A matrix big bang

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Plan

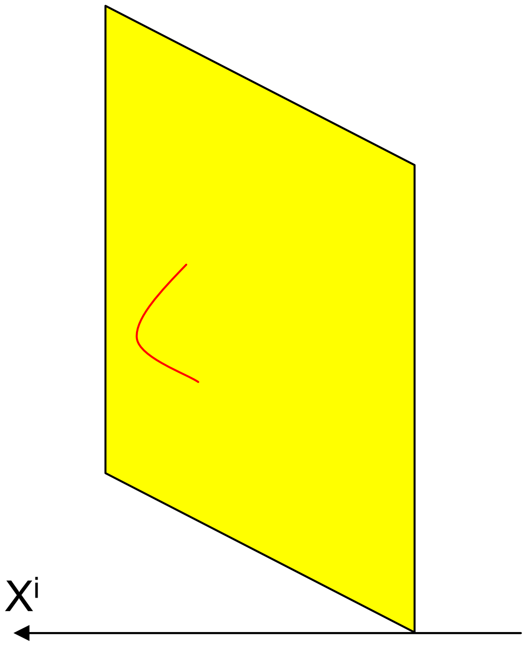
- Introduction: D-branes and matrix degrees of freedom
- Review of matrix (string) theory
- A matrix big bang
- Conclusions

D-branes: non-perturbative objects in string theory

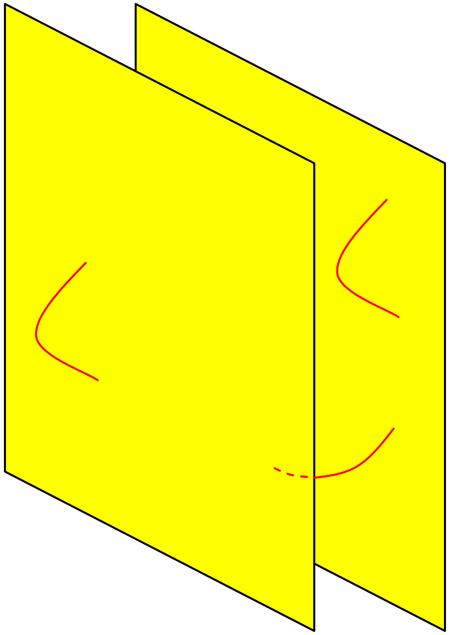
D-branes are extended objects on which open strings can end.

The oscillation modes of the open strings are the degrees of freedom of the brane. They include scalar fields X^i describing the location/profile of the brane in its transverse dimensions.

The tension of a D-brane is proportional to $1/g_s$, which is very large at weak coupling.



Multiple D-branes have matrix degrees of freedom



For two D-branes, one expects fields $(X^i)_{11}$ and $(X^i)_{22}$, describing the transverse positions/profiles of the two branes.

However, one finds more fields, corresponding to open strings stretching between the two branes: $(X^i)_{12}$ and $(X^i)_{21}$.

The fields combine in a 2 x 2 matrix $X^i = \begin{pmatrix} X^i_{11} & X^i_{12} \\ X^i_{21} & X^i_{22} \end{pmatrix}$

It turns out that there is a potential $V \sim \mathrm{Tr}[X^i, X^j]^2$

This implies that the off-diagonal modes (stretched strings) are very massive when the branes are well-separated. Then only the diagonal modes (brane positions/profiles) are light. When the branes are close to each other, all the matrix degrees of freedom are light!

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Type IIA string theory is M-theory on a circle

Type IIA string theory:

- perturbative string theory: asymptotic series in g_s
- what happens for large g_s?

Important tool: supersymmetry (BPS states,...)

$\tau_{D0} = \frac{1}{g_s\sqrt{\alpha'}} \rightarrow$ D0 becomes light at strong coupling

There exist BPS bound states of N D0-branes with masses $N\tau_{D0}$

This matches the spectrum of KK modes for a periodic dimension of radius $R_{11} = g_s\sqrt{\alpha'}$

Conjecture: 10d type IIA string theory is a circle compactification of an 11d theory, called M-theory

The low energy effective field theory for M-theory is 11d supergravity

Witten

Type IIA string theory is M-theory on a circle (continued)

$$
\begin{aligned}
ds^2_{11} &= G^{11}_{MN}(x^\mu)dx^M dx^N \\
&= \exp\left(-\frac{2}{3}\underset{\overset{\uparrow}{\text{dilaton}}}{\phi}(x^\mu)\right) \underset{\overset{\uparrow}{\text{10d metric}}}{G^{10}_{\mu\nu}} dx^\mu dx^\nu + \exp\left(\frac{4}{3}\phi(x^\mu)\right)\left[dx^{10} + \underset{\overset{\uparrow}{\text{RR 1-form potential}}}{C_\nu}(x^\mu)dx^\nu\right]^2
\end{aligned}
$$

Dimensional reduction keeps only modes with $p_{10} = 0$. The KK modes with nonzero p_{10} correspond to D0-branes and their bound states.

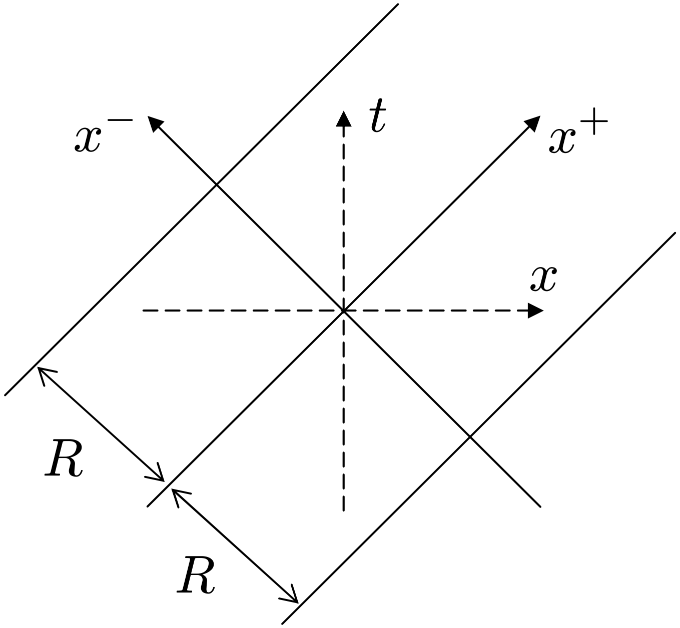
What is M-theory?

- We know what it is when compactified on a small circle
- We know its low energy limit
- What is its microscopic description?

Matrix theory is a non-perturbative description of M-theory in 11d asymptotically Minkowski background (and some compactifications)

Matrix theory from DLCQ

Discrete light-cone quantization (DLCQ)



$$\begin{pmatrix} x \\ t \end{pmatrix} \sim \begin{pmatrix} x - R/\sqrt{2} \\ t + R/\sqrt{2} \end{pmatrix}$$

i.e. $x^- \sim x^- + R$

Momentum quantization: $p^+ = \dfrac{2\pi N}{R}$

Focus on sector with fixed total p^+, i.e. fixed N

Define DLCQ as limit of spacelike compactification:

$$\begin{pmatrix} x \\ t \end{pmatrix} \sim \begin{pmatrix} x - \sqrt{\frac{R^2}{2} + R_s^2} \\ t + R/\sqrt{2} \end{pmatrix}$$

with $\underset{\uparrow}{R_s} \to 0$

size of spacelike circle

Banks, Fischler, Shenker, Susskind; Susskind; Seiberg

Matrix theory from DLCQ (continued)

$$\begin{pmatrix} x \\ t \end{pmatrix} \sim \begin{pmatrix} x - \sqrt{\frac{R^2}{2} + R_s^2} \\ t + R/\sqrt{2} \end{pmatrix}$$

with $R_s \to 0$

size of spacelike circle

Lorentz boost:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh\beta & \sinh\beta \\ \sinh\beta & \cosh\beta \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix}$$

with $\cosh\beta = \sqrt{1 + \frac{R^2}{2R_s^2}}$

Then $\begin{pmatrix} x' \\ t' \end{pmatrix} \sim \begin{pmatrix} x' - R_s \\ t' \end{pmatrix}$

M-theory on lightlike circle ⟺ M-theory on spacelike circle with radius $R_s \to 0$

Banks, Fischler, Shenker, Susskind; Susskind; Seiberg

Matrix theory from DLCQ (continued)

M-theory on lightlike circle ⟺ M-theory on spacelike circle with radius $R_s \to 0$

But this is Type IIA string theory with $g_s = (R_s M_p)^{3/2}, \quad \frac{1}{\sqrt{\alpha'}} \equiv M_s = \sqrt{R_s M_p^3}$

11d Planck mass

In the $R_s \to 0$ limit, we get weakly coupled IIA strings ($g_s \to 0$), but the string length becomes large ($\alpha' \to \infty$), which would seem problematic.

However, in M-theory on a lightlike circle, we are interested in states with $p^+ = N/R$ and lightcone energy P^-

After the boost:

$$\begin{cases} \text{momentum} \quad P' = \dfrac{N}{R_s} \quad \to \quad N \;\mathrm{D0-branes} \\ \text{energy} \quad E' = \dfrac{N}{R_s} + \Delta E' \end{cases}$$

with $$P^- = \frac{1}{\sqrt{2}}(E-P) = \frac{1}{\sqrt{2}} e^{\beta}(E'-P') \approx \frac{R}{R_s}\Delta E'$$

So $$\Delta E' \approx \frac{R_s P^-}{R} \quad \Longrightarrow \quad \frac{\Delta E'}{M_s} \approx \frac{\sqrt{R_s} P^-}{R M_p^{3/2}} \to 0$$

Matrix theory from DLCQ (continued)

M-theory on lightlike circle with radius R in sector with $p^+ = N/R$

$$\Updownarrow$$

Type IIA string theory in the presence of N D0-branes with

$$g_s \to 0,\ \sqrt{\alpha'}\Delta E' \to 0$$

In this limit, the only non-decoupled degrees of freedom are the NxN matrices X^i of the D0-brane worldvolume theory, which reduces to the dimensional reduction of 10d super-Yang-Mills theory: “matrix theory”.

Eventually, one wants to decompactify the lightlike circle:

$$R \to \infty,\ N \to \infty \quad \text{with} \quad p^+ = N/R \quad \text{fixed}$$

Taking the matrix theory Lagrangian as a starting point, spacetime emerges from the moduli space of vacua, corresponding to flat directions for diagonal matrix elements. The large N model contains the Fock space of 11d supergravitons in its spectrum. Supersymmetry is essential to protect the flat directions.

Banks, Fischler, Shenker, Susskind; Susskind; Seiberg

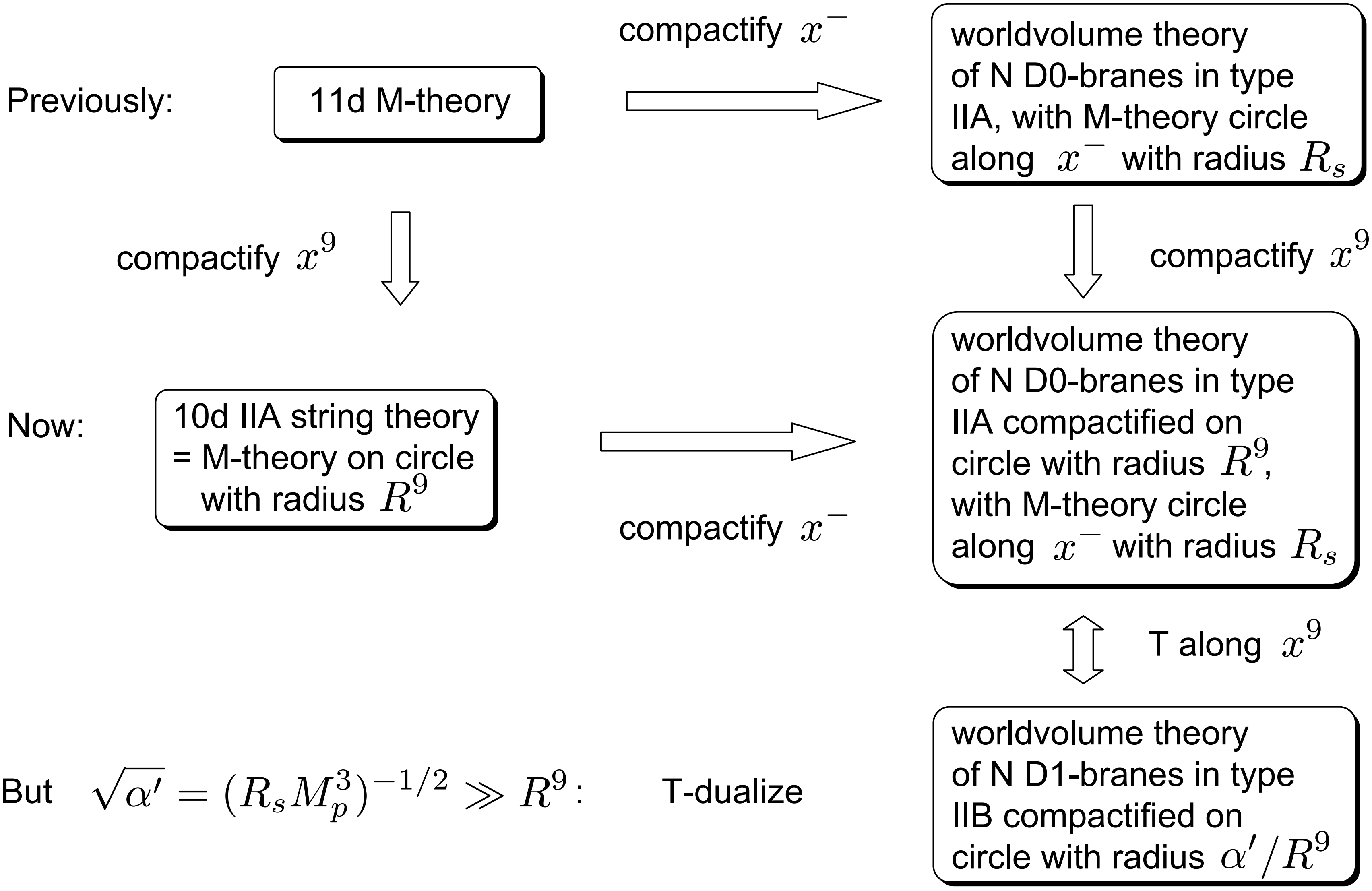
Conclusion: M-theory as matrix quantum mechanics

The DLCQ of M-theory in a sector with N units of lightcone momentum is given by the low-energy limit of the worldvolume theory of N D0-branes.

This is the dimensional reduction of 9+1 dimensional SYM theory to 0+1 dimensions: matrix quantummechanics.

To get uncompactified M-theory, one has to take a large N limit.

Matrix description of type IIA strings: matrix string theory



Previously:

11d M-theory

compactify x^-

worldvolume theory of N D0-branes in type IIA, with M-theory circle along x^- with radius R_s

compactify x^9

compactify x^9

Now:

10d IIA string theory = M-theory on circle with radius R^9

compactify x^-

worldvolume theory of N D0-branes in type IIA compactified on circle with radius R^9, with M-theory circle along x^- with radius R_s

T along x^9

But $\sqrt{\alpha'} = (R_s M_p^3)^{-1/2} \gg R^9$: T-dualize

worldvolume theory of N D1-branes in type IIB compactified on circle with radius α'/R^9

Motl; Banks, Seiberg; Dijkgraaf, Verlinde, Verlinde

Matrix description of type IIA strings: matrix string theory (continued)

10d IIA string theory
with N units of
lightcone momentum

$\Rightarrow$

worldvolume theory of
N D1-branes in type IIB

In the previous derivation, the original IIA string theory (with N units of lightcone momentum) was related to an auxiliary IIA string theory (with N units of D0-brane charge) by a 9-11 flip (i.e. viewing two different circles as the M-theory circle).

The 9-11 flip is equivalent to a sequence of T-duality, S-duality and T-duality:

$$\text{momentum} \xrightarrow[T]{} \text{F1 winding} \xrightarrow[S]{} \text{D1 winding} \xrightarrow[T]{} \text{D0 charge}$$

Thus the original IIA theory is related to the auxiliary IIB theory above by a sequence of T-duality and S-duality.

Motl; Banks, Seiberg; Dijkgraaf, Verlinde, Verlinde

Matrix string theory: non-perturbative string theory in Minkowski space

Matrix string theory is a non-perturbative formulation of type IIA superstring theory in (9+1)-dimensional Minkowski space. It is described by the low-energy effective action of N D1-branes in type IIB string theory, which is $\mathcal{N} = 8$ super-Yang-Mills theory in 1+1 dimensions with gauge group U(N), in a large N limit:

$$S = \int d\tau d\sigma \, \mathrm{Tr} \left((D_\mu X^i)^2 + \theta^T \not{D} \, \theta + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2}[X^i, X^j]^2 + \frac{1}{g_s}\theta^T \gamma_i [X^i, \theta] \right)$$

$X^i, \theta^\alpha, \theta^{\dot\alpha}$ are N x N hermitean matrices, transforming in the $\mathbf{8}_v, \mathbf{8}_s, \mathbf{8}_c$ representations of the SO(8) R-symmetry group of transverse rotations.

The worldsheet is an infinite cylinder with coordinates (τ, σ), where $\sigma \sim \sigma + 2\pi.$

These are the same fields as in the light-cone Green-Schwarz formulation of the superstring, except that now they are matrix-valued.

Relation: eigenvalues of the matrices X^i correspond to coordinates of (pieces of) superstring, and similarly for the fermions.

Banks, Fischler, Shenker, Susskind; Motl; Banks, Seiberg; Dijkgraaf, Verlinde, Verlinde

Matrix strings reduce to perturbative strings at weak string coupling

$$S = \int d\tau d\sigma \, \mathrm{Tr} \left((D_\mu X^i)^2 + \theta^T \not{D} \, \theta + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2} [X^i, X^j]^2 + \frac{1}{g_s} \theta^T \gamma_i [X^i, \theta] \right)$$

The YM coupling constant is $g_{YM} = 1/g_s \ell_s,$ with $\ell_s \equiv \sqrt{\alpha'}$ the string length, which we usually set equal to 1.

To compare with perturbative string theory, one takes $g_s \to 0.$ (This is the infrared limit of the Yang-Mills theory.) Then the potential is very strong: the matrices are forced to commute and can be simultaneously diagonalized.

The off-diagonal matrix elements X^i_{ab} have very large masses and can be integrated out:

$$m \sim \frac{||X_{aa} - X_{bb}||}{g_s}$$

Supersymmetry ensures that no potential is generated for the diagonal elements!
Spacetime then arises dynamically from the moduli space of vacua.

Perturbative string interactions have been reproduced from the small g_s limit of matrix string theory.

Dijkgraaf, Verlinde, Verlinde

Conclusion: IIA string theory as 1+1 super-Yang-Mills theory

The DLCQ of type IIA string theory in a sector with N units of lightcone momentum is given by the low-energy limit of the worldvolume theory of N D1-branes.

This is the dimensional reduction of 9+1 dimensional SYM theory to 1+1 dimensions.

To get uncompactified type IIA string theory, one has to take a large N limit.

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The model: light-like linear dilaton

An extremely simple time-dependent solution of ten-dimensional (type IIA) string theory is flat space with a light-like linear dilaton (it preserves 16 supersymmetries):

$$ds_{10}^2 = -2dX^+dX^- + (dX^i)^2$$

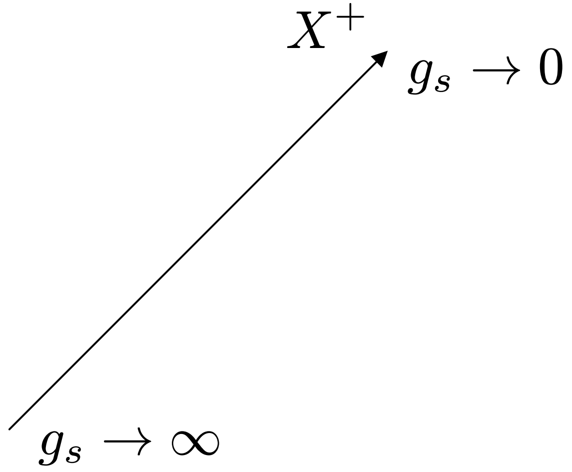
$$\Phi = -QX^+$$

The dilaton Φ is a scalar field that appears in the low-energy effective action as

$$S \sim \int d^{10}x \sqrt{G}\, e^{-2\Phi}\, (R + 4\partial_\mu \Phi \partial^\mu \Phi + \ldots)$$

Therefore, the exponential of the dilaton can be viewed as the string coupling "constant":

$$g_s = e^\Phi$$



BC, Sethi, Verlinde

The light-like linear dilaton model in Einstein frame: a light-like big bang

In the Einstein conformal frame, where the dilaton factor in front of the Ricci scalar is absent, the metric is non-trivial and exhibits a big bang singularity for $X^+ \to -\infty$:

$$ds_E^2 = e^{QX^+/2}\left[-2dX^+dX^- + (dX^i)^2\right]$$

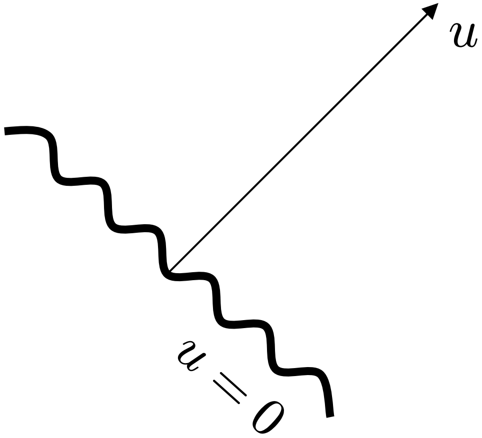
$$\Phi = -QX^+$$

New coordinate $u = e^{QX^+/2}$:

$$ds_E^2 = -\frac{4}{Q}du\,dX^- + u(dX^i)^2$$

Riemann tensor:

$$R_{iuiu} = \frac{1}{4u}$$



BC, Sethi, Verlinde

DLCQ leads to matrix description of the light-like linear dilaton

$$ds_{10}^2 = -2dX^+dX^- + (dX^i)^2$$

$$\Phi = -QX^+$$

Discrete Light-Cone Quantization: $\begin{cases} X^- \sim X^- + R \\ \text{consider sector with } p^+ = \dfrac{N}{R} \end{cases}$

Eventually: $N \to \infty,\, R \to \infty,\, p^+$ fixed

Define DLCQ as $\epsilon \to 0$ limit of $\begin{pmatrix} X^+ \\ X^- \\ X^1 \end{pmatrix} \sim \begin{pmatrix} X^+ \\ X^- \\ X^1 \end{pmatrix} + \begin{pmatrix} 0 \\ R \\ \epsilon R \end{pmatrix}$

BC, Sethi, Verlinde

Matrix description of the light-like linear dilaton (continued)

$$\begin{pmatrix} X^+ \\ X^- \\ X^1 \end{pmatrix} \sim \begin{pmatrix} X^+ \\ X^- \\ X^1 \end{pmatrix} + \begin{pmatrix} 0 \\ R \\ \epsilon R \end{pmatrix}$$

Lorentz transformation:

$$\begin{cases} X^+ = \epsilon x^+ \\ X^- = \dfrac{x^+}{2\epsilon} + \dfrac{x^-}{\epsilon} + \dfrac{x^1}{\epsilon} \\ X^1 = x^+ + x^1 \end{cases}$$

Then

$$\begin{cases} ds^2_{10} = -2dx^+dx^- + (dx^i)^2 \\ \Phi = -\epsilon Q x^+ \end{cases}$$

with

$$\begin{cases} x^1 \sim x^1 + \epsilon R \\ p^1 = \dfrac{N}{\epsilon R} \end{cases}$$

BC, Sethi, Verlinde

Matrix description of the light-like linear dilaton (continued)

$$\begin{cases} ds^2 = -2dx^+dx^- + (dx^i)^2 \\ \Phi = -\epsilon Q x^+ \end{cases} \quad \text{with} \quad \begin{cases} x^1 \sim x^1 + \epsilon R \\ p^1 = \dfrac{N}{\epsilon R} \end{cases}$$

T-duality along x^1 and S-duality leads to type IIB with N D1-branes wrapped around x^1 in the background

$$\begin{cases} ds^2 = \epsilon R \, e^{\epsilon Q x^+} \left[-2dx^+dx^- + (dx^i)^2\right] \\ \Phi = \epsilon Q x^+ + \log \epsilon R \end{cases}$$

with

$$x^1 \sim x^1 + \frac{1}{\epsilon R}$$

BC, Sethi, Verlinde

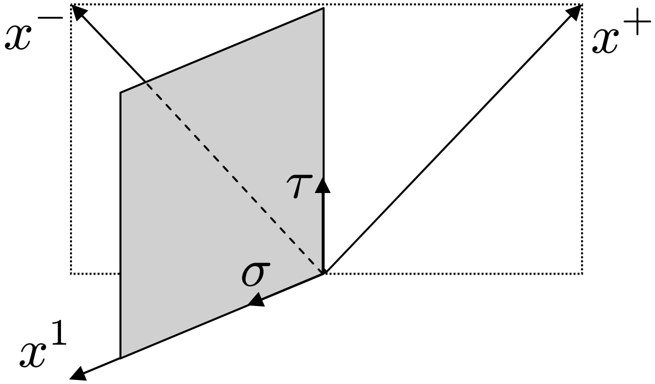
Matrix description of the light-like linear dilaton (continued)

$$\begin{cases} ds^2 = \epsilon R\, e^{\epsilon Q x^+} \left[-2dx^+dx^- + (dx^i)^2\right] \\ \Phi = \epsilon Q x^+ + \log \epsilon R \end{cases}$$

with $x^1 \sim x^1 + \frac{1}{\epsilon R}$

$$S_{D1} = -\int d\tau d\sigma\, e^{-\Phi}\sqrt{-\det(\partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu} + F_{\alpha\beta})}$$ + non-abelian

Ground state:



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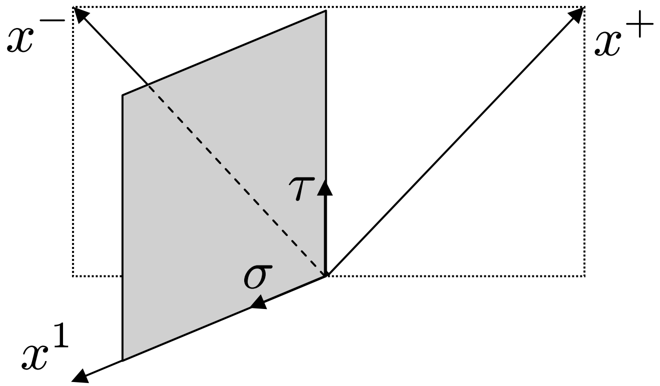
Matrix description of the light-like linear dilaton (continued)

Gauge choice:
$$\begin{cases} x^1 = \dfrac{\sigma}{\epsilon R} \\ x^+ = \dfrac{\tau}{\epsilon R} \end{cases}$$



with $\sigma \sim \sigma + 2\pi$

New coordinates:
$$\begin{cases} x^- = \dfrac{\tau}{\epsilon R} + y^1 \\ x^i = y^i \quad (i = 2, \ldots 8) \end{cases}$$

Then
$$S_{D1} = \int d\tau d\sigma \, \mathrm{Tr} \left\{ (\partial_\alpha y^i)^2 + \frac{1}{g_{YM}^2} F_{\alpha\beta}^2 - g_{YM}^2 [y^i, y^j]^2 \right\}$$

+ fermions + tension + higher derivative

with
$$g_{YM} = \frac{1}{\ell_s} \exp\left(\frac{Q \tau \ell_s}{R} \right) = \frac{1}{\ell_s g_s^{IIA}}$$

BC, Sethi, Verlinde

Matrix description of the light-like linear dilaton: result

$$S = \int d\tau d\sigma \, \mathrm{Tr} \left((D_\mu X^i)^2 + \theta^T \not{D} \, \theta + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2}[X^i, X^j]^2 + \frac{1}{g_s}\theta^T \gamma_i [X^i, \theta] \right)$$

Result: simply plug in $g_s = e^{-QX^+} = e^{-\frac{Q\tau}{R}}$, leading to (1+1)-dimensional SYM

on the cylinder, with coupling $g_{YM} = \frac{1}{\ell_s} \exp\left(\frac{Q\ell_s \tau}{R}\right)$

This is equivalent to (1+1)-dimensional SYM with constant coupling on the 2d Milne space:

$ds^2 = e^{2\tilde{Q}\tau}(-d\tau^2 + d\sigma^2)$ with $\tilde{Q} \equiv Q\ell_s/R$

BC, Sethi, Verlinde

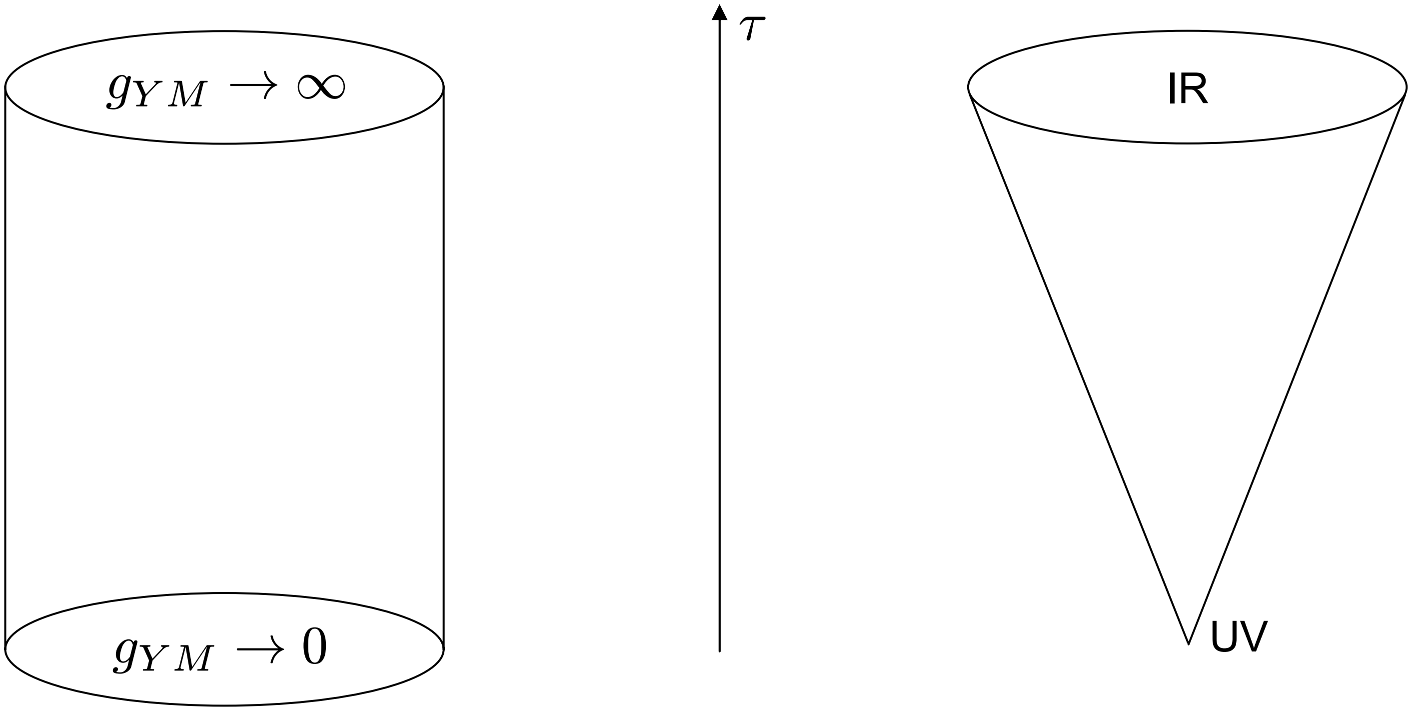
Cosmological evolution as RG flow

SYM on the cylinder with

$$g_{YM} = e^{\tilde{Q}\tau}$$

SYM with constant coupling on

$$ds^2 = e^{2\tilde{Q}\tau}(-d\tau^2 + d\sigma^2)$$



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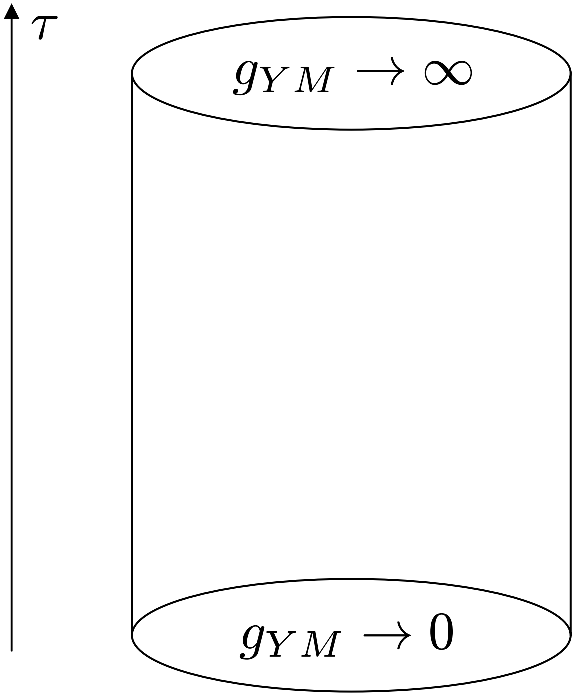
Cosmological evolution: emergence of spacetime



[X^i, X^j] = 0: spacetime emerges
(weakly coupled strings)

free SYM: non-commuting matrices
(new light degrees of freedom)

BC, Sethi, Verlinde

Regime of validity of the matrix string description

Type IIA with $ds^2 = -2dX^+dX^- + (dX^i)^2$ and $\Phi = -QX^+$

with lightcone energy E^- w.r.t. X^+

$$\Updownarrow$$

Type IIB with $ds^2 = \epsilon R\, e^{\epsilon Q x^+} \left[-2dx^+dx^- + (dx^i)^2\right]$ and $\Phi = \epsilon Q x^+ + \log \epsilon R$

with lightcone energy ϵE^- w.r.t. x^+

Thus $\ell_s^{\text{eff}} = \dfrac{\ell_s^{3/2}}{\sqrt{\epsilon R}} \exp\left(-\dfrac{\epsilon Q x^+}{2}\right)$ and $G_N^{\text{eff}} = \dfrac{\ell_s^{10}}{\epsilon^2 R^2} \exp(-2\epsilon Q x^+)$

In the limit $\epsilon \to 0$: $\begin{cases} \epsilon E^- \ell_s^{\text{eff}} \to 0 \\ (\epsilon E^-)^8 G_N^{\text{eff}} \to 0 \end{cases}$

Therefore, closed and massive open strings decouple and the matrix string description is valid all the way to the singularity!

BC, Sethi, Verlinde

Is the SYM theory weakly coupled?

Lightcone energy ϵE^- w.r.t. x^+

$$\Updownarrow$$

Worldsheet energy $\dfrac{E^-\ell_s}{R}$ w.r.t. τ

Dimensionless coupling:

$$g_{YM}\left(\frac{E^-\ell_s}{R}\right)^{-1} \sim \frac{R}{E^-\ell_s^2}\,\exp\left(\frac{Q\tau\ell_s}{R}\right) \sim \frac{N}{p^+E^-\ell_s^2}\,\exp\left(\frac{Q\tau\ell_s p^+}{N}\right)$$

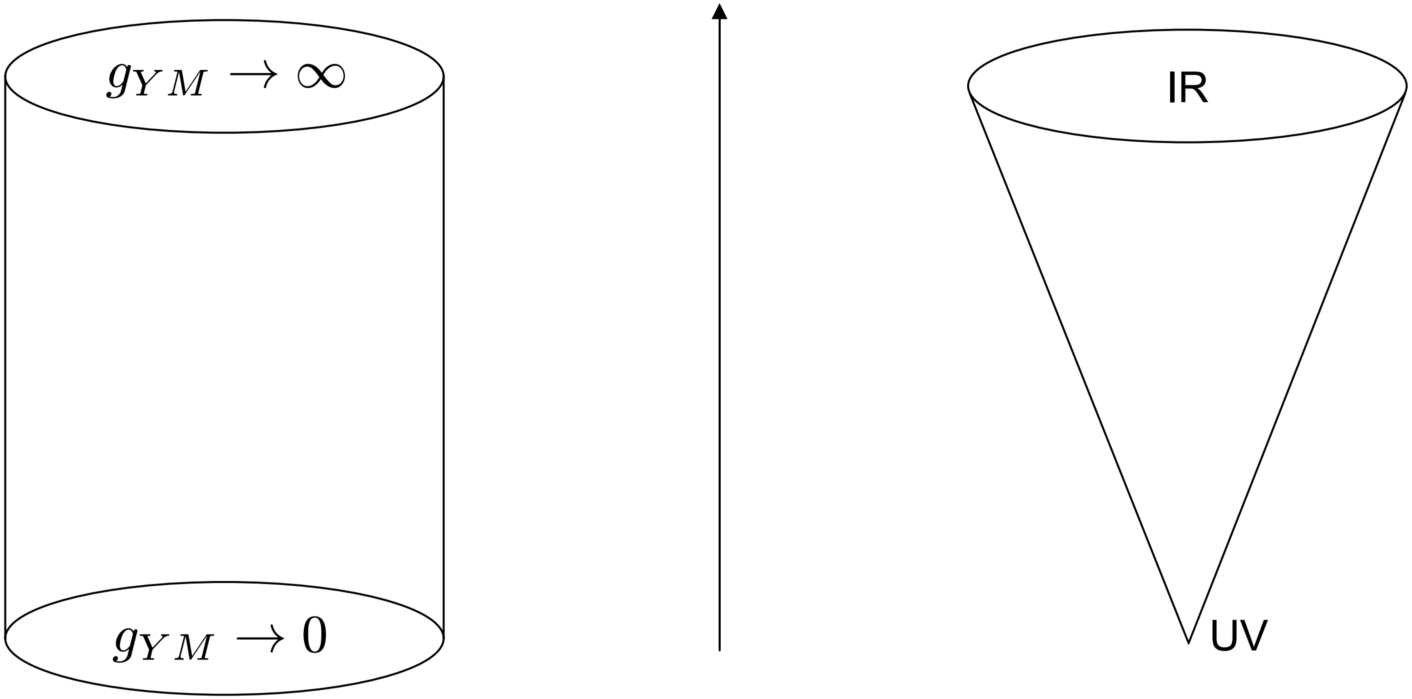
Thus for any finite N, the theory is weakly coupled at early times and strongly coupled at late times.

If N is strictly infinite, the theory is always strongly coupled.

The question of how to take the large N limit is important. For now, let us assume it is OK to work with finite N.

BC, Sethi, Verlinde

Can the matrix description be continued through the Milne singularity?



The Milne description suggests that the tip of the Milne cone is a finite time away: excitations follow straight lines on the Minkowski covering space. What happens at the Milne singularity? Can time be continued further?

In the cylinder description, the time coordinate extends to minus infinity. However, interactions turn off at early times. This presumably means that “clocks” will run slower and tick only a finite number of times since the beginning of time.

Quantum corrections: effective potential for diagonal matrix elements?

$$S = \int d\tau d\sigma \, \mathrm{Tr} \left((D_\mu X^i)^2 + \theta^T \not{D} \, \theta + g_s^2 F_{\mu\nu}^2 - \frac{1}{g_s^2} [X^i, X^j]^2 + \frac{1}{g_s} \theta^T \gamma_i [X^i, \theta] \right)$$

A vacuum configuration of the classical theory corresponds to constant matrices X^i that mutually commute. By a gauge transformation, we can make the matrices diagonal.

The quartic potential then gives masses to the off-diagonal matrix elements:

X^i_{ab} has mass squared $\quad m^2 = \frac{1}{g_s^2} ||X_{aa} - X_{bb}||^2$

The diagonal matrix elements correspond to flat directions, at least classically. The flat directions are identified with spacetime positions.

What happens quantum-mechanically, when we integrate out the (massive) off-diagonal modes? Does that generate an effective potential for the (massless) diagonal modes?

Importantly, in cases with enough supersymmetry, this doesn't happen because the contribution from integrating out the bosons is cancelled by the contribution from integrating out the fermions.

Is this true for the matrix big bang model as well?

BC, Rajaraman, Sethi

Supersymmetry breaking: do the flat directions survive?

The light-like linear dilaton background preserves 16 supersymmetries. However, for any N>0 (i.e. for any $p^+>0$) these are all spontaneously broken.

Interpretation: matrix theory does not describe the (susy) vacuum, but a (non-susy) sector with N>0 units of lightcone momentum. Or from another point of view, it describes the system in a highly boosted frame.

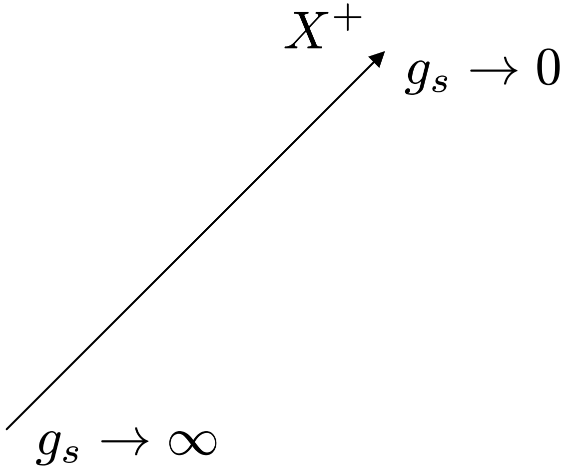
The worldsheet theory has no unbroken supersymmetry and one expects a potential to be generated quantum mechanically.

It turns out (for separation b between two eigenvalues and for late times):

$$\int \sqrt{g}\, V_{1-\mathrm{loop}}(b) \sim \int \left(\frac{b}{g_s}\right)^{1/2} \exp\left(-\frac{Cb}{g_s}\right)$$

with C a constant.

This suggests that the potential turns off at late times, where a spacetime description was indeed expected!



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Summary: the one-loop effective potential

- Late times: potential turns off fast

Spacetime description emerges

- Early times: attractive potential

Spacetime replaced by non-abelian gluon phase

BC, Rajaraman, Sethi

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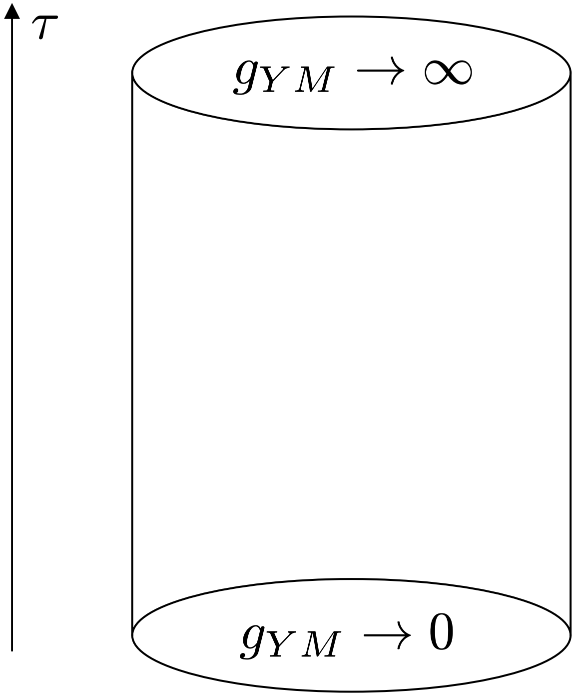
Conclusions

The matrix big bang is a proposal for a non-perturbative description of a light-like singularity. Near the big bang, the model describes weakly coupled matrices. At late times, spacetime emerges dynamically.

A potential problem for the emergence of spacetime, related to the absence of unbroken supersymmetry, appears to be harmless in this model.



[X^i, X^j] = 0: spacetime emerges
(weakly coupled strings)

free SYM: non-commuting matrices
(new light degrees of freedom)