \ {\rm MeV} \ . \tag{23}$$

The small-δ expansion of $E^{\rm FFG}$ reads then

$$E^{\rm FFG}(n_{\rm b},\delta) = \frac{3}{5}\,\epsilon_{\rm F}(n_{\rm b}) + \frac{1}{3}\,\epsilon_{\rm F}(n_{\rm b})\,\delta^2\ , \tag{24}$$

which gives the symmetry energy for the free Fermi gas model in the form

$$S^{\rm FFG} \approx 12.3\left(\frac{n_{\rm b}}{n_0}\right)^{2/3}\ {\rm MeV}\ . \tag{25}$$

It is easy to check that the quadratic approximation, Eq. (21), is very precise even at $\delta = 1$. From Eq. (22) applied to a pure neutron matter we obtain $E^{\rm FFG}(n_{\rm b},1) = (3/5)\,2^{2/3}\,\epsilon_{\rm F}(n_{\rm b}) \approx 0.9524\,\epsilon_{\rm F}(n_{\rm b})$, while Eq. (24) gives $14\epsilon_{\rm F}(n_{\rm b})/15 \approx 0.9333\,\epsilon_{\rm F}(n_{\rm b})$ which is only 2% smaller!
The simple form of the dependence of $E_{\rm N}$ on x_p enables us to clarify the relation between the symmetry energy and the composition of the npe matter at beta-equilibrium. Using Eq. (21) we can easily calculate the difference between the chemical potentials of neutrons and protons,

$$\mu_n - \mu_p = 4(1-2x_p)S(n_{\rm b})\ . \tag{26}$$

Symmetry energy and the proton fraction - 3

The beta-equilibrium in the npe matter (where $x_e = x_p$) implies, therefore,

$$\frac{x_p^{1/3}}{1-2x_p} = \frac{4S(n_{\rm b})}{\hbar c\,(3\pi^2 n_{\rm b})^{1/3}}~. \tag{27}$$

Accordingly, the proton fraction at a given $n_{\rm b}$ is determined by the symmetry energy. Under typical conditions, the proton fraction is small, $x_p \ll 1$, and

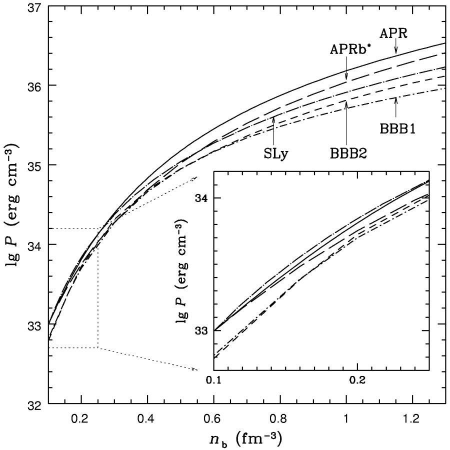
$$x_p(n_0) \approx \frac{64\,[S(n_{\rm b})]^3}{3\pi^2(\hbar c)^3\,n_{\rm b}} \approx 4.75\times 10^{-2}\left(\frac{n_0}{n_{\rm b}}\right)\left(\frac{S(n_{\rm b})}{30~\text{MeV}}\right)^3~, \tag{28}$$

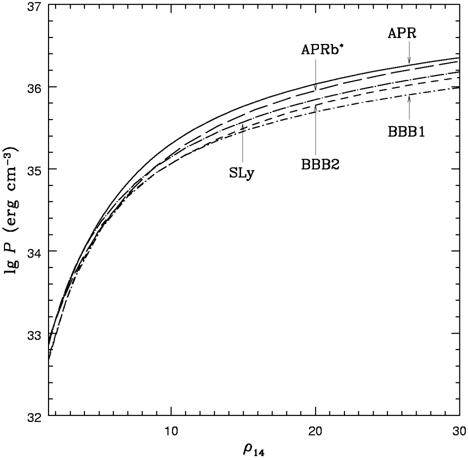
As the experimental value of S_0 is $S^{\rm exp} \simeq 30$ MeV, the proton fraction in the neutron-star matter at the normal nuclear density should be $x_p(n_0) \simeq 5\%$, independently of any specific EOS of dense matter. On the other hand, Eq. (28) tells us that the actual value of $x_p(n_0)$ for a given model of dense matter is very sensitive to the value of S_0 of that model. In particular, the free Fermi-gas model yields a very small S_0, Eq. (25), and gives an unrealistically low value $x_p^{\rm FFG}(n_0) \approx 0.0033$.

Table: **1. Selected EOSs of neutron star cores.** **Boldface - EOSs based on realistic NN+NNN; which yield good saturation parameters for nuclear matter. BPAL12 and BGN2 are "extremists": extremely soft and stiff, respectively.**

EOS	model	reference
BPAL12	$npe\mu$ energy density functional	Bombaci (1995)
BGN1H1	$np\Lambda\Xi e\mu$ energy density functional	Balberg & Gal (1997)
FPS	$npe\mu$ energy density functional	Pandharipande & Ravenhall (1989)
BGN2H1	$np\Lambda\Xi e\mu$ energy density functional	Balberg & Gal (1997)
BGN1	$npe\mu$ energy density functional	Balberg & Gal (1997)
BBB2	$npe\mu$ Brueckner theory, Paris NN plus Urbana UVII NNN potentials	Baldo et al. (1997)
BBB1	$npe\mu$ Brueckner theory, Argonne A14 NN plus Urbana UVII NNN potentials	Baldo et al. (1997)
SLy	$npe\mu$ energy density functional	Douchin & Haensel (2001)
APR	$npe\mu$ variational theory, Argonne A18 NN plus Urbana UIX NNN potentials	Akmal et al. (1998)
APRb*	$npe\mu$ variational theory, Argonne A18 NN with boost correction plus adjusted Urbana UIX* NNN potentials	Akmal et al. (1998)
BGN2	$npe\mu$ effective nucleon energy functional	Balberg & Gal (1997)

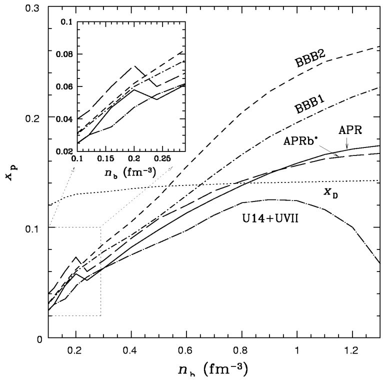
Outer and inner core - EOS - $npe\mu$ matter





Pressure vs. baryon density (left panel) and vs. mass density (right panel) for several selected EOSs of the $npe\mu$ matter. Labels are the same as in Table 1.

Selected models: proton fraction



x_p in $npe\mu$ matter at beta equilibrium for different EOSs (Table 1). Negative-slope segments for the APR and APRb* models correspond to a mixed-phase region. Dotted line: the threshold x_p above which the direct Urca process is allowed. *See next slide*

$x_p > x_{\rm D}$ direct Urca allowed;
$x_p < x_{\rm D}$ direct Urca prohibited

Compare the values of x_p given by different theories at $n_{\rm b} = n_0$. They range from 0.035 for the U14+UVII model to 0.06 for the APRb* one. A difference by a factor ~ 2 stems from using $S_0 = 28$ MeV and $S_0 = 35$ MeV, respectively; see Eq. (28). These values of the symmetry energy are still within extreme experimental values of S_0.

Proton fraction and the Direct Urca process

Direct Urca (Lattimer et al. 1991) processes are:

$$n \longrightarrow p + e + \overline{\nu}_e \ , \qquad p + e \longrightarrow n + \nu_e \ , \tag{29a}$$

$$n \longrightarrow p + \mu + \overline{\nu}_\mu \ , \qquad p + \mu \longrightarrow n + \nu_\mu \ . \tag{29b}$$

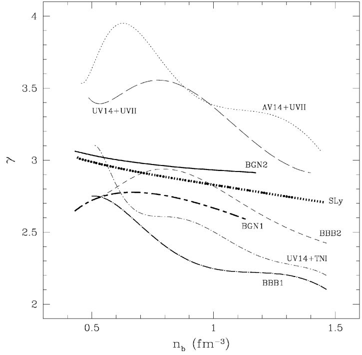
Direct Urca reactions were **not considered before 1991**, see, e.g., Shapiro & Teukolsky (1983)! They are allowed only at rather high $n_{\rm b}$ at which $x_p(n_{\rm b})$ exceeds a threshold value $x_{\rm D}(n_{\rm b}) \approx 0.11 - 0.14$ (Lattimer et al. 1991). The reason: neutrons, protons, and electrons form degenerate Fermi liquids, only the states close to the Fermi surfaces (within a shell of the thickness $\sim k_{\rm B}T$ around the Fermi are involved in the processes (29). Therefore, $p_j \approx p_{{\rm F}j}$ $(j = n, p, e, \mu)$, while the neutrino momentum $p_\nu \sim k_{\rm B}T/c \ll p_{{\rm F}j}$. Neglecting $k_{\rm B}T/c \ll p_{{\rm F}j} \Longrightarrow$ momentum conservation imposes the triangle rule:

$$p_{{\rm F}n} < p_{{\rm F}p} + p_{{\rm F}e} \ , \tag{30}$$

which is satisfied for $x_p > x_{\rm D}$. In the absence of muons, $x_{\rm D} = 1/9$; their presence slightly increases $x_{\rm D}$ above $1/9$, and $x_{\rm D}$ may become as large as 0.14. Replacing electrons by muons in Eqs. (30) one can get the threshold proton fraction which opens the muon direct Urca process. This process becomes allowed at a slightly higher density than the electron one.

If $x_p < x_{\rm D} \simeq 0.11 - 0.14$ neutrino emission proceeds via the so called **modified Urca** (Chiu & Salpeter 1964) processes with an additional nucleon in initial and final states of Eq. (29). $N + n \longrightarrow N + p + e + \overline{\nu}_e$ etc. where $N = n, p$ is a "spectator nucleon" "Spectator nucleon" does not participate in weak beta processes but only opens it via a momentum transfer mediated by strong interactions. This strongly suppresses the neutrino emission rate. If direct Urca processes operate, then a non-superfluid neutron-star core cools to 10^9 K in a minute, and to 10^8 K in a year. If they are not allowed, the timescales will be one year and 10^5 years, respectively.

Adiabatic index



Adiabatic index of the $npe\mu$ matter versus baryon density for selected EOSs. All but three EOSs are from Table 1, the remaining ones are the EOSs of Wiringa et al. (1988).

Calculated using analytic fits of $P(n_{\rm b})$ to tabulated EOSs.

Adiabatic index characterizes the stiffness of the EOS with respect to density perturbations

$$\gamma = \frac{n_{\rm b}}{P}\,\frac{{\rm d}P}{{\rm d}n_{\rm b}} = \frac{P+\mathcal{E}}{P}\,\frac{{\rm d}P}{{\rm d}\mathcal{E}}\ . \qquad (31)$$

Calculated at fixed (frozen) composition
γ enters the speed of sound:

$$\frac{v_{\rm s}}{c} = \left(\frac{{\rm d}P}{{\rm d}\mathcal{E}}\right)^{1/2}_{\rm fr} = \left(\frac{\gamma_{\rm fr}P}{\mathcal{E}+P}\right)^{1/2}\ . \qquad (32)$$

Hyperons in the inner core

physics of baryons - see, e.g., Perkins (2000). A presupernova core at the brink of a collapse contains atomic nuclei (of the Fe-Ni group), alpha particles, free nucleons, electrons and positrons, but not hyperons. However, a huge gravitational compression can initiate the transformation of nucleons into hyperons, as soon as such transformation lowers the energy density at a given $n_{\rm b}$. This process is mediated by the strangeness-changing weak interaction and may become possible at $\rho \gtrsim 2\rho_0$ (Cameron 1959; Ambartsumyan & Saakyan 1960; Salpeter 1960). Energy density $\mathcal{E}_{\rm B}(\{n_B\})$, where $\{n_B\}$ is a set of number densities of baryon species $\{B\}$,

$$\sum_B n_B = n_{\rm b} \ . \tag{33}$$

At baryon densities $n_{\rm b} \lesssim 10\, n_0$, relevant for neutron-star cores, it is sufficient to consider the octet of lightest baryons (next page).
The electric charge density and the strangeness per baryon are given by

$$q_{\rm b} = \sum_B n_B Q_B \ , \qquad s_{\rm b} = \sum_B n_B S_B / n_{\rm b} \ . \tag{34}$$

Table: 2. Masses, electric charges, strangeness, and e-folding (mean) lifetimes of the baryon octet, measured in laboratory. The baryon number, spin, and parity of all these baryons are 1, $1/2$, and $+1$, respectively.

baryon name	mc^2 (MeV)	Q (e)	S	τ (s)
p	938.27	1	0	$> 10^{32}$
n	939.56	0	0	886
Λ^0	1115.7	0	-1	2.6×10^{-10}
Σ^+	1189.4	1	-1	0.80×10^{-10}
Σ^0	1192.6	0	-1	7.4×10^{-20}
Σ^-	1197.4	-1	-1	1.5×10^{-10}
Ξ^0	1314.8	0	-2	2.9×10^{-10}
Ξ^-	1321.3	-1	-2	1.6×10^{-10}

Hyperons in the inner core

Consider electrically neutral matter composed of baryons B (nucleons and hyperons) and leptons ℓ (electron and muons) at a given baryon number density $n_{\rm b}$

$$\sum_B n_B = n_{\rm b} \ . \tag{35}$$

Charge neutrality condition is

$$\sum_B n_B Q_B - \sum_{\ell=e,\mu} n_\ell = 0 \ , \tag{36}$$

where Q_B is the electric charge of a baryon B in units of e. The energy density depends on the number densities of baryons $\{n_B\}$ and leptons (n_e, n_μ), $\mathcal{E} = \mathcal{E}(\{n_B\}, n_e, n_\mu)$. The equilibrium state has to be determined by minimizing $\mathcal{E}$ under the constraints given by Eqs. (35) and (36). To this aim, we will use the method of Lagrange multipliers (see section on $npe\mu$ matter).

In analogy with Eq. (14) we define the auxiliary energy density $\widetilde{\mathcal{E}}$

$$\widetilde{\mathcal{E}} = \mathcal{E} + \lambda_{\rm b} \left(\sum_B n_B - n_{\rm b} \right) + \lambda_{\rm q} \left(\sum_B Q_B n_B - \sum_{\ell = e, \mu} n_\ell \right) \ . \tag{37}$$

Let $N_{\rm B}$ be the number of the baryon species. Minimizing $\widetilde{\mathcal{E}}$, we get a set of $N_{\rm B} + 2$ equations

$$\partial \widetilde{\mathcal{E}} / \partial n_B = \mu_B + \lambda_{\rm b} + \lambda_{\rm q} \, Q_B = 0 \qquad (B = 1, \ldots, N_{\rm B}), \tag{38a}$$

$$\partial \widetilde{\mathcal{E}} / \partial n_\ell = \mu_\ell - \lambda_{\rm q} = 0 \qquad (\ell = e, \ \mu), \tag{38b}$$

where $\lambda_{\rm b}$ and $\lambda_{\rm q}$ are Lagrange multipliers and $\mu_j = \partial \mathcal{E} / \partial n_j$.

Thermodynamic equilibrium in hyperonic matter

Eliminating Lagrange multipliers, we get a system of $N_{\rm B}$ relations for $N_{\rm B}+2$ chemical potentials. We have two additional relations, Eqs. (35) and (36), so that the total number of equations is equal to $N_{\rm B}+2$. The relations involving the chemical potentials of nucleons and leptons are equivalent to Eqs. (16) obtained for the $npe\mu$ matter: $\mu_e=\mu_\mu$, $\mu_n=\mu_p+\mu_e$. However, we have now additional equations which describe equilibrium with respect to weak interactions. The equilibrium equations depend on Q_B. In our case $Q_B=-1,0,1$:

$$Q_B=-1 \quad : \quad \mu_{B^-}=\mu_n+\mu_e \ , \tag{39a}$$

$$Q_B=0 \quad : \quad \mu_{B^0}=\mu_n \ , \tag{39b}$$

$$Q_B=+1 \quad : \quad \mu_{B^+}=\mu_n-\mu_e \ . \tag{39c}$$

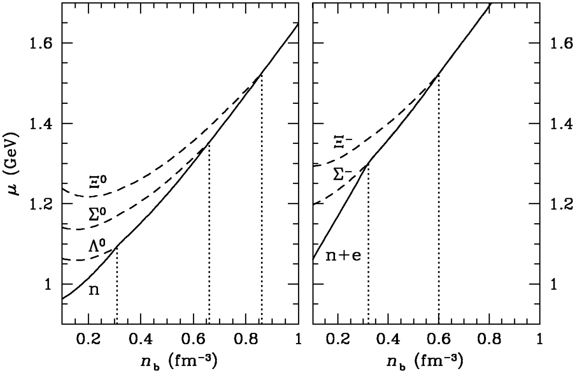
The lightest baryons form an octet, containing nucleons and Λ, Σ, Ξ hyperons (Table 2).

Thresholds for hyperons

One can calculate the threshold densities ρ_H of hyperons $(H = \Lambda, \Sigma, \Xi)$ by checking the threshold condition at various ρ. One can start at $\rho = 0.5\,\rho_0$, where hyperons are certainly absent. Nevertheless, one can always calculate the minimum increase of the energy of the matter produced by adding a single hyperon H at a fixed pressure P. This can be done by considering the energy of the matter with an admixture of given hyperons and by calculating numerically the limit of the derivative

$$\lim_{n_H \longrightarrow 0} (\partial \mathcal{E}/\partial n_H)_{\rm eq} \equiv \mu_H^0 \ . \tag{40}$$

To be specific, consider the lightest Λ hyperon. As long as $\mu_\Lambda^0 > \mu_n$, this hyperon cannot survive because the system will lower its energy via an exothermic reaction $\Lambda + N \longrightarrow n + N$. However, μ_n increases with growing $n_{\rm b}$ and the functions $\mu_\Lambda^0(n_{\rm b})$ and $\mu_n(n_{\rm b})$ intersect at some $n_{\rm b} = n_{\rm c}^\Lambda$ (the left panel in next slide). For $n_{\rm b} > n_{\rm c}^\Lambda$, the Λ hyperons become stable in dense matter: their decay is blocked by the Pauli principle (neutron, proton, and electron states are occupied).



Threshold chemical potentials of neutral hyperons and neutron (*left*) and of negatively charged hyperons and the sum $\mu_e + \mu_n$ (*right*) versus baryon number density for model C of Glendenning (1985). Vertical dotted lines mark the thresholds for the creation of new hyperons; dashed lines show minimum enthalpies μ_H^0 of unstable hyperons before the thresholds.

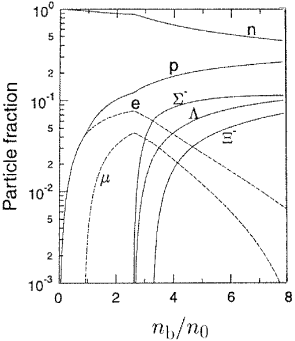
Usually Λ is not the first one to appear in a neutron-star core, because Σ^- appears at lower density. This is visualized in figures on next three slides. The threshold condition for the Σ^- hyperon creation is

$$\mu^0_{\Sigma^-} = \mu_n + \mu_e \ . \tag{41}$$

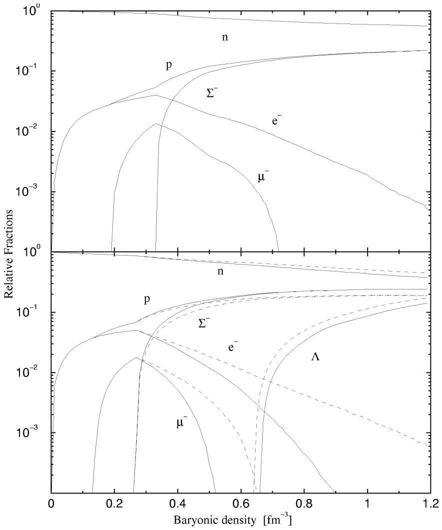
μ_e adds to μ_n and the threshold condition is usually satisfied at lower density (and at lower μ_n) than for Λ. However, this is not a strict rule, as one can see by comparing the left and right panels of Fig. on the previous slide. The figure shows also examples of the appearance of other hyperons (for $Q = 0$ and $Q = -1$). Large μ_e may prohibit the appearance of $Q = +1$ hyperons in neutron-star cores. For example, consider Σ^+, the lightest positively charged hyperon. As follows from the bottom line of Eqs. (39), the condition of its appearance is

$$\mu^0_{\Sigma^+} = \mu_n - \mu_e \ . \tag{42}$$

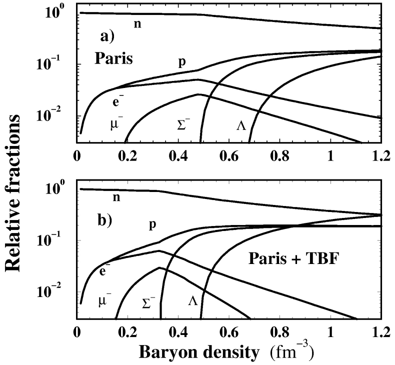
The subtraction of μ_e can easily lead to $\mu^0_{\Sigma^+} > \mu_n - \mu_e$ in dense matter, making Σ^+ unstable (because the process $\Sigma^+ + e \longrightarrow n + \nu_e$ is exothermic). Accordingly, Σ^+ hyperons do not appear in neutron-star cores in some models. However, there is no strict rule to forbid their presence.



Fractions of particles $x_j = n_j/n_\mathrm{b}$ versus baryon number density n_b (in units of $n_0 = 0.16$ fm^{-3}) calculated by Hanauske et al. (2001) for two relativistic models of baryonic interactions. *Left*: Effective chiral model of Hanauske et al. (2001). *Right*: Relativistic mean field model TM1 of Sugahara & Toki (1994).



Particle fractions $x_j = n_j/n_\mathrm{b}$ versus n_b as calculated by Vidana et al. (2000) in the Brueckner theory for two models of baryonic interactions. *Upper panel*: Nijmegen model E of Rijken et al. (1999): only Σ^- hyperon is present in neutron-star cores. *Lower panel*: APR model for the nucleon sector (Table 1, Akmal et al. 1998) and Nijmegen model E of Rijken et al. (1999) for NH and HH interactions. In contrast to the upper panel, Λ is present in dense matter. Solid lines in both panels: all baryon-baryon (NN, NH, HH) interactions are included. Dashed lines in the lower panel: HH interaction is (artificially) switched off.



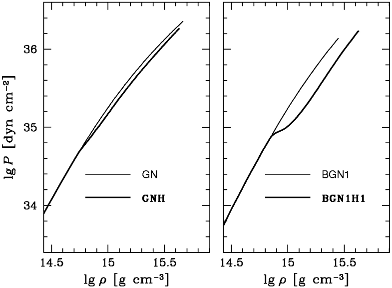
Effect of the three-body forces between nucleons on particle fractions $x_j = n_j/n_{\rm b}$ in dense matter; from Baldo & Burgio (2001). NNN forces in dense matter at $\rho \gtrsim 2-3\rho_0$ are repulsive (in contrast to $^3{\rm H}$, $^3{\rm He}$,$^4{\rm He}$, and nuclear matter at ρ_0 where they give an additional binding of nucleons). Therefore NNN leads to lowering of threshold densities for hyperons, compared with no NNN case.

Figures on the previous three slides show clearly that in the presence of hyperons proton fraction $x_p = n_p/n_{\rm b}$ becomes very easily greater than 0.14, the triangle condition $p_{{\rm F}n} < p_{{\rm F}p} + p_{{\rm F}e}$ is satisfied, so that direct Urca process is open, allowing for a very rapid neutrino cooling. Moreover, so called **hyperon Urca processes** become easily open (Prakash et al. 1991). Examples of hyperon Urca processes are

$$\Sigma^- \longrightarrow n + e^- + \overline{\nu}_e \ , \quad \Sigma^- \longrightarrow \Lambda^0 + e^- + \overline{\nu}_e \ , \ \ldots \tag{43}$$

However, neutrino emissivity from hyperon Urca processes is weaker than in the direct Urca with nucleons only (because matrix elements for weak interactions with strange baryons are smaller than for nucleons). Curiously, possibility of direct Urca and hyperon Urca in hyperonic neutron star core was **not considered before 1991!**

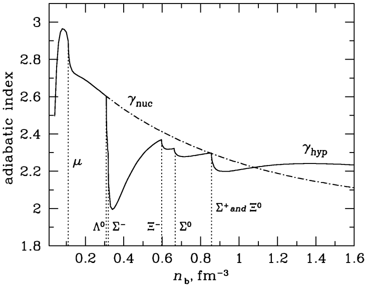
Hyperon softening of the EOS



Softening of EOSs by the presence of hyperons. Each panel shows an EOS with (thick line) and without (thin line) hyperons. *Left*: model EOSs of Glendenning (1985). *Right*: the BGN1 and BGN1H1 EOSs of Balberg & Gal (1997) (labeled as in Table 1)

The formation of hyperons softens the EOS because high-energy neutrons are replaced by more massive low-energy hyperons (producing lower pressure). The softening is a generic effect independent of models of NH and HH interactions, as illustrated in the figure above, but its magnitude is model-dependent.

Adiabatic index for hyperonic matter



γ versus $n_{\rm b}$. Calculations for an EOS of Glendenning (1985). $\gamma_{\rm hyp}$ - hyperonic matter (vertical dotted lines indicate thresholds for the appearance of muons and hyperons); $\gamma_{\rm nuc}$ corresponds to the EOS in which the appearance of hyperons is artificially forbidden. From Haensel et al. (2002).

Adiabatic index defines the linear response of local pressure P to the local perturbation of baryon number density, $\delta P/P = \gamma \times \delta n_{\rm b}/n_{\rm b}$. If the perturbation is *quasistatic*, then $\gamma = \frac{n_{\rm b}}{P} \left(\frac{{\rm d}P}{{\rm d}n_{\rm b}}\right)_{\rm eq} = \frac{P+\mathcal{E}}{P} \left(\frac{{\rm d}P}{{\rm d}\mathcal{E}}\right)_{\rm eq}$.

However, neutron star pulsations have periods ~ 0.1 ms, and therefore the hyperon matter is partially off-equilibrium during pulsations, because full equilibration requires weak processes which are too slow.

Notice: equilibration between Λ^0 and Σ^- is produced by **strong interaction**, because $n + \Lambda^0 \leftrightarrows \Sigma^- + p$ is sufficient (no leptons, no strangeness change). However, other processes are so slow that $\gamma_{\rm puls} > \gamma_{\rm eq}$. See Haensel et al. (2002b).

With v_{NH} and v_{HH} used for hypernuclei and for NH scattering etc. one gets very often $M_{\rm max} < 1.44\ M_\odot$ - which contradicts observations. Also recently neutron star mass was measured with $M > 1.6\ M_\odot$ (at 97% confidence level) in WD(=white dwarf)+NS binary (the central value of NS mass is $2.1\ M_\odot$!).

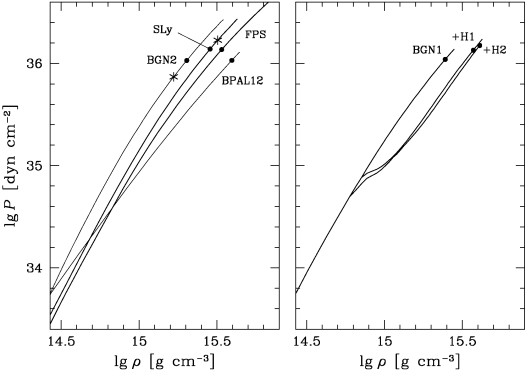
Maybe (unknown) HHH, NNH, NHH are sufficiently repulsive to support $M > 1.6\ M_\odot$? But if NH etc. are so repulsive, then hyperons will have no chance to appear in NS! $\Longrightarrow$ neutron star core have plain $npe\mu$ composition.

Bludman & Ruderman (1968,1970); Caporaso & Brecher (1979); Olsson (2000) Since the EOS at $n_{\rm b} \gg n_0$ is very uncertain, it is important to impose model-independent bounds. The basic requirement is that any EOS should respect *Lorentz invariance* and *causality*. These requirements can be formulated in different (and not equivalent) ways. For instance, it has been claimed that a physically correct EOS can be neither *superluminal* nor *ultrabaric*, where

$$\text{ultrabaric EOS:} \quad P > \mathcal{E}\ ; \tag{44}$$

$$\text{superluminal EOS:} \quad \mathrm{d}P/\,\mathrm{d}\mathcal{E} > 1\ . \tag{45}$$

An excited medium can be superluminal without breaking Lorenz invariance.



Selected model EOSs of neutron-star cores (denoted as in Table 1). Filled circles show maximum densities in stable neutron stars, while asterisks indicate the densities above which EOSs are superluminal ($v_s > c$).

Superluminal beyond a point indicated by an asterisk: BGN2, SLy. For the SLy EOS, the superluminality occurs at densities which are not realized in stable neutron stars. BGN2: most massive neutron stars have a superluminal central core with $v_s > c$. Both EOSs are derived within a non-relativistic many-body theory - superluminality results from the lack of Lorentz invariance.

Effect of baryon superfluidity on the EOS

$n_n = k_F^3/3\pi^2$, $\mathcal{E}$ - energy density. For neutron matter $n_n = n_b$. Critical temperatures are $T_c \sim 10^9 - 10^{10}$ K, superfluidity occurs soon after the neutron star birth (about one year if direct Urca processes are not allowed). Consider the simplest model of a superfluid neutron gas and use the Bardeen-Cooper-Schrieffer (BCS) model with an isotropic ${}^1S_0$ neutron pairing (see, e.g., §51 of Fetter & Walecka 1971). $T \ll T_c \Longrightarrow \Delta(T) \simeq \Delta(0) \equiv \Delta_0$, where $\Delta_0 \approx 1.76\, k_B\, T_c$. Gain in $\mathcal{E}$

$$\mathcal{E}_n - \mathcal{E}_s = \frac{m k_F}{4\pi^2 \hbar^2} \Delta_0^2 \ , \tag{46}$$

The relative change in $\mathcal{E}$ resulting from superfluidity is therefore

$$\frac{\mathcal{E}_n - \mathcal{E}_s}{\mathcal{E}_n} = \frac{5}{8} \left(\frac{\Delta_0}{\epsilon_F} \right)^2 \approx 1.83 \times 10^{-4} \left(\frac{\Delta_0}{\text{MeV}} \right)^2 \left(\frac{n_0}{n_b} \right)^{2/3} \ , \tag{47}$$

where ϵ_F is the neutron Fermi energy. One can also show that $P_n - P_s = -\mathcal{E}_n + \mathcal{E}_s$. For gaps $\Delta \lesssim 1$ MeV, the relative effect of superfluidity on the EOS at supranuclear density is less than $10^{-4} - 10^{-2}$.

Effect of strong magnetic field on the EOS

Magnetic field strongly affects the EOS if particles occupy only a few lowest energy Landau states. Otherwise, the effects of the magnetic field on the EOS are minor. Most affected by $\boldsymbol{B}$ are **electrons**. P_e significantly affected if

$$B \gtrsim (3.8 \times 10^{19}\ \mathrm{G})\,(x_e n_\mathrm{b}/\mathrm{fm}^{-3})^{2/3}, \tag{48}$$

where x_e is the number of electrons per baryon. We have $n_\mathrm{b} \sim 3n_0 \approx 0.5\ \mathrm{fm}^{-3}$, $x_e \sim 0.05 \Longrightarrow P_e$ affected if $B \gtrsim 3 \times 10^{18}$ G. However, $P_e \ll P_N$.
One can easily generalize Eq. (48) for other fermions (μ-mesons, nucleons). In this case, x_e should be replaced by the number of given particles per baryon, and the right-hand side should be multiplied by $m_\mu/m_e = 206.77$ (muons) and $\sim 10^3$ (protons). Accordingly, the nucleon pressure of the $npe\mu$ gas in the core cannot be affected by a magnetic field unless $B \gtrsim 10^{21}$ G - but such fields in neutron stars are out of question (hydrostatic equilibrium impossible!) **Conclusion: effects of $\boldsymbol{B}$ become important well above maximum allowable $B_\mathrm{max}(NS) = 10^{18}$ G.**

Meson condensations, quark deconfinement could lead to **phase transitions** in the inner neutron star core

hadronic Bose-Einstein condensation

A B-E condensate does not contribute to pressure (macroscopic occupation of a single quantum state).

π-condensation *Migdal 1972, Sawyer 1972, Scalapino 1972*

K-condensation *Kaplan & Nelson 1986*

Question: do they occur at all?

quark deconfinement

Collins & Perry 1975 Simplicity of the EOS for $\rho \longrightarrow \infty$ - free Fermi gas of quarks - results from the Asymptotic Freedom property of QCD.

Question: does it occur at $\rho < \rho_{\rm c,max}$?

Generic effect: softening of the EOS $\Longrightarrow$ decrease of $M_{\rm max}$

Strange matter hypothesis Self-bound state of quark matter - true ground state of hadronic matter *Bodmer 1971; Terasawa 1979; Witten 1984*

quark stars = strange stars *Haensel et al. 1986, Alcock et al. 1986*

huge bags containing $\sim 10^{58}$ $u - d - s$ quarks

EOS $P \simeq ac^2\rho - B = ac^2(\rho - \rho_{\rm s})$, $\quad a = 0.3 - 0.5$

quark matter at the surface

$$\rho_{\rm s} = \rho(R) \simeq B/ac^2 \sim 10^{15}\ g\ cm^{-3}$$

$$B = \mathcal{E}_{\rm vac}^{\rm QCD} - \mathcal{E}_{\rm vac}^{\rm our}$$

$$M_{\rm max} \propto \rho_{\rm s}^{-1/2} \propto B^{-1/2}$$

for "experimental" B one gets $M_{\rm max} \sim 2\ M_\odot$

Abnormal matter. A superdense self-bound state of huge binding energy, in which nucleons become ≈ massless *Lee & Wick 1974*

Q-stars *Bahcall et al. 1991*
A solution for the ground state of hadronic matter, with a self-bound state of large binding energy, which could have subnuclear energy density! Q-stars of small $\rho_{\rm s} \sim 5 \times 10^{13}\ g\ cm^{-3}$. Are we really so ignorant about the actual structure of matter at $\rho \sim 10^{13}$ g cm^{-3}?

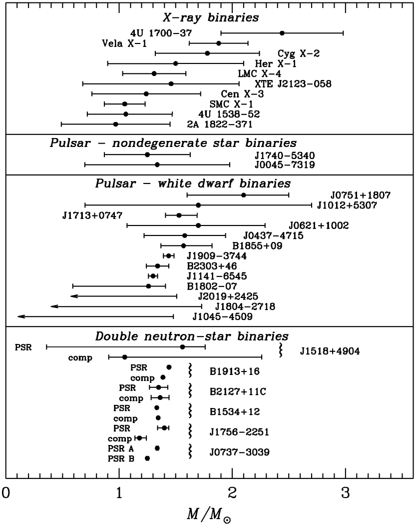
It is easy to see that "sensationally high" $M_{\rm max} \sim 10 M_\odot$ are just a simple consequence of nearly maximally stiff EOS of Q-stars $P \approx ac^2(\rho - \rho_{\rm s})$, with $a \approx 1$, combined with $\rho \sim 10^{13}$ g cm^{-3}.

We have a scaling $M_{\rm max} \approx 3 M_\odot \times (\rho_{\rm s}/5 \times 10^{14}$ g cm$^{-3})^{-1/2}$ (write hydrostatic equilibrium for Q-stars in dimensionless form, and you get this scaling; see Shapiro & Teukolsky 1983) so that: $\rho_{\rm s} \sim 5 \times 10^{13}\ g\ cm^{-3} \quad \Longrightarrow M_{\rm max} \sim 9\ M_\odot$

Rotation: $P_{\rm min} = 1.4\ ms$ $(716Hz)$ Increases $M_{\rm max}$ by $\Delta M_{\rm max} \sim 3\%$. Static approximation is OK.

Condition on EOS:
$M_{\rm max}({\rm EOS}) > M_{\rm obs}^{\rm max}$

Haensel, Potekhin, & Yakovlev 2007 $\longrightarrow$



What do we get from NS mass measurements in October 20

- Very precise neutron star masses in three NS-NS binaries with a radio pulsar and in one binary with two pulsars. Hulse - Taylor pulsar still the most massive. *Evolutionary bias* $\Longrightarrow$ *not helpful to constrain EOS.*
- NS-WD are promising, and their number will increase rapidly. Different formation than that of NS-NS. Significant accreted mass. PSR J0751+1807 $2.1^{+0.4}_{-0.5}\ M_\odot$ *Luck needed ...*
- Three X-ray binaries: Cyg X-2, Vela X-1, 4U 1700-37 contain good candidates for high-mass ($\sim 2M_\odot$) neutron stars. *If only the errors could be lower ...*

If $2.1\ M_\odot$ is confirmed then hyperons and phase transitions seem to be unlikely, while most realistic "minimal model" ($npe\mu$) survives. Is Nature so simple?