VI. horizons

An horizon is a boundary separating observable events from non-observable.

Two such concept must be distinguished

Event horizon:

> For an observer O, it divides all events in 2 classes:
>
> 1. those that have been or will be observable by O
> 2. those who will never be observable

[e.g. Schwarzschild spacetime.

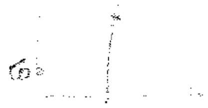
For the FLRW a necessary and sufficient condition for an event horizon to exist is

radial geo : $dt = a\,d\chi$

$$\boxed{\int^{\infty} \frac{dt}{a(t)} < \infty}$$

under this condition, $\forall t_0$ there exists a world line

$$\chi = \chi_0 = \int_{t_0}^{\infty} \frac{dt}{a(t)}$$ such that the photon emitted at t_0 in χ_0 toward $\chi = 0$ reaches $\chi=0$ at $t = \infty$



All photons emitted at t_0 in $\chi > \chi_0$ will never reach $\chi = 0$

$\chi < \chi_0$ will reach $\chi = 0$ in finite time

ex • de Sitter $a = e^{Ht}$: $\chi_0 = \int_{t_0}^{\infty} \frac{dt}{a} = \frac{1}{H}\left(1 - e^{-Ht_0}\right) < \infty$

• $a \propto t^n$ $\int t^{-n} dt < \infty$ iff $n > 1$

For $k=0$ and $w = c^{t}$: $3n = \frac{2}{1+w}$ so that

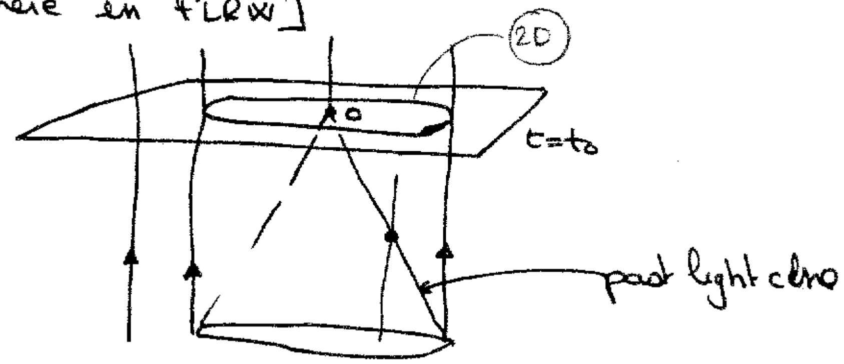
there is an event horizon if $w < -\frac{1}{3} \Leftrightarrow (\rho + 3P) < 0$

Particle horizon:

For an observer O at t_0, it is the surface of the $\{t = t_0\}$ hypersurface that divides the events in 2 families

1- those who have not been observed at t_0
2- those that have been observed at t_0

at each t_0, it is the intersection of the geodesics of the furthest particles observed at t_0 with $t = t_0$, that is a 2-dimensional space hypersurface [which reduces to a sphere in FLRW]



A sufficient and necessary condition for a particle horizon to exist is

$$\boxed{\int_{0 \text{ or } -\infty} \frac{dt}{a} < \infty}$$

under this condition, $\forall t_0$, any particle such that $X > \int_0^{t_0} dt/a$ has not been observed by O @ t_0.

and

$$X = \phi(t_0) = \int_0^{t_0} \frac{dt}{a(t)}$$ defines the particle horizon @ t_0

and divides particles observed fr non-observed

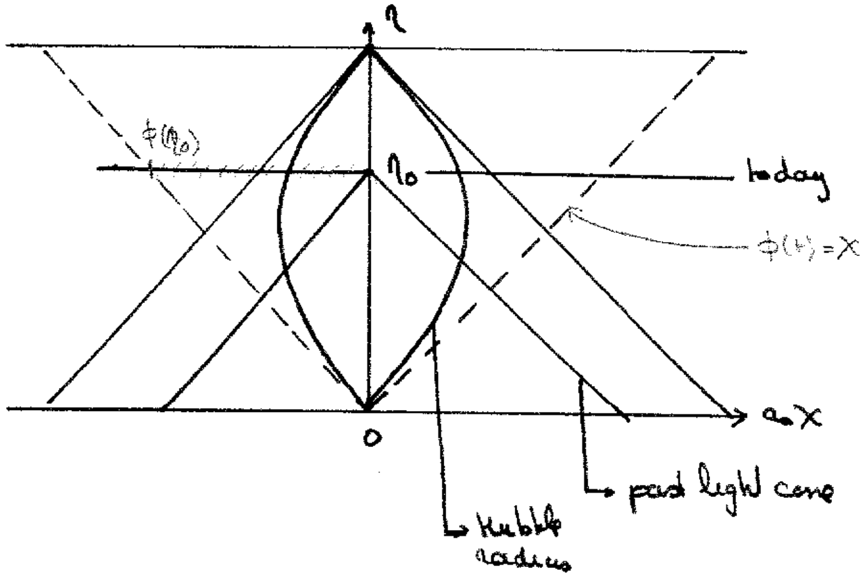
It can be seen as the section of the 3-dim surface $r = \phi(t)$ which is the future light cone of the observer at $t = 0$

If $\dot{a}(t) > 0$, $\phi(t) \nearrow$ more and more particles can be observed

If $\phi(t) < \infty$ when $t \to \infty$ then the universe also have an event horizon

- when $a \propto t^n$ $\int_0 t^{-n} dt < \infty$ iff $n < 1$

that is when $\rho + 3P > 0$ ($K = 0$; $w = c^t$)

- 

- consider an event @ t_1, the physical diameter of the its particle horizon at $t_2 > t_1$ is

$$D_{hp}(t_1, t_2) = 2\, a(t_2) \int_{t_1}^{t_2} \frac{dt}{a(t)}$$

$$= \frac{6(1+w)}{1+3w}\, t_2 \left[1 - \left(\frac{t_1}{t_2}\right)^{\frac{1+3w}{3+3w}} \right] \qquad w = c^t.$$

$\sim \frac{4}{\text{[illegible]}} D_H(t_2)$ $\qquad t_2 \gg t_1$

K=0 FLRW is conformal to M_4. Two conformal spaces have same causal structure

$$ds^2 = a^2(\eta)\left[-d\eta^2 + \delta_{ij}\, dx^i dx^j\right]$$

using null coordinate $v = \eta + r \; ; \; w = \eta - r \qquad -\infty < v, w < \infty$

$$ds^2 = a^2(\eta)\left[-dv\, dw + \tfrac{1}{4}(v-w)^2 d\Omega^2\right]$$

To study the structure at infinity, we introduce coordinates that bring ∞ to finite value

$$\tan p = v \; ; \; \tan q = w \qquad -\pi/2 \leq p, q \leq \pi/2$$

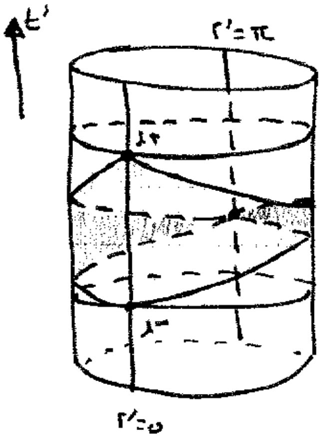
Then $t' = p+q \; ; \; r' = p-q \qquad -\pi < t'+r' < \pi$

$-\pi < t'-r' < \pi \qquad r' \geq 0$

conformal to $ds^2 = -dt'^2 + dr'^2 + \sin^2 r'\, d\Omega^2$ which is locally identical to the Einstein static space

$$\begin{cases} 2\eta = \tan\frac{t'+r'}{2} + \tan\frac{t'-r'}{2} \\ 2r = \tan\frac{t'+r'}{2} - \tan\frac{t'-r'}{2} \end{cases}$$

suppressing θ, φ, Ei stat is a cylinder $x^2 + y^2 = 1$ embedded in M_3



$p = \frac{1}{2}\pi \quad \mathscr{I}^+$
$q = -\frac{1}{2}\pi \quad \mathscr{I}^-$ } null surf.

$(\frac{1}{2}\pi, \frac{1}{2}\pi) \quad i^+$
$(\frac{1}{2}\pi, -\frac{1}{2}\pi) \quad i^0$
$(-\frac{1}{2}\pi, -\frac{1}{2}\pi) \quad i^-$

concrete example of an hor. in FLRW (?)

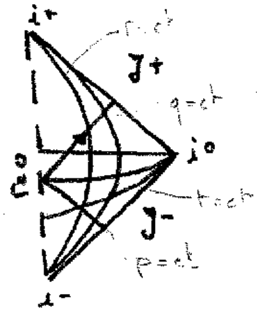
Any future timelike geodesic → i^+ } future/past timelike infinity
past → i^-

null geodesic $\mathcal{J}^- \longrightarrow \mathcal{J}^+$ null infinity

i^0 : spacelike infinity

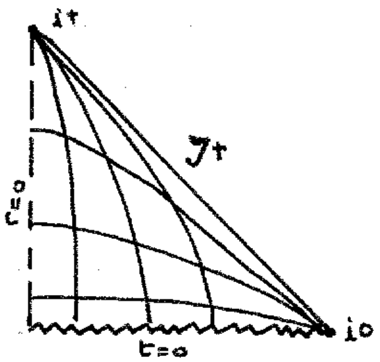
cauchy surface intersect all timelike & null geodesics : cross section really i_0

in the (t', r') plane



FLRW is different because $\eta > 0$ $[\Lambda = 0;\ \rho + 3P > 0]$

⇒ $K=0;\ \Lambda=0;\ \rho+3P>0$



$\mathcal{J}^-$ spacelike

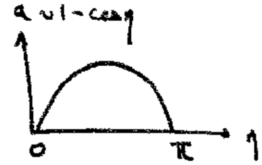
because of the behaviour of $a(\eta)$ we have various sections of V_4 possible.

K = +1

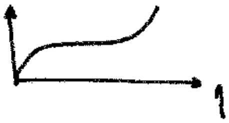
$$ds^2 = a^2 \left[-d\eta^2 + d\chi^2 + \sin^2\chi \, d\Omega^2 \right]$$

it is conformal to Eds but $a(\eta)$ implies that we cover just part of it

e.g. (I) $\Lambda = 0$ $p = 0$

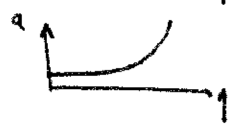


(II) $\Lambda \neq 0$ $p = 0$
$\Lambda > \Lambda_{crit}$



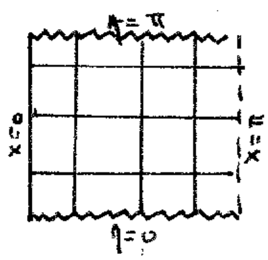
~~and we can get the two diagrams~~

(III) Also possible



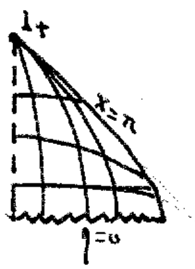
K = +1

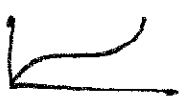
(I) $\Lambda = 0$



$\mathcal{I}^-$ & $\mathcal{I}^+$ spacelike

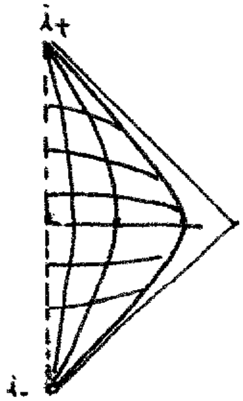
(II) $\Lambda > \Lambda_{crit}$



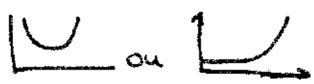


$\mathcal{I}^-$ spacelike
$\mathcal{I}^+$ timelike

(III)



$\mathcal{I}^-$ & $\mathcal{I}^+$ timelike



For $\kappa = -1$;

one can obtain EdS conformal with

$$\begin{cases} t' = \arg tg\left[th \frac{1+\chi}{2} \right] + \arg tg\left[th \frac{1-\chi}{2} \right] \\ r' = \qquad\qquad\qquad\qquad - \end{cases}$$

$$-\pi/2 \leq r' + t' \leq \pi/2 \quad ; \quad -\pi/2 \leq t' - r' \leq \pi/2$$

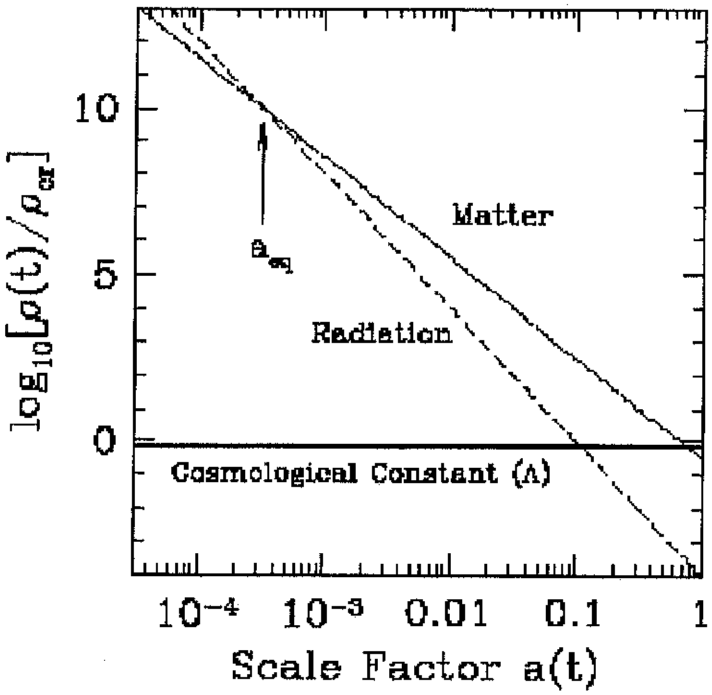
Again it is a diamond part of EdS whose size depends on the range of η.

VII . The HOT BIG BANG

- The Hubble diagram
  - H_0
  - q_0
- Age of the universe
- Thermal history
- BBN
- CMB
  - T
  - $T \propto (1+z)$

$\Rightarrow$ slides - less details

FRIEDMANN EQUATIONS: CONSEQUENCE



The universe was dominated by radiation in the past
HOT BIG BANG MODEL

SUCCESSES: EXPANSION

Hubble diagram:

1- cépheids
2- Type Ia supernovae
3- Tully-Fischer relation

4- gravitational lensing
5-Sunyaev-Zel'dovich effect

H_0(km · s^{-1}/Mpc)	Méthode
71 ± 2 ± 6	SN Ia
71 ± 3 ± 7	Tully-Fischer
70 ± 5 ± 6	brillance de surface
72 ± 9 ± 7	SN II
82 ± 2 ± 6	plan fondamental
72 ± 8	méthode combinée

H_0(km · s^{-1}/Mpc)	Méthode	Référence
73 ± 15	photosphère	Schmidt *et al.* (1994)
72 ± 7 ± 15	effet de lentilles	Tonry et Franx (1998)
60 ± 20	effet de lentilles	Fassnacht *et al.* (2000)
60 ± 4	effet Sunyaev-Zel'dovich	Reese *et al.* (2002)
72 ± 5	WMAP	Spergel *et al.* (2003)

Cepheids: pulsating atmosphere

$L \propto P^{-1.6}$ with 20% dispersion

$\longrightarrow D_L$ @ 10% up to ~ 20 Mpc

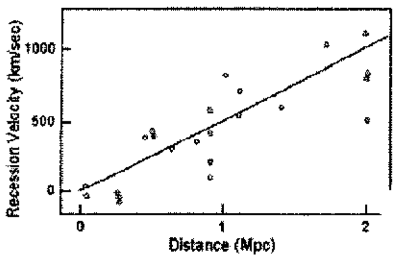
SNIa: result from explosion of white dwarf / accretion from companion up to $1.4\,M_\odot$

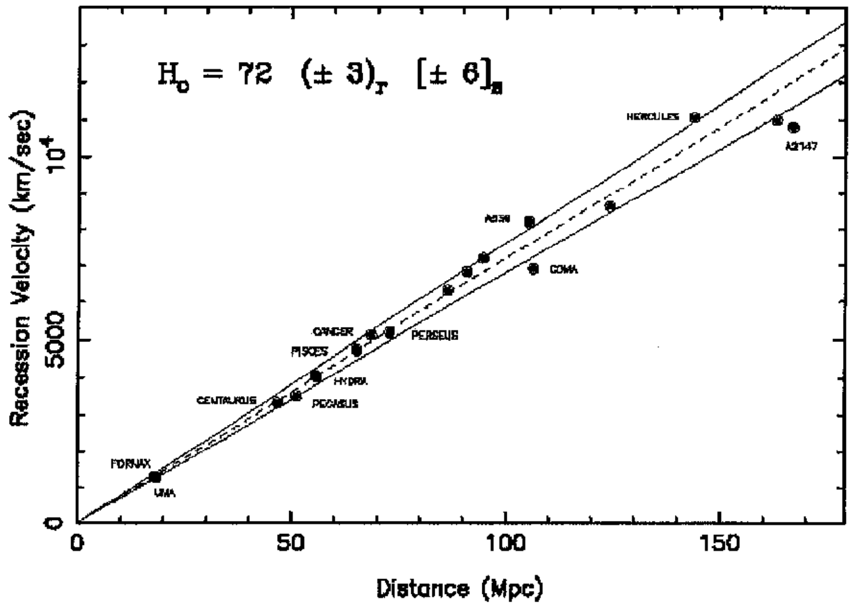
up to # 100 Mpc

$L \sim 12\% \Rightarrow D_L \sim 6\%$

SUCCESSES: EXPANSION

Hubble's Data (1929)



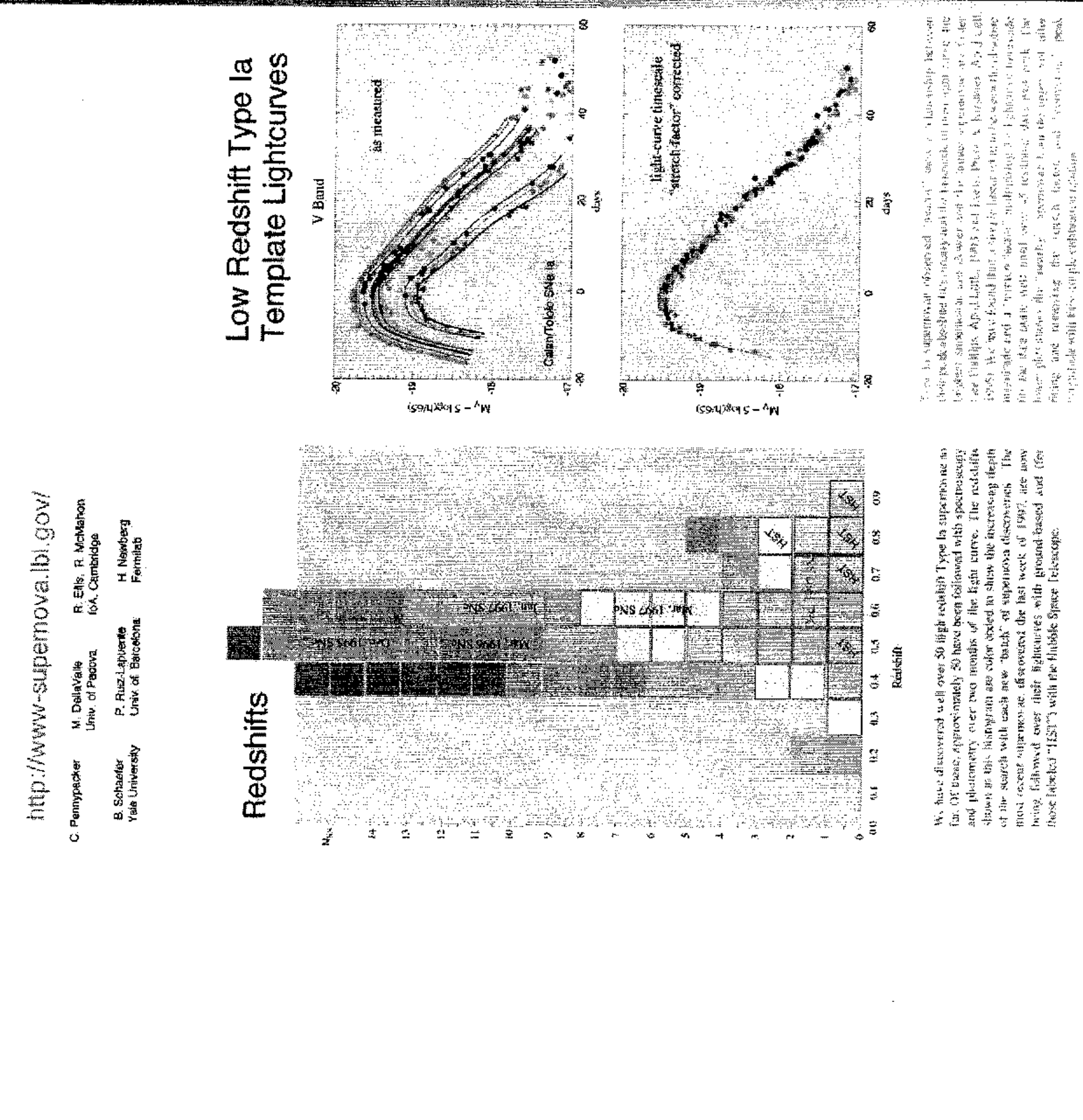


light curves can be calibrated

SCP: 42 SN . 18 $z < 0.101$
24 $0.18 < 0.83$

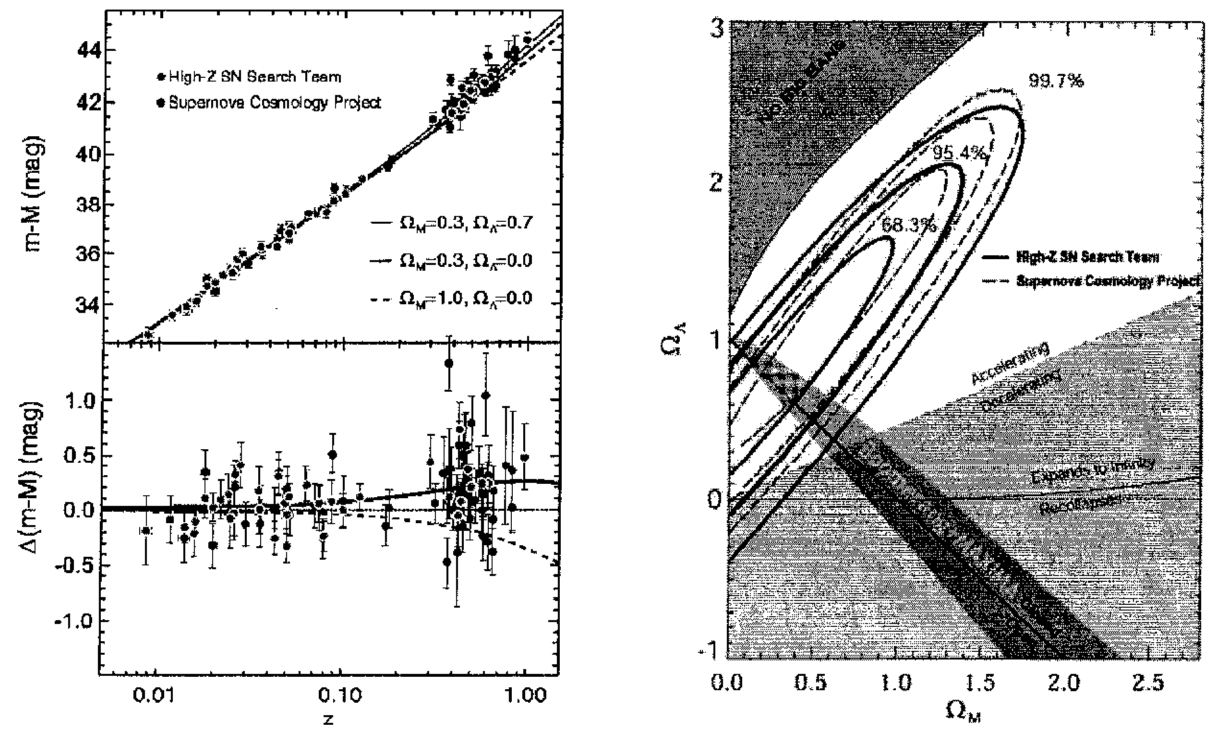
HZT 34 SN

SUCCESSES: EXPANSION



② - standard candles
- absorption
- X-ap oscillation

SUCCESSES: EXPANSION



①

[$q_0 < 0$ if $\Omega_m > 0$

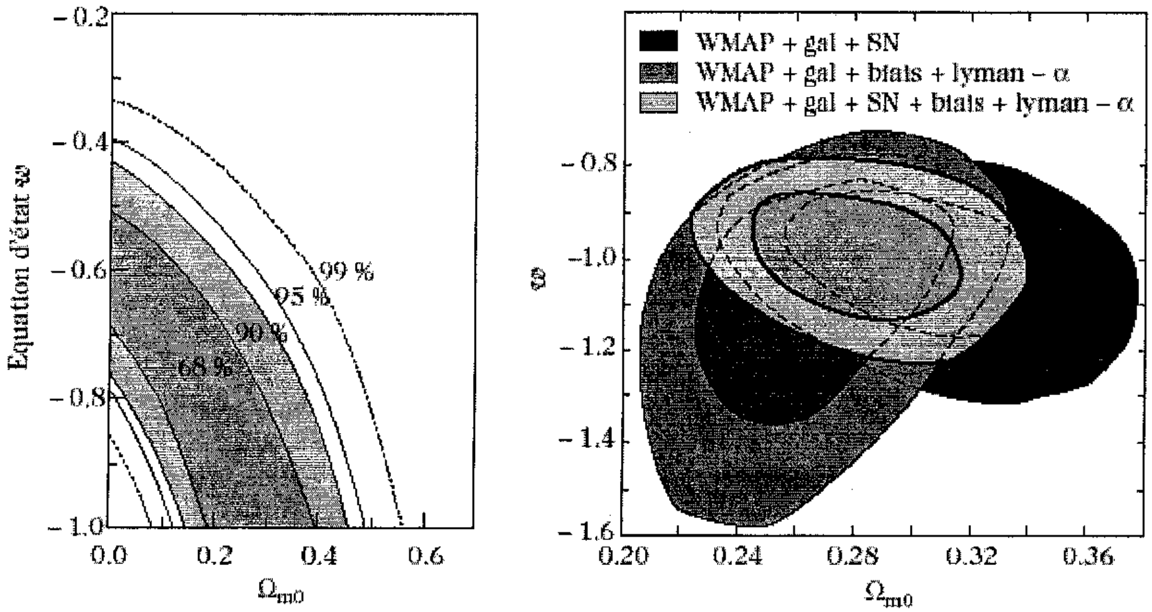
SN: $8\Omega_{m0} - 6\Omega_{\Lambda 0} \simeq -2 \pm 1$

flat $\Omega_{m0} + \Omega_{\Lambda 0} \sim 1.02 \pm 0.02$ } $\Omega_{\Lambda 0} \sim 0.7$, $\Omega_{m0} \sim 0.3$

conclusions

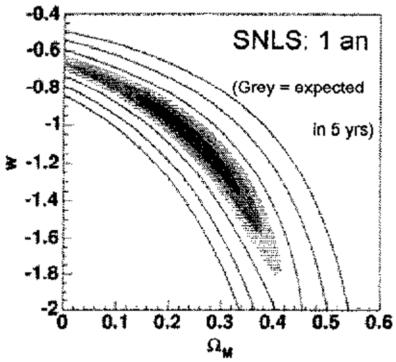
$q_0 < 0$ just from FLRW from [CP].

SUCCESSES: EXPANSION



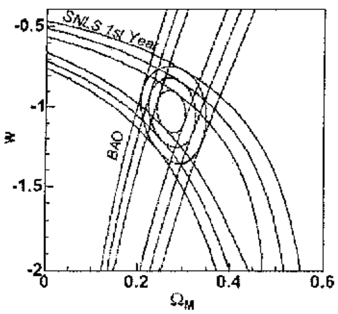
Constraints on a constant equation of state

SN



Astier et al 2005

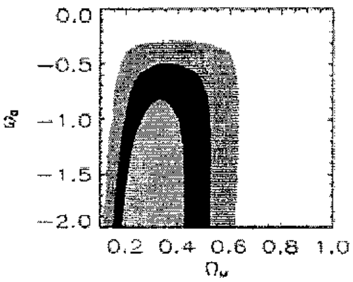
BAO-SN



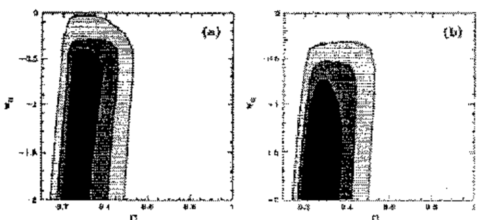
Astier et al 2005

Einsestein et al. 2005

Lensing



Semboloni et al 2005



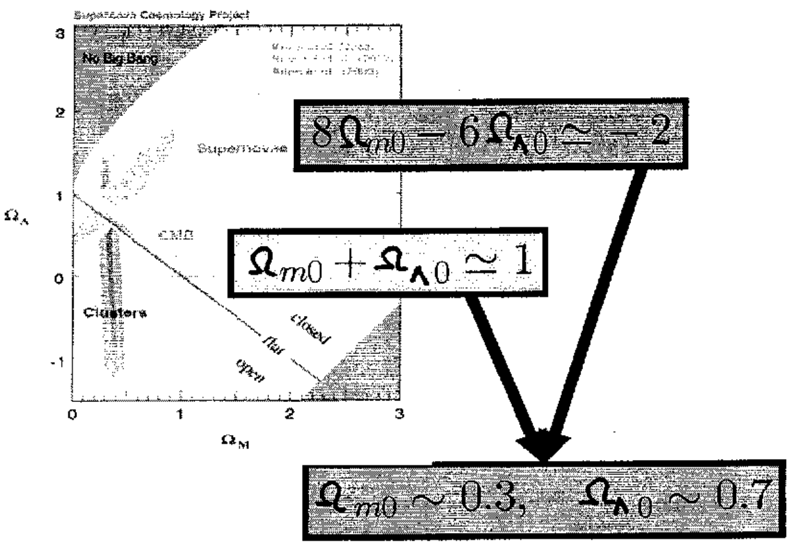
Hoekstra et al 2005

THE ΛCDM MODEL

The simplest extension is the introduction of a cosmological constant

- Einstein (1917)
- interpretation as vacuum quantum energy
- constant energy density
- well-defined and predictif model.

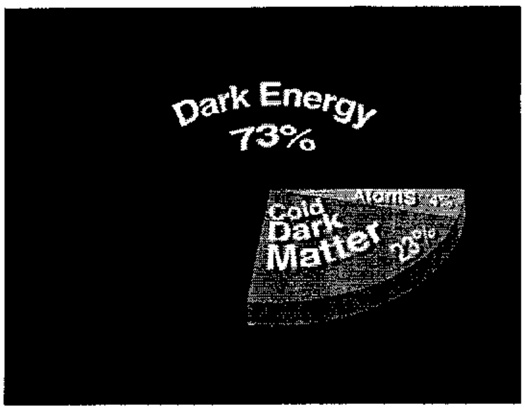
$$\rho_{\Lambda} = \frac{\Lambda}{8\pi G} = -P_{\Lambda}$$



$$8\Omega_{m0} - 6\Omega_{\Lambda 0} \simeq -2$$

$$\Omega_{m0} + \Omega_{\Lambda 0} \simeq 1$$

$$\Omega_{m0} \sim 0.3, \quad \Omega_{\Lambda 0} \sim 0.7$$



THE ΛCDM MODEL

Observationnally, OK with all data

Phénomenologically, very simple (1 parameter)

But

$$\rho_{\Lambda,obs} = \frac{\Lambda}{8\pi G} = H_0^2 M_p^2 = 10^{-47}\mathrm{GeV}^4$$

$$\rho_{\Lambda,th} = M^4_{\mathrm{fondamental}} > 10^{12}\mathrm{GeV}^4$$

Cosmological constant problem

$$\rho > 10^{59} \rho_{\Lambda,obs} \;!!$$

Today, no solution
Critical problem of fundamental physics

POSSIBILITIES

The observed acceleration implies that

$$(\rho + 3P) < 0$$

si general relativity and the Copernician principle hold on cosmological scales

One must change one of the 3 assumptions of the model

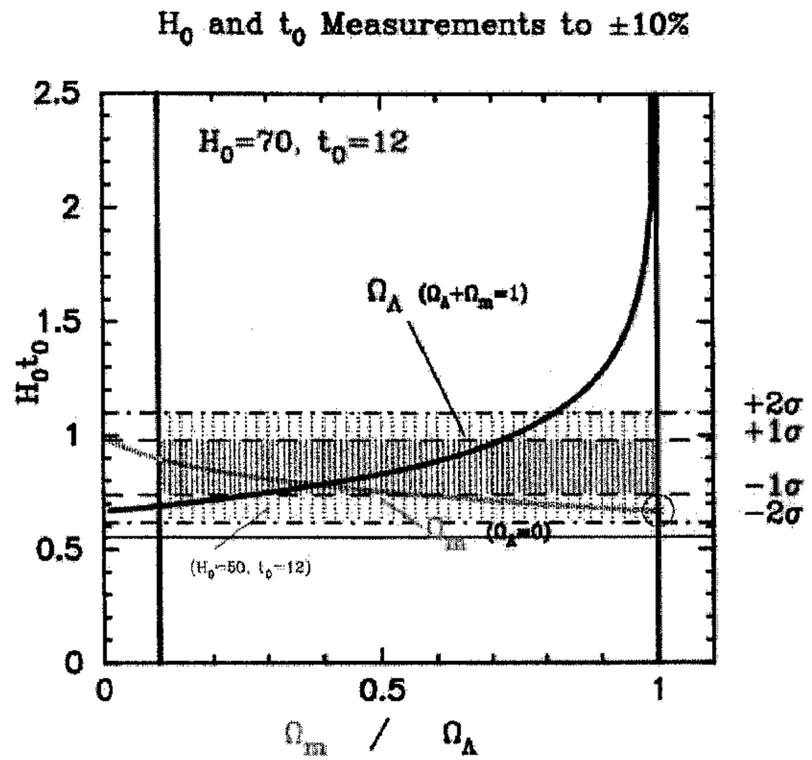
1- The Copernician principle is not valid

2- It exists matter such that $\rho+3P<0$

3- Gravitation is not described by general relativity on large scales

Nature of the dark energy

SUCCESSES: AGE



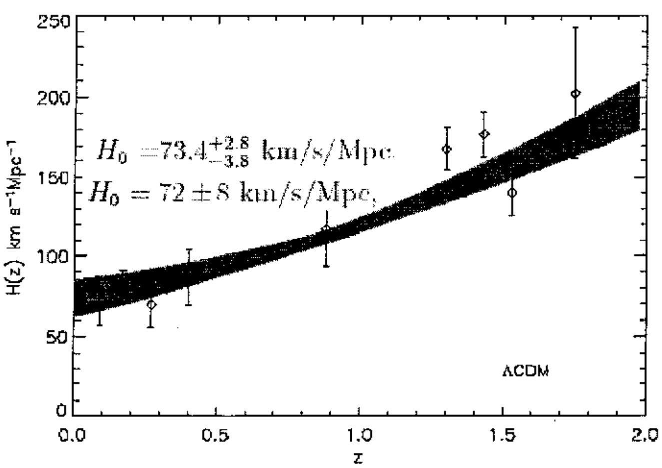
Âge (10⁹ ans)	Méthode	Référence
15.2 ± 3.7	nucléochronologie (étoile CS 22892)	Snéden *et al.* (1996)
12.5 ± 3	nucléochronologie (étoile CS 31082-0018)	Cayrel *et al.* (2001)
11.5 ± 1.3	5 méthodes (amas globulaire)	Chaboyer *et al.* (1998)
11.8 ± 1.2	séquence principale (amas globulaire)	Gratton *et al.* (1997)
14 ± 1.2	séquence principale (amas globulaire + binaires)	Pont *et al.* (1998)
13.7 ± 0.2	WMAP + grandes structures	Spergel *et al.* (2003)

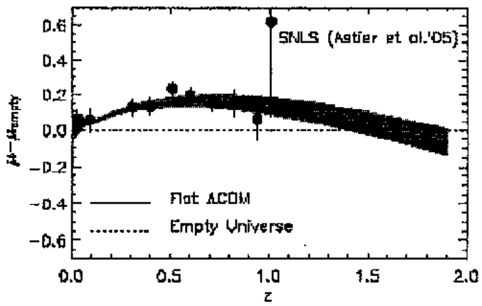
WMAP- DYNAMICS OF THE UNIVERSE

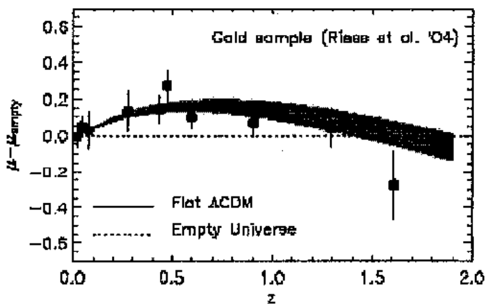
Analysis of a ΛCDM model with 6 parameters (h, Ω_m, Ω_b, τ, n_s, σ_8)

Spergel et al. , astro-ph/0603449

$$t_0 = 13.73^{+0.13}_{-0.17}\ \mathrm{Gyr}$$







SUCCESSES: THERMAL HISTORY

Consider a particle of mass M.

T>M It is in thermal equilibrium with its anti-particle

T<M Annihilation implies that it disappears $X + \bar{X} \rightarrow l + \bar{l}$

But, particles are diluted by expansion

Competition between dilution and annihilation → $H < \Gamma$ Interaction effective; $H > \Gamma$ Interaction frozen

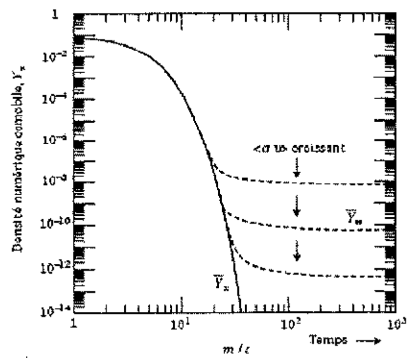
H ↗ ↖ $\Gamma \sim n\langle \sigma v \rangle$

In radiation era $H \propto T^2$

If $\Gamma \propto T^{n+3}$ (n=2 for weak interaction)

There ALWAYS a temperature below which an interaction is frozen !

The universe has a THERMAL HISTORY



$$L(f) = C[f]$$

$$L = \frac{d}{ds}$$

$$L[f] = p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha}$$

$$\alpha \quad f = f(E, t)$$

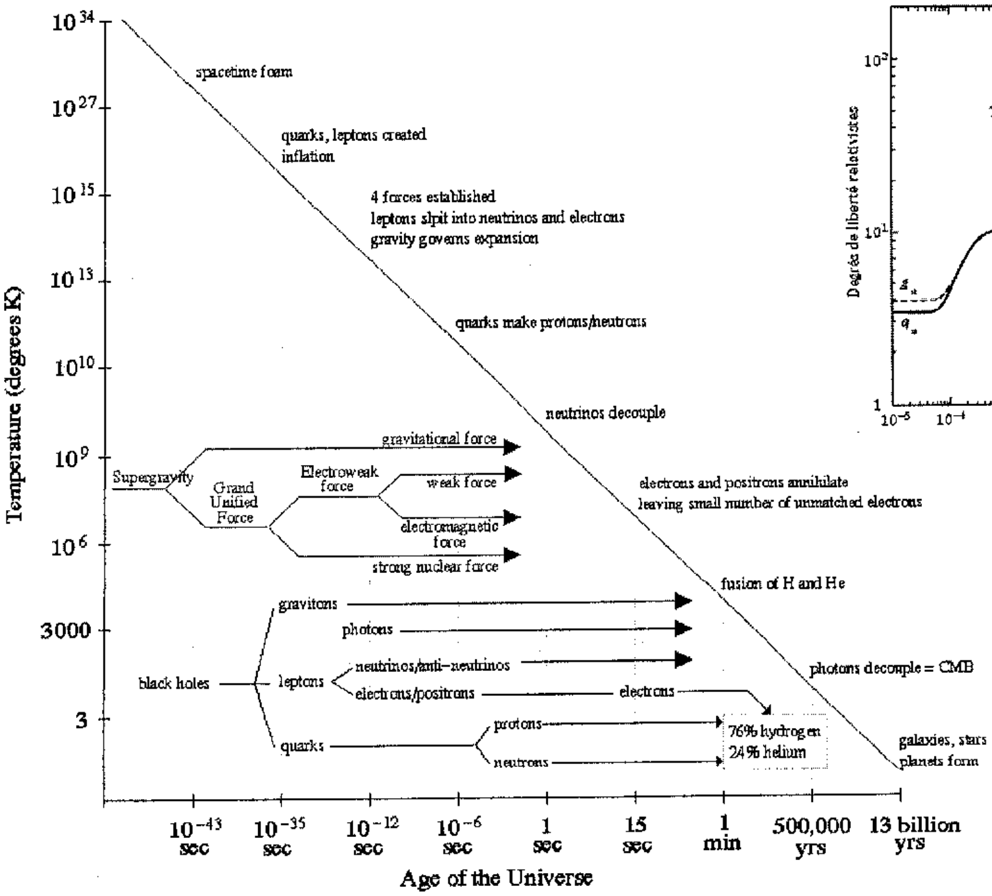
$$\Rightarrow \quad L[f] = E \frac{\partial f}{\partial t} - Hp^2 \frac{\partial f}{\partial E}$$

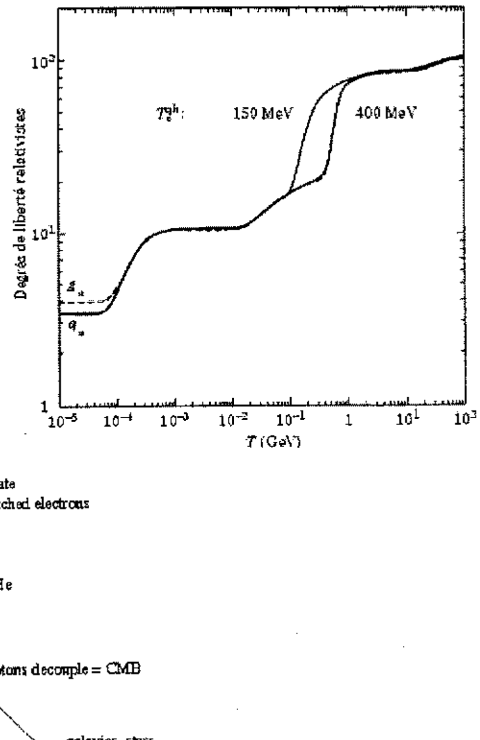
$$n = g_i \int f \frac{d^3 p}{(2\pi)^3}$$

$$\dot{n}_i + 3Hn_i = C_i$$

$$C_i = \frac{g_i}{(2\pi)^3} \int C[f_i] \frac{d^3 p_i}{2E_i}$$

SUCCESSES: THERMAL HISTORY





$$\Gamma_w = \frac{7\pi}{60}\left(1+3g_A^2\right) G_F^2\, T^5$$

SUCCESSES: BIG-BANG NUCLEOSYNTHESIS

T>> 100 Mev Electron, positron, neutrinos and photons: UR / proton, neutron: NR

T>> 1 Mev Neutron and proton in equilibrium by weak interaction

$$(n/p)_{eq} \sim \exp(-Q/T) \sim 1, \quad Q = m_n - m_p \sim 1.3\,\mathrm{MeV}$$

T=1-0.7 MeV Weak interaction freezes

$$\Gamma_{\mathrm{weak}} \sim H \qquad T_f \sim 0.8\,\mathrm{MeV}$$

$$(n/p)_f \sim \exp(-Q/T_f) \sim 1/5$$

Free neutrons decay in 887 sec

T=0.7-0.05 MeV Light nuclei are formed by a series of nuclear reactions

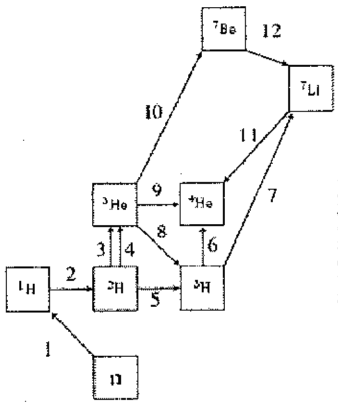
$$n + p \rightarrow \mathrm{D} + \gamma$$

D can be produced only when T<0.066MeV is low enough so that photo-dissociation negligible is negligible

$$(n/p)_N \simeq 1/7$$

Helium-4

$$Y = \left.\frac{2n}{n+p}\right|_N \simeq 0.25$$



1. $p \leftrightarrow n$
2. $p(n, \gamma)d$
3. $d(p, \gamma)^3$He
4. $d(d, n)^3$He
5. $d(d, p)t$
6. $d(d, n)^4$He
7. $t(\alpha, \gamma)^7$Li
8. ^{3}He$(n, p)t$
9. ^{3}He$(d, p)^4$He
10. ^{3}He$(\alpha, \gamma)^7$Be
11. ^{7}Li$(p, \alpha)^4$He
12. ^{7}Be$(n, p)^7$Li

$$\left.\frac{n}{p}\right)_N = \left.\frac{n}{p}\right)_f e^{-\frac{t_{nuc}}{\tau_n}}$$

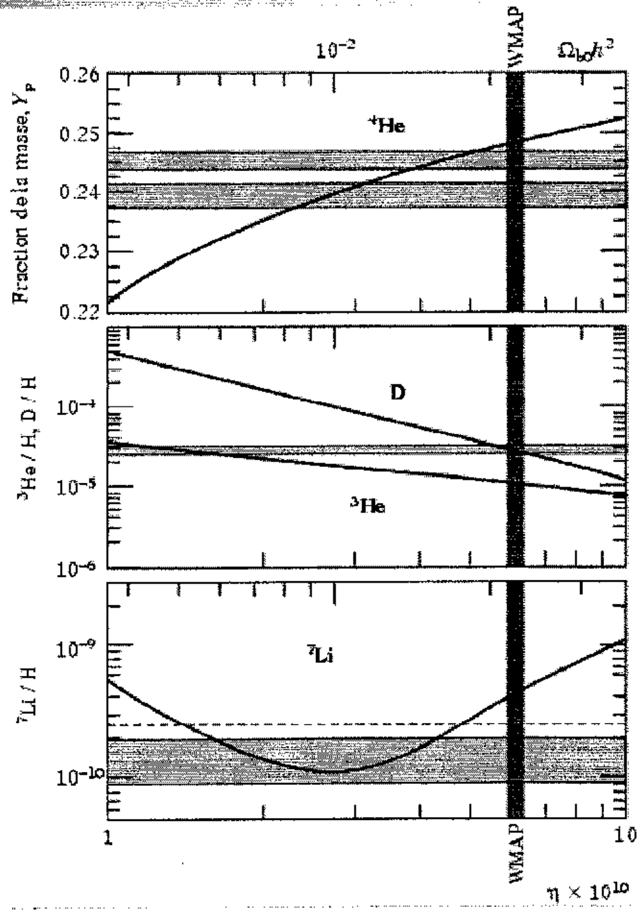
SUCCESSES: BIG-BANG NUCLEOSYNTHESIS

Parameters:

- -Number of relativistic particles
- - lifetime of neutron
- - $\eta = n_{baryon}/n_{\gamma}$
- - G, G_F, α,...

Allows to test extensions of the standard model of particle physics

$N_\nu = 3$



$$p + e \longleftrightarrow H + \gamma$$

$$n_b = n_p + n_H$$

$$n_e = n_p = X_e n_b$$

$$n_H = (1 - X_e) n_b$$

13.6 eV

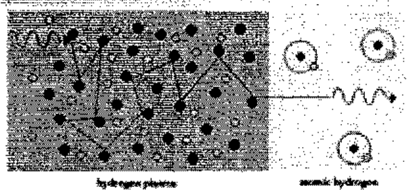
$$\frac{X_e^2}{1 - X_e} = \left(\frac{m_e T}{2\pi}\right)^{3/2} \frac{e^{-E_I/T}}{n_b}$$

$T \longrightarrow 2.725\,(1+z)$ K

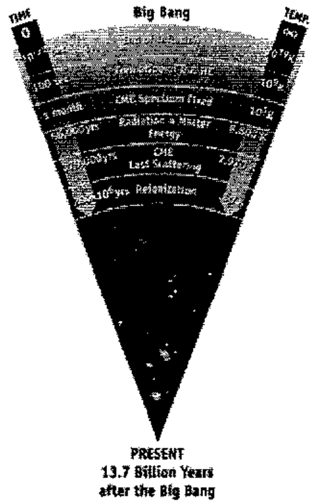
$n_b \longrightarrow n_b = \eta\, n_{\gamma_0} (1+z)^3$ cm^{-3}

SUCCESSES: COSMIC MICROWAVE BACKGROUND

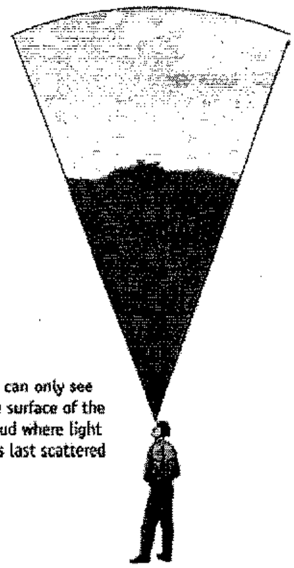
The universe cools during expansion



Around **T~4000 K**, protons and électrons Combine to form hydrogene. The universe becomes *transparent*



The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.



Gamow argument

Knowing the baryonic density today

$$n_{b0} \sim 10^{-7}\,\mathrm{cm}^{-3}$$

and at BBN

$$n_b \sim 10^{18}\,\mathrm{cm}^{-3}$$

he inferred the redshift at BBN

$$1 + z_{BBN} \sim (n_b/n_{b0})^{1/3} \sim 2 \times 10^8$$

and the temperature of the photon bath today

$$T_{\gamma 0} = \frac{T_{BBN}}{1 + z_{BBN}} \sim 5K$$

$$\log\left(\frac{X_e^2}{1 - X_e}\right) = 20.98 - \log\left[\Omega_{b0} h^2 (1+z)^{3/2}\right] - \frac{25163}{1+z}$$

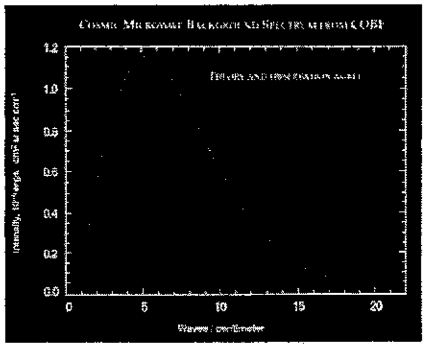
$T = E_I$ nh.s $\sim 10^{15}$ $\Rightarrow$ $X_e(T \sim E_I) \sim 1$

starts at $T \ll E_I$

X_e	z	
.5	13[illegible]	$z_{rec} \sim 1200 - 1400$

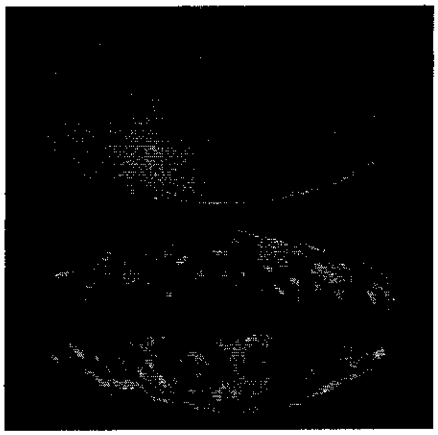
SUCCESSES: COSMIC MICROWAVE BACKGROUND

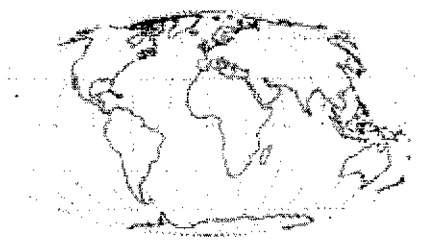
Emission d'un fond de photons avec un spectre de corps noir à une température de 2.725K aujourd'hui.



COBE observation

Dipole after
Monopole substraction

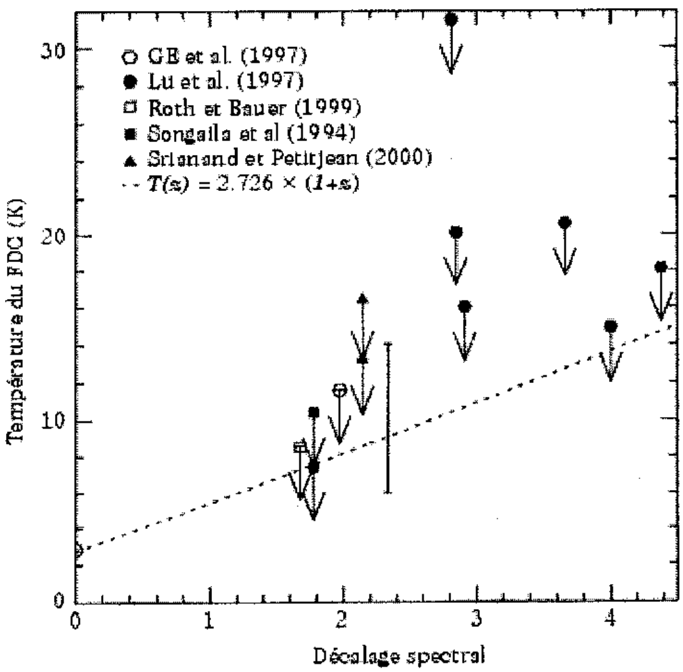




After dipole
Substraction:
Fluctuation of order µK

SUCCESSES: COSMIC MICROWAVE BACKGROUND

Temperature of CMB scales as 1/(1+z)



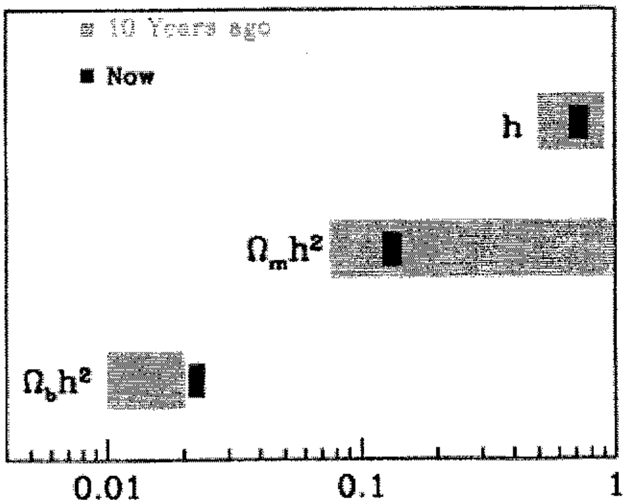
PARAMETERS OF THE MODEL

4 numbers to describe the dynamics of the universe

$$\Omega_m, \ \Omega_r, \ \Omega_K, \ \Omega_\Lambda \ , \ H_0$$

They start to be accurately measured
- we shall see how later
- ``precision cosmology''
- This is a very successful model

But...



The universe cools down from a hot thermal equilibrium state

<u>Successes:</u>
-expansion observed (Hubble law and redshift)
- light nuclei abundances (RG and weak interaction)
- CMB (RG and electromagnetism)

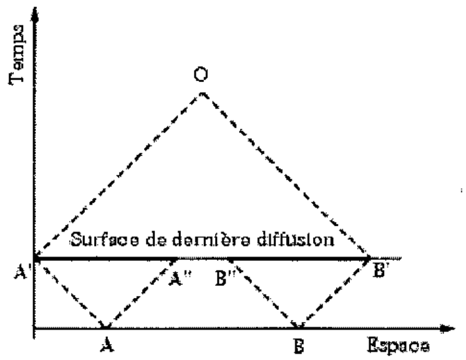
PROBLEMS AND QUESTIONS

Flatness

$|\Omega_{K0}| < 0.1 \quad |\Omega_K(z_{pl})| < 10^{-60}$

Horizon

CMB isotropic but corresponds to 10^{87} causal zones.
How do they reach thermal equilibrium?



Origin of structures

The universe is obviously not smooth. Where do the structure come from?

Dark sector $\Omega_r : \Omega_b : \Omega_m : \Omega_\Lambda \sim 10^{-3} : 1 : 5 : 14$

Good description up to approx. 10^{16} GeV

- Effect of the GUT unification scheme on the particle content
- Topological defects...
- Close to 10^{19} GeV, we expect to have effect of quantum gravity
- We have a window on energies not accessible in accelerator!

WMAP- CURVATURE OF SPACE

Non-flat ΛCDM *models are* compatible with WMAP