Scale invariant cosmological perturbations without inflation?

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based upon

"Tensor perturbations in quantum cosmological backgrounds"

{ JCAP 07, 014 (2005). [hep-th/0509232] }

"Gravitational wave background in perfect fluid quantum cosmology"

{ PRD 73, 104017 (2006) . [gr-qc/0605060] }

"A non inflationary model with scale invariant cosmological perturbations"

{ PRD ??, ???? (2006 - 7). [hep-th/0610205 }

P.P., E. Pinho and N. Pinto Neto

Inflation:

- ☺ **solves cosmological puzzles**
- ☺ **uses GR + scalar fields [(semi-)classical]**
- ☺ **can be implemented in high energy theories**
- ☺ **makes falsifiable predictions**
- ☺ **is consistent with all known observations**

Alternative model???

- **string based ideas (PBB, branes, string gas, …)**
- **singularity and initial conditions**
- **bounces**
- **provide challengers!**

Approach: Quantum cosmology (4D)

$$\mathcal{S} = -\frac{1}{6\ell_{\rm pl}^2}\int\sqrt{-g}\,R\mathrm{d}^4x + \int\sqrt{-g}\,p\mathrm{d}^4x$$

Perfect fluid: $p = \omega\rho$ → **bounce**

- **no horizon problem**
- **homogeneity = anthopic solution?**
- **flatness = time?**
- **monopoles = ???**

Results:

$$n_{_{\rm T}} = n_{_{\rm S}} - 1 = \frac{12\omega}{1+3\omega}$$

$$\frac{T}{S} \simeq 4\times 10^{-2}\sqrt{n_{_{\rm S}} - 1}$$

Digression: about QM

Schrödinger

$$i\frac{\partial \Psi}{\partial t} = \left(-\frac{\nabla^2}{2m} + V\right)\Psi$$

Polar form of the wave function $\Psi = A\mathrm{e}^{iS}$

Hamilton-Jacobi

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V + Q = 0$$

quantum potential $= -\frac{1}{2m}\frac{\nabla^2 A}{A}$

Ontological interpretation (BdB) $\exists \; x(t)$

Trajectories satisfy $$m\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\nabla\left(V+Q\right)$$

Properties:

- **strictly equivalent to Copenhagen QM**
  - **probability distribution (attractor)**

$$\exists t_0; \rho(x,t_0) = |\Psi(x,t_0)|^2$$

- **classical limit well defined** $Q \to 0$
- **state dependent**
- **intrinsic reality**
  - **non local …**
- **no need for external classical domain!**

Quantum cosmology

$$\mathrm{d}s^2 = N^2(\tau)\mathrm{d}\tau - a^2(\tau)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Perfect fluid: Schutz formalism ('70)

$$p = p_0 \left[\frac{\dot{\varphi}+\theta\dot{s}}{N(1+\omega)}\right]^{\frac{1+\omega}{\omega}} \exp\left(-\frac{s}{s_0\omega}\right)$$

$(\varphi, \theta, s) =$ **Velocity potentials**

canonical transformation:

$$T = -p_s \mathrm{e}^{-s/s_0} p_\varphi^{-(1+\omega)} s_0 \rho_0^{-\omega} \qquad \ldots$$

+ rescaling (volume …) + units … : simple Hamiltonian:

$$H = \left(-\frac{p_a^2}{4a} - \mathcal{K}a + \frac{p_T}{a^{3\omega}} \right) N \quad N \to a^{3\omega}$$

Wheeler-De Witt $\qquad H\Psi = 0$

$$i\frac{\partial\Psi}{\partial T} = \frac{1}{4} a^{(3\omega-1)/2} \partial_a \left[a^{(3\omega-1)/2} \partial_a \right] \Psi + \mathcal{K}a\Psi$$

$$\mathcal{K} = 0 \implies \chi \equiv \frac{2a^{3(1-\omega)/2}}{3(1-\omega)} \Rightarrow i\frac{\partial\Psi}{\partial T} = \frac{1}{4}\frac{\partial^2\Psi}{\partial\chi^2}$$

space defined by $\chi > 0$ → **constraint (cf. Schrödinger & well)**

$$\bar{\Psi}\frac{\partial\Psi}{\partial\chi} = \Psi\frac{\partial\bar{\Psi}}{\partial\chi}$$

+ Gaussian initial wave function …

alternative way of getting the solution:

WKB exact superposition: $\Psi = \int \mathrm{e}^{iET}\rho(E)\psi_{_E}(T)\mathrm{d}E$

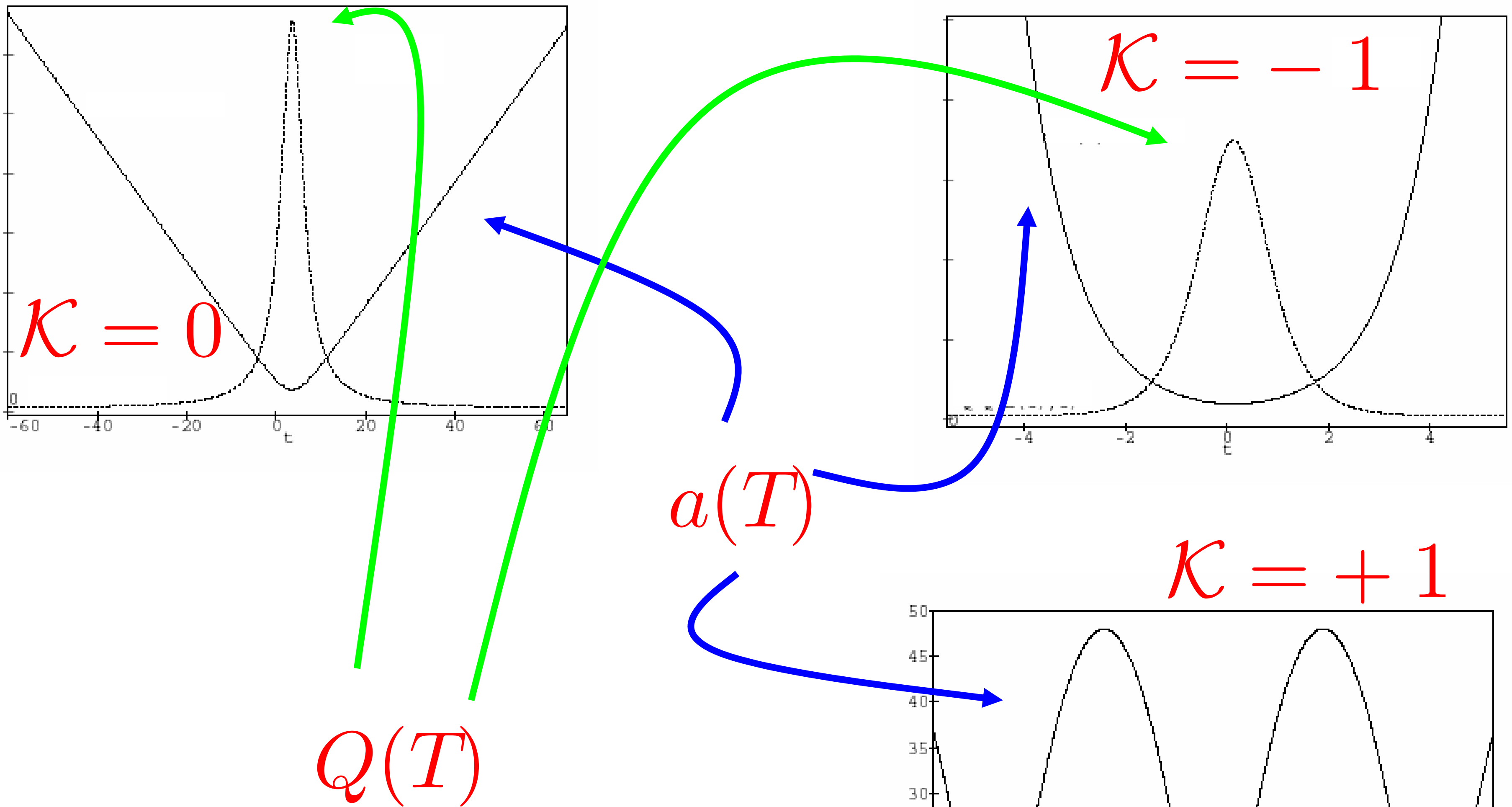
Gaussian wave packet ← $\propto \mathrm{e}^{-(ET_0)^2}$

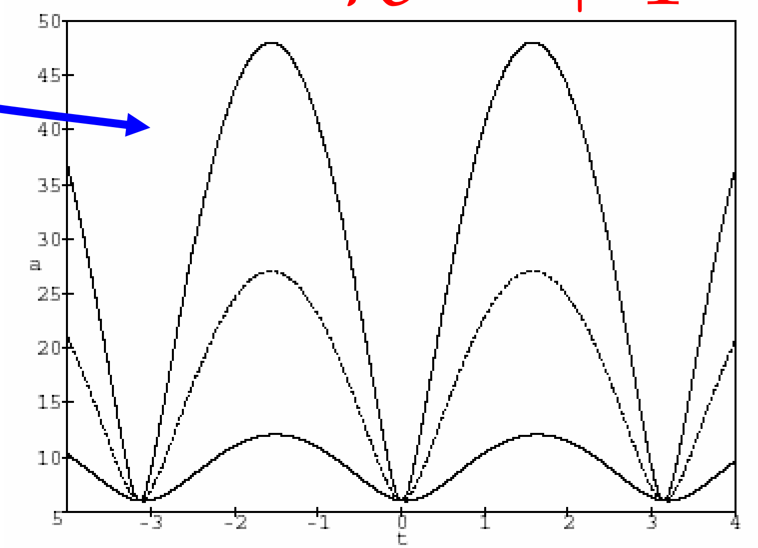
$$\Psi = \left[\frac{8T_0}{\pi\left(T_0^2+T^2\right)^2}\right]^{\frac{1}{4}} \exp\left(-\frac{T_0\chi^2}{T_0^2+T^2}\right)\mathrm{e}^{-iS(\chi,T)}$$

phase $$S = \frac{T\chi^2}{T_0^2+T^2} + \frac{1}{2}\arctan\frac{T_0}{T} - \frac{\pi}{4}$$

Bohmian trajectory $\dot{a} = \{a, H\}$

$$a = a_0\left[1+\left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-\omega)}}$$





quantum potential

[Acacio de Barros, Pinto-Neto & Sagioro-Leal, Phys. Lett. A **241**, 229 (1998)]

What about the perturbations?

Hamiltonian up to 2^{nd} order $H = H_{(0)} + H_{(2)} + \cdots$

factorization of the wave function

$$\Psi = \Psi_{(0)}(a, T)\Psi_{(2)}\left[v, T; a(T)\right]$$

comes from 0^{th} order

$$\Delta\Phi = -\frac{3\ell_{\text{Pl}}^2}{2}\sqrt{\frac{\rho+p}{\omega}}a\left(v/a\right)'$$

Bardeen gravitational potential

conformal time $\mathrm{d}\eta = a^{3\omega-1}\mathrm{d}T$

+ canonical transformations: (Pinho & Pinto-Neto, hep-th/0610192)

$$i\frac{\partial\Psi_{(2)}}{\partial\eta} = \int \mathrm{d}^3x \left(-\frac{1}{2}\frac{\delta^2}{\delta v^2} + \frac{\omega}{2} v_{,i}v^{,i} - \frac{a''}{2a} \right) \Psi_{(2)}$$

Fourier mode

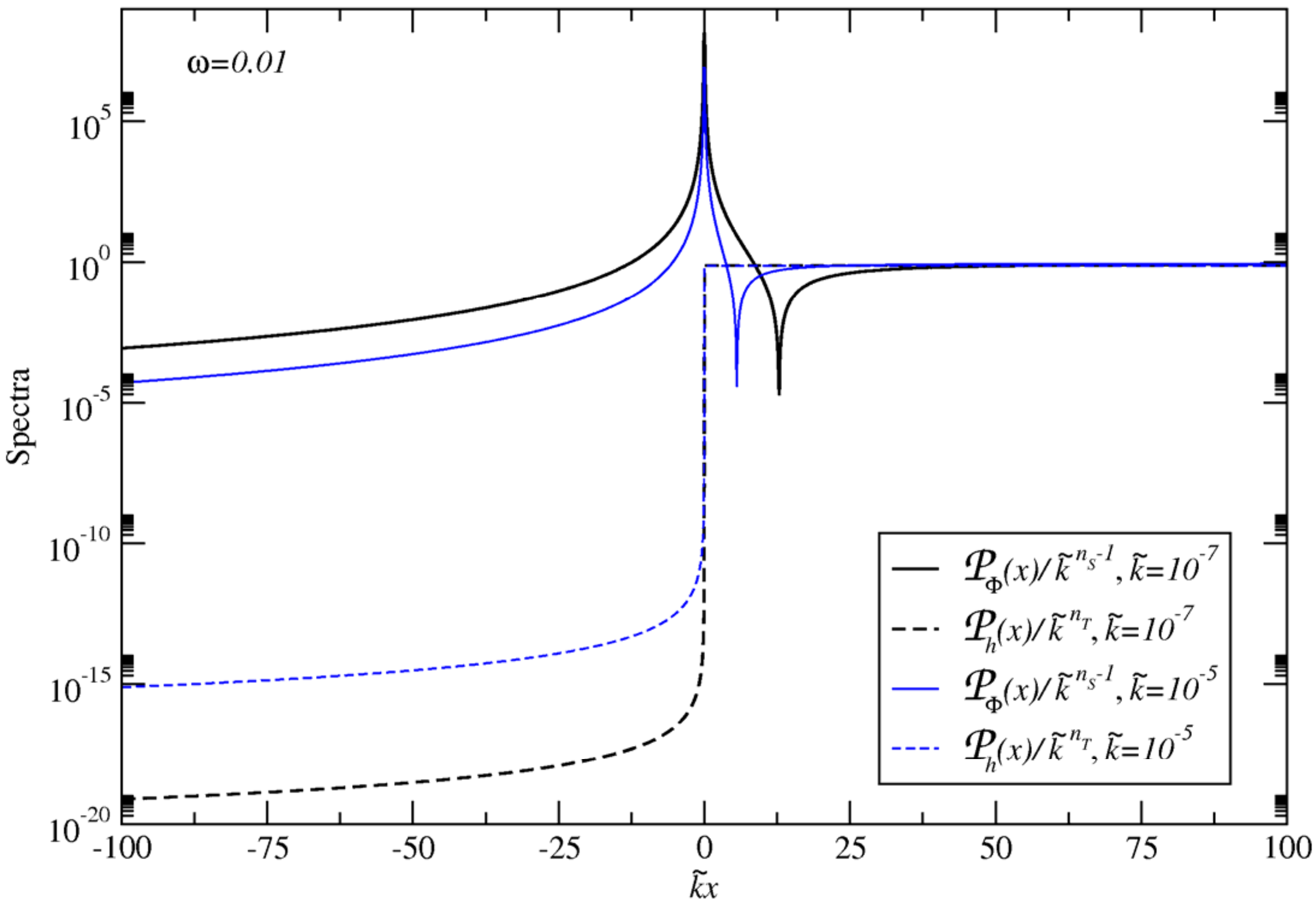
$$v_k'' + \left(c_{_\mathrm{S}}^2 k^2 - \frac{a''}{a} \right) v_k = 0$$

$$c_{_\mathrm{S}} = \sqrt{\omega} \neq 0$$

vacuum initial conditions

$$v_k \propto \frac{\exp(-\,i c_{_\mathrm{S}} k\eta)}{\sqrt{2 c_{_\mathrm{S}} k}}$$

+ evolution (matchings and/or numerics)


ω=0.01
Spectra
10^5
10^0
10^{-5}
10^{-10}
10^{-15}
10^{-20}
-100
-75
-50
-25
0
25
50
75
100
$\tilde{k}x$
$\mathcal{P}_\Phi(x)/\tilde{k}^{n_S-1}, \tilde{k}=10^{-7}$
$\mathcal{P}_h(x)/\tilde{k}^{n_T}, \tilde{k}=10^{-7}$
$\mathcal{P}_\Phi(x)/\tilde{k}^{n_S-1}, \tilde{k}=10^{-5}$
$\mathcal{P}_h(x)/\tilde{k}^{n_T}, \tilde{k}=10^{-5}$

spectrum $\mathcal{P}_{_\Phi} \propto k^3|\Phi|^2 \propto A_{_\mathrm{S}}^2 k^{n_{_\mathrm{S}}-1}$

id. grav. waves: $\mu'' + \left(k^2 - \frac{a''}{a}\right)\mu = 0$

$$\mu = \frac{h}{a} \qquad \mu_{\mathrm{ini}} \propto \frac{\exp(-ik\eta)}{\sqrt{k\eta}}$$

$$\mathcal{P}_h \propto k^3|h|^2 \propto A_{_\mathrm{T}}^2 k^{n_{_\mathrm{T}}}$$

same dynamics + initial conditions ⇨ **same spectrum**

$$n_{_\mathrm{T}} = n_{_\mathrm{S}} - 1 = \frac{12\omega}{1+3\omega}$$

CMB normalisation $A_{_\mathrm{S}}^2 = 2.08 \times 10^{-10}$

⇨ **bounce curvature** $T_0 a_0^{3\omega} \simeq 1500\,\ell_{_\mathrm{Pl}}$

WMAP3 constraint

$$n_{\rm S} = 0.96 \pm 0.02 \Longrightarrow n_{\rm S} < 1.01 \Longrightarrow \omega < 8 \times 10^{-4}$$

predictions

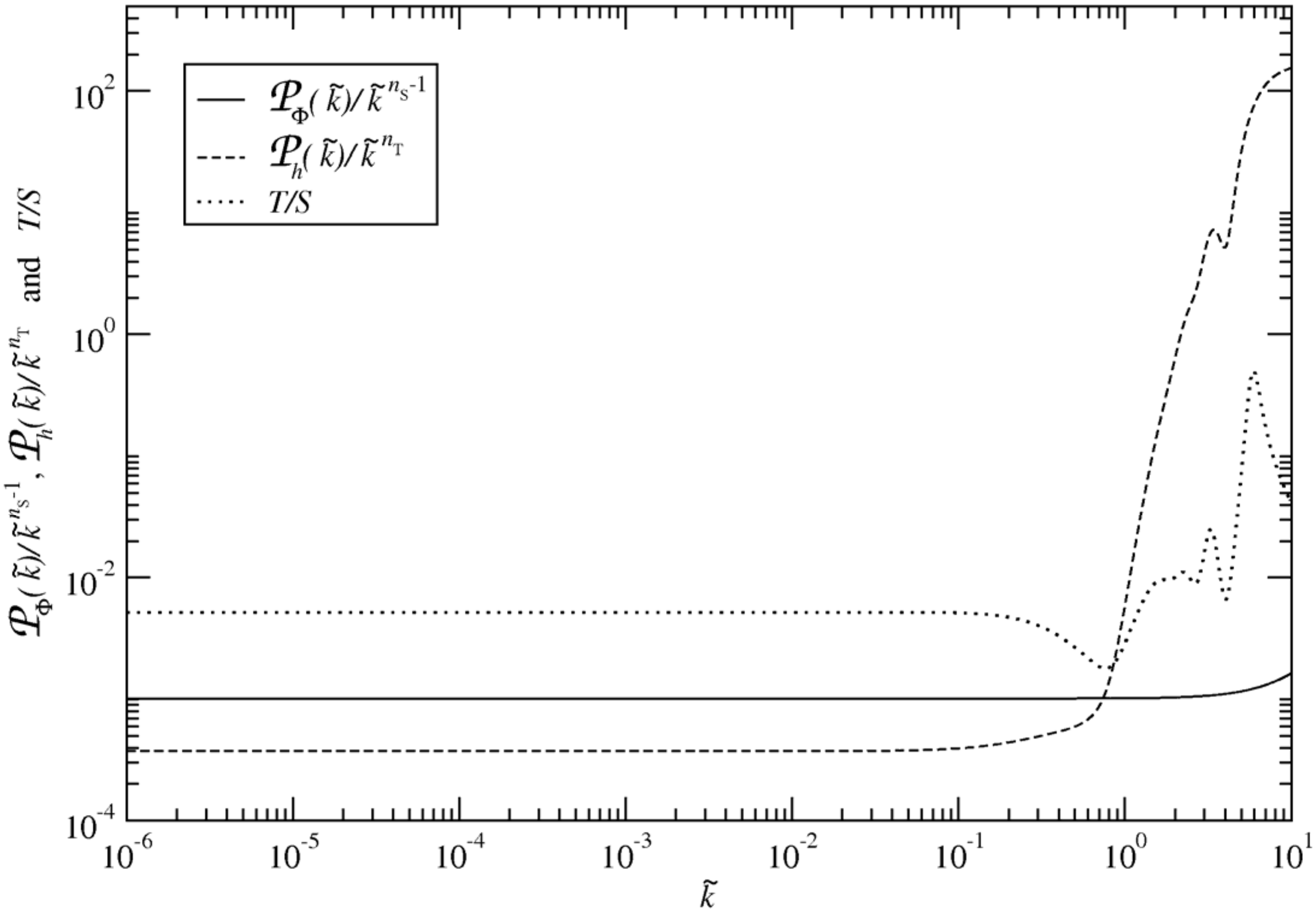
- **spectrum slightly blue**

power-law + concordance

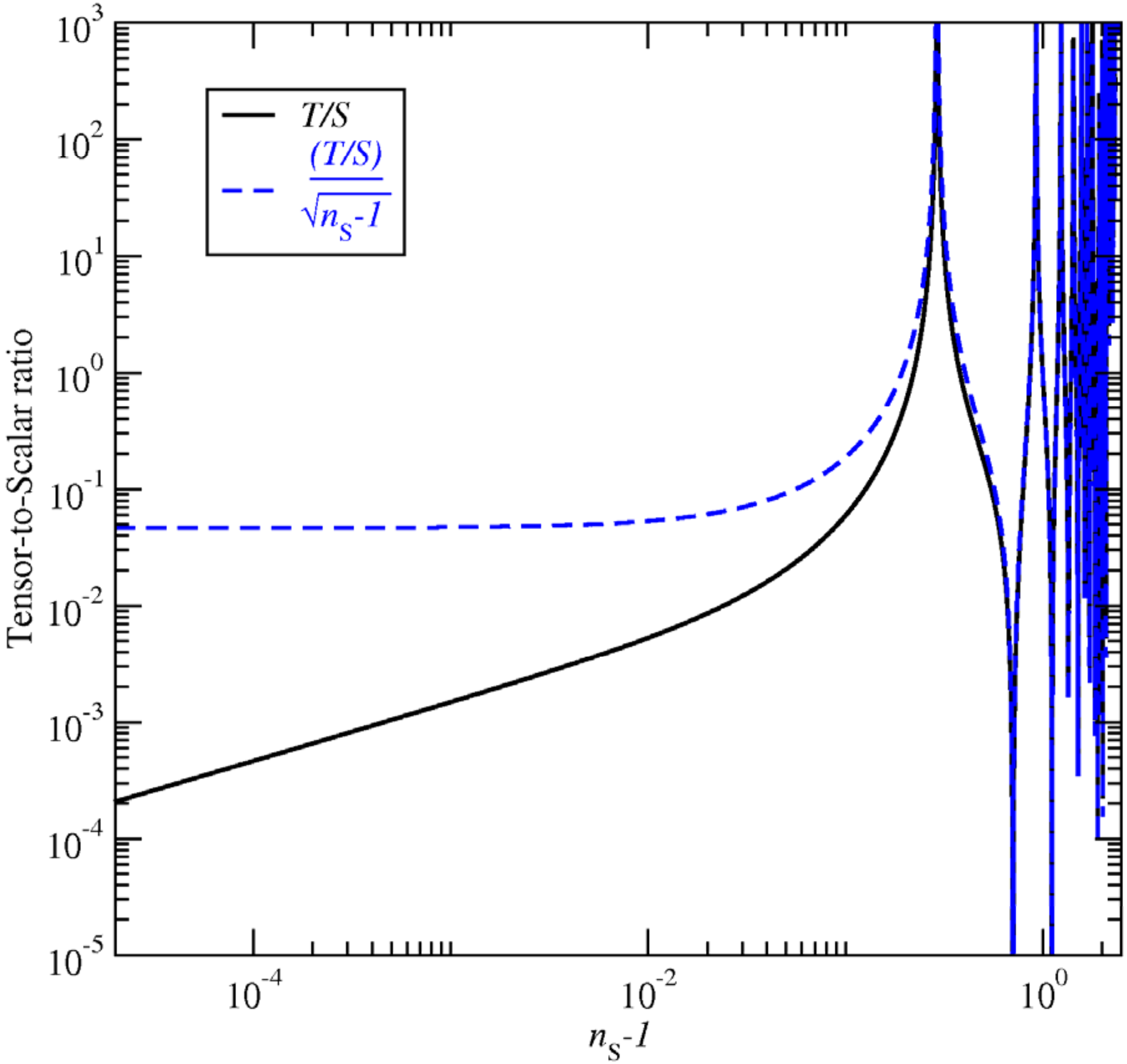
$$\simeq 0.62$$

$$\frac{T}{S} = \frac{c_{10}^{(\rm T)}}{c_{10}^{(\rm S)}} = \mathcal{F}(\Omega, \cdots) \frac{A_{\rm T}^2}{A_{\rm S}^2} \propto \sqrt{\omega}$$

$$\frac{T}{S} \simeq 4 \times 10^{-2} \sqrt{n_{\rm S} - 1}$$



$\mathcal{P}_\Phi(\tilde{k})/\tilde{k}^{n_S-1}$
$\mathcal{P}_h(\tilde{k})/\tilde{k}^{n_T}$
T/S
$\mathcal{P}_\Phi(\tilde{k})/\tilde{k}^{n_S-1}$, $\mathcal{P}_h(\tilde{k})/\tilde{k}^{n_T}$ and T/S
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1}
$\tilde{k}$


T/S
(T/S)
√nS-1
Tensor-to-Scalar ratio
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-4
10^-2
10^0
nS-1

In short: alternative model, QC in 4D

$$\mathcal{S} = -\frac{1}{6\ell_{\rm pl}^2}\int\sqrt{-g}\,R\,{\rm d}^4x + \int\sqrt{-g}\,p\,{\rm d}^4x$$

Perfect fluid: $p = \omega\rho$ → **bounce**

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