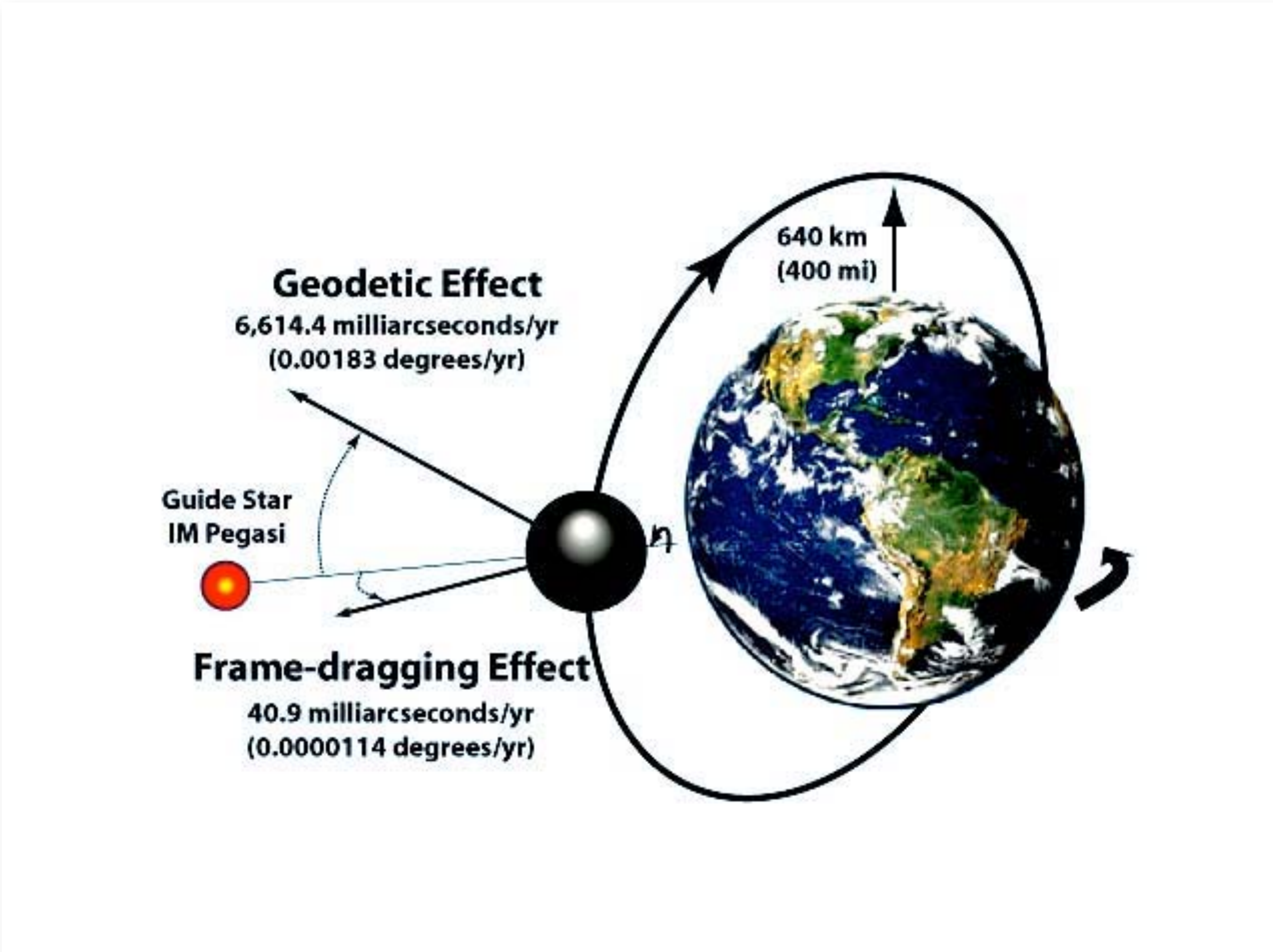
Institut Henri Poincaré

The Gravity Probe B Relativity Mission:

Theory Background

John Mester
Stanford University

The Relativity Mission Concept



Acknowledgements

This work was accomplished under NASA funding, Contract No. NAS8-39225.

• **Stanford University** *C.W.F. Everitt PI, GP-B team*	**Development, Science Instrument, Management Mission Operations, Data Analysis**
• **Lockheed Martin** *GP-B team*	**Probe, Dewar, Satellite, Flight Software**
• **Science Advisory Committee** *Clifford Will chair*	
• **Harvard Smithsonian** *Irwin Shapiro*	**Guide Star and Star Proper Motion Studies**
· **JPL** *John Anderson*	**Independent Science Analysis**
· **York University** *Norbert Bartell*	**Guide Star and Star Proper Motion Studies**
• **Purdue University** *Steve Collicot*	**Helium Ullage Behaviour**
• **San Francisco State** *Jim Lockhart*	**Gyroscope Read-out Topics**
· **National University of Ireland** *Susan M.P. McKenna-Lawlor*	**Proton Monitor**
• **University of Aberdeen** *Mike Player*	**High Precision Homogeneity Measurement of Quartz**

Tests of General Relativity: Background

Einstein Field Equation $G_{\mu\nu} = R_{\mu\nu} - 1/2\, g_{\mu\nu}R = (8\pi G/C^4)T_{\mu\nu}$

"matter tells spacetime how to curve, and curved space tells matter how to move"

No adjustable parameters

– G directly measurable Newtonian gravitational constant

Schwarzschild solution: static, spherically symmetric field of a point mass

– weak field expansion to first order

$$ds^2 = (1-2GM/C^2R)C^2dt^2 - (1+2GM/C^2R)dr^2$$

$$\Phi = GM/R$$

Metric Outside Rotating Earth

The metric outside the rotating earth to lowest non trivial order in Φ and in v - the relevant parameters- involved is given by:

$g_{00} = (1 - 2\Phi/C^2)$

$g_{ij} = - d_{ij} (1 + 2\Phi/C^2)$

$g_{i0} = z_i$ where $\mathbf{z} = 2GI_e/C^2R^3 (\mathbf{R} \times \mathbf{w_e})$

I_e and w_e are the moment of inertia and rotation velocity of the earth

Metric Outside Rotating Earth 2

Plausibility argument and interpretation

Working to first order (and setting C=1) we start with the line element for a stationary point mass $ds^2 = (1–2GM/R)dt^2 – (1+2GM/R)dr^2$

Generalize to moving point mass by transforming to moving system w/ 1st order Lorentz xform t = t-vx x_r = x-vt. (for motion in x direction) —> $ds^2 = (1–2GM/R)dt^2 – (1+2GM/R)d\mathbf{r}^2 + 4m/r\ vdxdt$

So for motion in general direction:

$ds^2 = (1–2GM/R)dt^2 – (1+2GM/R)d\mathbf{r}^2 + 4m/r\ \mathbf{v}\cdot d\mathbf{r}\ dt$

Since the theory is linear to this order, can superpose distribution of point masses to get $ds^2 = (1–2\Phi)dt^2 – (1+ 2\Phi)d\mathbf{r}^2 + \mathbf{h}\cdot d\mathbf{r}\ dt$ where,

$$\Phi(r) = G\int \frac{\rho dr'}{r'} \qquad h(r) = 4G\int \frac{\rho v dr'}{r'}$$

Thus Φ(r) and **h**(r) are scalar and vector potentials

in analogy with EM, coulombs law + Special Relativity yields magnetism

See Adler and Silbergleit, IJTP 39 pg 1291, 2000.

Effects of Gravitation on Spin

Particle in Free Fall U^α = dx^α/dt = four velocity $dU^\alpha{}_\alpha dt$ = 0

S_α = four spin dS_α/dt = 0 in Special Rel

S_α = {**S**,0} in rest frame $S_\alpha U^\alpha$ = 0 in all frames

To go from Special Relativity to General Relativity make equations covariant
—> derivatives go over to:

$dU^\mu/dt + \Gamma^\mu{}_{\nu\lambda} U^\nu U^\lambda = 0$

$dS_\alpha/dt - \Gamma^\mu{}_{\alpha\nu} U^\nu S_\mu = 0$ Parallel transport

$\Gamma^\sigma{}_{\lambda\mu} = 1/2 g^{\nu\sigma} [\, dg_{\mu\nu}/dx^\lambda + dg_{\lambda\nu}/dx^\mu - dg_{\mu\lambda}/dx^\nu \,]$ Affine Connection

See Weinberg *Gravitation and Cosmology* and Misner, Thorn and Wheeler *Gravitation*.

Precession of a Gyro in Earth Orbit

In orbit —> no forces on gyro

Therefore Four spin will undergo Parallel transport

$dS_\alpha/dt - \Gamma^\mu{}_{\alpha\nu} S_\mu dx^\nu/dt = 0$

$S_\mu dx^\mu/dt = 0$ —> $S_0 = -V^i S_i$

So can eliminate S_0 to get:

$dS_i/dt = \Gamma^j{}_{i0} S_j - \Gamma^0{}_{k0} V^k S_i + \Gamma^j{}_{ik} V^k S_j - \Gamma^0{}_{ik} V^k V^j S_j$

Define precession vector **W** by d**S**/dt = **W x S**

Then using $g_{\mu\nu}$ of earth to calculate Γs and eliminating terms which vanish for periodic orbits one finds:

$$\overline{\Omega} = \overline{\Omega}_G + \overline{\Omega}_{FD} = \frac{3GM}{2c^2R^3}\left(\overline{R} \times \overline{\mathrm{v}}\right) + \frac{GI}{c^2R^3}\left[\frac{3\overline{R}}{R^2}\left(\overline{\omega}_e \cdot \overline{R}\right) - \overline{\omega}_e\right]$$

Where R is the vector from the earth center to the gyro, v is the gyro velocity

Precession of a Gyro in Earth Orbit 2

The first term, the geodetic precession, also called the deSitter effect, is caused by the interaction of the mass or the earth with the gyro frame and the distortion of spacetime by the mass of the earth and depends on the orbital velocity of the gyro

$$\overline{\Omega}_G = \frac{3GM}{2c^2R^3}\left(\overline{R} \times \overline{\mathrm{v}}\right)$$

The latter term, the Frame Dragging or Lense-Thirring term depends on the angular momentum of the earth. The earth is the prime solar system body for the source since it's angular momentum is well known.

$$\overline{\Omega}_{FD} = \frac{GI}{c^2R^3}\left[\frac{3\overline{R}}{R^2}\left(\overline{\omega}_e \cdot \overline{R}\right) - \overline{\omega}_e\right]$$

Since its an interaction between spin angular momenta of the earth and gyro this term is sometimes referred to as 'hyperfine interaction', in analogy with atomic physics. But does not depend on magnitude of gyro spin.
Any vector that points a fixed direction in the local inertial frame will precess in frame of distant stars.

Precession of a Gyro in Earth Orbit 3

For a polar orbit the Frame dragging and geodetic precession are perpendicular and can therefore be resolved independently

Both terms are maximized by taking R_{orbit} close to R_{earth}

For 642km altitude and gyro spin near the equatorial plane

The magnitude of the effects averaged over an orbit are

Geodetic W_G = 6.6 arcsec/year   Frame Dragging W_{FD} = 0.041 arcsec/year

GP-B goal is to test these to parts in 10^5 and to better then 1% resp.

Leading Corrections are due to:

Geodetic effect due to the sun, W_G^s = 19 marcsec/year

(in plane of the ecliptic)

Oblateness correction of Geodetic Effect, W_G^{J2} = –7marcsec/year

(parallel to W_G)

Frame dragging due to the sun and effects from the moon are negligible

Interpretation of Predicted Effects

As noted in the introduction, General Relativity is a theory with no adjustable parameters. So there is no need for interpretation - all of the science of GP-B is revealed by the precession equation

$$\overline{\Omega} = \overline{\Omega}_G + \overline{\Omega}_{FD} = \frac{3GM}{2c^2R^3}\left(\overline{R} \times \overline{\text{v}}\right) + \frac{GI}{c^2R^3}\left[\frac{3\overline{R}}{R^2}\left(\overline{\omega}_e \cdot \overline{R}\right) - \overline{\omega}_e\right]$$

Still it can be interesting to form a physical picture of how the precessions arise. But these pictures are probably a matter of taste. So if a particular interpretation doesn't appeal to you, just go back to the equations.

Parameterized Post-Newtonian Formalism

Eddington, Nordtvedt, Will

Useful parameterization for a range of metric theories of gravity

Low velocity, weak field limit

- Solar system

Metrics of various theories similar form

- Expansion about flat (Minkowski) space
- Gravitational potentials provide deviation from flat space
- Various theories provide different coefficients

Parameter	Meaning	GR value
g	Space curvature	1
b	Non-linearity in gravity superposition	1
x	Preferred location	0
a_{1-3}	Preferred frame	0
G_{1-4}	Conservation of momentum	0

$$\overline{\Omega}_G = \left(\gamma + \frac{1}{2}\right)\frac{GM}{c^2R^3}\left(\overline{R} \times \overline{\mathrm{v}}\right)$$

$$\overline{\Omega}_{FD} = \left(\gamma + 1 + \frac{\alpha_1}{4}\right)\frac{GI}{2c^2R^3}\left[\frac{3\overline{R}}{R^2}\left(\overline{\varpi}_e \cdot \overline{R}\right) - \overline{\varpi}_e\right]$$

Caveat: Danger in viewing all GR tests in terms of PPN parameter limits. Metric theories that fit within PPN framework span only a part of the space of possible alternative theories. Most recent attempts at unification of the forces of nature require non-metric revisions of GR.

Significance of Frame Dragging

In GR gravitational effects are due to the geometry of spacetime.

A non-rotating body produces a static spacetime curvature due to Mass. A rotating body produces an additional dragging of external inertial frames via the body's angular momentum, analogous to the body being surrounded by a viscous fluid. The instantaneous direction change of the gyro over the poles in opposite in direction to that when the gyro is above the equator. This is just what a test "straw" oriented perpendicular to the immersed in a fluid would do.

Frame Dragging is conceptually related to Mach's principle: Inertial forces arise from accelerations and rotations with resp. to total mass of the universe.   Under this interpretation the gyro spin direction reaches a compromise between following the distant stars and the rotating earth.

Lense and Thirring in 1918 considered the metric inside a rigidly rotating hollow sphere. In 1966 it was shown that as the shell's gravitational radius, $2GM/c^2$, approaches its physical radius, the precession of a gyro at the center approaches the angular velocity of the shell. That is, the gyro spin becomes locked to the rotating distant matter. Water would climb up the sides of a non-rotating bucket at the center of the shell.

Gravitoelectric / Gravitomagnetic Viewpoint

Space-time metric	Newtonian analog	EM analog	Gravito EM analog	Rotational effect
g_{00}	ϕ	V	E_g	$1/3\Omega_G$
g_{0i}	No analog	A_i	B_g	Ω_{FD}
g_{ij}	No analog	No analog	No analog (space curvature)	$2/3\Omega_G$

This view point arises in analogy with electromagnetism or from treating GR as a spin 2 field theory to lowest order. Static matter generates a gravito-electric potential and space curvature. Rotation produces a gravito-magnetic dipole field. Frame dragging arises form the interaction of the gyro spin with a vector potential g_{0i}.

A moving mass (mass current)generates a gravito-magnetic field; it is related to gravito-electrostatics through Lorentz xform (see slide 11). However, a gravito-magnetic field of a rotating mass cannot be derived by rotation of coordinates from the static field of a non-rotating mass. Frame Dragging as tested by GP-B does not have a trivial relationship to static gravitation.

Evidence of Frame Dragging Effects

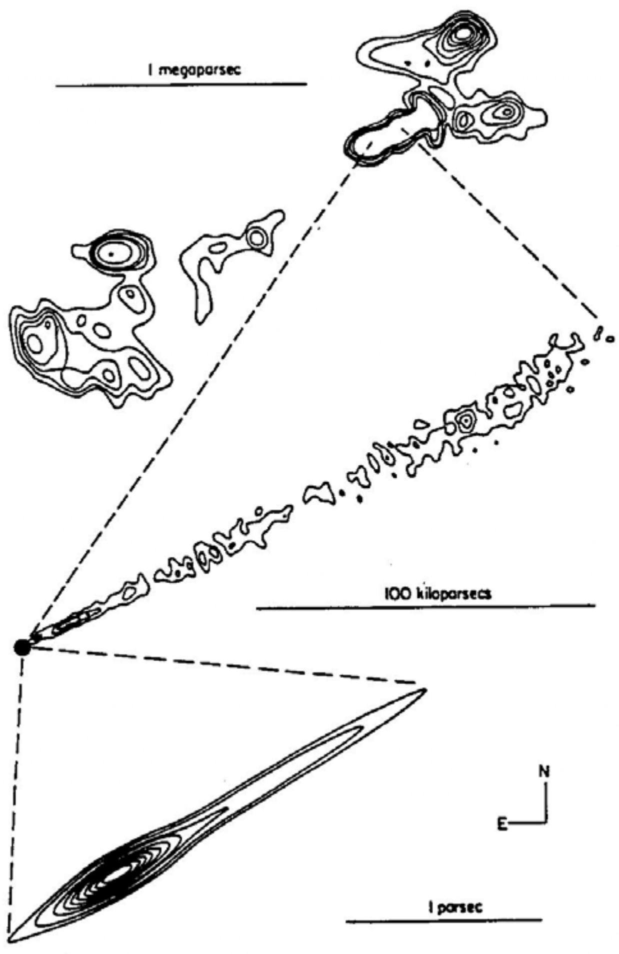
Within GR, frame dragging should produce a precession of the orbital plane of a body in earth orbit (the original prediction of Lense and Thirring) and a precession of the pericenter of the orbit. Cuifolini et al. have analyzed laser ranging data of the LAGEOS I and LAGEOS II satellites and claim a confirmation of FD effects to a 25% level. Results are controversial due to the need to subtract out Newtonian perturbations that are > 10^7 times larger than the GR predictions, as well as understanding non-gravitational disturbances due to radiation pressure and thermal effects.

In the Taylor-Hulse binary pulsar B1913+16, uncertainties in the constituent properties are too large to be used for precision tests of gravitomagnetic effects. A pulsar orbiting a fast rotating black hole could provide an appropriate test system; no definitive candidates are yet know.

Some effects of frame dragging by translational motion are present (in combination with other effects) in Lunar Laser ranging tests of the Nordtvedt effect.

Within the PPN framework, Shapiro time delay and LLR measurements place tight limits on the γ and α_1 parameters.

Frame-Dragging Evidence in Astrophysics



Radio Map of Galaxy NGC 6251

Observed jets from Galaxy NGC 6251
Power output 10^{38} watts!
(A trillion suns)

Compact object holds jet direction aligned millions of years. Plausible cause of the alignment is the effect of frame dragging from a Supermassive Black Hole. "Frame dragging imprints the angular momentum of the source on the distant spacetime." Evidence for FD but does not allow for quantitative measurement.

Interpretation of Geodetic Precession

The Geodetic effect is a combination of precession due the gravito-electric field and precession due curved spacetime.

The Missing Inch(and a half)

The gravito-electric part, 1/3 of the total effect, is due to special relativistic corrections to the motion of the gyro in the g_{00} potential.

Consider the path traveled by the gyro as it goes around the earth at velocity v. From special relativity the distance traveled is contracted:

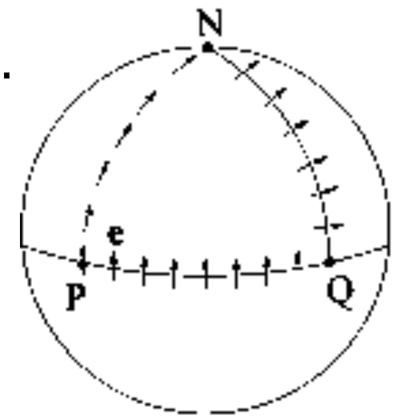
$$L = L_0\sqrt{1-\frac{v^2}{C^2}}$$

So for assumed circumference of the orbit,
$L_0 = 2\pi(R_e+Alt) = 2\pi$ x7010km = 44,228km
And gyro velocity $v \sim L_0/97.6$ min = 7553 m/sec

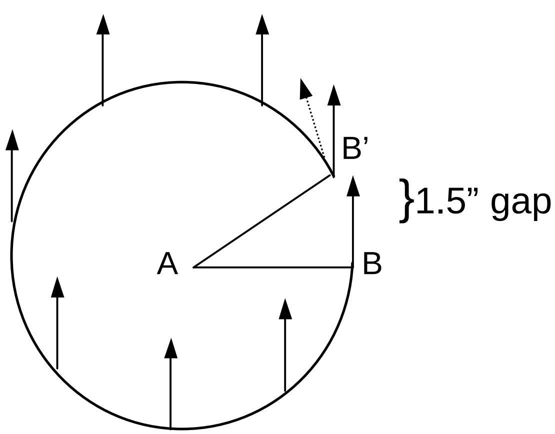
So $L_0 - L = 0.014$m (about half an inch)

Interpretation of Geodetic Precession 2

The other 2/3 of the precession is due the curvature of space around the earth. Change in vector orientation under parallel transport is a hallmark of curvature as seen in 2 dimensional example. Transporting from points Q-P-N-Q the vector is rotated.



What about the missing Inch?
For the GP-B case the curvature is due to the mass of the earth distorting the space around it. This produces an orbit circumference smaller than what one would calculate using $2\pi r$ in flat space, smaller by about 1 inch over 27000 miles.



The effect this has on gyro spin direction can be visualized with the diagram due to Kip Thorn. As the gyro orbits about point A the spin direction stays fixed in the local flat sheet geometry. Cutting out the gap wedge and connecting points B and B' to create a cone one sees a change in spin direction.

For GP-B the 1.5 inch gap corresponds to ~1marcs per orbit ==> 6.6 arcsec over a year.

Evidence of Geodetic Effects

For The Earth-Moon system can be considered as a ``gyroscope'', with its axis perpendicular to the orbital plane. The predicted precession, first calculated by de Sitter, is about 19 marcseconds per year. This effect has been measured to about 0.7 percent using Lunar laser ranging data.

Within the PPN framework, light deflection and Shapiro time delay measurements around the sun set limits on the curvature parameter g.

The current limits established by telemetry data from the Cassini spacecraft are a few parts in 10^5.

Several alternative theories, such as Damour-Nordtvedt "attractor mechanism" tensor-scalar theories of gravity, predict deviations from zero of the PPN parameter g of up to $\gamma - 1 \leq 4\times10^{-5}$)

References

Will, *Confrontation between General Relativity and Experiment,* Living Reviews in Relativity, lrr-2001-4 **http://relativity.livingreviews.org**

Adler and Silbergleit, *General Treatment of Orbiting Gyroscope Precession*, IJTP 39 pg 1291, 2000

The National research Council Space Science Board, *Review of Gravity Probe B*, National Academy Press, 1995

Weinberg, *Gravitation and Cosmology.* J Wiley and Sons

Misner, Thorn and Wheeler, *Gravitation*.