Holographic Cosmology

IHP

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Holography

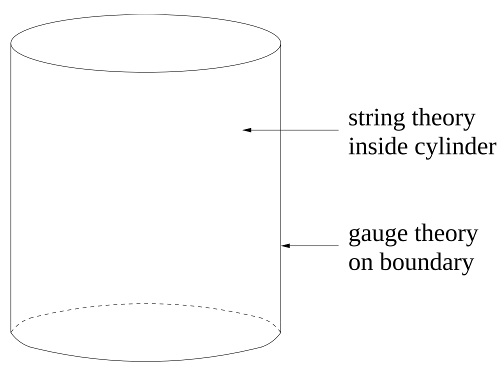
Singularity Theorems: quantum origin

$\longrightarrow$ predictive cosmology needs quantum gravity.

String theory: natural framework

$\longrightarrow$ dual quantum description of cosmology?

Gauge/Gravity Duality: [Maldacena '97]

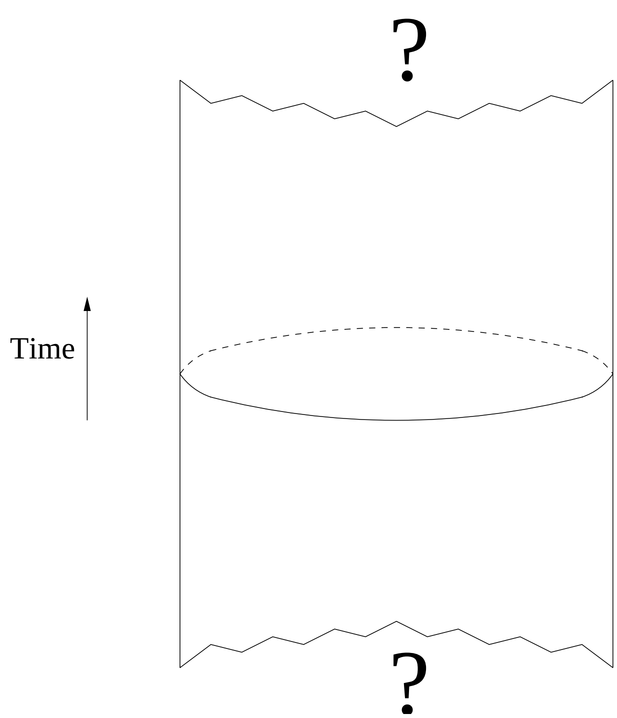


$$l_{AdS} = (4\pi g_s N)^{1/4} l_s = \lambda^{1/4} l_s$$

$\longrightarrow$ Finite N gauge theory viewed as *nonperturbative definition* of string theory on asympt AdS spacetimes.

Holographic (AdS) Cosmology

Generalization: SUGRA solutions where smooth asymptotically AdS initial data emerge from a big bang in the past and evolve to a big crunch in the future.



The dual finite N gauge theory evolution should give a fully quantum gravity description of the singularities!

Outline

- Cosmology with AdS boundary conditions
- Dual Field Theory Evolution
- To Bounce or not to Bounce?

Setup

We consider a consistent truncation of the low energy regime of string theory compactified on S^7 ,

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 + 2 + \cosh(\sqrt{2}\phi)\right]$$

$\longrightarrow$ string theory with $AdS_4 \times S^7$ boundary conditions.

Scalar, $\quad m^2 = -2 > m^2_{BF} = -9/4$

AdS in global coordinates,

$$ds^2 = -(1+r^2)dt^2 + \frac{dr^2}{1+r^2} + r^2 d\Omega_2$$

In all asymptotically AdS solutions, ϕ decays as

$$\phi(t,r,\Omega) = \frac{\alpha(t,\Omega)}{r} + \frac{\beta(t,\Omega)}{r^2}$$

Boundary Conditions

Standard (susy) boundary conditions on ϕ: $\beta = 0$

$$\phi = \frac{\alpha(t,\Omega)}{r} + \mathcal{O}(1/r^3)$$

$$g_{rr} = \frac{1}{r^2} - \frac{(1+\alpha^2/2)}{r^4} + O(1/r^5)$$

More generally: $\beta(\alpha) \neq 0$

$$\phi = \frac{\alpha(t,\Omega)}{r} + \frac{\beta(\alpha)}{r^2}$$

Conserved total energy remains finite, but acquires an explicit contribution from ϕ.

e.g. with spherical symmetry

$$M = 4\pi(M_0 + \alpha\beta + \int_0^{\alpha} \beta(\tilde{\alpha}) d\tilde{\alpha})$$

AdS-invariant boundary conditions

One-parameter class of functions $\beta_k(\alpha)$ that define AdS-invariant boundary conditions,

$$\beta_k = -k\alpha^2$$

$$M = 4\pi(M_0 - \tfrac{4}{3}k\alpha^3)$$

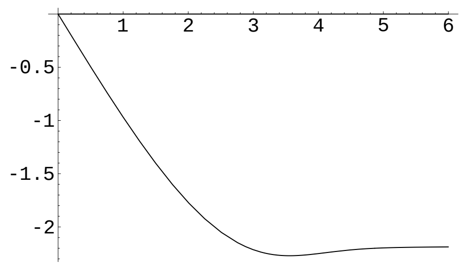
Claim: For all $k \neq 0$, there exist smooth asymptotically AdS initial data that evolve to a singularity which extends to the boundary of AdS in finite global time.

Example: Solutions obtained by analytic continuation of Euclidean instantons.

AdS Cosmology

$O(4)$ symmetric Euclidean instanton,

$$ds^2 = \frac{d\rho^2}{b^2(\rho)} + \rho^2 d\Omega_3, \qquad \phi(\rho) \sim \frac{\alpha}{\rho} + \frac{\beta}{\rho^2}$$



Lorentzian cosmology by analytic continuation:

- Inside lightcone from $\phi(0)$: FRW evolution to big crunch that hits boundary as $t \to \pi/2$.

- Asymptotically (at large r) one has

$$\phi = \frac{\alpha(t)}{r} - \frac{k\alpha^2(t)}{r^2} + O(r^{-3}), \qquad \alpha(t) = \frac{\alpha(0)}{\cos t}$$

Dual Field Theory

M Theory with $AdS_4 \times S^7$ boundary conditions is dual to the 2+1 CFT on a stack of M2 branes.

- With $\beta = 0$, $\phi \sim \alpha/r$ is dual to $\Delta = 1$ operator $\mathcal{O}$,

$$\mathcal{O} = \frac{1}{N} Tr T_{ij} \varphi^i \varphi^j$$

  and

$$\alpha \leftrightarrow \langle \mathcal{O} \rangle$$

- Taking $\beta(\alpha) \neq 0$ corresponds to adding a multitrace interaction $\int W(\mathcal{O})$ to the CFT, such that [Witten '02, Berkooz et al. '02]

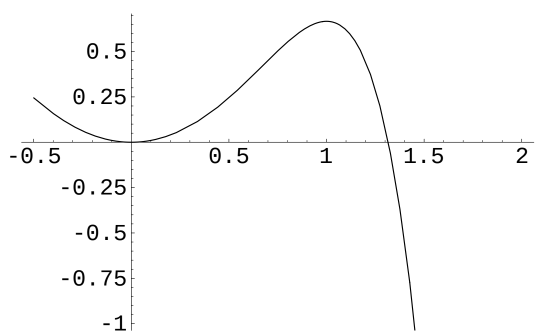
$$\beta = \frac{\delta W}{\delta \alpha}$$

Dual Field Theory

With $\beta_k = -k\alpha^2$,

$$S = S_0 - \frac{k}{3}\int \mathcal{O}^3$$

The dual description of AdS cosmologies involves field theories that always contain an operator $\mathcal{O}$ with an effective potential that (at large N) is unbounded from below.



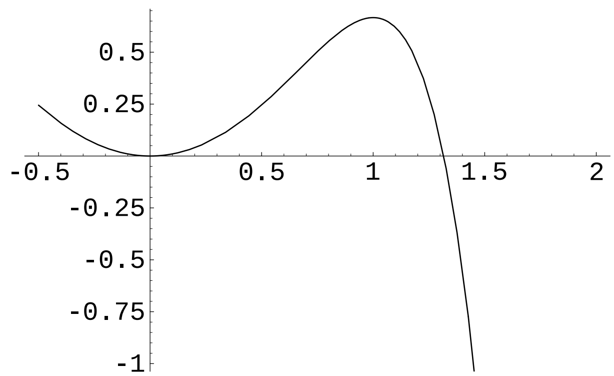
What is the CFT evolution dual to AdS cosmologies?

To leading order in $1/N$, $\quad < \mathcal{O} > \to \infty$

Semiclassical Evolution

Neglecting the nonabelian structure ($\mathcal{O} \leftrightarrow \varphi^2$),

$$V = \frac{1}{8}\varphi^2 - \frac{k}{3}\varphi^6$$



Exact homogeneous classical (zero energy) solution,

$$\varphi(t) \sim \frac{1}{k^{1/4}\cos^{1/2} t}$$

reproduces time evolution of SUGRA solutions.

$\longrightarrow$ semiclassical analysis suggests CFT evolution ends in finite time...

Quantum Mechanics

Consider first homogeneous mode $\varphi(t) = x(t)$.

"Quantum mechanics with unbounded potentials."

A right-moving wave packet in $V(x)$ reaches infinity in finite time.

To ensure probability is not lost at infinity one constructs a self-adjoint extension of the Hamiltonian, by carefully specifying its domain. [Carreau et al. '90]

The center of a wave packet follows essentially the classical trajectory. When it reaches infinity, however, it bounces back.

$\longrightarrow$ Quantum mechanics indicates evolution continues for all time, with an immediate big crunch/big bang transition.

Quantum Field Theory

In the full field theory inhomogeneities develop as ϕ rolls down, in a process similar to "tachyonic preheating".

Does this significantly change evolution?

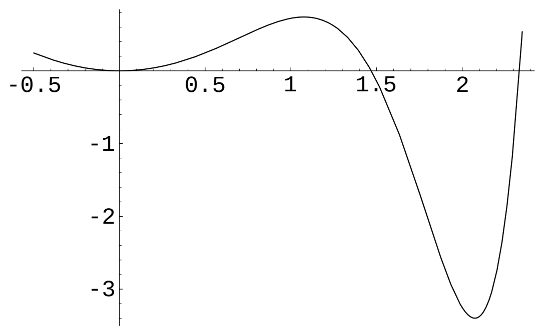
If tachyonic preheating efficiently converts most of the potential energy in gradient energy, then a bounce through the singularity would be extremely unlikely...

Whether or not this happens depends on what are the $1/N$ corrections to the potential.

1. Regularization at Finite N

Regularize by adding quartic interaction $\epsilon \mathcal{O}^4$,

$$V = \frac{1}{8}\varphi^2 - \frac{k}{3}\varphi^6 + \frac{\epsilon}{4}\varphi^8$$



Does this change nature bulk singularity?

With bulk boundary conditions

$$\beta_{k,\epsilon} = -k\alpha^2 + \epsilon\alpha^3,$$

- small change instanton initial data, $M_i \sim -\epsilon$
- potentially significant change bulk evolution in regime $\alpha^2 > k/\epsilon$, i.e. near the singularities

Black Holes with Scalar Hair

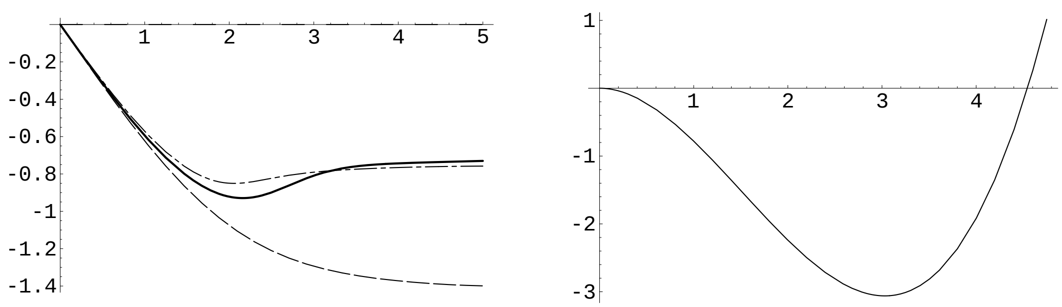
Metric; $ds_4^2 = -h(r)e^{-2\delta(r)}dt^2 + h^{-1}(r)dr^2 + r^2 d\Omega_2^2$

Asymptotic scalar profile; $\phi(r) = \frac{\alpha}{r} + \frac{\beta}{r^2}$

Regularity at horizon R_e determines $\phi_{,r}(R_e)$.

Integrating field equations outward yields a point in (α, β) plane for each pair (R_e, ϕ_e).

Repeating for all ϕ_e gives curves $\beta_{R_e}(\alpha)$ for each R_e:



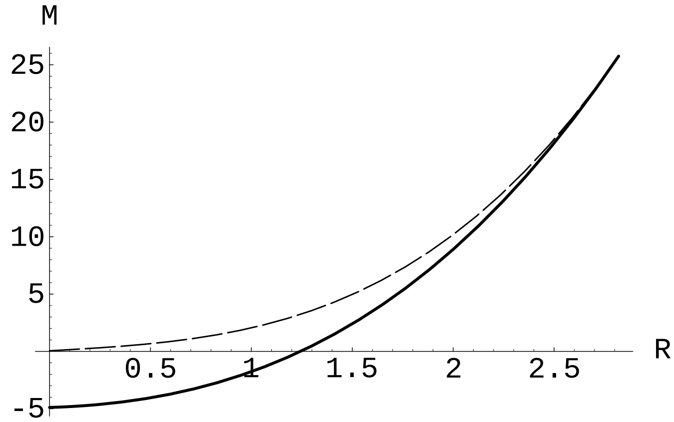
Black hole solutions are given by intersection points

$$\beta_{R_e}(a) = \beta_{k,\epsilon}(\alpha)$$

$\longrightarrow$ two branches of black holes with scalar hair!

Back to Cosmology

Mass of hairy black holes:



$\rightarrow$ Finite N regularization of the dual field theory modifies bulk dynamics, turning the big crunch into a giant hairy black hole. This is dual to an equilibrium field theory state around the global minimum that arises from the regularization.

What would it mean?

Conjecture: *Evolution would continue for all times, but cosmological singularities would be quantum gravitational equilibrium states, described in terms of dual variables.*

$\longrightarrow$ minisuperspace approximation would miss key physics

$\longrightarrow$ asymmetry between past and future singularities.

A note on predictive cosmology:

Testing the theory would require the evaluation of conditional probabilities for observables, as well as a good understanding of the quantum state
$\longrightarrow$ major challenge

2. No Regularization at Finite N

- Black hole formation even without global minimum, as long as ϕ does not reach infinity in finite time. Equilibration happens when inhomogeneous modes 'unfreeze'.

- By contrast, when V" remains negative, inhomogeneities remain frozen, no black hole forms and the homogeneous evolution may in fact be accurate.

Conjecture: A big crunch/big bang transition does happen, and cosmological singularities are qualitatively different from black hole singularities.

What are the 1/N corrections?

String theory with $AdS_5 \times S^5$ boundary conditions may offer guidance,

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 + 2e^{2\phi/\sqrt{3}} + 4e^{-\phi/\sqrt{3}}\right]$$

Scalar has $\quad m^2 = -4 = m^2_{BF}$

Asymptotically, ϕ decays as

$$\phi(t,r,\Omega) = \frac{\alpha(t,\Omega)\ln r}{r^2} + \frac{\beta(t,\Omega)}{r^2}$$

One again finds instantons for boundary conditions

$$\beta_k = -\lambda\alpha$$

Dual field theory action is given by

$$S = S_{YM} - \frac{\lambda}{2}\int\psi^4$$

which remains unbounded ...