Null Singularities and their Gauge Theory duals

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Open-closed duality

- Much of the success of string theory in understanding puzzling gravitational phenomena can be traced to open-closed duality – particularly in situations in which this leads to a holographic correspondence.
- The open string description of phenomena does not involve a dynamical space-time, and the quantum mechanics of open strings is conventional.
- Dynamical space-time and gravity are **emergent concepts** which are useful only in a certain regime.
- In this talk we will explore recent work which indicates that AdS/CFT duality may be useful in understanding null singularities.

AdS/CFT

- String theory on $AdS_5 \times S^5$ dual to N=4 gauge theory on the boundary of AdS_5

$$R^4 = 4\pi(g_{YM}^2 N) l_s^4 \qquad\qquad g_{YM}^2 = g_s$$

- The gauge theory provides a fundamental definition of the theory – this is the open string description.
- The string theory description is useful only in the ‘t Hooft limit $g_{YM} \to 0, \quad N \to \infty, \quad {g_{YM}}^2 N = fixed$
- Supergravity is valid in the regime of strong ‘t Hooft coupling – this is the limit in which conventional ten dimensional space-time emerges out of the original 3+1 dimensional space-time of the gauge theory.

- The standard solution is

$$ds^2 = \frac{r^2}{R^2}[\eta_{\mu\nu}dx^\mu dx^\nu] + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

$$F_{(5)} = R^4(\omega_5 + *_{10}\omega_5) \quad \mu,\nu = 0,1,\cdots 3$$

- The gauge theory is then on a flat 3+1 dimensional space.
- We want to *deform* this solution in a time-dependent fashion and explore whether this is dual to a deformed gauge theory.

Time dependent deformations

- Starting with the usual background

$$ds^2 = (\frac{r^2}{R^2})\eta_{\mu\nu}dx^\mu dx^\nu + (\frac{R^2}{r^2})dr^2 + R^2 d\Omega_5^2$$

$$F_{(5)} = R^4(\omega_5 + *_{10}\omega_5),$$

Time dependent deformations

- The following form an infinite number of deformations

$$ds^2 = (\frac{r^2}{R^2})\tilde{g}_{\mu\nu}dx^\mu dx^\nu + (\frac{R^2}{r^2})dr^2 + R^2 d\Omega_5^2$$

$$F_{(5)} = R^4(\omega_5 + *_{10}\omega_5),$$

$$\Phi = \Phi(x^\mu).$$ ← dilaton

- These are solutions provided

Ricci of the metric $\tilde{g}_{\mu\nu}$ → $\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi,$

$$\partial_\mu(\sqrt{-\det(\tilde{g})}\ \tilde{g}^{\mu\nu}\partial_\nu\Phi)$$

The Proposed Duals

- In fact these are near-horizon limits of asymptotically flat 3-brane solutions

$$ds^2 = Z^{-1/2}(x)\tilde{g}_{\mu\nu}dx^\mu dx^\nu + Z^{1/2}(x)\tilde{g}_{mn}dx^m dx^n$$

$Z(x)$ is a *harmonic* function
$\tilde{g}_{mn}$ is Ricci-flat

- We may guess the dual gauge theory by following the same logic which led to standard AdS/CFT

The Proposed Duals

- In fact these are near-horizon limits of asymptotically flat 3-brane solutions

$$ds^2 = Z^{-1/2}(x)\tilde{g}_{\mu\nu}dx^\mu dx^\nu + Z^{1/2}(x)\tilde{g}_{mn}dx^m dx^n$$

$$F_{(5)} = -\frac{1}{4\cdot 4!}\tilde{\epsilon}_{\mu\nu\rho\sigma}\frac{\partial_m Z(x)}{Z(x)^2}dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \wedge dx^m + \frac{1}{4\cdot 5!}\tilde{\epsilon}_{m_1 m_2 m_3 m_4 m_5}{}^{m_6}\partial_{m_6}Z(x)dx^{m_1} \wedge dx^{m_2} \wedge dx^{m_3} \wedge dx^{m_4} \wedge dx^{m_5}$$

$Z(x)$ is a *harmonic* function $\tilde{g}_{mn}$ is Ricci-flat

- We may guess the dual gauge theory by following the same logic which led to standard AdS/CFT

- These geometries are deformations of the AdS geometry by non-normalizable operators
- Therefore their duals should be the gauge theory with sources.

- **Conjecture** : In this case the dual is the gauge theory defined on a metric $\tilde{g}_{\mu\nu}$ and a time dependent coupling $\Phi = \Phi(x^\mu)$

$$S = \int d^4x \sqrt{\tilde{g}} e^{-\Phi} \, \mathrm{Tr} \tilde{g}^{\mu\alpha} \tilde{g}^{\nu\beta} F_{\mu\nu} F_{\alpha\beta}$$

- This is quite evident for small departures from AdS solution – the metric deformation couples to the energy-momentum tensor, and the dilaton couples to the correct operator.
- For finite departures, this is well motivated by the fact that these solutions are near-horizon geometries of deformed 3-brane solutions

Null cosmologies

Normally such deformations introduce *curvature singularities* at the Poincare horizon r=0.

This does not happen when the functions depend on a null direction.

In the following we will concentrate on solutions of the form

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = e^{f(X^+)}(-2dX^+dX^- + dx_2^2 + dx_3^2)$$

$$\Phi = \Phi(X^+)$$

$$\frac{1}{2}(f')^2 - f'' = \frac{1}{2}(\partial_+\Phi)^2$$

Start with any $f(X_i^+)$

Determine $\Phi(X^+)$

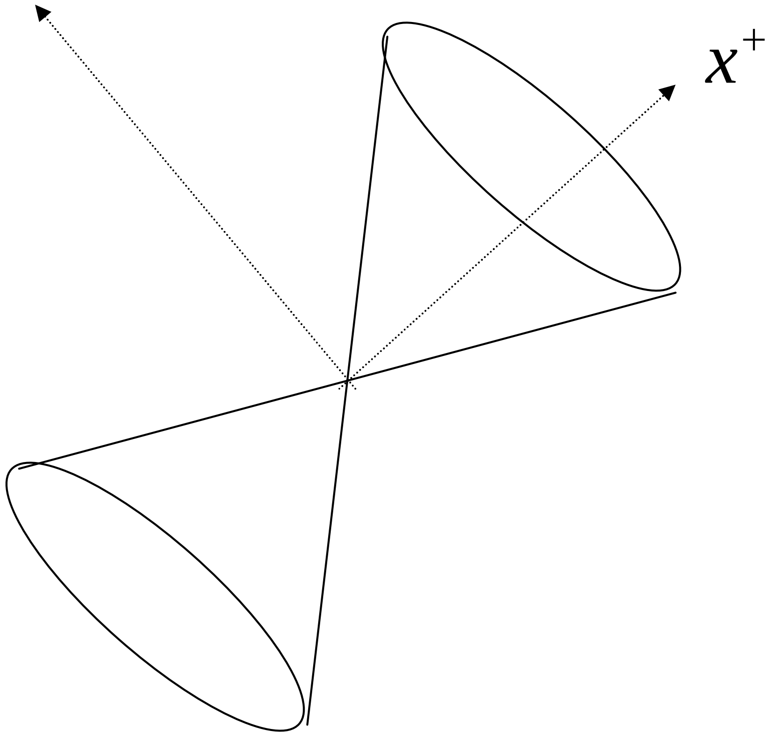
- These preserve half of the super-symmetries $\Gamma^+\epsilon = 0$
- Some of these solutions independently found by Chu and Ho

- An interesting solution has asymptotic $AdS_5 \times S^5$ with a null singularity at $X^+ = 0$

$$e^f = \tanh^2 X^+$$

$$e^\Phi = g_s \left| \tanh \frac{X^+}{2} \right|^{\sqrt{8}}$$

- The point $X^+ = 0$ can be reached in finite affine parameter – this is a singularity even though all curvature invariants are bounded here.



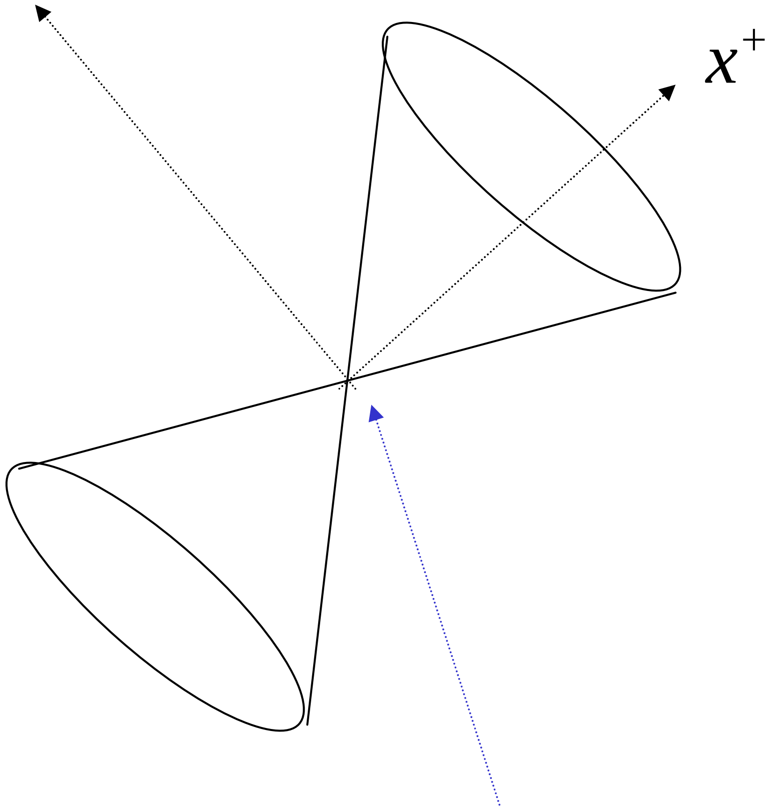
- The affine parameter along a geodesic along X^+ is given by

$$\lambda = \int e^{f(X^+)} dX^+$$

- The invariant quantity along geodesic $R_{ab}\xi^a\xi^b$ diverges.

$$R_{\lambda\lambda} = \frac{4}{\sinh^2 X^+ \tanh^4 X^+}$$

- The string coupling is, however bounded everywhere and weak at the singularity



Tidal forces diverge

Dual Theory near the singularity

- Since the brane metric is conformally flat, the factor $e^{f(X^+)}$ decouples in the classical action.
- In the quantum theory, however, this is spoiled by conformal anomalies. The one loop anomaly is

$$T_\mu{}^\mu = \frac{c}{16\pi^2}(C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}) - \frac{a}{16\pi^2}(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2)$$

- For these null backgrounds this expression vanishes.
- In the N=4 theory this one loop expression is exact because of supersymmetry. But now we have reduced supersymmetry due to a (null) time dependent dilaton.

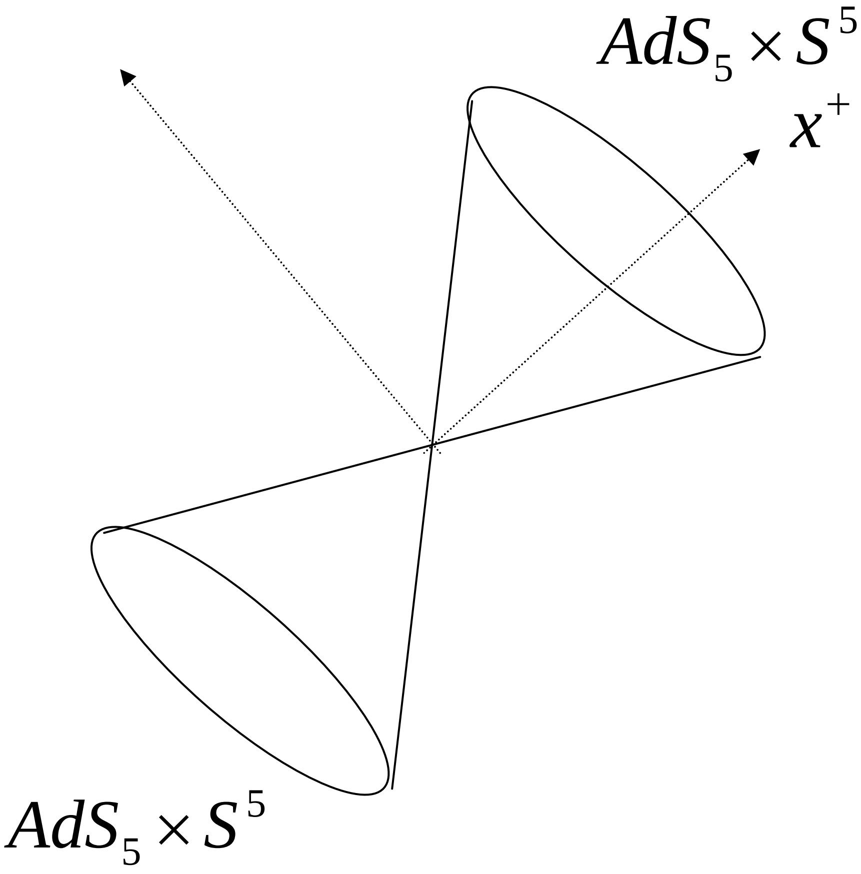
- However the dilaton leads to a vanishing coupling near the singularity – with vanishing derivatives

$$e^{\Phi} = g_s \left| \tanh \frac{X^+}{2} \right|^{\sqrt{8}}$$

- Therefore, near the singularity the corrections to the trace anomaly vanish– basically because the coupling vanishes here.
- Close to the singularity, the conformal factor decouples and correlation functions can be related to those in flat space, albeit with a varying dilaton

$$< 0| \prod_i \mathcal{O}_i(x_i)|0 >_{f(X^+),\Phi(X^+)} = \prod_i (e^{f(X_i^+)})^{\frac{\Delta_i}{2}} < 0| \prod_i \mathcal{O}_i(x_i)|0 >_{0,\Phi(X^+)}$$

- In the asymptotic region, the coupling variation vanishes and one has the standard $AdS_5 \times S^5$
- We want to prepare the system in the usual conformally invariant vacuum state at $X^+ = -\infty$ and examine its time evolution.
- At arbitrary times the gauge theory is strongly coupled.
- However, near the singularity the coupling vanishes – and one can treat the gauge theory perturbatively.



Back reaction controlled

Particle Production ?

- Generically in such backgrounds there could be particle production, even in the free theory.
- Consider for example the scalar sector of the theory, written heuristically as

$$S = -\int d^4x \; e^{-\Phi(X^+)}[(\partial\varphi)^2 - \lambda\varphi^4]$$

- The kinetic term for the canonically normalized field is standard – a field redefinition in fact moves all X^+ dependence to the coupling term $\tilde{\varphi} = e^{-\Phi(X^+)/2}\varphi$

$$S = -\int d^4x[(\partial\tilde{\varphi})^2 - \lambda e^{\Phi(X^+)}\tilde{\varphi}^4]$$

The null nature of the background is crucial for this

- Standard arguments in light front quantization then imply that there can be no particle production – once again because the background depends on X^+ only.
- The interaction picture state is

$$|s\rangle = T_+ \; e^{-i\int d^4x \;\, e^{\Phi(X^+)} \;\, \varphi^3(x)}|0\rangle$$

- In each term in a perturbation expansion the total momentum k_- must be zero, since coefficients are functions of X^+ alone
- However this cannot happen since in light front quantization all creation operators have positive k_-

- The correlation functions of course depend on the background.
- However in our case - since the interaction term vanishes near the singularity - there is correlators are non-singular everywhere.
- This may be verified by calculating these quantities perturbatively.
- Thus, smooth wave packets made out of Fock space states evolve smoothly through the singularity and there is a well-defined S-Matrix.

The gauge field sector

- There is a similar field redefinition in the gauge field sector.
- First fix a light cone gauge $A_{-} = 0$
- Now define new fields $\tilde{A}_i = e^{-\Phi/2} A_i$
- The gauge part of the action now becomes

$$S_{\rm GF} = -\frac{1}{4}\int d^4x \, [\, {\rm Tr}(\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu)^2 - 2ie^{\Phi/2}\,{\rm Tr}\{(\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu)[\tilde{A}^\mu, \tilde{A}^\nu]\}$$

- This is of the same form as in the previous slide – and the conclusion is the same – correlators of $\tilde{A}_i$ are non-singular.
- The $\tilde{A}_i$ form a complete set of gauge invariant observables. In any case these are the fields which are correctly normalized

- Note that all gauge invariant operators are not smooth.
- In fact correlators of $\mathrm{Tr}\ e^{-\Phi} F^{\mu\nu} F_{\mu\nu}$ - which is the operator dual to the dilaton mode are singular. The weak coupling answer for this does not agree with the supergravity calculation.
- The fact that there is a complete set of gauge invariant operators which are non-singular implies that one has to choose the correct complete set of dynamical variables to realize that one can evolve smoothly across the "singularity"

Stringy nature of *singularity*

- The fact that the gauge theory becomes weakly coupled at the singularity implies that stringy effects should be large.
- In fact the world-sheet action displays this. Writing the ten dimensional metric as

$$ds^2 = e^{\Phi/2}\left[\frac{e^{f(x^+)}}{Y^2}[2dX^+dX^- + d\vec{X}^2] + \frac{1}{Y^2}d\vec{Y}^2\right]$$

- The bosonic part of the light cone gauge worldsheet action

$$S = \frac{1}{2}\int d\sigma d\tau\Big[(\partial_\tau \vec{X})^2 + e^{-f(\tau)}(\partial_\tau \vec{Y})^2 - \frac{1}{Y^4}e^{2f(\tau)}e^{\Phi(\tau)}(\partial_\sigma \vec{X})^2 - \frac{1}{Y^4}e^{f(\tau)}e^{\Phi(\tau)}(\partial_\sigma \vec{Y})^2\Big]$$

- Near "singularity", $e^{\Phi(\tau)} = 0$ and all the modes of the string become light.
- We do not know yet whether the full world-sheet theory makes sense.

Penrose Limits and Matrix Theory

- The Penrose limit of the geometry has an Einstein Frame pp-wave metric given by

$$ds^2 = 2dUdV - [H(U)\vec{X}^2 + \vec{Y}^2](dU)^2 + d\vec{X}^2 + d\vec{Y}^2$$

- The singularity is now at $U \to \pi/2$ and near this

$$H(U) \sim \frac{1}{(U - \frac{\pi}{2})^2} \qquad e^{\Phi(U)} \sim (U - \frac{\pi}{2})^{\frac{\sqrt{8}}{3}}$$

- This is a generic singularity of the pp-wave.
- It appears that worldsheet string theory can be solved in this background – we have not yet studied the details. *(see e.g. Papadopolous, Russo and Tseytlin)*

- Compactifying x^{-} and one of the transverse directions, one may write down the DLCQ Matrix theory for this background – this is a 2+1 dimensional YM theory.

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$$\mathcal{L} = \mathrm{Tr}\,\frac{1}{2}\{[(D_\tau\chi^\alpha)^2 - e^{\Phi(\tau)}(D_\sigma\chi^\alpha)^2 - e^{-\Phi(\tau)}(D_\rho\chi^\alpha)^2] + \frac{1}{G_{\mathrm{YM}}^2}[e^{\Phi(\tau)}F_{\sigma\tau}^2 + e^{-\Phi(\tau)}F_{\rho\tau}^2 - F_{\rho\sigma}^2] - H(\tau)[(\chi^1)^2 + (\chi^2)^2] - (\chi^3)^2\cdots(\chi^6)^2 - 4(\chi^7)^2 + \frac{G_{\mathrm{YM}}^2}{2}[\chi^\alpha,\chi^\beta]^2 + 2iG_{\mathrm{YM}}\chi^7[\chi^5,\chi^6] + \frac{4}{G_{\mathrm{YM}}}\chi^7F_{\sigma\rho}\},$$

$$\alpha = 1,\cdots 7$$

- where $$0 < \sigma < 2\pi\frac{l_s^2}{R},\ 0 < \rho < 2\pi g_s\frac{l_s^2}{R}$$

$$G_{YM}^2 \sim \frac{1}{g_s}$$

- Compactifying x^- and one of the transverse directions, one may write down the DLCQ Matrix theory for this background – this is a 2+1 dimensional YM theory.
- When the original string coupling ρ is small the YM theory is strongly coupled and the fields become diagonal
- The gauge field strength can be then dualized into a scalar and now we have 8 scalars
- Naively the ρ direction becomes small – so that we have a 1+1 dimensional theory
- This theory is in fact identical to the worldsheet light cone gauge theory of the fundamental string in the pp-wave background.
- However the time dependent gradient terms imply that there is particle production of modes with momenta in the ρ direction. These are modes of a D-string

- This is similar to x^- actions obtained in pp-wave backgrounds with null linear dilatons *(S.R.D. and J. Michelson)*, which generalize the work of *Craps, Sethi and Verlinde*.
- However, unlike these cases, the matrix membrane theory we obtain has

(1) Constant couplings

(2) Time dependent space gradients

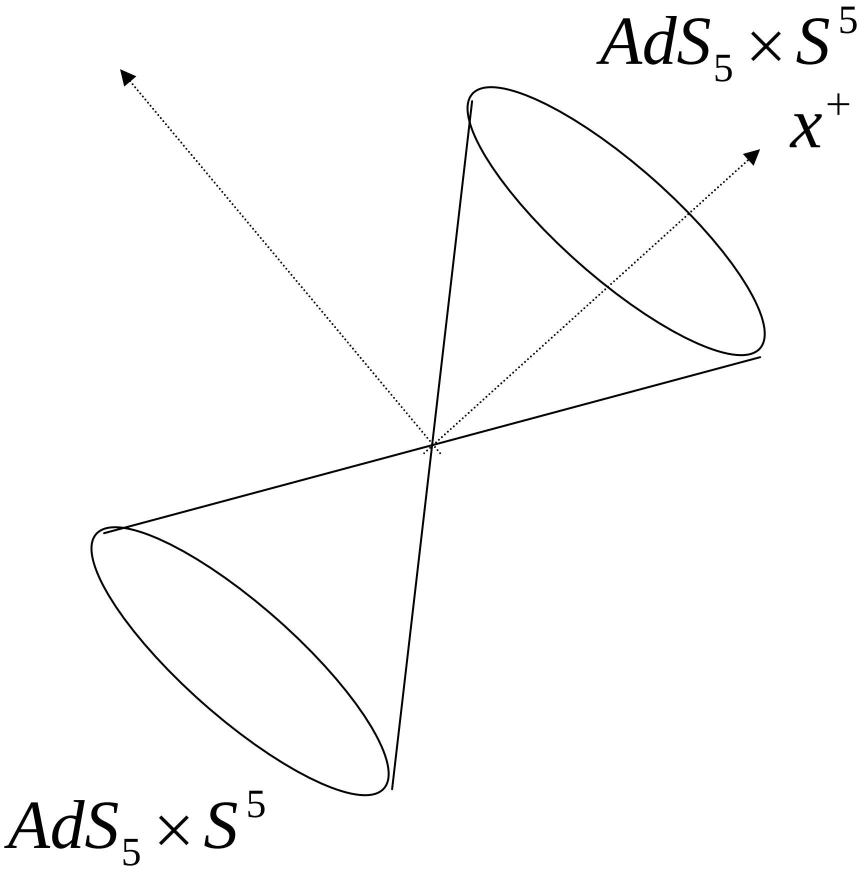
(3) Time dependent masses

In the appropriate limit $g_s \to 0$ this reproduces the F-string worldsheet action.

Near the singularity, excited modes of both F-strings and D-strings are produced copiously.

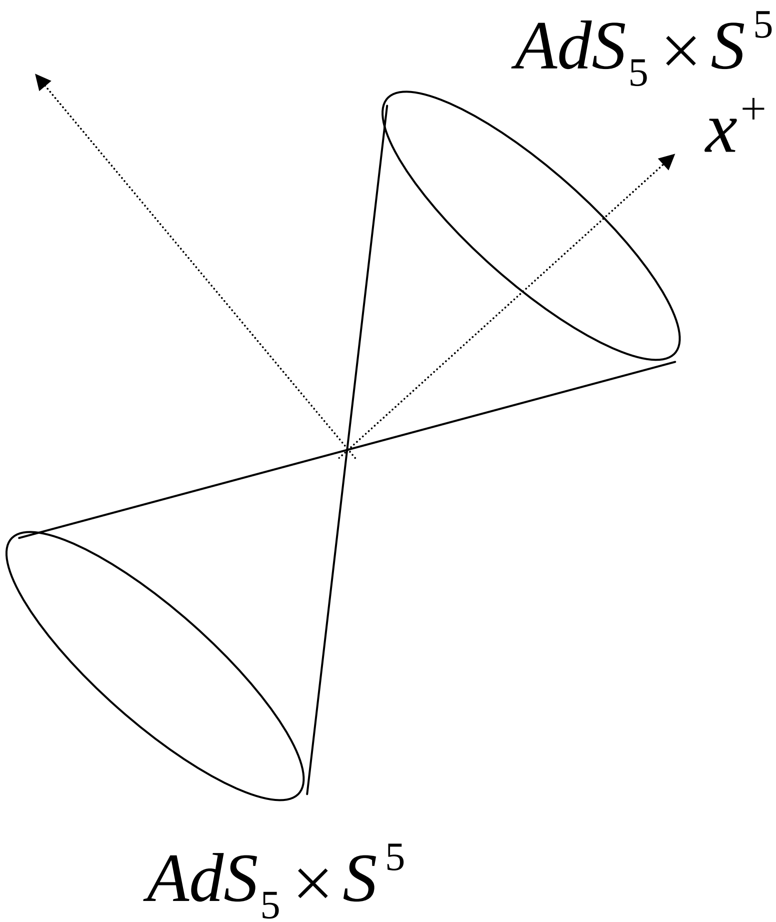
The overall Picture

- At early light cone times, the geometry is the standard $AdS_5 \times S^5$ and the dilaton is a constant.
- For large values of the 't Hooft coupling, curvatures are small and supergravity - and hence conventional space-time - is a good description



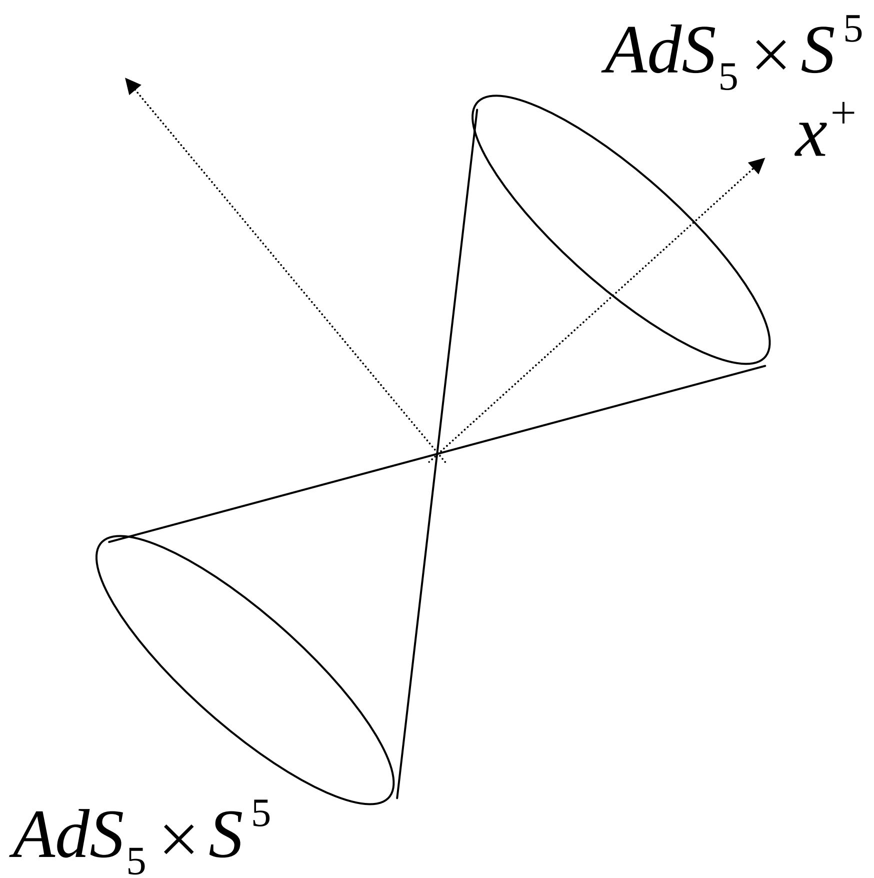
The overall Picture

- If we continue this description to $X^+ = 0$ we encounter a null singularity.
- Here curvature components and tidal forces diverge even though invariants remain bounded.
- This occurs at finite affine parameters along geodesics.
- e^{Φ} becomes small and vanishes at $X^+ = 0$



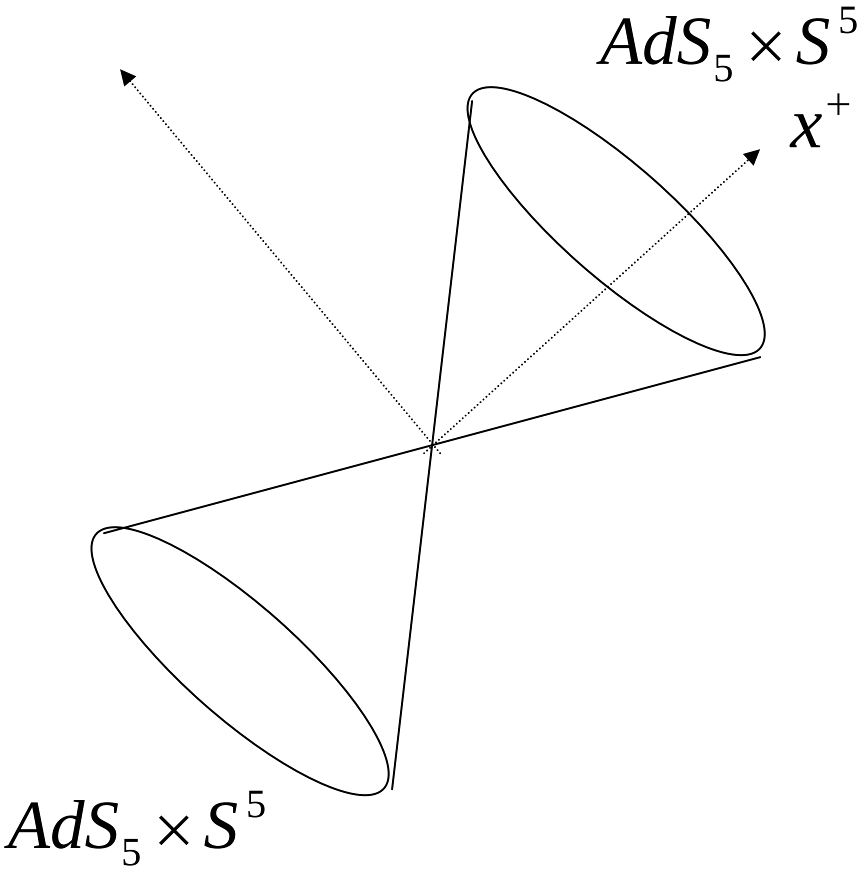
The overall Picture

- The fact that e^{Φ} becomes small, however, means that the dual gauge theory becomes weakly coupled – and the supergravity description should not be good in any case
- The gauge theory is well behaved here – there are no singularities in the correlators of normalized fields



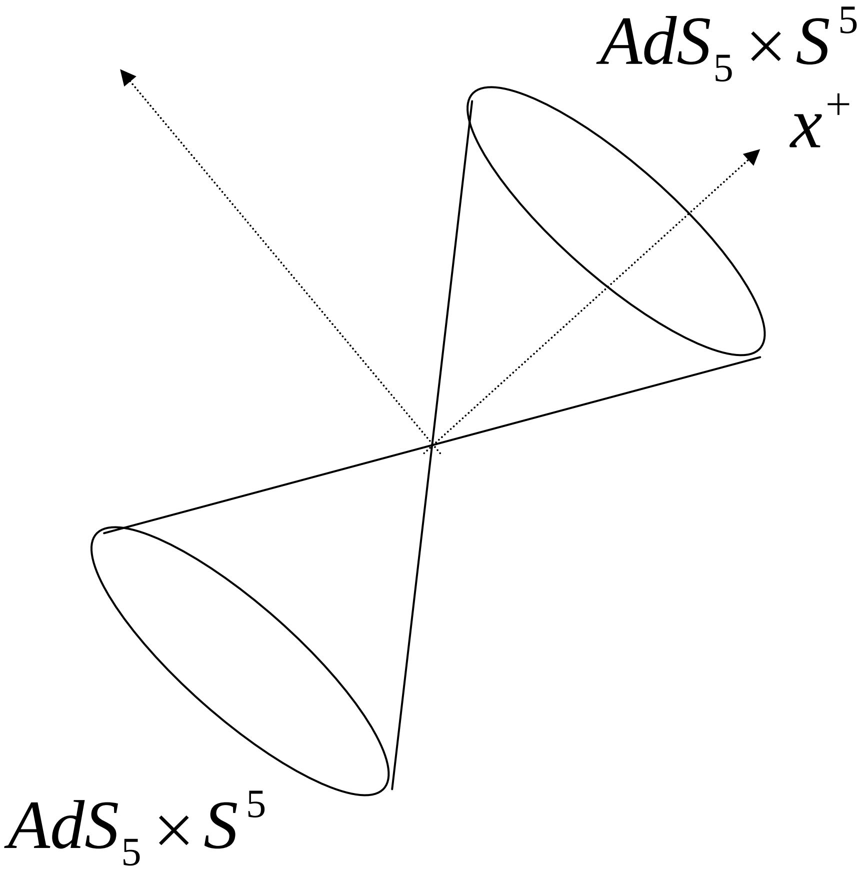
The overall Picture

- This means that a smooth wave packet made of standard fock space states at early times evolves smoothly across the singularity.
- There is no conventional ten dimensional space-time interpretation here.
- Rather one should replace this by a weakly coupled gauge theory



The overall Picture

- There is a possibility that perturbative string theory could also be well defined here.
- However Matrix Theory descriptions of the Penrose limit seem to indicate that D-brane states are excited as well.



- It has been suspected for a long time that near singularities the notions of space and time break down and have to be replaced with something else
- In these toy models of cosmology we have some idea of what structure should replace space-time – though this is by no means generic.