Self-accelerating cosmology from Lorentz symmetry violation on the brane

High Energy, Cosmology and Strings

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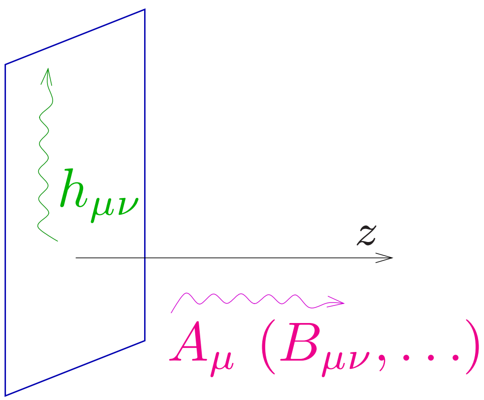
(hep-th/0506067, work in progress ...) by D.G. and S. Sibiryakov

Outline

- Description of the model
- Infrared modification of gravity potentials
- Dissipation of gravity waves
- Cosmological ansatz
- Self-accelerating solution
- AdS/CFT picture

The main idea: gravity modification @ IR

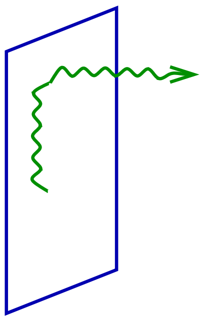
Consider brane world with infinite extra dimension and localized gravity (Randall, Sundrum):
brane with positive tension immersed in 5-dimensional AdS bulk



$$ds^2 = -dz^2 + \mathrm{e}^{-2|z|/l}\eta_{\mu\nu}dx^\mu dx^\nu$$

$$\eta_{\mu\nu} = (+,-,-,-)$$

Make graviton mix with bulk fields, e.g. vectors, 2-forms, ...
$\Longrightarrow$ graviton gets quasilocalized: goes away to extra dimensions at large distances and time scales



N.B. good chance to obtain naturally large r_c

Concrete model

RS setup + 3 massless vectors A_M^a , $a = 1, 2, 3$, living in the bulk

- standard bulk action

$$S_{A,bulk} = -\frac{1}{4}\int d^5x\sqrt{g}\, F_{MN}^a F^{aMN} \;, \qquad F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a$$

- quartic potential on the brane

$$S_{A,brane} = -\frac{\varkappa^2}{2}\int d^4x\sqrt{-\bar{g}}\,\left(\bar{g}^{\mu\nu}A_\mu^a A_\nu^b + \upsilon^2\delta^{ab}\right)^2$$

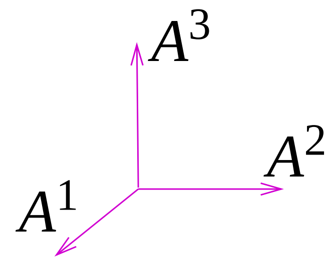
summation over repeated indices a, b

Symmetries of the action:

- 5d diff-invariance
- global $SO(3)$ acting on indices a, b
- gauge $[U(1)]^3$ in the bulk; broken explicitly on the brane

First: static solution

$$ds^2 = -dz^2 + \mathrm{e}^{-2|z|/l}\eta_{\mu\nu}dx^\mu dx^\nu$$
$$A^a_M = \upsilon\delta^a_M$$



Vectors form a spacelike orthogonal triad

Pattern of symmetry breaking: internal $SO(3)\times$Lorentz symmetry down to $SO(3)$ of spatial rotations + internal space rotations

Linearized analysis:

$$ds^2 = -dz^2 + \left(\mathrm{e}^{-2|z|/l}\eta_{\mu\nu} + h_{\mu\nu}(x,z)\right)dx^\mu dx^\nu$$
$$A^a_\mu = \upsilon\delta^a_\mu + a^a_\mu(x,z)$$

To linear order energy-momentum tensor of vector fields is present only on the brane

$$T^{vect}_{00} = T^{vect}_{0i} = 0$$
$$T^{vect}_{ij} = 2\varkappa^2\upsilon^4\left(h_{ij} + \frac{1}{\upsilon}(a^i_j + a^j_i)\right)$$

It violates weak energy condition

Propagating degrees of freedom

Parameters: $\varkappa^{-1} \sim \upsilon^{2/3} < M_5 = \left(M_{Pl}^2/l\right)^{1/3} \sim 1/l \sim M_{Pl}$

No localized modes

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tt symmetric tensor perturbation (graviton) is a collection of massive modes

$$G_g(x,x') = \int_0^{1/l} \frac{dm}{m} \frac{G_m(x,x')}{\left(1 + \frac{\log ml}{\log m_c l}\right)^2}$$

$m_c = l^{-1}\exp(-M_5^3/\upsilon^2)$ naturally exponentially small:

$\upsilon \approx (M_5/5)^{3/2} \Longrightarrow m_c \approx (10^{28} cm)^{-1} \sim H_0$

Continuum spectrum of completely delocalized modes; interaction with matter on the brane is suppressed

All perturbations are stable

Field of external matter source

For simplicity: a point mass M @ IR: $r \gg l, 1/(\varkappa\upsilon)^2$

$$\bar{h}_{00}(r) = -\frac{2G_N M}{r}\left(1 - \frac{\log[r/l]}{\log[r_c/l]}\right)\;, \quad \bar{h}_{ij}(r) = -\frac{2G_N M}{r}\delta_{ij}$$

$$r_c = 1/m_c = l \cdot \mathrm{e}^{M_5^3/\upsilon^2}$$

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Good news: no vDVZ discontinuity

Antigravity at distances $r > r_c$

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Antigravity at distances $r > r_c$

Bad news: phenomenology (light deflection by the Sun) requires

$$\log[r_c/l] > 10^5$$

Cure: couple vectors to a dilaton,
consider form-fields of higher degrees, ...

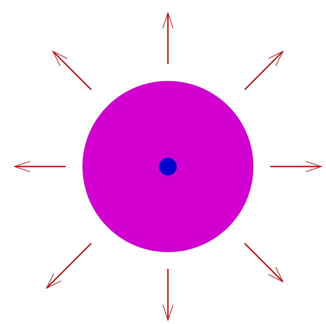
Origin of log-enhanced antigravity

PUZZLE Common wisdom: gravitons dissipate at large distances

$\Longrightarrow$ gravitational potential should become weaker $\quad \varphi \sim \frac{e^{-m_g r}}{r}$

In Lorentz breaking theories that's not true

External energy-momentum tensor $T^{ext}_{\mu\nu}$ gives rise to perturbations of vector fields



Vector fields produce energy-momentum tensor $T^{vect}_{\mu\nu}$

$T^{vect}_{\mu\nu}$ dominates at large distances

$T^{vect}_{\mu\nu}$ violates the weak energy condition

Gravity waves (GW)

External periodic tt source on the brane $T_{ij}(x) \propto \mathrm{e}^{-i\omega t}$

$$G(\mathbf{x}-\mathbf{x}';\omega) = -\frac{4G_N l}{r}\int_0^\infty dm \sum_{s=1,2}\left(\chi_{1m}^{(s)}(0)\right)^2 \mathrm{e}^{ip_\omega r} \; ,$$

$p_\omega = \sqrt{\omega^2 - m^2}$ when $m < \omega$ (and $p_\omega = i\sqrt{m^2-\omega^2}$ when $m > \omega$: not radiated).
In the regime $m_c \ll \omega \ll l^{-1}$,

$$G(\mathbf{x}-\mathbf{x}';\omega) \propto -\frac{4G_N}{r}\,\mathrm{e}^{i\omega r}\cdot\int_0^\omega \frac{dm}{m}\frac{\mathrm{e}^{-i\frac{rm^2}{2\omega}}}{\left(1+\frac{\log ml}{\log m_c l}\right)^2} \; ,$$

$r \ll \omega/{m_c}^2$: saturated by $m \sim m_c$ resulting in the usual 4-dim expression for GW
$r \gg \omega/{m_c}^2$: the integral is damped by the rapidly oscillating exponent

$$G(\mathbf{x}-\mathbf{x}';\omega) \propto -\frac{4G_N}{r}\frac{1}{\ln\frac{r}{2\omega l^2}}\,\mathrm{e}^{i\omega r} \; .$$

GW gradually dissipate into the bulk

Static solution: Conclusions and open questions

- A self-consistent model describing quazilocalized (massive) gravitons is proposed. The characteristic mass is naturally small
- At classical level the model is free from instrabilities and the vDVZ discontinuity
- There is antigravity at ultra-large distances
- A way to avoid constraints imposed by tests of the Einstein's relativity in the Solar system ?
- The scale of strong quantum coupling ?
- ...
- ...
- Cosmology ?

Cosmological ansatz

$$S_{A,brane} = -\frac{\varkappa^2}{2}\int d^4x\sqrt{-\bar{g}}\,\left(\bar{g}^{\mu\nu}A^a_\mu A^b_\nu + \upsilon^2\delta^{ab}\right)^2$$

3-dim rotations and translations:

$$ds^2 = F(\zeta,t)(dt^2 - d\zeta^2) - r^2(\zeta,t)d\mathbf{x}^2;$$

$$A^a_i = \upsilon\delta^a_i A(\zeta,t)\;, \quad A^a_0 = A^a_5 = 0.$$

the boundary conditions (brane moves in the external AdS_5):

$$F \to \frac{1}{(k\zeta)^2}\;,$$

$$r \to \frac{1}{k\zeta}\;, \qquad\qquad \zeta \to 0$$

$$A \to a(t)\;,\text{inflationary expansion on the brane :}\qquad a(t) = -\frac{1}{Ht}$$

$$k \equiv 1/l\;, \qquad t < 0\;, \qquad \zeta > 0\;.$$

Ansatz for self-accelerating cosmology

$$A = -\frac{1}{\sqrt{\lambda}u}\alpha\left(\frac{v}{u}\right)\;,\quad r = -\frac{1}{u}\rho\left(\frac{v}{u}\right)\;,\quad F = \frac{1}{u^2}f\left(\frac{v}{u}\right)\;,\qquad x \equiv \frac{v}{u}\;.$$

simple analytical approximation for $\alpha_0 \equiv \frac{\sqrt{\lambda}k}{H} = \frac{k}{H}\frac{\upsilon}{\sqrt{2M_5^3}} \gg 1$:

$$f = \rho^2 - \frac{\rho_h^4}{\rho^2} \qquad \text{AdS -- Schwartzshield}\;,$$

$$\alpha = \alpha_0\left(1 - C_1\cdot\alpha_0^{-2/3}x^{2\rho_h+1}\right)\;,\quad \rho_h = \left(\frac{\alpha_0^2}{2}\right)^{1/3}$$

$$\rho = \rho_h\left(1 + C_2\cdot\alpha_0^{-2/3}x^{2\rho_h+1}\right)\;.$$

Next step: growth of Black Hole

propagation of the vector fields in the background of a static BH in AdS_5:

$$ds^2 = F(r)dt^2 - \frac{dr^2}{F(r)} - (kr)^2 d\mathbf{x}^2\;,\quad F(r) = k^2r^2 - \frac{M}{6\pi^2 M_5^3 r^2}\;.$$

$$\frac{dr}{F} = -d\zeta$$

$$\partial_t M \propto T_0^\zeta\;:\quad \partial_t M \simeq \frac{6\pi^2 r_h \upsilon^2(\partial_t a)^2}{k^2}\;,\quad r_h = \left(\frac{M}{6\pi^2 M_5^3 k^2}\right)^{1/4}$$

Friedman equation: Self-acceleration

To the leading order in $\alpha_0 \equiv \frac{k}{H} \frac{\upsilon}{\sqrt{2M_5^3}} \gg 1$:

$$H^2 = \frac{8\pi G_N}{3}\rho_{mat} + H^2 \left(\frac{\lambda^2 k}{4H}\right)^{2/3} .$$

fixed point:

$$H = H_c \equiv \frac{\lambda^2 k}{4} = \frac{k}{16}\left(\frac{\upsilon^2}{M_5^3}\right)^2$$

$H^{-1} \sim 10^{28}$ cm for $\upsilon \sim 10^9$ GeV, if $M_5 \sim k \sim M_{Pl}$

Other stages, $a = a(t)$

perturbations about brane moving in AdS_5

Matter dominated stage:

$$H^2 = \frac{8\pi G_N}{3}\rho + \text{const} \cdot H^2 \left(\frac{\lambda^2 k}{H}\right)^{2/3}$$

fixed point again! $H_c = \lambda^2 k$ ... corrections:

$$H \left| \ln\left[\frac{\lambda H}{k}\right] \right|^{3/2} \ll \sqrt{\lambda} k \,,$$

Conjecture: this equation is valid at all stages... up to corrections

Effective equation of state

$$H^2 = \frac{8\pi G_N}{3}\rho + H^2\left(\frac{H_c}{H}\right)^{2/3}$$

Cosmological observations confine a viable region in (Ω_M, ω_{DE}) space

$$\Omega_M \equiv \frac{\rho_M(t_0)}{\rho_{tot}(t_0)}\,, \omega_{DE} \equiv \frac{\rho_{DE}(t_0)}{\rho_{tot}(t_0)}\,,$$

$$H^2 = \frac{8\pi G_N}{3}(\rho_M + \rho_{DE})$$

$$\dot{\rho}_M + 3H\rho_M = 0\,,$$

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1+\omega_{DE}) = 0\,.$$

$$\omega_{DE} = -\frac{1}{1-\Omega_M}\left(1+\frac{2\dot{H}}{3H^2}\right)\,,$$

$$H_c = H_0 \cdot (1-\Omega_M)^{3/2}\,.$$

$$\omega_{DE} = -\frac{1}{1+2\Omega_M}$$

AdS/CFT picture: Dual description

vectors in the bulk $\longleftrightarrow$ global currents in CFT
black hole in the bulk $\longleftrightarrow$ non-zero temperature of plasma

$$\text{Dirichlet} \quad \longleftrightarrow \int d^4x\sqrt{-g}A_{(4)\mu}j^\mu \;, \qquad A_{(4)\mu} = \frac{1}{\sqrt{k}}A_\mu|_{\zeta=0}$$

in our setup: electric fields in physical coordinate frame

$$E_i^a = \frac{1}{a^2}F_{ti} = \frac{\upsilon H}{\sqrt{k}}\delta_i^a$$

heat conformal plasma:

$$W = \sum_a \mathbf{E}^a \mathbf{j}^a \;.$$

Bulk: the energy carried away from the brane by the vector fields

$$W = 2T_0^\zeta \;,$$

Heated plasma (SYM, N^2 degrees of freedom):

$$\mathscr{E} = N^2 T^4 \;, \qquad T = \frac{k^3\zeta_B r_h}{\pi} \;, \quad N = \frac{1}{\sqrt{G_5 k^3}} \;,$$

AdS/CFT picture: Dual description

the Friedman equation:

$$H^2 = \frac{8\pi G_N}{3}(\rho + \mathscr{E}) = \frac{8\pi G_N}{3}\rho + \frac{T^4}{k^2}$$

$$\dot{\mathscr{E}} + 4\frac{\dot{a}}{a}\mathscr{E} = W\;, \quad \mathbf{j} = (\alpha\omega + \beta T)\mathbf{E}$$

$$\alpha = \begin{cases} \pi - 2i\ln\left[\frac{k}{\omega}\right]\;, & \omega \gg T \\ -2i\ln\left[\frac{k}{T}\right]\;, & \omega \ll T \end{cases}\;,$$

$$\beta = 2\pi\;.$$

near the self-accelerated fixed point $\omega \ll T$

$$T^2\dot{T} + \frac{\dot{a}}{a}T^3 = G_5 k^3 E^2 = \lambda k^2 H^2\;, \longrightarrow T = \mathrm{const}\cdot(\lambda k^2 H)^{1/3}$$

This gives the same Friedman equation — self-acceleration

1) pile up of energy into the conformal matter produced by Electric fields compensates for cooling of the plasma due to the cosmological expansion

2) Lorentz symmetry breaking prevents Electric fields from rapid decay which is usually caused by the cosmological expansion.

Conclusions

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  consistent with cosmologycal observations