"High Energy, Cosmology and strings"
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Graviton cloning, light massive gravitons and gauge theory/gravity correspondence

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Bibliography

- The work has appeared in

  E. Kiritsis **hep-th/0608088**

- Related work by:

  O. Aharony, A. Clark and A. Karch **hep-th/0608089**

Introduction

• Gravity is the oldest known interaction.

• There is widespread feeling that it is probably the least understood.

• The first signals stem from failed attempts to construct the quantum theory due to non-renormalizability.

• Further signals emerged from the presence of black-hole solutions, the associated thermodynamics, and the ensuing information paradox

• The cosmological constant problem hounds physicists for the past few decades.

• And the latest surprise is that the universe seems to accelerate due a 70% component of dark energy.

These are good reasons to advocate that we do not understand gravity very well.

The gauge theory/string-theory correspondence

One of the most promising approaches to such problems has been the gauge-theory/string theory correspondence.

- It provides a set of microscopic degrees of freedom for gravity

- It defines a non-perturbative quantum theory of gravity

- It explains BH thermodynamics and provides a resolution to the information paradox.

- It has not provided a breakthrough on the cosmological constant yet, but the verdict is still out.

Some questions for gravity

• Are there consistent and UV complete theories of multiple interacting massless gravitons?

• Are there consistent and UV complete theories of multiple interacting massive gravitons?

In string theory there are massive stringy modes that are spin-2 but their mass cannot be made light without bringing down the full spectrum

A similar remark applies to KK gravitons.

• Is it always, the gravitational dual of a large-N CFT$_d$, a string theory on $AdS_{d+1} \times X$ or a warped product?

♠The plan is to answer these questions using the tools of gauge-theory/gravity correspondence

The quick answers

• No more than one interacting massless gravitons are possible. This is in agreement with previous studies in field theory and string theory.

♠There can be many massive interacting gravitons in a theory. The light ones can have masses proportional to the string coupling $\mathcal{O}(g_s)$, or equivalently in the large N theory , N_c^{-1}.
This provides an UV completion to theories with light massive gravitons

♣There are conformal large-N gauge theories, whose gravitational duals are defined on a product of two (or more) AdS_5 manifolds (baring internal manifolds).

The associated theories are tensor products of large N theories coupled by multiple-trace deformations.

This is probably the most general type of geometry that can describe the duals of large-N conformal theories.

Massive gravitons at low energy

• Massive gravitons have been effectively described very early.

Fierz+Pauli

$$\frac{\mathcal{L}_{\rm FP}}{M_P^2}=\int d^4x\left[\sqrt{-g}\;R+\sqrt{-\eta}\left[k_1 h^{\mu\nu}h_{\mu\nu}+k_2(h^{\mu}{}_{\mu})^2\right]\right]\quad,\quad \sqrt{-g}g^{\mu\nu}=\sqrt{-\eta}(\eta^{\mu\nu}+h^{\mu\nu})$$

$$m_g^2=4k_1\quad,\quad m_0^2=-\frac{2k_1(k_1+4k_2)}{k_1+k_2}\quad (ghost\to k_1+k_2=0)$$

• Effectively massive gravitons (resonances) arise in induced brane gravity.

Dvali+Gabadadze+Porrati

♠All such theories are VERY sensitive in the UV. There are intermediate thresholds where the theory is strongly coupled or depends on UV details.

Vainshtein, Kiritsis+Tetradis+Tomaras, Luty+Porrati+Ratazzi,Rubakov

♠In the FP theory, there is a strong coupling threshold at

$$\Lambda_V\sim\left(\;m_g^4M_P\;\right)^{\frac{1}{5}}\quad,\quad\Lambda_{\rm tuned}\sim\left(\;m_g^2M_P\;\right)^{\frac{1}{3}}$$

Arkani-Hamed+Georgi+Schwartz

• It is suspected that the most improved threshold is $\Lambda\sim\sqrt{\;m_gM_P}$

• If one aspires to use 4d gauge theoriues to desribe observable gravity, he then is forced to have at best a massive, albeit VERY LIGHT 4d graviton.

Kiritsis+Nitti

Massive graviton cosmology

• We consider the cosmology of a Fierz-Pauli theory $\mathcal{L}=\mathcal{L}_{\rm FP}+\mathcal{L}_{\rm matter}$ and a cosmological ansatz

Babak+Grishchuk, Damour+Kogan+Papazoglou

$$g_{00}=-b^2\quad,\quad g_{ij}=a^2\delta_{ij}\quad,\quad d\tau=b\;dt$$

$$G_{\mu\nu}+M_{\mu\nu}=T_{\mu\nu}\quad,\quad M_{\mu\nu}=\frac{m_g^2}{4}\left(2\delta^\alpha_\mu\delta^\beta_\nu-g^{\alpha\beta}g_{\mu\nu}\right)\left[h_{\alpha\beta}-h^\gamma{}_\gamma\eta_{\alpha\beta}\right]$$

• The equations map to
$$\left(\frac{\dot a}{a}\right)^2=\frac{\rho}{3M_P^2}+\rho_{\rm m}$$

$$\rho_{\rm m}=\frac{m_g^2}{4}\left(\frac{2b}{a}+\frac{1}{b^2}-\frac{3}{a^2}\right)\quad,\quad \dot\rho+3\frac{\dot a}{a}(\rho+p)=0\quad,\quad a^2b^3-(a^4+2)b+2a^3=0$$

• Solving we find a late-time positive effective cosmological constant

Kiritsis

$$\rho_{\rm m}=\frac{m_g^2}{2}+\mathcal{O}\left(\frac{1}{a^2}\right)$$

♠Assuming $m_g\sim H_0^{-1}$, the effective vacuum energy is what we measure today. But.... the cutoffs are very low, except $\sqrt{m_gM_P}\sim 10^{-3}-10^{-4}$ eV.

♣There are still signals of the peculiar UV-IR effects here also: higher terms in the potential for the graviton give very sensitive IR contributions.

Massive gravitons in AdS$_{d+1}$/CFT$_d$

The massless gravitons are typically dual to the CFT stress tensor

$$e^{-W(h)} = \int \mathcal{D}A \; e^{-S_{CFT}+\int d^4x \; h_{\mu\nu}T^{\mu\nu}}$$

Energy conservation translates into (linearized) diffeomorphism invariance:

$$x^\mu \to x^\mu + \epsilon^\mu \qquad \to \qquad \partial_\mu T^{\mu\nu} = 0 \qquad \to \qquad W(h_{\mu\nu} + \partial_\mu \epsilon_\nu + \partial_\nu \epsilon_\mu) = W(h_{\mu\nu})$$

$h_{\mu\nu}$ is promoted to a massless 5d graviton. If

$$\partial_\mu T^{\mu\nu} = J^\nu \neq 0$$

then $\Delta_T > d$ and J^ν corresponds to a bulk vector A^ν. This will be massive

$$\partial_\mu J^\mu = \Phi \neq 0 \qquad \qquad \Delta(\Phi) = d + 2$$

in order to the degrees of freedom to match. This is the gravitational Higgs effect

$$M^2_{grav} = d(\Delta_T - d)$$

There is no vDVZ discontinuity for gravitons in AdS

Porrati, Kogan+Mouslopoulos+Papazoglou

Conserved and non-conserved stress tensors

• An example of a non-conserved stress tensor can be obtained by introducing a $(d-1)$-dimensional defect in a CFT$_d$

Karch+Randall

The graviton is massive due to the fact that energy is not conserved (it can leak to the bulk via the defect).

This theory however is not translationally invariant.

• Other (trivial) examples exist typically in any CFT. In $\mathcal{N}$=4 SYM all operators of the type

$$Tr[\Phi^i\Phi^j\cdots\Phi^k D_\mu D_\nu \Phi^l]$$

give rise to massive gravitons, albeit with large (string-scale) masses.

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- Non-trivial examples appear in perturbations of product CFTs

In $CFT_1 \times CFT_2$ both stress tensors are conserved.

$$\partial_\mu T_1^{\mu\nu} = \partial_\mu T_2^{\mu\nu} = 0$$

This should correspond to two massless gravitons that are however non-interacting.

- The dual theory is gravity on $(AdS_{d+1} \times C_1) \times (AdS_{d+1} \times C_2)$

- The two spaces are necessarily distinct

♠The central idea in the following will be to consider products of large-N CFTs that are coupled in the UV.

Massless interacting gravitons

- Have been argued to be impossible in the context of FT
  Aragone+Deser, Boulanger+Damour+Gualtieri+Henneaux

- Have been argued to not be possible in the context of asymptotically flat string theory
  Bachas+Petropoulos

Assume that we have a CFT_2 (dual to an asymptotically AdS_3 theory of gravity) with two conserved stress tensors. This was analyzed in 2d in detail with the following results:

- It is at the heart of the coset construction
  Goddard+Kent+Olive

- It is the key to the generalizations, that use this to factorize the CFT into a product:
  Kiritsis, Dixon+Harvey
  Halpern+Kiritsis

The strategy is to diagonalize the two commuting hamiltonians as well as the action of the full conformal group.

- The product theory can have discrete correlations between the two factors.
  Douglas, Halpern+Obers

- These remarks generalize to other dimensions although they are less rigorous.

- We conclude: two or more massless gravitons are necessarily non-interacting

Interacting product CFTs

It is now obvious that if we couple together (at the UV) two large-N CFTs, one of the two gravitons will became massive

$$S = S_{CFT_1} + S_{CFT_2} + h \int d^d x \; O_1 O_2$$

with $O_i \in CFT_i$ be scalar single-trace operators of dimension Δ_i, with $\Delta_1 + \Delta_2 = d$, and $\langle OO \rangle \sim \mathcal{O}(1)$

- This is necessarily a double-trace perturbation

- When $h \sim \mathcal{O}(1)$, $\beta_{O^2} \sim \mathcal{O}\left(\frac{1}{N}\right)$ and the perturbation is marginal to leading order in $1/N_c$.

- When $h \sim \mathcal{O}(N)$, generically $(O_1)^2$ and $(O_2)^2$ perturbations are also generated, and the perturbation is marginally relevant

Witten, Dymarksy+Klebanov+Roiban

The relevant perturbations are:

$$\delta\langle T^1(x)T^1(y)\rangle = \frac{h^2}{2!}\int d^4z_1 d^4z_2\ \langle T^1(x)T^1(y)O(z_1)O(z_2)\rangle_c\ \langle\tilde{O}(z_1)\tilde{O}(z_2)\rangle_c$$

$$\delta\langle T^1(x)T^2(y)\rangle = \frac{h^2}{2!}\int d^4z_1 d^4z_2\ \langle T^1(x)O(z_1)O(z_2)\rangle_c\ \langle T^2(y)\tilde{O}(z_1)\tilde{O}(z_2)\rangle_c$$

$$\delta\langle T^2(x)T^2(y)\rangle = \frac{h^2}{2!}\int d^4z_1 d^4z_2\ \langle T^2(x)T^2(y)\tilde{O}(z_1)\tilde{O}(z_2)\rangle_c\ \langle O(z_1)O(z_2)\rangle_c$$

• All are of order $\mathcal{O}\left(\frac{h^2}{N^2}\right)$. The subleading corrections scale as higher powers of h but are always $\sim\ N^{-2}$. Therefore

$$M^2_{grav} = \frac{h^2}{N^2}\left[a_1 + a_2 h + a_3 h^2 + \cdots\right] + \mathcal{O}\left(\frac{h^3}{N^4}\right)$$

• The graviton mass is a one-loop effect on the gravitational side.

• $\delta\langle T^1(x)T^2(y)\rangle$ is a trivial correction because it is spacetime independed. The same applies to $\delta\langle O_1(x)O_2(x)O_1(y)O_2(y)\rangle$.

• The corrections to the higher couplings are $\delta\langle T^n\rangle \sim \frac{h^2}{N^2}\langle T^n\rangle$

The graviton mass

The conserved stress tensor is

$$T^{\mu\nu} = T_1^{\mu\nu} + T_2^{\mu\nu} - \frac{h}{2}\, g_{\mu\nu}\ O_1 O_2$$

The orthogonal linear combination is

$$\tilde{T}^{\mu\nu} = c_2 T_1^{\mu\nu} - c_1 T_2^{\mu\nu} - \frac{h}{2d}\left[c_1\Delta_2 - c_2\Delta_1\right]\ g_{\mu\nu}\ O_1 O_2$$

with $\langle T^i_{\mu\nu} T^i_{\rho\sigma}\rangle = c_i \left(g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho} - \frac{2}{d} g_{\mu\nu}g_{\rho\sigma}\right)$ To leading order in h it satisfies

$$\partial^\mu \tilde{T}_{\mu\nu} = h\ (c_1 + c_2)\left[\frac{\Delta_2}{d}(\partial_\nu O_1)O_2 - \frac{\Delta_1}{d}(\partial_\nu O_2)O_1\right]$$

Using

$$|\partial_\mu O|^2 = 2\Delta\ |O|^2 \quad , \quad |\partial_\mu T^{\mu\nu}|^2 = 2c\frac{(d+2)(d-1)}{d}\left(\Delta_{\tilde{T}} - d\right) > 0$$

we finally obtain

$$M^2_{grav} = d\left(\Delta_{\tilde{T}} - d\right) = h^2\left(\frac{1}{c_1} + \frac{1}{c_2}\right)\frac{d}{(d+2)(d-1)}\Delta_1\Delta_2 \quad \sim \quad \mathcal{O}\left(\frac{h^2}{N^2}\right)$$

Aharony+Clark+Karch

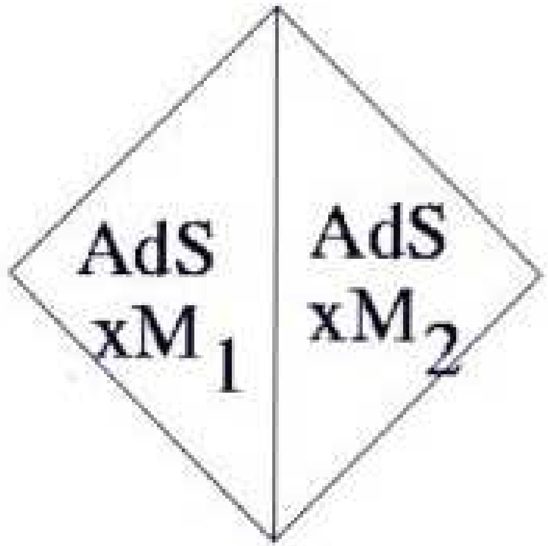
The spacetime picture

What is the spacetime picture?

• A small deformation of the product geometry

$$(\ AdS_{d+1} \times M_1 \ ) \quad \times \quad (\ AdS_{d+1} \times M_2 \ )$$

• As the two CFTs are defined on the same spacetime R^d, the boundaries of the two AdS_{d+1} should be identified. In particular the two holographic directions are distinct.



• In the non-conformal (relevant) case, the AdS spaces are replaced by asymptotically AdS spaces.

Correlated boundary conditions

Description of the perturbation $O_1 O_2$, that couples the two CFTs?

• It is a double-trace perturbation and it is implemented by the "canonical" formalism

Witten

$$\Phi_1 \leftrightarrow O_1 \quad , \quad \Phi_2 \leftrightarrow O_2 \quad , \quad m_1^2 \ell_1^2 = m_2^2 \ell_2^2$$

because $\Delta_1 + \Delta_2 = d$. Their asymptotic behavior is ($\Delta_1 < d/2$)

$$\Phi_{\Delta_1} \sim q_1(x)\ r_1^{\Delta_1} + p_1(x)\ r_1^{d-\Delta_1} \quad , \quad \tilde{\Phi}_{4-\Delta} \sim p_2(x)\ r_2^{\Delta_1} + q_2(x)\ r_2^{d-\Delta_1}$$

$p_1(x)$ and $p_2(x)$ correspond to the expectation values of the associated operators while $q_1(x)$ and $q_2(x)$ correspond to sources.

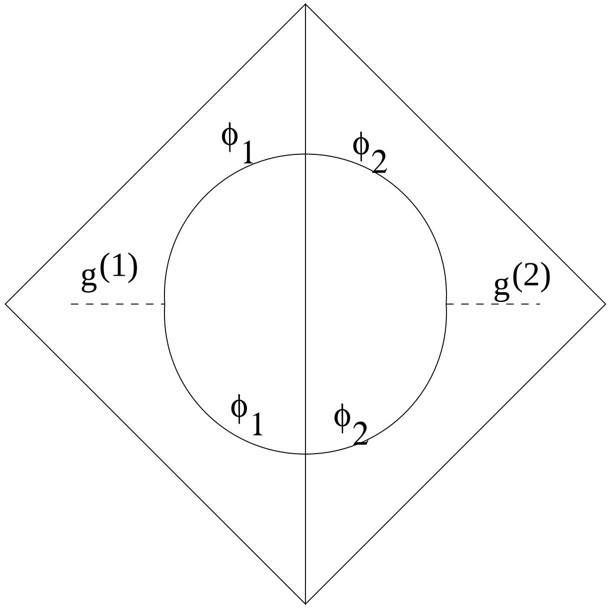
• The perturbation is generated by the bc

$$q_1(x) + h\ p_2(x) = 0 \quad , \quad q_2\ (x) + h\ p_1(x) = 0$$

The full canonical formalism

The gravitational loop calculation

• On the gravity side the graviton mass arises from the loop corrections of the scalars Φ_1 and Φ_2 associated to the perturbing opeartors $O_{1,2}$.



• The "half calculation" corresponds to giving Φ_1 "transparent" boundary conditions and was done already by Porrati, and Duff+Liu+Sati. It does give the graviton a mass.

• The induced mass agrees with the CFT formula

Aharony+Clark+Karch

Power counting

Reinstating dimensionfull quantities we have

$$M^3\ \ell_{\text{AdS}}^3 \sim N^2 \quad , \quad m_g \sim \frac{1}{N\ell_{\text{AdS}}} \quad , \quad \ell_{\text{string}} = \frac{\ell_{\text{AdS}}}{\lambda^{\frac{1}{4}}}$$

- Define the following "cutoff" lengths

$$L_n \equiv (M m_g^n)^{-\frac{1}{n+1}} = \ell_{\text{AdS}}\ N^{\frac{3n-2}{3(n+1)}}$$

- $L_0 \to L_P$, $L_4 \to$ Vainshtein cutoff, $L_2 \to$ Arkani-Hamed, Georgi, Schwartz

cutoff, $L_1 \to$ maximal cutoff.

$$L_P \ll \ell_{\text{string}} \ll \ell_{\text{AdS}} \ll L_1 \ll L_2 \ll L_4$$

$\mathcal{N}=4$ d=4 Super Yang Mills

Consider CFT$_1$=CFT$_2$=$\mathcal{N}=4$ SYM

- The only operators that can be used to deform are the **20**-plet

$$O=\frac{1}{N}\sum_{I=1}^{6}Tr[\Phi^I\Phi^I]\quad,\quad O_{IJ}\equiv\frac{1}{N}\left[Tr[\Phi^I\Phi^J]-\frac{1}{6}\delta^{IJ}\ O\right]$$

so that

$$S_{\text{interaction}}=h_{IJ,KL}\int d^4x\ O_{IJ}\tilde{O}_{KL}$$

- This is generically a marginal perturbation when $h_{IJKL}\sim\mathcal{O}(1)$ but...

- It is non-perturbatively unstable: the resulting potential is unbounded below (easily visible on the Cartan $\Phi^I\to\Phi^I_i$, $i=1,2,\cdots,6$

$$S_{\text{interaction}}=\frac{h_{IJ,KL}}{N_1N_2}\int d^4x\ \left[\Phi^I\cdot\Phi^J-\frac{1}{6}\delta^{IJ}\Phi\cdot\Phi\right]\left[\tilde{\Phi}^I\cdot\tilde{\Phi}^J-\frac{1}{6}\delta^{IJ}\tilde{\Phi}\cdot\tilde{\Phi}\right]$$

The conifold CFT

• This is the $\mathcal{N}=1$ $SU(N)\times SU(N)$ quiver CFT dual to $AdS_5\times T^{1,1}$, with two bi-fundamentals A_i and two anti-bifundamentals B_i and an SU(2)×SU(2)×U(1)$_R$ global symmetry. There is a line of fixed points.

Klebanov+Witten

• It is known that the theory can be deformed keeping conformality and preserving the R-symmetry by

$$W = Tr[A_1B_1A_2B_2 - A_1B_2A_2B_1]$$

leading to a two-parameter family of CFTs

Klebanov+Witten

• It is also known that the double-trace perturbation generated by

$$W = Tr[A_1B_1]Tr[A_2B_2] - Tr[A_1B_2]Tr[A_2B_1]$$

preserves both conformal invariance and R-symmetry.

Aharony+Berkooz+Silverstein

This implies that the deformation of the product of two conifold CFTs (at the same moduli point): $CFT_c \times CFT'_c$ by the double-trace operator

$$W = Tr[A_1 B_1] Tr[A'_2 B'_2] - Tr[A_1 B_2] Tr[A'_2 B'_1]$$

is exactly marginal

- The R symmetry is broken to the diagonal one

$$(SU(2)^2 \times U(1)_R) \times (SU(2)^2 \times U(1)_R)' \to (SU(2)^2 \times U(1)_R)_{\text{diagonal}}$$

The fate of the axial combination is as the gravitons'. The bulk gauge bosons get masses at one-loop.

- The geometry remains $(AdS_5 \times T^{1,1})^2$ pasted back-to-back.

- Nothing is known about the non-perturbative stability of this deformation.

Multiply connected CFTs

- Several copies of a CFT can be coupled together two at a time

$$S = \sum_{i=1}^{M} S_i + \sum_{i<j}^{M} h_{ij} \int d^d x \; O_i O_j \quad , \quad \Delta_i + \Delta_j = d$$

- The combined theory is an asymptotically free theory and in special cases conformal.

- It contains M copies of an AdS_{d+1} coupled via boundary conditions in their common boundary.

- It contains 1 massless and M-1 massive gravitons.

Can we have more than two theories coupled together via a "cubic" or higher "vertex" eg.

$$S = \sum_{i=1}^{M} S_i + \int d^d x \; \prod_{i=1}^{M} O_i \quad , \quad \sum_{i=1}^{M} \Delta_i \leq d$$

• The answer to this question is dimension dependent and we need the unitarity bounds $\Delta_{scalar} \geq \frac{d-2}{2}$, $\Delta_{vector} \geq d-1$, $\Delta_{s=2} \geq d$.

♠In d=6, the maximum possible is a cubic vertex, and $Dim(O) = 2 \to$ O is a free scalar. This is leads to an unstable potential.

♣In d=4 a quartic vertex has a similar fate. But there can be a non-trivial cubic vertex using CFTs with scalar operators with $\Delta \leq 4/3$
SQCD in conformal window $\frac{1}{3} < \frac{N}{N_f} < \frac{2}{3}$.

$$\text{Meson operators} \to \Delta_{\text{meson}} = 3 - 3\frac{N}{N_f}$$

We take the Veneziano limit

$$N \to \infty \quad , \quad N_f \to \infty \quad , \quad x = \frac{N}{N_f} = \text{fixed} \quad , \quad \frac{1}{3} \leq x \leq \frac{2}{3}$$

We may now take the product $SQCD_{x_1} \times SQCD_{x_2} \times SQCD_{x_3}$ with $x_1 + x_2 + x_3 = \frac{5}{3}$

• This cubic vertex can be used also in tadem to connect several CFTs as advoicated earlier.

♠In d=2 the unitarity bound squeezes to zero, and this allows any possible vertex coupling these theories.

• In the example we studied, the 't Hooft couplings can be chosen so that $\Delta_{\Box,\bar{\Box};1} = \frac{1}{k}$, $k = 1, 2, 3 \cdots$

Moreover, fixed points can be found in weak coupling perturbation theory.

• Again the non-perturbative stability of such deformations is not understood.

The relationship to multi-throat geometries

- It is known that multi-throat geometries can arise in the IR of string compactifications

- A prototype of this is the breaking of $U(2N) \to U(N) \times U(N)$ by Higgs vevs

Klebanov+Witten

Here the two large-N throats, are coupled in the IR, but not in the UV

- The dual geometrical picture is very different: one space (with two throats), one graviton (with two localisations).

- There is a simplified RS-like picture where two AdS slices (with in general different cosmological constants) are separated by a RS brane.

Padilla, Gabadadze+Grisa+Shang

It involves a RS graviton and a massive DGP-like bound state.

However, in this case the two cutoff-AdS spaces communicate via the RS brane. This is not the case in the backgrounds that are coupled in the UV. There is an infinite barrier in between.

Directions and open problems

• Are products of AdS spaces the most general dual geometry of large-N CFTs?

• What are other "frame-independent" characteristics of perturbed product large-N CFTs? (beyond $\Delta_{\tilde{T}} = 4 + \mathcal{O}(N^{-2})$.)?

• Analysis of concrete examples where 3 or more CFTs are coupled together. Structure of graviton mass matrix.

• These are examples of UV complete theories of massive gravitons. It is interesting to see how they resolve the problems of Pauli-Fierz truncations, what is the effective UV cutoff, and what is the effective resolution of the strong coupling puzzles of massive graviton theories.

• The question of thermalization of coupled products is correlated with the existence of black-holes in the product space-times. This may shed light in the process of equilibration between coupled reservoirs.

• There seems to be a structure reminding cobordism, but it is certainly distinct. What are the precise rules, and is that interesting mathematically?

• Are such product geometries non-perturbatively stable?

• How much of this survives at small N?

• There are indications that massive gravitons with $mass \sim H_0^{-1} \sim 10^{-33}$ eV can produce today's acceleration. Can the theories here help implement this idea?

• One may extend these ideas to asymptotically flat string backgrounds. This produces clone universes interacting at their asymptotic boundaries. Can this be responsible of what we see in our universe?

Double trace couplings and the RG flow

We normalize single trace operators as

$$\langle O(x)O(y)\rangle = \frac{1}{|x-y|^{2\Delta}} \quad , \quad \langle O^n\rangle \quad \sim \quad \frac{1}{N^{n-2}}$$

We label by O_i operators in CFT$_1$ and O_I operators in CFT$_2$ (all of dimension $d/2$) and perturb

$$\delta S = f_{ij}\int O_iO_j + \tilde{f}_{IJ}\int O_IO_J + g_{iI}\int O_iO_I$$

We may now compute the flow equations by considering

$$\langle O_iO_j\ \delta S\ \delta S\rangle \quad , \quad \langle O_IO_J\ \delta S\ \delta S\rangle \quad , \quad \langle O_iO_I\ \delta S\ \delta S\rangle$$

to obtain

$$\dot{f}_{ij} = -8(f^2)_{ij} - 2(gg^T)_{ij}$$

$$\dot{\tilde{f}}_{IJ} = -8(\tilde{f}^2)_{IJ} - 2(g^Tg)_{IJ}$$

$$\dot{g}_{iI} = -2(g\tilde{f})_{iI} - 2(fg)_{iI}$$

Generically, the couplings are asymptotically free (marginally relevant).

RETURN

The full canonical formalism

Witten, Mück

The perturbed CFT action:

$$I^W = I_{CFT} + \int d^4x \; W(O) \quad , \quad W(O) \to \text{local}$$

The CFT action is related to the bulk supergravity action as

$$\langle \text{exp} \left[- \int d^4x \; \alpha \; O \right] \rangle = \text{exp} \left[-I_{sugra}(q) \right]$$

The source $\alpha(x)$ is related to the asymptotic form of the bulk field Φ

$$\lim_{r \to 0} \Phi(x,r) \sim r^{\Delta} q(x) + r^{4-\Delta} p(x) + \cdots \quad , \quad q(x) + \alpha(x) = 0$$

In the Hamilton-Jacobi formalism, p and q are conjugate variables with

$$p = -\frac{\delta I_{sugra}(q)}{\delta q} \quad , \quad q = \frac{\delta J(p)}{\delta p} \quad , \quad J(p) = I_{sugra} - \int d^4x \; qp$$

The bulk generating functional for the perturbed theory is

$$I^W_{sugra}(\alpha) = I_{sugra}(q) + \int d^4x \; \left(W(p) - p \frac{\delta W}{\delta p} \right) \quad , \quad \frac{\delta I^W_{sugra}}{\delta p} = q + \frac{\delta W(p)}{\delta p} + \alpha = 0$$

The bulk/boundary correspondence translates to:

$$\langle \text{exp} \left[- \int d^4x \; \alpha \; O \right] \rangle_W = \text{exp} \left[-I^W_{sugra}(\alpha) + I^W_{sugra}(0) \right]$$

For the case of interest the perturbed CFT action is

$$I^W = I_{CFT_1} + I_{CFT_2} + \int d^4x\ W(O_\Delta, \tilde{O}_{4-\Delta}) \quad , \quad W(O_\Delta, \tilde{O}_{4-\Delta}) = h\ O_\Delta \tilde{O}_{4-\Delta}$$

The canonical variables are

$$p_i = -\frac{\delta I^i_{sugra}(q_i)}{\delta q_i} \quad , \quad q_i = \frac{\delta J^i(p_i)}{\delta p_i} \quad , \quad J^i(p) = I^i_{sugra} - \int d^4x\ q_i p_i \quad , \quad i = 1, 2$$

The bulk generating functional for the perturbed theory is

$$I^W_{sugra}(\alpha_1, \alpha_2) = I^1_{sugra}(q_1) + I^2_{sugra}(q_2) + \int d^4x\ \left(W(p_1, p_2) - \sum_{i=1}^{2} p_i \frac{\delta W}{\delta p_i} \right)$$

with p_i, q_i determined by the sources α_i

$$\frac{\delta I^W_{sugra}}{\delta p_i} = q_i + \frac{\delta W(p_1, p_2)}{\delta p_i} + \alpha_i = q_i + g\ (\sigma^1)^{ij} p_j + \alpha_i = 0$$

The bulk/boundary correspondence recipe is

$$\langle \exp\left[-\int d^4x\ \left(\alpha_1\ O_\Delta + \alpha_2 \tilde{O}_{4-\Delta} \right) \right] \rangle_W = \exp\left[-I^W_{sugra}(\alpha_1, \alpha_2) + I^W_{sugra}(0, 0) \right]$$

RETURN

Transversality and the graviton mass

Porrati, Duff+Liu+Sati

Most general graviton self-energy in AdS satisfying the Ward identities is

$$\Sigma_{\mu\nu;\alpha\beta} = \beta(\Delta)\Pi_{\mu\nu;\alpha\beta} + \gamma(\Delta)K_{\mu\nu;\alpha\beta} \quad , \quad \Delta \to \text{Lichnerowicz}$$

$$\Pi_{\mu\nu}{}^{\alpha\beta} = \delta^\alpha_\mu\delta^\beta_\nu - \frac{1}{3}g_{\mu\nu}g^{\alpha\beta} + 2\nabla_\mu\left(\frac{\delta^\beta_\nu + \nabla_\nu\nabla^\beta/2\Lambda}{\Delta - 2\Lambda}\right)\nabla^\alpha - \frac{\Lambda}{3}\left(g_{\mu\nu} + \frac{3}{\Lambda}\nabla_\mu\nabla_\nu\right)\frac{\left(g_{\alpha\beta} + \frac{3}{\Lambda}\nabla_\alpha\nabla_\beta\right)}{3\Delta - 4\Lambda}$$

$$K_{\mu\nu}{}^{\alpha\beta} = \frac{\Delta - \Lambda}{3\Delta - 4\Lambda}d_{\mu\nu}d^{\alpha\beta} \quad , \quad d_{\mu\nu} = g_{\mu\nu} + \frac{1}{\Delta - \Lambda}\nabla_\mu\nabla_\nu \quad , \quad \Lambda = -\frac{3}{\ell^2_{AdS}}$$

Consider the kinetic graviton operator, and the linearized equation of motion

$$\left[\frac{1}{16\pi G}D_{\mu\nu}{}^{\alpha\beta} + \Sigma_{\mu\nu}{}^{\alpha\beta}\right]h_{\alpha\beta} = 0 \quad , \quad \Sigma_{\mu\nu}{}^{\alpha\beta} = \frac{c}{2\ell^4_{AdS}}\Pi_{\mu\nu}{}^{\alpha\beta}$$

Using $K * h = -\frac{M^2}{2}h$ we obtain

$$M^2_{grav} = (16\pi G)\frac{c}{\ell^4_{AdS}}$$

Ad$_{d+1}$ Scalar propagators

We will use homogeneous coordinates, X^μ, to embed AdS$_{d+1}$ in $R^{(2,d-1)}$:

$$X \cdot X = -\ell_{AdS}^2$$

The propagator from X to Y is a function of $Z = X^\mu Y_\mu$ and satisfies

$$\left[(1 - Z^2)\partial_Z^2 - (d+1)Z\partial_Z + L(L-d)\right] D_L = 0 \quad , \quad m^2\ell_{AdS}^2 = L(L-d)$$

Boundary conditions are parametrized by α and β as follows

$$D_{1,d-1}(Z) = \frac{1}{(Z^2-1)^{\frac{d-1}{2}}}\left[\alpha + \beta Z \;\; F\left(\frac{1}{2}, \frac{3-d}{2}, \frac{3}{2}, Z^2\right) \;\right]$$

Here $\alpha = 1$, $\beta = 0$ are the boundary conditions conserving energy and momentum across rthe bounadary. On the other hand $\alpha = \beta = 1$ are "transparent" boundary conditions.

In the case of the double trace perturbation the full propagator is

Mück, Aharony+Berkooz+Katz

$$G = \frac{1}{1+\hat{h}^2}\begin{pmatrix} D_1+\hat{h}^2 D_2 & \hat{h}(D_1-D_2) \\ \hat{h}(D_1-D_2) & D_2+\hat{h}^2 D_1 \end{pmatrix} \quad , \quad \hat{h} = (2\Delta_1 - d)\ h$$

for $CFT_1 = CFT_2$

• This can be used to calculate the 2×2 matrix $\langle g_i^{\mu\nu}(x) g_j^{\rho\sigma}(y)\rangle$ and from this extract the graviton mass

RETURN

Examples in two dimensions

The simplest coupling between two distinct CFTs in 2d is a current-current coupling

$$S = S_1 + S_2 + g\int d^2z\ J_1\bar{J}_2 \quad , \quad \partial\bar{J}_2 = \bar{\partial}J_1 = 0$$

and this is always an exactly marginal perrturbation. It provides a boost of the Charge lattice $Q_1 \times Q_2$

• This may not have a large-N interpretation generically, but it has the basic property of the double-trace perturbation: only disconnected correlators survive.

• A good example of a solvable large-N CFT in 2d is the conformal coset

$$\frac{SU(N)_{k_1} \times SU(N)_{k_2}}{SU(N)_{k_1+k_2}}$$

• It is the IR limit of an SU(N) gauge theory. The 't Hooft coupling constants are

$$\lambda_1 = \frac{N}{k_1} \quad , \quad \lambda_2 = \frac{N}{k_2}$$

- The large N limit involves

$$N \to \infty \quad , \quad \lambda_{1,2} = \text{fixed}$$

$$c = \frac{(\lambda_1 + \lambda_2 + 2\lambda_1\lambda_2)}{(1+\lambda_1)(1+\lambda_2)(\lambda_1+\lambda_2+\lambda_1\lambda_2)}(N^2-1)$$

- One single trace operator is $\Phi_{\Box,\bar{\Box};1}$ with

$$\Delta_{\Box,\bar{\Box};1} = \frac{1}{2}\left[\frac{\lambda_1}{(1+\lambda_1)} + \frac{\lambda_2}{(1+\lambda_2)}\right] + \mathcal{O}\left(\frac{1}{N}\right)$$

It can be used to couple together two such theories, provided the λ_i are appropriately chosen.

- For $\lambda_i = 1 - \epsilon_i$, $\epsilon_i \ll 1$, there is a fixed point in perturbation theory.

Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 3 minutes
- The gauge theory/string-theory correspondence 4 minutes
- Some questions for gravity 6 minutes
- The quick answers 8 minutes
- Massive gravitons at low energy 10 minutes
- Massive graviton cosmology 13 minutes
- Massive gravitons in $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ 15 minutes
- Conserved and non-conserved stress tensors 18 minutes
- Massless interacting gravitons 21 minutes
- Interacting product CFTs 23 minutes
- The graviton mass 26 minutes
- The spacetime picture 28 minutes
- Correlated boundary conditions 30 minutes
- The gravitational loop calculation 32 minutes
- Power counting 34 minutes

- $\mathcal{N} = 4$ d=4 Super Yang Mills 36 minutes
- The conifold CFT 39 minutes
- Multiply connected CFTs 44 minutes
- The relationship to multithoat geometries 46 minutes
- Directions and open problems 51 minutes
- The double trace couplings and the RG flow 53 minutes
- The full canonical formalism 55 minutes
- Transversality and the graviton mass 57 minutes
- AdS Scalar Propagators 59 minutes
- Examples in 2d 63 minutes