IHP 2006

Experimental Tests of General Relativity

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Ranging to Remote Benchmarks

- **Lunar Laser Ranging**
- **Radar Ranging to Planets**
- **Satellite Ranging**
  - **Shapiro Time Delay**
    - **Viking Mission**
    - **Cassini Mission**
  - **Pionear Anomaly**

Lunar Laser Ranging

In the late 1950’s researchers at Princeton , incl. Robert Dicke, proposed using optical ranging form earth to the moon to probe gravity

Laser invented circa 1958 seemed tool of choice due to:

- Precise wavelength – low beam divergence
- Q switching made possible pulses of a few nanoseconds

Concept is to measure the round trip time of flight of photons between earth and reflectors on the moon to determine the earth moon distance as a function of time as both bodies orbit each other and the sun.

Due to large distance and non zero beam divergence the concept requires that retro-reflectors be landed.

In July 1969 Niel Armstrong placed the first reflector array on the moon.

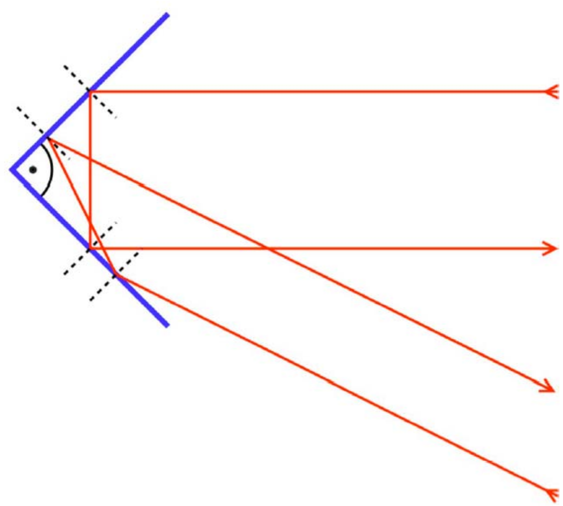
By August 1969 reflected photons were detected by the McDonald Observatory making Lunar Laser ranging a reality and beginning a nearly continuous suite of measurements over the last 37 years.

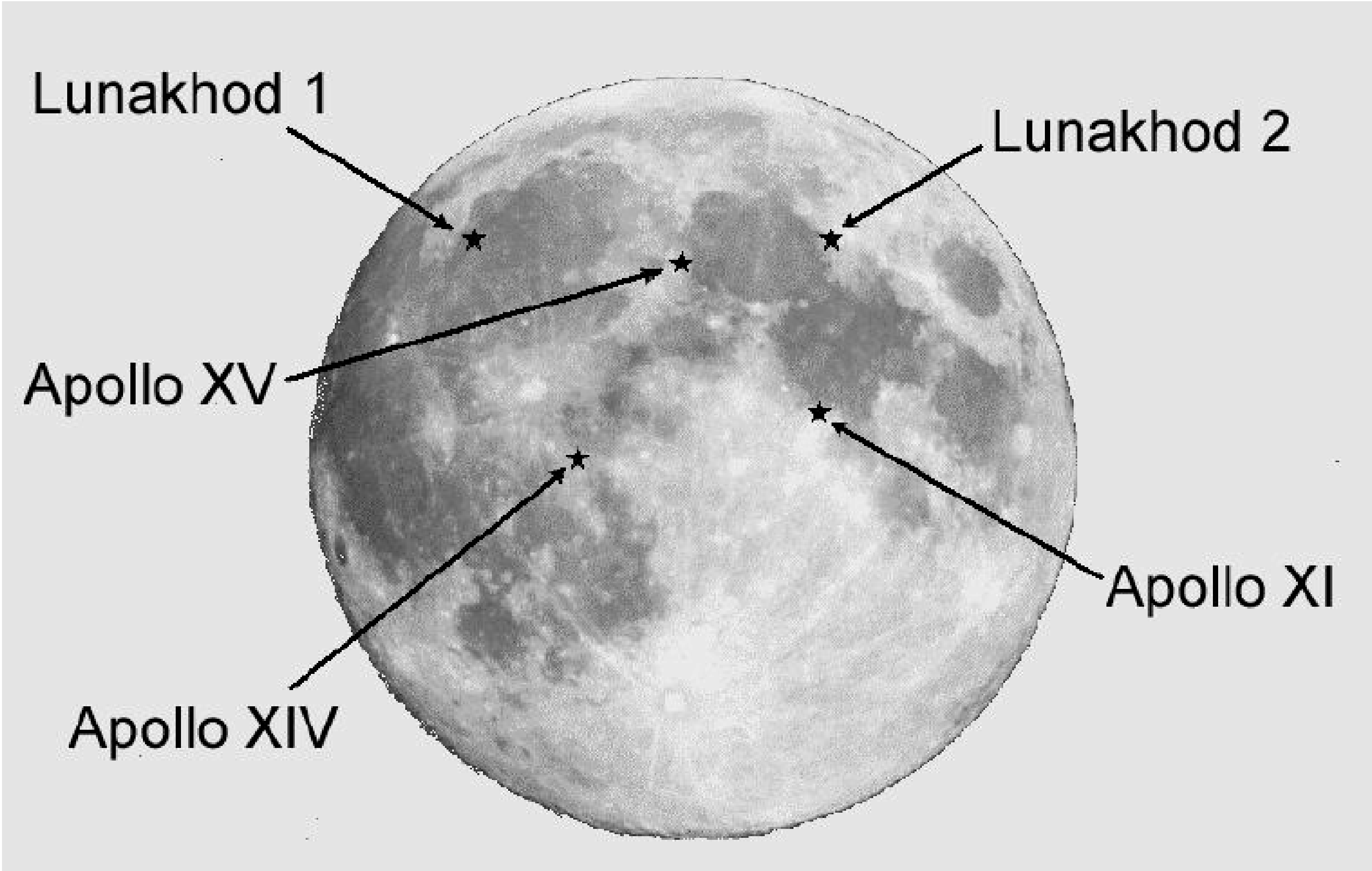
Retro-reflectors

After Apollo 11 delivered the first reflector array 4 more arrays have been landed. 3 by NASA Apollo missions and 2 French built arrays delivered by the Soviet Luna missions.

1969 Apollo 11		array of 100 3.8cm prism corner cubes
1970 Lunakhod 1	(Luna 17 mission)	array of 14 11 cm corner cubes
1971 Apollo 14		array of 100 3.8cm prism corner cubes
1971 Apollo 15		array of 300 3.8cm prism corner cubes
1973 Lunakhod 2	(Luna 21 mission)	array of 14 11 cm corner cubes

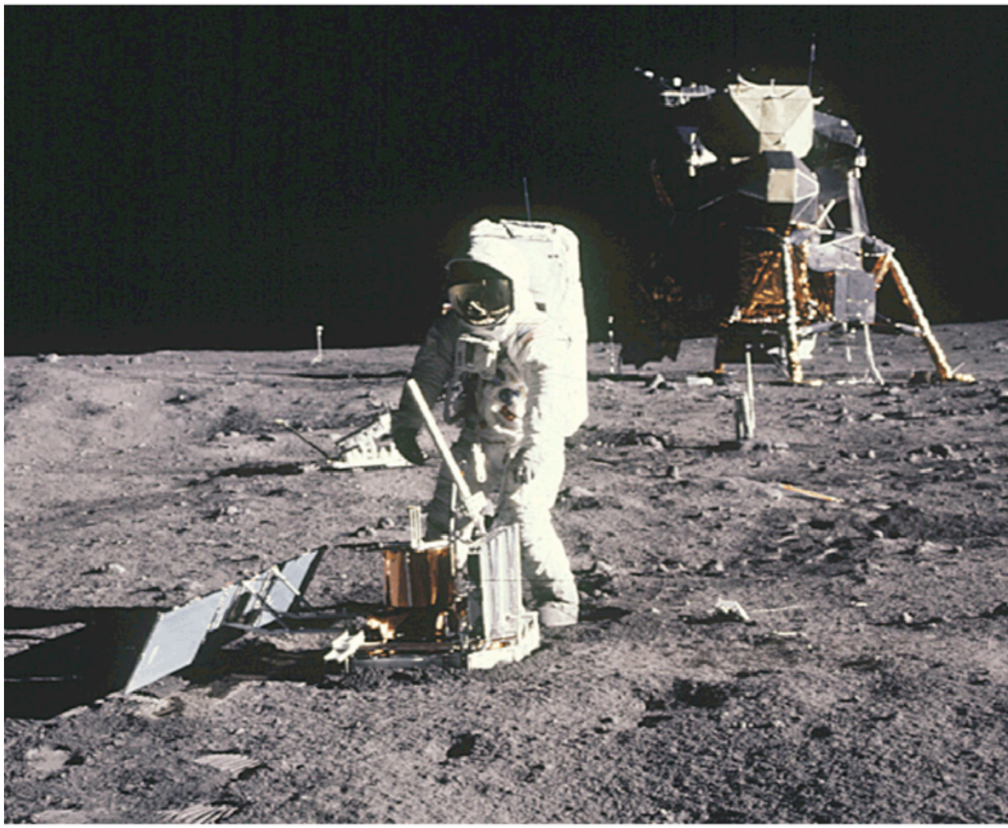
The corner cube reflector concept is show in the figure. Light is reflected back in the same direction as it arrives.



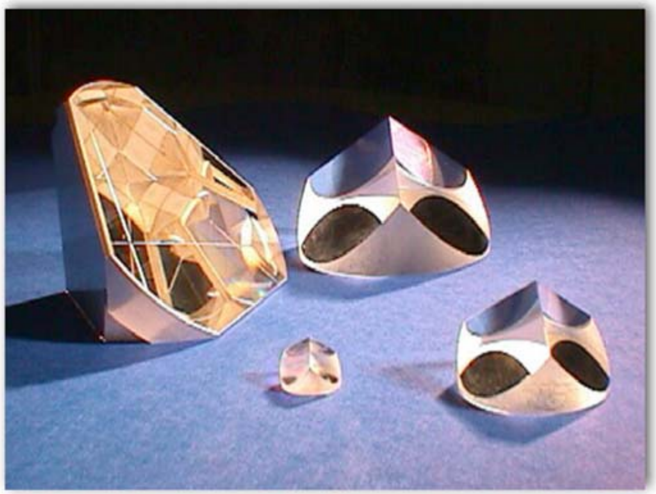


Reflector array locations. Signal from Lunakhod 1 was detected soon after placement but has since been lost.

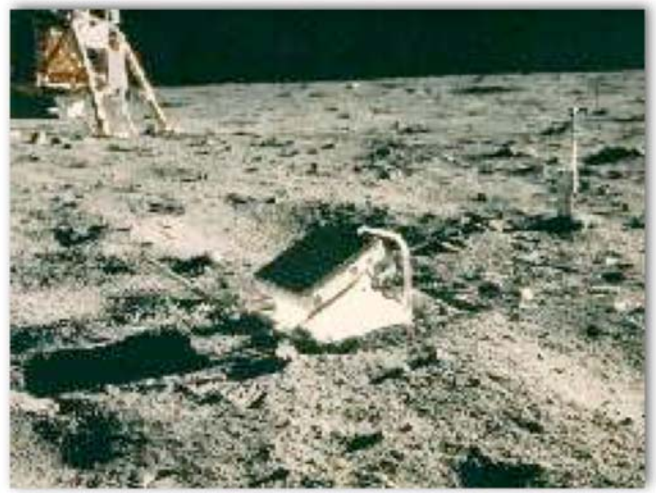
Apollo 11 and Luna 17



Retroreflector arrays

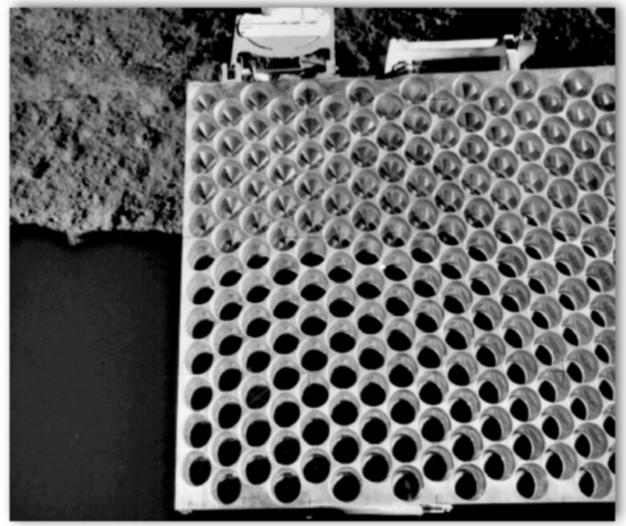


Corner cubes



Apollo 11 retroreflector array

Apollo 14 retroreflector array



Apollo 15 retroreflector array

Several laser ranging stations around the world have observed LLR signals.

Virtually all of the data used to test GR has come from 3 stations

McDonald Observatory near Fort Davis, Texas (USA).
1969 to mid 80's (2.7-m telescope) ruby laser
Mid 80's to present (0.76-m telescope) YAG laser

The Haleakala Observatory on Maui, Hawaii (USA)
0.4 m telescope late 80's to 1990. YAG laser

The CERGA station
(Centre d'Etudes et de Recherche en G´eodynamique et Astronomie)
Observatoire de Cˆote d'Azur (OCA), 'Plateau de Calern near Grasse France
Since 1982 1.54-m Cassegrain telescope
Rubis laser replaced by a YAG in 1987

CERGA

McDonald

Lunar Laser Ranging Concept

Average Earth Moon distance 385 000 km
➔ round trip time of flight 2.5 sec
So to measure the distance to 1 cm requires an accuracy of ~100 picoseconds

Time measurement based on Cs atomic clock with frequency accuracy of 1 part in 10^{12} so over 2.5 sec this can yield a measurement of better than 10 picosec

But at this level other factors come into play

Atmospheric time delay ~ 100 picosec depending on pressure temp humidity
Libration of moon – oscillation of array orientation ~ several hundred picosec
Photo diode detection uncertainties

There are also changes in the distance that have to be modeled out
Earth tide (30cm on ground)
Moon libration
Seismic disturbances

Though these limit the GR tests, these disturbances yield information on earth – moon planetary science. e.g the hypothesis of the moon having a liquid core is supported by LLR measurements.

Laser Detection

The main challenge is the detection of photons.

For the YAG laser at CERGA operating a 532 nm produces 10 pulses per second of 300 mJ per pulse. This corresponds to ~ 10^{18} photons per pulse

Now the beam divergence is 3 to 4 seconds of arc (limited not by the telescope optics but by the earths atmosphere)

So by the time the laser pulse hits the moon the diameter of the light spot is ~7km. This ➔ 1 photon out of 10^8 to 10^9 hits the array

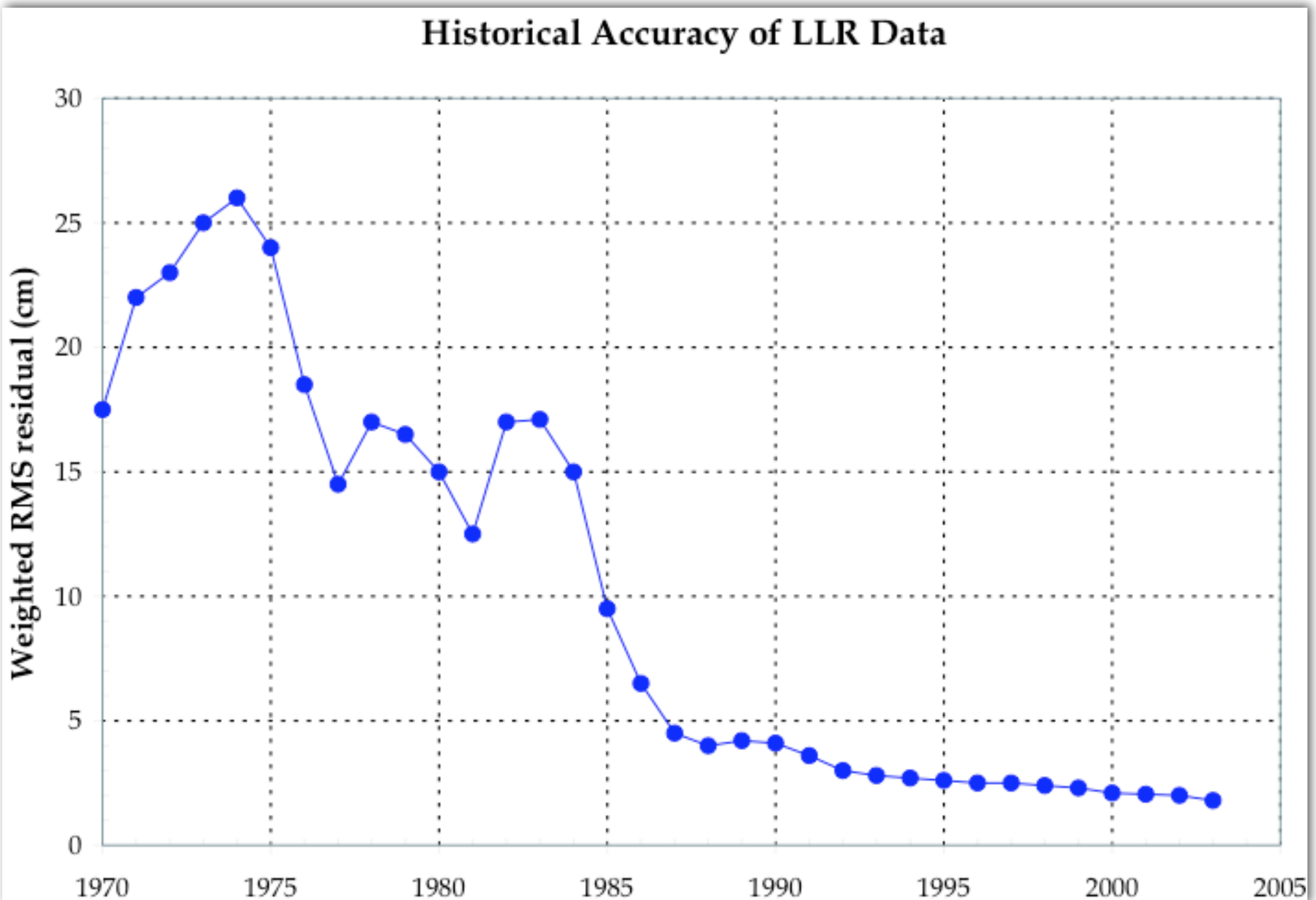
The divergence of the reflected beam is worse, about 12 arcsec, so when the light beam reaches the earth the diameter is ~ 25 km

So for a 1m telescope about 1 in 10^9 reflected photon has a chance

There are also other loses in the optics atmosphere and detection systems so that about 1 in 10^{20} photons is detected.

That is less than one photon per pulse! So the measurements rely on high rep rates (10 pulses per sec) and long observation times

Accuracy of Earth Moon Distance Measurement Over Time



Before LLR the distance was known to about 100m.

Measurements of the Earth – Moon distance have enabled the following tests of GR

- Weak Equivalence Principle: $\Delta a/a \approx$ parts in 10^{-13}
- Strong Equivalence Principle: $|\ \eta\ | \leq 5\times10^{-4}$
- time-rate-of-change of *G:* $\leq 10^{-12}$ per year
- geodetic precession: 0.35%
- $1/r^2$ force law: 10^{-10} times force of gravity at earth moon distance

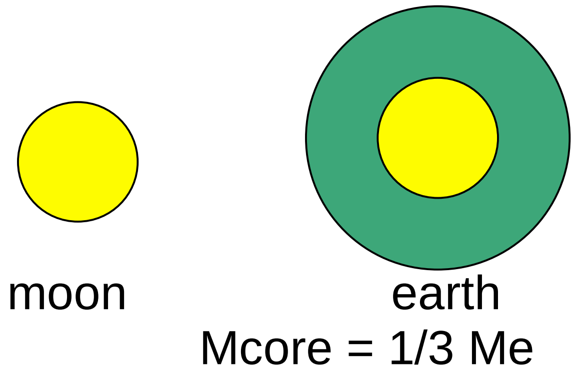
Equivalence Principle via LLR

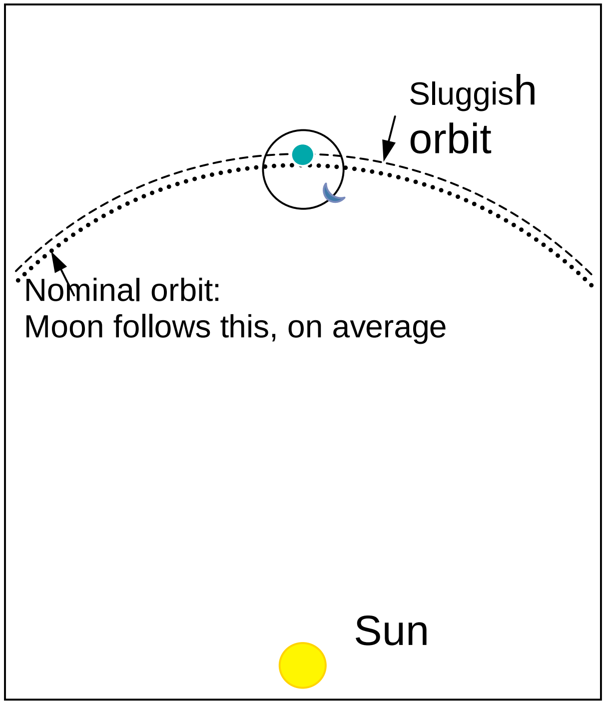
Weak EP – Universality of Free Fall

Composition dependence?

The moon is made up of silicates

The earth has two main constituents
Silicate mantel
Iron Core





If $M_{\text{Inertial earth}} > M_{\text{gravitational earth}}$ in a way that differs from the moon, then the earth would fall around the sun with a larger orbit than the moon

This would be detected as a polarization of the lunar orbit that is as a signal proportional to cosD where D is the lunar phase angle

Putting in the details
The range signal becomes $\Delta r = 12.9\ \Delta a/a \cos(D)$ meters
D = the moons synodic phase, period = 29.5 days

For this particular frequency and phase, LLR places limits of 4mm

➔ $\Delta a/a < 1.5 \times 10^{-13}$

That is, the earth and moon fall toward the sun with the same acceleration to parts in 10^{13}

This is comparable to the best ground based measurements of the Equivalence Principle

At this level the LLR EP measurement is also sensitive to the strong equivalence principle

The mass energy of the earth is comprised of all the normal constituents,
nucleons and electrons, electromagnetic, weak and strong force energy
And gravitational self-energy (consider the potential energy one would have to overcome by moving all the particles of the earth out to infinity)

The gravitational self-energy of the earth comprises a fraction of about $4.6\text{x}10^{-10}$ of the total mass of the earth. For the moon this fraction is nearly a factor of 20 lower. So if gravity 'gravitates' differently than matter, one would detect a difference in the range signal.

If we assume there is no composition dependence then this means
That this strong EP measurement is good to a precision of $3\text{x}10^{-4}$

Of course we should not make that assumption, and fortunately we don't have to.

I have said that the precision of the LLR EP measurement is on the same order as ground based tests (we will cover these in the next lecture)

The Eotwash group has tested the free fall of a moon like material against and earth core like material in precise torsion balance experiments.

They measure no deviation to the level $\Delta_{\text{Core-moon}} < 5\text{x}10^{-13}$

Therefore the total deviation from LLR gives

$4.5\text{x}10^{-10}\,\eta + 1/3\,\Delta_{\text{Core-moon}} < 1.5\text{ x}10^{-13}$

Where η is the fractional deviation of the Strong EP condition that the earths gravitational self energy contribute fully to the earths gravitational mass.

So LLR places the limit $\eta < 5\text{x}10^{-4}$

In the PPN formalism $\eta = 4\beta - 3 - \gamma$ And equals 0 in GR
So LLR places strict limits on this combination of parameters.
Note β is interpreted as a the non-linearity parameter which make sense since in GR gravitational energy gravitates.

Geodetic Precession

In 1916, Wilhelm de Sitter calculated the precession rate for the lunar perigee as the earth-moon system moves through the gravitational field of the sun.

He found that according to General Relativity. The perigee should precess at a rate give by

$$\Omega_{dS} = (\gamma + \frac{1}{2})\frac{GM_{\odot}}{c^2 r^3}\,|\vec{r} \times \vec{v}|$$

where M is the mass of the sun, v is the velocity of the earth-moon system orbiting the sun and r is the distance vector of the earth-moon system with respect to the sun. We will revisit this when we discuss the GP-B experiment.

The GR prediction amounts to a precession rate of 19.2 milli-arcsec per year.

This has to be distinguished form 40.7 degrees per year due to classical perturbations.

LLR enabled the first measurement of this effect. It has now been tested to ~ 4 parts in 10^3 corresponding to an accuracy of ±70 microarcseconds per year.

Deviations from $1/r^2$

Deviations from the Newtonian $1/r^2$ force law results in perigee precession of orbits.

LLR tests of precession rates therefore can set limits on Yukawa type couplings.

That is consider a possible interaction potential of the form

$$V(r) = \frac{Gm_1m_2}{r^2}\left(1 + \alpha e^{r/\lambda}\right)$$

The present limits set by LLR are $\alpha < 5\text{x}10^{-11}$ time the strength of gravity at a length scale λ, of10^8 meters.

The Future of Lunar Laser Ranging

A new Lunar Laser ranging operation has recently come on line.

APOLLO – Apache Point Observational Lunar Laser-ranging Operation

Goal to provide order of magnitude improvements over present state of the art

Apache Point Observatory - 3.5 meter telescope in southern New Mexico at 9,200 ft (2800 m) elevation

Operated by 7-university consortium (UW, U Chicago, Princeton, Johns Hopkins, Colorado, NMSU, U Virginia)

APOLLO uses a Nd:YAG mode-locked laser, frequency-doubled to 532 nm

90 ps pulse width (FWHM)
115 mJ per pulse
20 Hz repetition rate
2.3 Watt average power
GW peak power!!

Detectors can resolve multiple reflected photons per pulse

Peak rate averages of >0.6 photons per shot (12 per second)
compared to typical 1/500 for McDonald, 1/100 for CERGA

This will allow Ranging during at full moon – not possible at other stations due to high background

Became operational in July 2005

Expected to achieve Millimeter range precision

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Shapiro Time Delay

In 1964, Irwin Shapiro proposed a new test of general relativity based on radar signals reflected from planets or spacecraft passing behind the Sun. This distinct test of GR, which Shapiro titled Fourth Test of General Relativity (PRL **13** 26 1964), exploits the prediction that "the speed of light depends on the gravitational potential along it path".

In particular if a radar signal is sent from the earth and travels past the sun on its way to reflect off a planet or spacecraft, then the round trip time of flight should be delayed by

$$\delta t = 2(1+\gamma)(M_{sun} G / c^3)\ln(\frac{(r_{sun} + \vec{X}_{sun} \bullet \vec{n})(r_{earth} + \vec{X}_{earth} \bullet \vec{n})}{d^2})$$

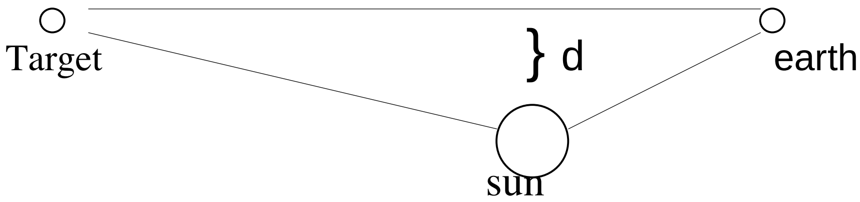
where the r's are the distance from the target to the earth or sun, and the x's are the corresponding position vectors (see Will LRR 9 2001).

For a beam grazing the sun this reduces to

$$\delta t = 2(1+\gamma)(\frac{M_{sun} G}{c^3})\ln(\frac{4 r_{earth} r_{t\arg et}}{d^2})$$

where d is the closest approach distance to the sun.

The PPN parameter γ is included in both expressions to show the dependence on curvature; in GR $\gamma = 1$)



Putting in the numbers, if Venus is the target the maximum delay (obtained when d = radius of the sun) is 240 micro sec (and not much different for Mercury or Mars due to the log term).

So this is fairly large in comparison to the state of the art of timing measurements, even by 1960's standards. However to do a measurement one needs something to compare the signal to see if its delayed.

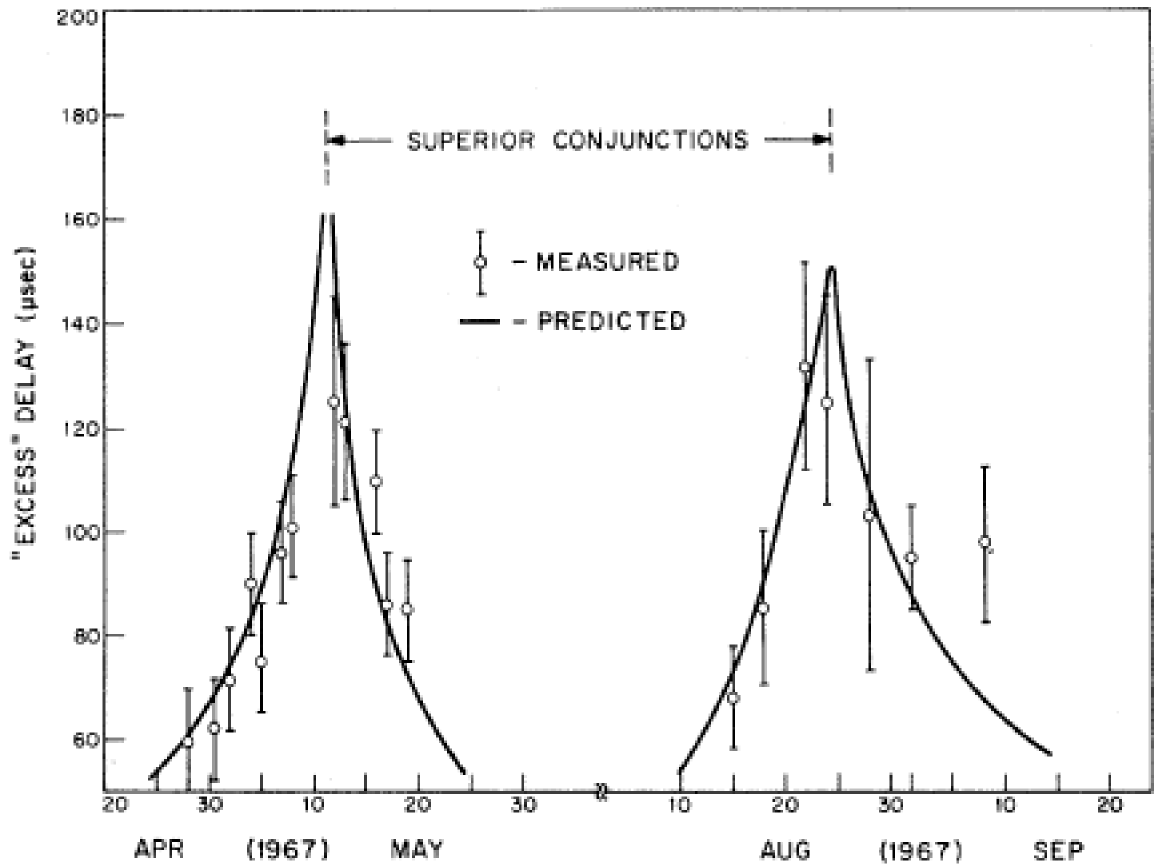
Consider a ranging measurement to Venus at superior conjunction.
One would need to know the earth – Venus distance to a fraction of
Cx240 μsec or a fraction of 72 km. This level of metrology was not available.

The way around this is to make measurements over a long period of time and use the radar data to fit an ephemeris. Then during the days near superior conjunction an excess delay signal should be manifest.

The first measurements of Shapiro Time delay were made in 1966 and 1967 to Mercury and Venus using the MIT Lincoln Laboratory Haystack radar station.

The station was upgraded for this purpose to include a 7.84 GHz transmitter which was time coded by linkage to a hydrogen maser.
Transmission signal power was 300kW.

The receiver system for detecting the reflected (echo) signals was capable of detection signals of $<10^{-21}$ W, almost 27 orders lower than the transmission.

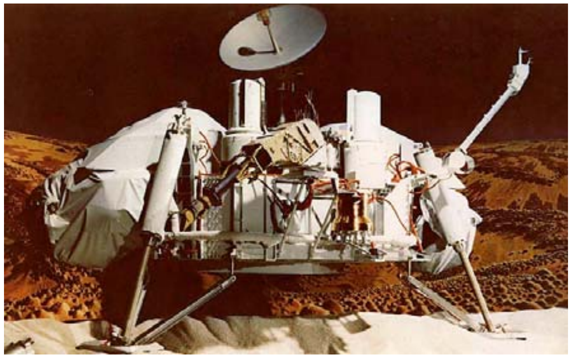


Some the first measurements are shown in the figure from Shapiro, et al. PRL 20 22 (1968) These initial measurement confirm the GR prediction to a level of 20%.

Viking Relativity Experiment

The decade following the first radar echo experiments saw steady but modest improvements in time delay tests.

A big advance was realized in 1976 with the Viking Relativity Experiment.



Viking Lander

The Viking spacecraft reached Mars in the summer of 1976. The program consisted of two orbiter spacecraft and two landers. The landers had S band transponders which allowed high accuracy radar ranging to the Mars surface.

Systematic errors caused by the solar corona plasma were reduced using the one way coherent X (8.4 GHz) band and S band (2.5GHz) communication links from the orbiters to the earth. Plasma delays are frequency dependent and are greater for the S band.

Ranging data were taken over 14 months including the superior conjunction of Nov 25 1976. The data were analyzed within a solar system ephemeris model Which included the PPN γ as a fitting parameter. The RMS of the fit residuals was 50 ns although the averages near conjunction were higher due to the plasma corrections. Overall a test precision of γ to 0.1 % was obtained

Solar Corona Effects

The Solar Corona consists of hot plasma. EM properties are dominated by free electrons

This produces an effective dielectric constant of

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2},$$

Where ω_p is the plasma frequency defined by:

$$\omega_p = \sqrt{\frac{n_e\, e^2}{\epsilon_0\, m_e}}$$

For high frequency waves (i.e. w > wp)
The group velocity is given by

$$v_g = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}},$$

So this gives rises to possible error in the time delay measurements.

Between about 4 and 20 solar radii the electron density is

$\sim n_e = 5x10^5\ (r_{sun}/r)^2$

So this implies plasma frequencies on the order of 10Mhz

With perturbations that change with position and time.

The Cassini Mission

Our present limit on γ { $(1-\gamma) = 2.1 \pm 2.3 \times 10^{-5}$ } was obtained from ranging measurements to the Cassini spacecraft.

These measurements were made from June 6 through July 7, 2002 as the spacecraft was on it was to Saturn, bracketing the time of a superior conjunction. During this time the spacecraft was about 8.5 AU form the sun and the closest approach of the radio beam was 1.6 solar radii.

At this close range previous experiments would have had large error due to effects of the solar corona. Cassini overcame these challenges by employing mult-band communication links in the high frequency range.

The communication system consisted two carriers at 7,175MHz (X-band) and 34,316MHz (Ka-band) transmitted from the ground and three carriers transmitted from the spacecraft.

Two of these carriers, at 8,425MHz and 32,028MHz, are locked to the received X and the Ka signals sent from the ground. The third carrier is a nearby Ka-band

downlink coherent with the X-band uplink.

This unique comm. system enabled the a new method for the time delay measurement, Doppler tracking.

The fractional frequency Doppler shift is found by differentiating the time delay equation with respect to time.

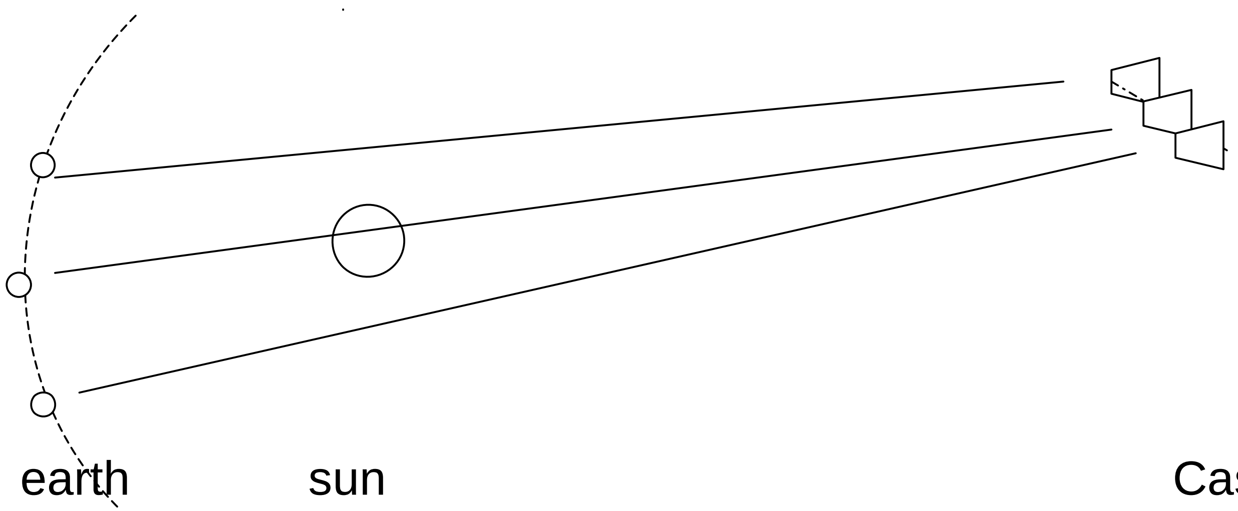
$$\frac{\Delta\nu}{\nu} = \frac{d\delta t}{dt}$$

$$\frac{\Delta\nu}{\nu} = 2(1+\gamma)(\frac{M_{sun}G}{c^3})\ln(\frac{4r_{earth}r_{target}}{d^2})$$ define $b^2 = d^2/4r_{earth}r_{sun}$

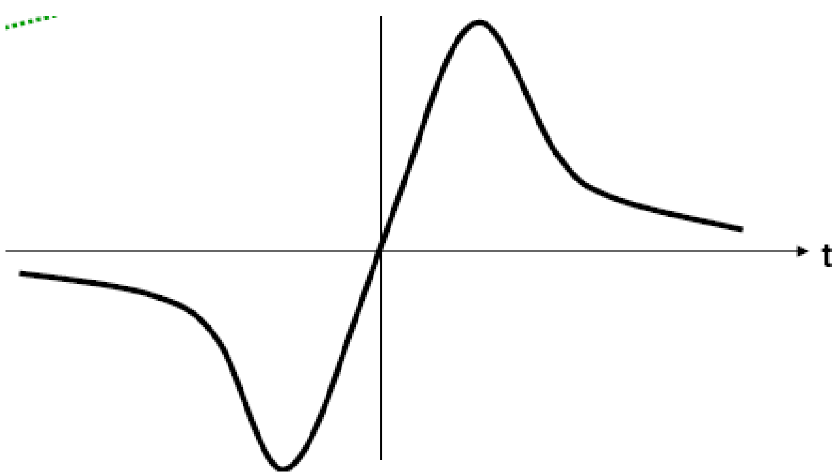
then $$\frac{\Delta\nu}{\nu} = 2(1+\gamma)(\frac{M_{sun}G}{c^3})\frac{1}{b}\frac{db}{dt} = -1x10^{-5}\sec\cdot(1+\gamma)\frac{1}{b}\frac{db}{dt}$$

for the Cassini experiment geometry the rate of change of the impact parameter is dominated by the velocity of the earth, v_{earth} = 30 km/s. This yields a peak value of

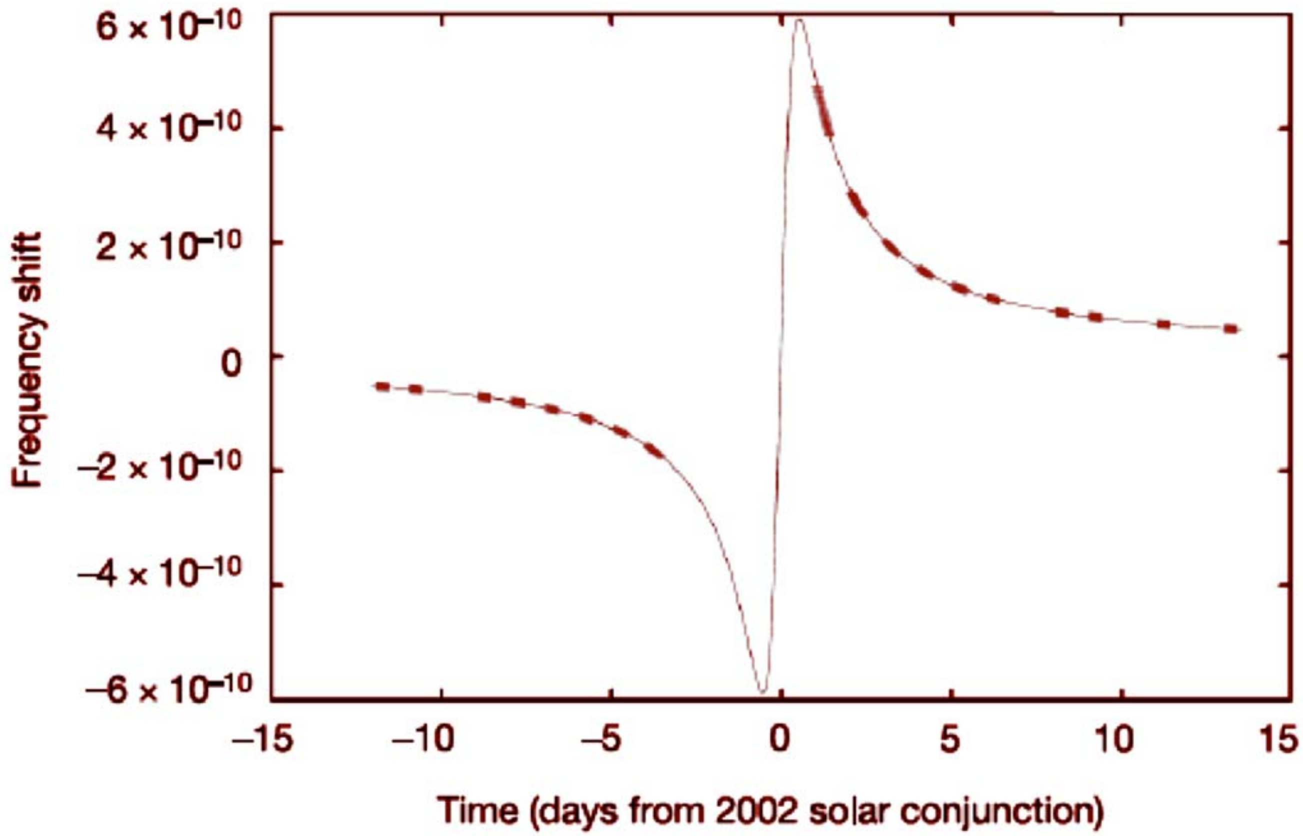
$$\frac{\Delta\nu}{\nu} = 6x10^{-10}$$ for the GR case of $\gamma = 1$



As the radar beams go through closest approach ($1.6R_{sun}$ out of plane) the Doppler signal should move form a minimum through zero and through a max.



This is precisely what the Cassini experiment measured.


6 × 10−10
4 × 10−10
2 × 10−10
0
−2 × 10−10
−4 × 10−10
−6 × 10−10
Frequency shift
−15
−10
−5
0
5
10
15
Time (days from 2002 solar conjunction)

The Pioneer Anomaly

See Dittus, Lämmerzahl, Turyshev, and Anderson COSPAR 2006

Navigational data from Pioneer 10 and11 indicate presence of a small, anomalous, blue-shifted Doppler frequency drift uniformly changing with the rate of

$$\dot{f}_p = (5.99 \pm 0.01) \cdot 10^{-9} \text{ Hz / s}$$

The drift can be interpreted as a constant acceleration of

$a_p = (8.74 \pm 1.33) \cdot 10^{-10} \text{ m/s}^2$ in the direction toward the sun

Pioneer Anomaly has the following properties:

–A line of sight constant acceleration of the spacecraft toward the Sun.

–Analysis with data (1987–1998.5) shows anomalous acceleration for heliocentric distances 20–70 AU.

–Anomaly continues out as a constant after Saturn

–Constancy – temporal and spatial variations are ≤ 3%.

Pioneer 10 and 11 are identical spacecraft

	Pioneer 10	**Pioneer 11**
Launch	2.3.1972	5.4.1973
planetary fly-by	Jupiter: 4.12.1973	Jupiter: 2.12.1974 Saturn: 1. 9.1979
last data received	27.4.2002 (after 30 years of operation) @ 80.2 AU	1.10.1990 @ ca. 30 AU
direction of motion	star Aldebaran	constellation of Aquila

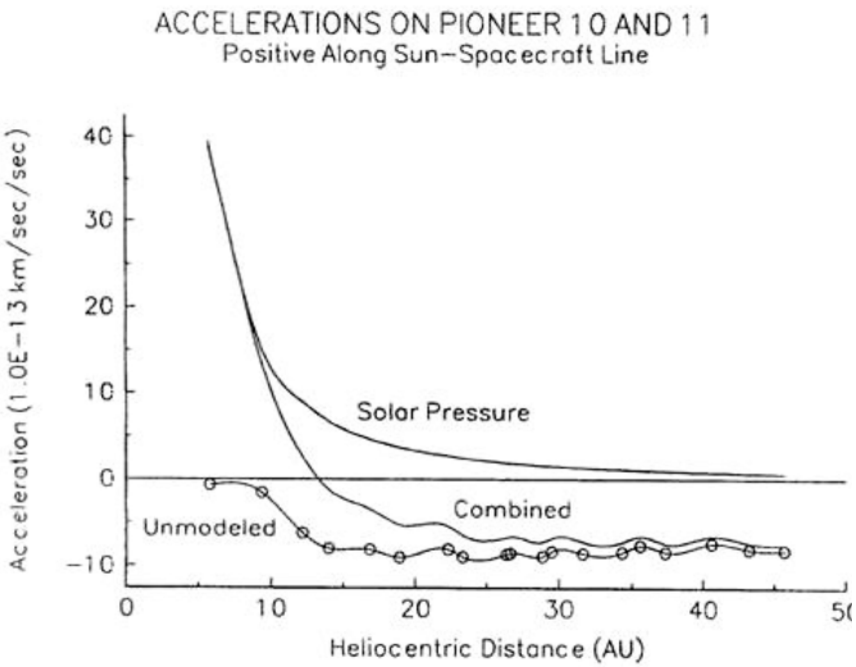
S/C total mass	259 kg
power supply: SNAP-19 RTGs	boom 3 m / mass 13.6 kg
magnetometer	boom 6 m / mass 5 kg
high gain antenna (diameter)	2.74 m
maximum cross section	5.914 m^2
spin stabilized (spin rate)	4.28 rpm
principal moment of inertia	588.3 $kg \cdot m^2$
up-link frequency	2.110 GHz (S-band)
down-link frequency	2.292 GHz (S-band)
wave length	ca. 13 cm
continous transmission	8 W

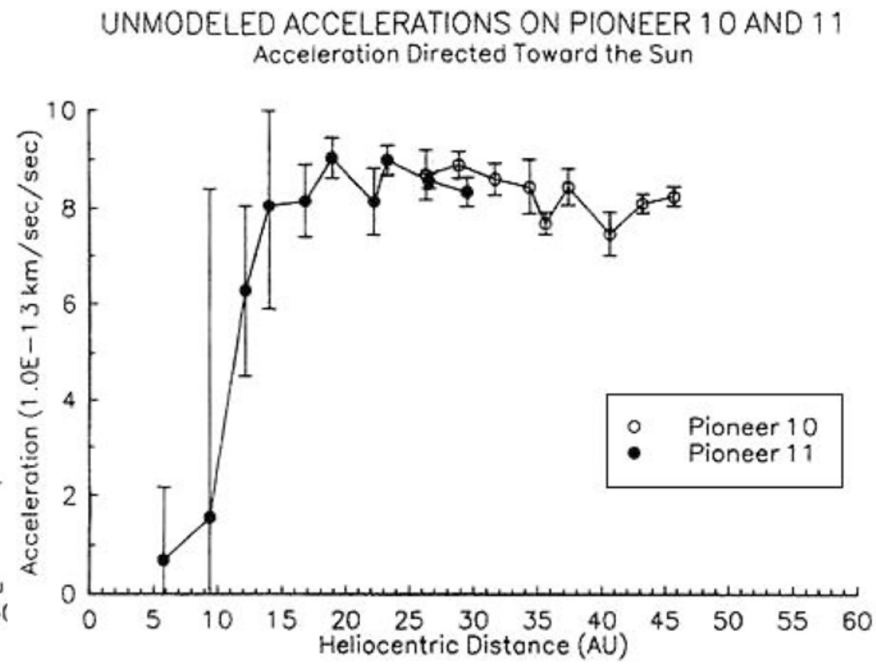
Search for unmodeled accelerations with Pioneers started in 1979:

- Motivation: search for Planet X – initiated when Pioneer 10 was at 20 AU;
- The solar-radiation pressure away from the Sun became < 5×10^{-8} cm/s^2

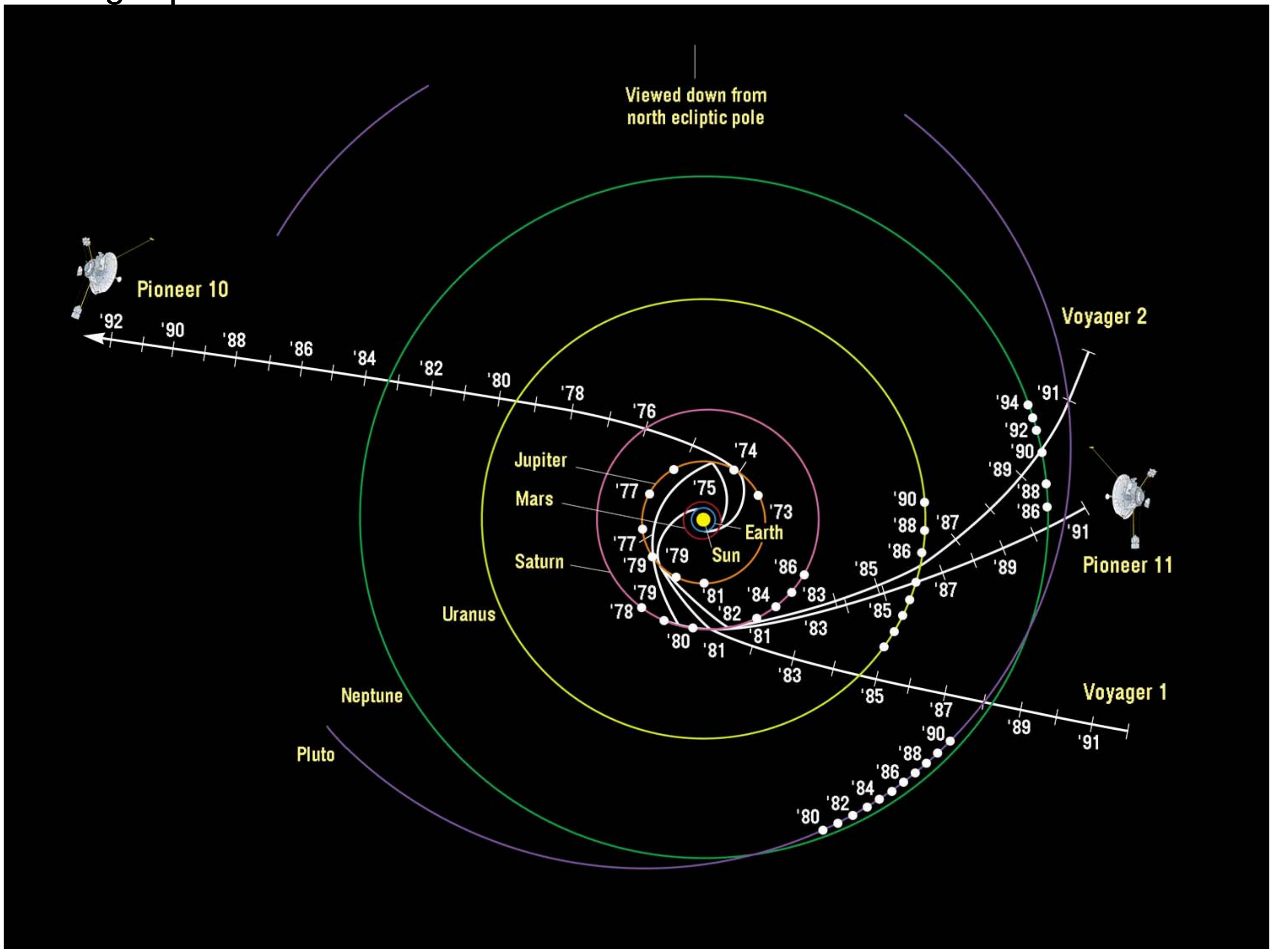
Original detection of the anomaly by JPL orbit determination in 1980:

- The analysis found the biggest systematic error in the acceleration residuals is a constant bias $a_P \sim (8 \pm 3) \times 10^{-8}$ cm/s^2 directed towards the Sun





Flight paths of the Pioneers



- Elliptical (bound) orbits before last fly-by
- Hyperbolic (escape) orbits after last fly-by

Studies conducted at JPL (1980-2000) focused on the the on-board systematics (most plausible cause of the anomaly):

mechanisms investigated include:

–External effects:

- Solar radiation pressure, solar wind, interplanetary medium, dust
- Viscous drag force due to mass distributions in the outer solar system
- Gravity from the Kuiper belt; gravity from the Galaxy
- Dark Matter distributed in a halo around the solar system
- Drifting clocks, mismodeling of the general relativistic effects
- Errors in the planetary ephemeris, in the values of the Earth Orientation Parameters, precession, and nutation
-

–On-board systematic & other hardware-related mechanisms:

- Precessional attitude control maneuvers and associated “gas leaks”
- Nominal thermal radiation due to $^{94}Pu^{238}$ decay [half life 87.74 years]
- Heat rejection mechanisms from within the spacecraft
- Hardware problems at the DSN tracking stations
- Identical design of Pioneer 10/11 spacecraft (supported by Galileo, Ulysses S/Cs)
-

–Phenomenological time models:

- Drifting clocks, quadratic time augmentation, uniform carrier frequency drift, effect due to finite speed of gravity, and many others

None of these can account for the effect.

The only one within an order of magnitude is the differential emissivity of the RTGs

There is a number of proposals invoking “New Physics” to explain the anomalous signal.

Newton´s $1/r^2$ – law has not been tested systematically for distances larger than 20 AU

Orbits of the outer planets have not been observed precisely enough / Pluto orbit has not been completed since the detection of the planet

MOND (Modified Newtonian Dynamics) theories can match these data and the galaxy spiral data only with the inclusion of many tuned parameters

Until new measurements of spacecraft or ranging to the outer planets this will probably remain a mystery