Cliff Will

Lecture 1 - Einstein Equivalence Principle

Conventions $c = G = 1$

$\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$

$,$ = partial $;$ covariant

$[\ ]$, $(\ )$

1. SRT

- Lorentz transformations
- line element, $\eta_{\mu\nu}$
- SR dynamics $\boxed{PPT}$

$$I = -\sum m_a \left(-\eta_{\mu\nu} \frac{dx^\mu}{dt}\frac{dx^\nu}{dt}\right) dt + \sum_a e_a \int A_\mu dx^\mu$$

$$-(16\pi)^{-1} \int \eta^{\alpha\mu} \eta^{\beta\nu} F_{\alpha\beta} F_{\mu\nu} \sqrt{-\eta}\, d^4x$$

$\Rightarrow$ Maxwell's equations, Lorentz Dirac eq. of motion

$$F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta}$$

- spacetime metric $\eta_{\mu\nu}$

2. Weak Equivalence Principle

Pisa 1589-92

Viviani 1622

- demo
- history $\boxed{PPT}$ $\boxed{PPT}$ - quiz
- Einstein's insight - acc = grav. or free fall = no gravity (1907)
- clue to add gravity: in free fall frame, SRT valid
- implies spacetime cannot be Minkowskian - why?

3. Dicke and EEP

Dicke viewpoint - broad, based on general principles

A. Physics takes place in spacetime - a N dim differentiable manifold

(doesn't need metric or affine connection a priori)

- laws of physics are expressed in covariant form

- coordinates arbitrary

B. Minimum viability criteria

- complete

- self consistent

- relativistic

- correct Newtonian limit

C. EEP - $\boxed{\text{PPT}}$

4. Tests of EEP

a. WEP $\quad \eta = 2\left(\frac{a_1 - a_2}{a_1 + a_2}\right)$

$F = m_I a, \quad F = m_p g$

$m_p = m_I + \eta^A E^A \quad \Rightarrow \quad \boxed{\eta = \sum_A \eta^A \left(\frac{E_1^A}{m_1} - \frac{E_2^A}{m_2}\right)}$

$\boxed{\text{PPT}}$

b. LLI

- Earth moves at 370 km/s

- Lorentz transform

$\boxed{\text{PPT}}$ $\boxed{\text{PPT}}$

c. LPI

Valid $\quad f_f = f_i(1+gh) \quad \boxed{PPT}$

No valid

Free fall standard comoving at top:

$$f_i = f_i^{\text{comoving}} = \frac{\alpha}{\tau(\phi_i)}$$

at bottom

$$f_f^{\text{comoving}} = \frac{\alpha}{\tau(\phi_f)} = f_i^{\text{coming}} \frac{\tau(\phi_i)}{\tau(\phi_f)}$$

$$f_f = f_f^{\text{comovim}}(1+gh)$$

$$= f_i^{\text{com}} \frac{\tau(\phi_i)}{\tau(\phi_f)}(1+gh)$$

Let $\tau(\phi_i) = \tau(\phi_f) + \tau' gh = \tau(\phi_f)(1+\alpha gh)$

Then

$$Z = \frac{f_f - f_i}{f_i} = (1+\alpha)gh$$

α may depend on kind of clock.

LPI $\Rightarrow$ $\alpha = 0$

Also compare different clocks

$$\frac{f_2}{f_1} = \frac{\tau_1(\phi)}{\tau_2(\phi)} = f(\phi) \sim \begin{cases} \phi \sim U & \boxed{PPT} \\ \phi \sim \text{cosmological} & \boxed{PPT} \end{cases}$$

5. Metric Theories of Gravity

- postulates – why a consq. of EEP?
- metric, connection Γ, $\nabla_{\vec{u}} \vec{V} = u^\gamma V^\alpha{}_{;\gamma} = u^\gamma (V^\alpha{}_{,\gamma} + \Gamma^\alpha{}_{\beta\gamma} V^\beta)$
- geodesic $\nabla_{\vec{u}} \vec{u} = 0$

$$\vec{u} = \frac{d\vec{x}}{d\lambda} \qquad \frac{d^2 x^\alpha}{d\lambda^2} + \Gamma^\alpha{}_{\beta\gamma} \frac{dx^\beta}{d\lambda}\frac{dx^\gamma}{d\lambda} = 0$$

λ unique upto linear transf$^{\underline{n}}$

- In nbhd of an event, can make

$$g_{\mu\nu} \to \eta_{\mu\nu}, \quad \Gamma^\alpha{}_{\beta\gamma} \to 0$$

ie $g_{\mu\nu} \to \eta_{\mu\nu} + O|\Delta x^\alpha|^2$

check numerology

$$g_{\mu\nu} = 10 \qquad x^{\alpha'} = x^{\alpha'}(x^\beta): \quad \frac{\partial x^{\alpha'}}{\partial x^\beta} = \Lambda^{\alpha'}{}_\beta = 16$$

$16 - 10 = 6$ degrees – what?

$$g_{\mu\nu,\alpha} = 40 \qquad \frac{\partial^2 x^{\alpha'}}{\partial x^\beta \partial x^\gamma} = 40 \qquad \text{OK}$$

$$g_{\mu\nu,\alpha\beta} = 100 \qquad \frac{\partial^3 x^{\alpha'}}{\partial x^\beta \partial x^\gamma \partial x^\delta} = 4 \times \left(4 + \frac{4!}{2!2!} + \frac{4!}{3!1!}\right) =$$

$$= 4 \times 20 = 80$$

$100 - 80 = 20$?

Universal coupling

$$\boxed{\begin{array}{l} g_{\mu\nu} \longrightarrow \eta_{\mu\nu} \\ \Gamma^\alpha{}_{\beta\gamma} \longrightarrow 0 \\ ; \longrightarrow , \end{array}}$$

In LIF no external fields
- <u>only</u> metric
- coupling universal
-

6 Non Metric Theories

- example $\boxed{PPT}$

a. THεμ framework

b. SME

$k_\phi^{\mu\nu}$ - real symmetric trace free - 9 components

$K_F^{\mu\nu\alpha\beta}$ - Riemann symmetries, double trace free $20 - 1 = 19$

c. Is string theory "non-metric"?

7. Physics in Curved Spacetime

Free body $\quad \frac{du^\alpha}{d\tau} = u^\alpha{}_{,\beta} u^\beta = 0 \qquad u^\alpha{}_{;\beta} u^\beta = 0$

EM $\quad F^{\alpha\beta}{}_{,\beta} = 4\pi J^\alpha \qquad F^{\alpha\beta}{}_{;\beta} = 4\pi J^\alpha$

$$F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta} \qquad F_{\alpha\beta} = A_{\beta;\alpha} - A_{\alpha;\beta}$$

EM action $\boxed{PPT}$

Energy momentum conservation from action

$$I = \int \mathcal{L}_{NG}\, d^4x \qquad \mathcal{L}_{NG} = \mathcal{L}_{NG}(g_{\mu\nu}, A_\mu, \mu^\mu, \ldots) = \mathcal{L}_{NG}(g_{\mu\nu}, q_A)$$

Variational deriv:

$$\frac{\delta\mathcal{L}}{\partial A} = -\frac{\partial\mathcal{L}}{\partial A} + \frac{\partial}{\partial x^\mu}\frac{\partial\mathcal{L}}{\partial A_{,\mu}} + \cdots$$

general covariance $\Rightarrow$

$$T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta\mathcal{L}_{NG}}{\delta g_{\mu\nu}}$$

Then

$$T_\mu{}^\nu{}_{;\nu} = -\frac{1}{\sqrt{-g}} \sum_A \left[q_{A,\mu} \frac{\delta\mathcal{L}_{NG}}{\delta q_A} + \left(d^\alpha_{A\mu} \frac{\delta\mathcal{L}_{NG}}{\delta q_A} \right)_{,\alpha} \right]$$

$= 0$ if $\delta\mathcal{L}_{NG}/\delta q_A = 0$ ie if EOM hold.

$$T^{\mu\nu} = \sum_a m_{0a} \frac{u^\mu u^\nu}{u^0\sqrt{-g}} \delta^3(\underset{\sim}{x} - \underset{\sim}{x}_a(t))$$

$$+ \frac{1}{4\pi}\left(F^{\mu\alpha}F_\alpha{}^\nu - \frac{1}{4} g^{\mu\nu} F_{\alpha\beta}F^{\alpha\beta}\right)$$

We will use perfect fluid often

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu + p g^{\mu\nu}$$

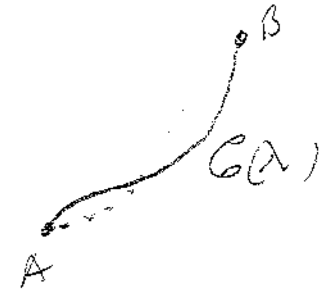
$$\rho, p, u^\mu = \ldots$$

geodesics — extremal proper time

$$S = \int_A^B ds$$

$$= \int_A^B \sqrt{g_{\mu\nu}\, dx^\mu dx^\nu}$$

$$= \int \sqrt{\left| g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} \right|}\, d\lambda$$



If $\delta S = 0$ for arbi. variations holding A + B fixed

Then path $G(\lambda)$ is a geodesic

$$\frac{d^2 x^\alpha}{d\lambda^2} + \Gamma^\alpha{}_{\beta\gamma} \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda} = 0$$

timelike — max
spacelike — min
null — saddle point