# Domain Walls as Probes of Gravity

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What: Gravitational effect of Domain Walls in IR-modified gravity

Why: Gravitating DW solutions in DGP can be found ***exactly***.
Useful information about intrinsically non-linear theory.

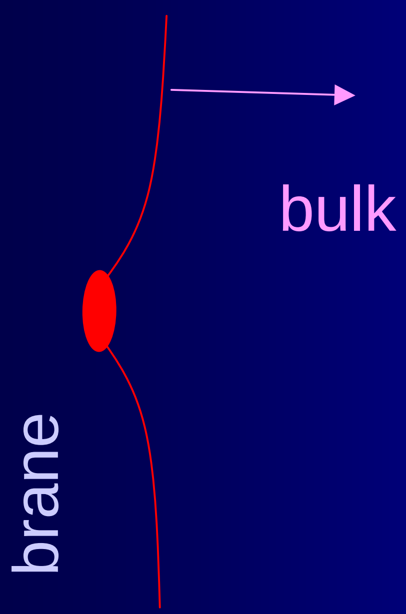
Main Result 1) Screening of 4D and 5D tension.
Analog to screening in Schwarzchild-like solutions.

Main Result 2) Domain Walls are short-distance probes
of large-distance modified gravity.

# Dvali Gabadadze Porrati (DGP)

$$S = \int d^5x \sqrt{-g}\, \frac{M_*^3}{2}\, R_5 + \int d^4x \sqrt{-h}\, \frac{m_P^2}{2}\, R_4$$

$$r_c = \frac{m_P^2}{2M_*^3}$$




$$G_{AB}^{(5)} = 0 \qquad \left( x_5 \neq 0 \right)$$

$$M_*^3 \left( K_{\mu\nu} - K\, h_{\mu\nu} \right) = T_{\mu\nu} - m_P^2\, G_{\mu\nu}^{(4)}$$

# vDVZ discontinuity

IR modified

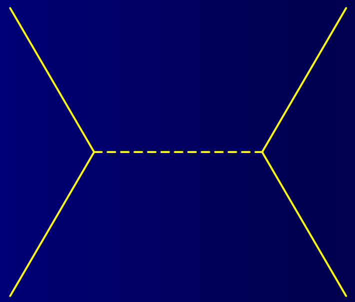
(more d.o.f.'s)

$$\mathcal{A} \;\propto\; G_N \, \frac{T_{\mu\nu}T'^{\mu\nu} - \frac{1}{3}TT'}{\Box - m^2(\Box)}$$

$$m^2(\Box) \;=\; \sqrt{\Box}/r_c$$

GR

$$\mathcal{A} \;\propto\; G_N \, \frac{T_{\mu\nu}T'^{\mu\nu} - \frac{1}{2}TT'}{\Box}$$



way out: higher orders in Perturbation Theory become important at short distances

# Non-perturbative continuity

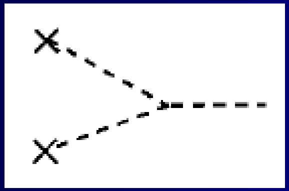
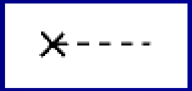
$$r_* = \left( r_g r_c^{\ 2} \right)^{1/3}$$

(DGP)

$r_g = 2 G_N M$ $\qquad$ $r_*$ $\qquad$ $r_c$

4D GR $\qquad$ 4D scalar-tensor $\qquad$ 5D

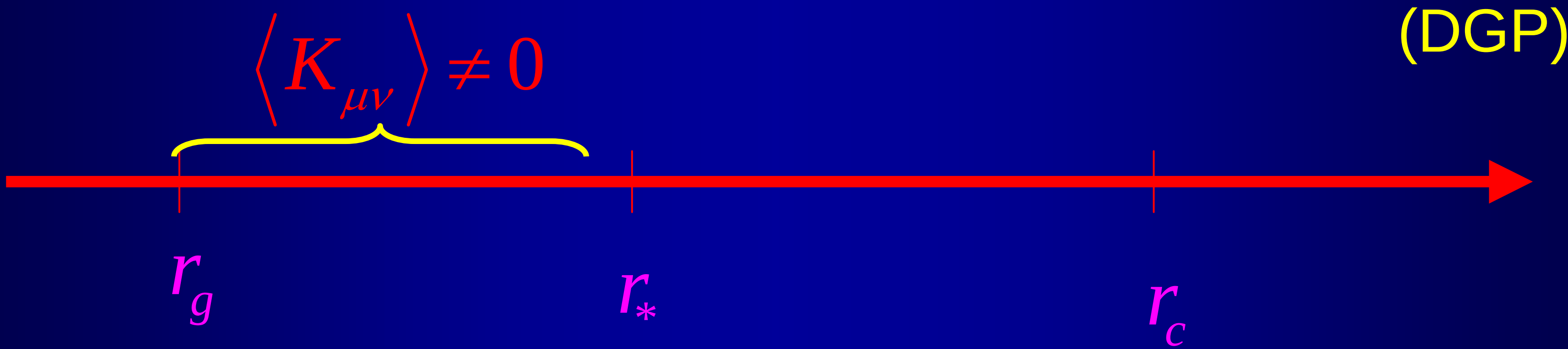
Non-linearities:

 $\gg$ 

Weak field expansion breaks down.

Large $r_c$ expansion is OK.

# Non-perturbative continuity



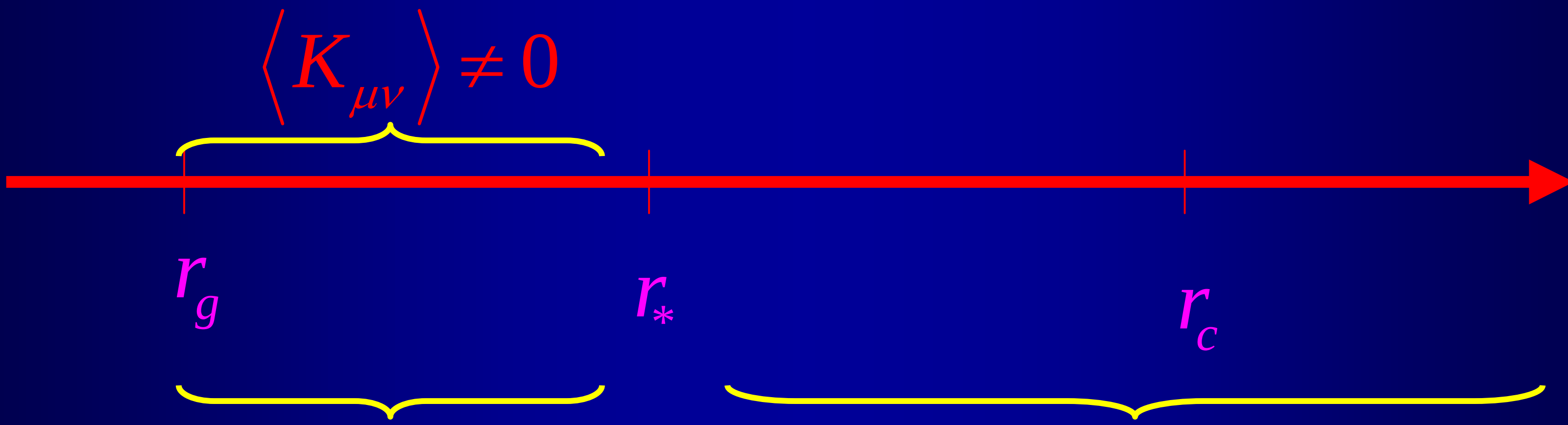

4D GR | 4D scalar-tensor | 5D

$$\phi_N \simeq -\frac{r_g}{r} + \frac{r^2}{r_c^2}\left[\frac{r_*}{r}\right]^{3/2} \qquad -\alpha\frac{r_g}{r} \qquad -\beta\frac{2G_5M}{r^2}$$

Schwarzchild-AdS like

# ‘Schwarzchild-DGP’

(Gabadadze-Iglesias)




$$\phi_N \simeq -\frac{r_g}{r} + \frac{r^2}{r_c^2}\left[\frac{r_*}{r}\right]^{3/2-\beta}$$

$$\beta \simeq 0.04$$

$$\phi_N \simeq -\frac{r_* r_g}{r^2}$$

4D Screening (Total)

5D Screening (Partial)

# ‘Schwarzchild-DGP’

(Gabadadze-Iglesias)

$$\phi_N \simeq -\frac{2G_4 M^{(4D)}}{r} - \frac{2G_5 M^{(5D)}}{r^2}$$

Screening:

| branch | $M^{(4D)}/M$ | $M^{(5D)}/M$ |
|---|---|---|
| CB | 0 | $0.56\,\frac{r_*}{r_c}$ |
| SAB | 0 | $-0.45\,\frac{r_*}{r_c}$ |

# OUTLINE

- Domain Walls in GR
- Strings in GR (sub-critical, super-critical)
- Domain Walls on a DGP brane
    - Conventional Branch
    - Self-Accelerated Branch
- Screening of the tension (both 4D and 5D)
- Extension to other IR-modified gravity

# Domain Walls in GR
# (co-dimension=1)

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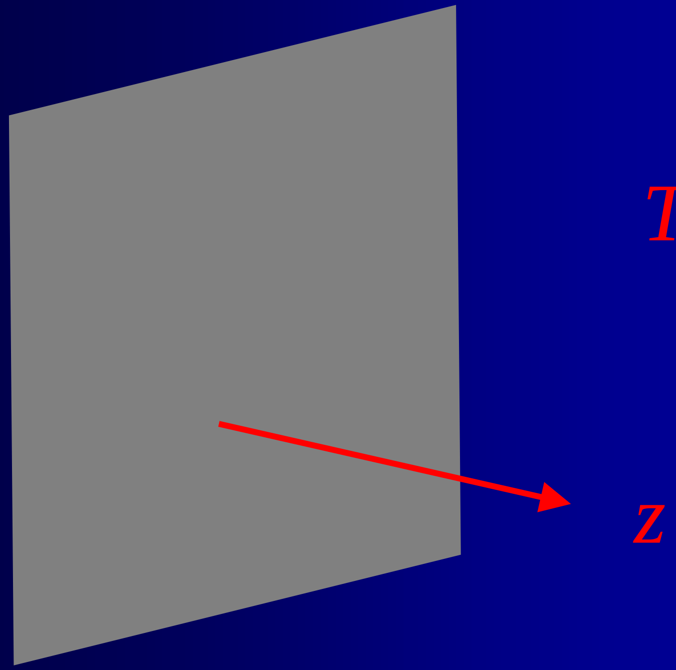
# (co-dimension=1)




$$T_{\mu\nu} = \sigma\ \delta(z)\ \mathrm{diag}\left(1,-1,-1,0\right)_{\mu\nu}$$

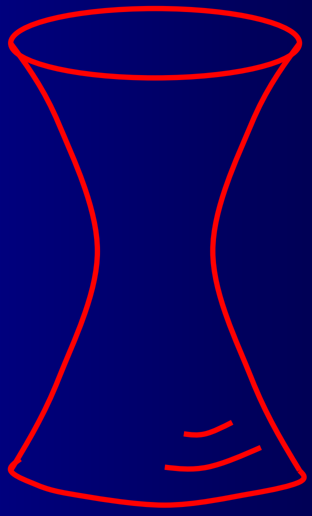
# Domain Walls in GR (co-dimension=1)

$$ds^2 = \left(1 - H\,|\,z\,|\right)^2 \left(-dt^2 + e^{2H\,t}\left[dx^2 + dy^2\right]\right) + dz^2$$

$$H = \frac{\sigma}{4m_P^2}$$ horizon

DW inflates.

Hubble rate= *H*



Vilenkin-Ipser-Sikivie

# Domain Walls in GR
# (co-dimension=1)

$$ds^2 = (1 - H|z|)^2 \left(-dt^2 + e^{2Ht}\left[dx^2 + dy^2\right]\right) + dz^2$$

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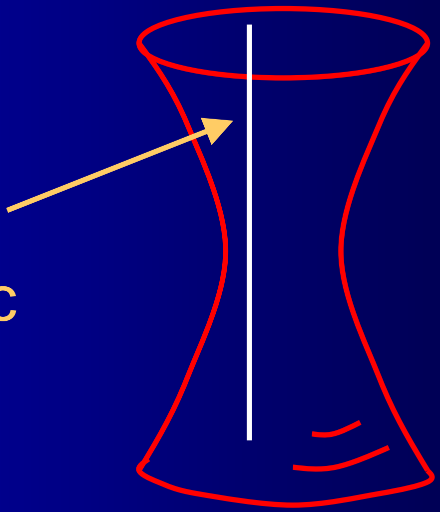
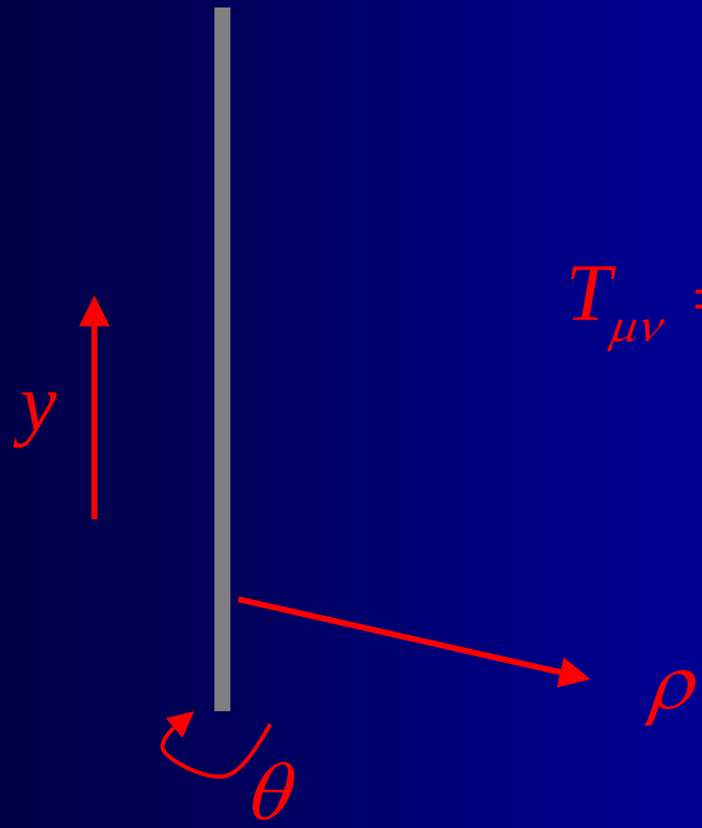


Repulsive force ⟵ geodesic

# Strings in GR
# (co-dimension=2)



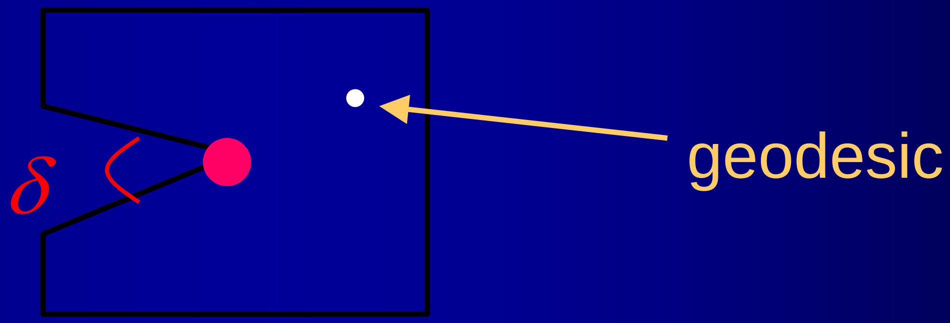

$$T_{\mu\nu} = \mu\ \delta^2(\rho)\ \mathrm{diag}\left(1,-1,0,0\right)_{\mu\nu}$$

# Strings in GR
# (co-dimension=2)

$$ds^2 = \left(-dt^2 + dy^2\right) + d\rho^2 + \left(1 - \frac{\mu}{m_P^2}\right)^2 \rho^2 d\theta^2$$

$$\delta = 2\pi \frac{\mu}{m_P^2}$$ Deficit angle

No force
(on static particles)

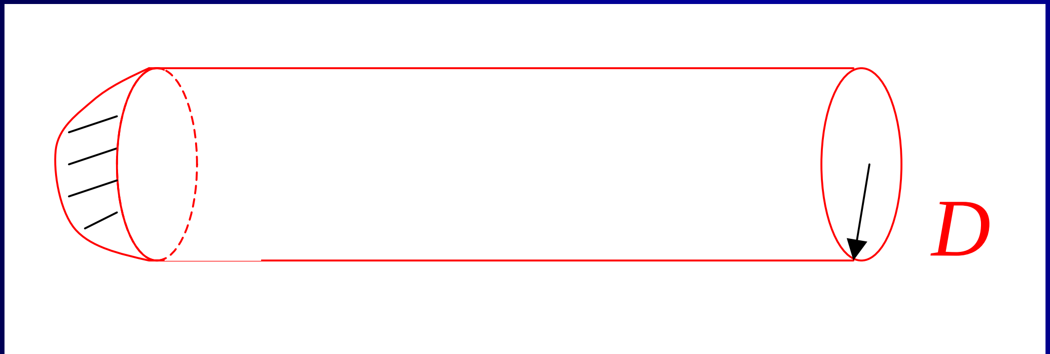


# Strings in GR
# (co-dimension=2)

$\mu = 2\pi\ m_P^2$ 'critical string' $\delta = 2\pi$

$$ds^2 = \left(-dt^2 + dx^2\right) + d\rho^2 + D^2 d\theta^2 \qquad D = \text{thickness}$$

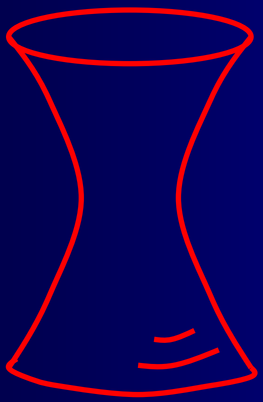
Transverse space compactified.




(flat worldsheet)

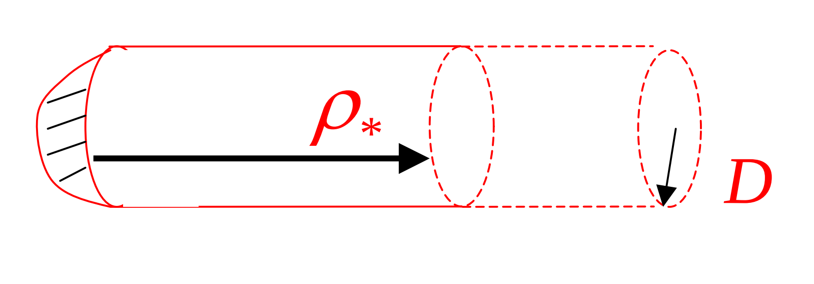
# Strings in GR
# (co-dimension=2)

$\mu > 2\pi \ m_P^2$ 'super-critical string' $\delta = 2\pi$



(Transverse space compactified)

***Topological Inflation*** $H_* \simeq \frac{\mu - 2\pi m_P^2}{4D \ m_P^2}$




Horizon at $\rho_* = 1/H_*$

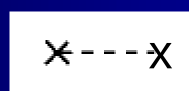
$$\frac{R_0}{r_c} \sim \frac{\sigma}{M_*^3}$$

Critical tension $\sigma_c \sim M_*^3$

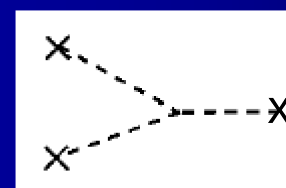
Deficit angle $\delta \sim 2\pi$

# Domain Walls in DGP: Perturbative arguments

What is $r_*$ for a DW??



$\approx$



$= 0!!$

$$(\partial\chi)^2 \qquad (\partial\chi)^2 r_c^2 \partial^2 \chi$$

$$(z \gg r_*)$$

$$\Box\chi = \frac{T}{m_P} = \frac{\sigma}{m_P}\delta(z) \qquad \chi \propto |z|$$

$$r_* = d$$

thickness

# Domain Walls in DGP: Exact Solutions

Domain wall with maximal symmetry → Flat $\kappa = 0$ ; (2+1) De Sitter $\kappa = 1$

Birkhoff Theorem:

$$ds^2 = f(R)dZ^2 + \frac{dR^2}{f(R)} + R^2 ds^2_\kappa$$

$$f(R) = \kappa - C/R^2$$

Non-singular

Non-compact bulk

→ $C = 0$ 5D Minkowski bulk

# Domain Walls in DGP: Exact Solutions

$$ds^2 = dZ^2 + dR^2 + R^{2\kappa} ds_\kappa^2$$

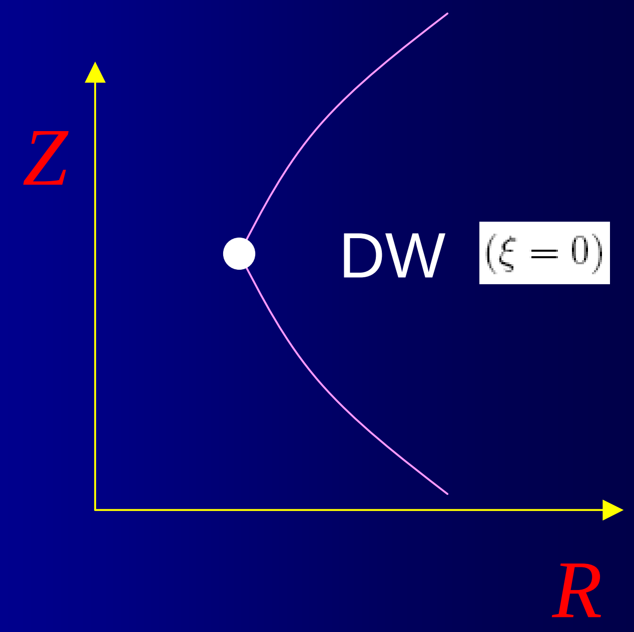
Cartesian $\kappa = 0$

Rindler $\kappa = 1$

Brane parametrization (embedding):

$$(R(\xi), Z(\xi))$$

$$Z'^2 + R'^2 = 1$$




Induced metric on the brane:

$$ds_4^2 = d\xi^2 + R^{2\kappa}(\xi)\, ds_\kappa^2$$

# Domain Walls in DGP: Exact Solutions

Equations for the embedding:
Israel junction conditions

$$M_*^3\left(K_{\mu\nu} - K\,h_{\mu\nu}\right) = T_{\mu\nu} - m_P^2\,G_{\mu\nu}^{(4)}$$

A)

$$\epsilon\,\kappa\,\frac{\sqrt{1-R'^2}}{r_c\,R} = -\kappa\,\frac{1-R'^2}{R^2} + \frac{\tau}{3m_P^2}$$

B)

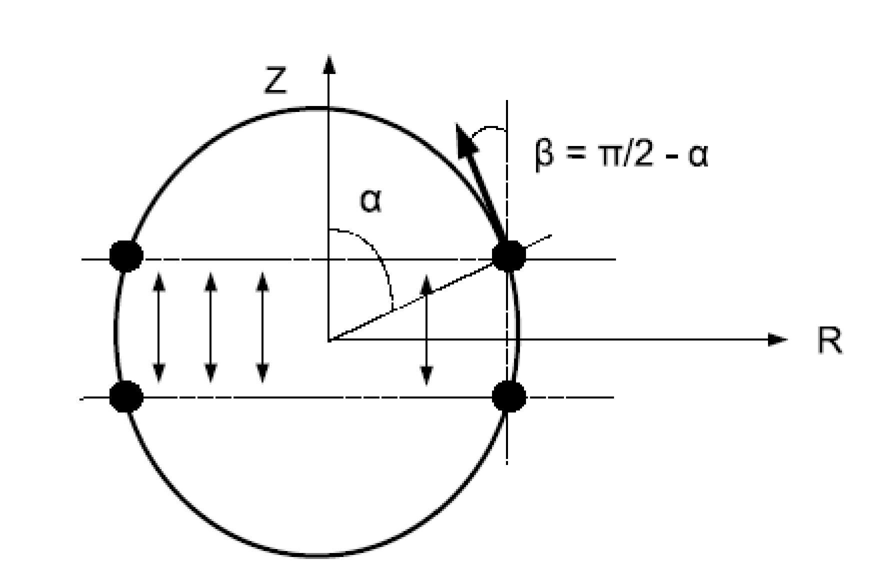
$$-\epsilon\frac{R''}{2r_c\sqrt{1-R'^2}} = \kappa\,\frac{R''}{R} + \frac{\sigma}{2m_P^2}\delta(\xi)$$

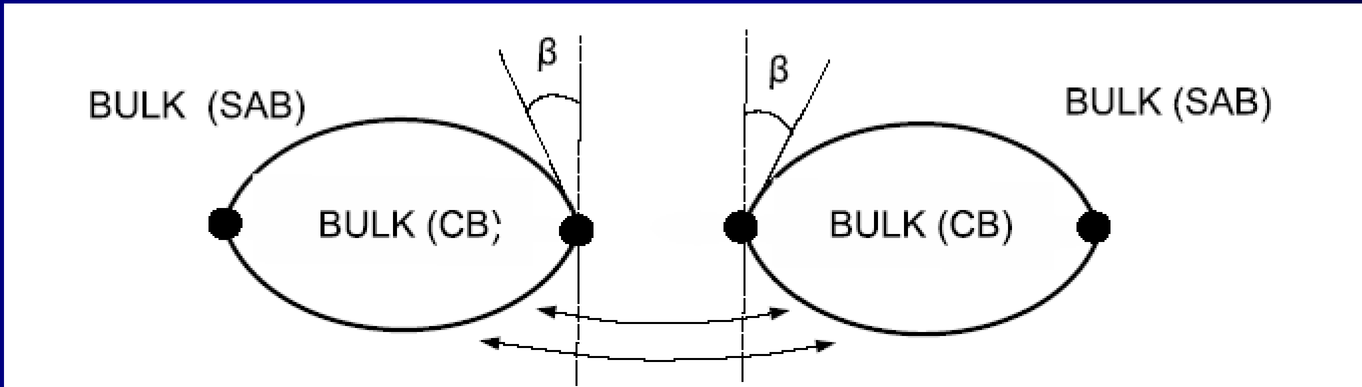
# Domain Walls in DGP: Exact Solutions

A) (Outside the DW)

$$R(\xi) = H^{-1} \sin\left[H(\xi_0 - |\xi|)\right]$$

$$\epsilon\ \kappa\ \frac{H}{r_c} = -\kappa\ H^2 + \frac{\tau}{3m_P^2}$$



# Domain Walls in DGP: Exact Solutions

B) Junction condition on the DW

$$-\kappa \, \frac{\Delta R_0'}{R_0} + \frac{\delta}{4 r_c} = \frac{\sigma}{2 m_P^2}$$

$$2\kappa \, \epsilon \, H r_c \tan\frac{\delta}{4} + \frac{\delta}{4} = \frac{\sigma}{4 M_*^3}$$

# B) Junction condition on the DW

$$-2\kappa \, m_P^2 \, \frac{\Delta R_0'}{R_0} + \delta \, M_*^3 = \sigma$$

4D viewpoint: $\sigma_{\rm eff}^{(4D)} \equiv \sigma - \delta \, M_*^3$ Extrinsic curvature

5D viewpoint: $\sigma_{\rm eff}^{(5D)} \equiv \sigma + \kappa \, m_P^2 \, \frac{\Delta R_0'}{R_0}$ DGP term

$$\sigma_{\rm eff}^{(4D)} + \sigma_{\rm eff}^{(5D)} = \sigma$$

# Domain Walls in DGP: Exact Solutions

Conventional Branch

Subcritical Wall $\sigma < 2\pi M_*^3$

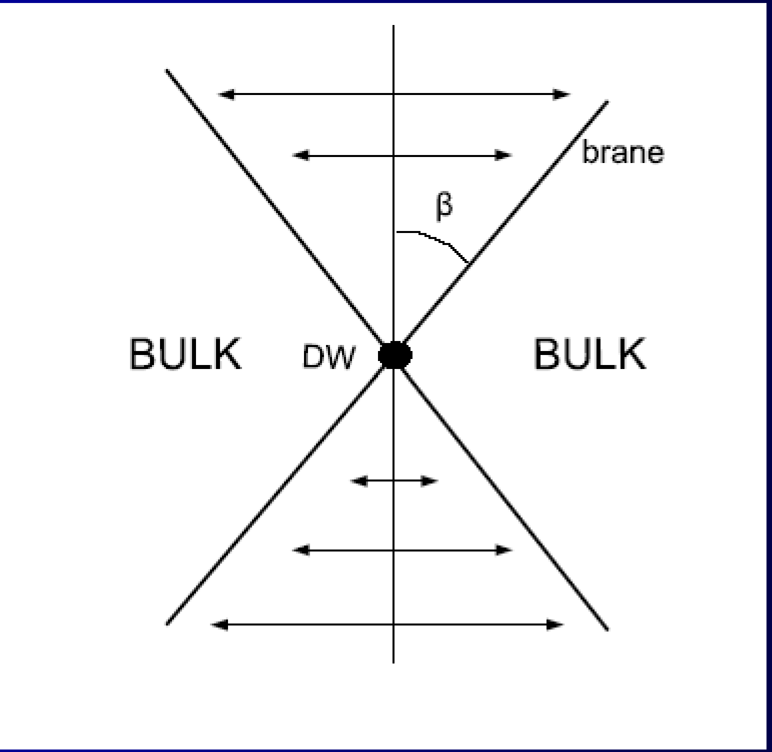
$$\kappa = 0$$

$$R(\xi) = |\xi| \ \sin\beta$$

$$Z(\xi) = \xi \ \cos\beta$$

'brane bending'

$$4\beta = \delta = \sigma / M_*^3$$



# Domain Walls in DGP: Exact Solutions

Conventional Branch

Subcritical Wall $\sigma < 2\pi M_*^3$

$\kappa = 0$

5D: $$\sigma_{\rm eff}^{(5D)} \equiv \sigma + \kappa\, m_D^2 \frac{\Delta R_0'}{R_0}$$

No 4D screening

4D: $$\sigma_{\rm eff}^{(4D)} \equiv \sigma - \delta\, M_*^3 = 0!!$$

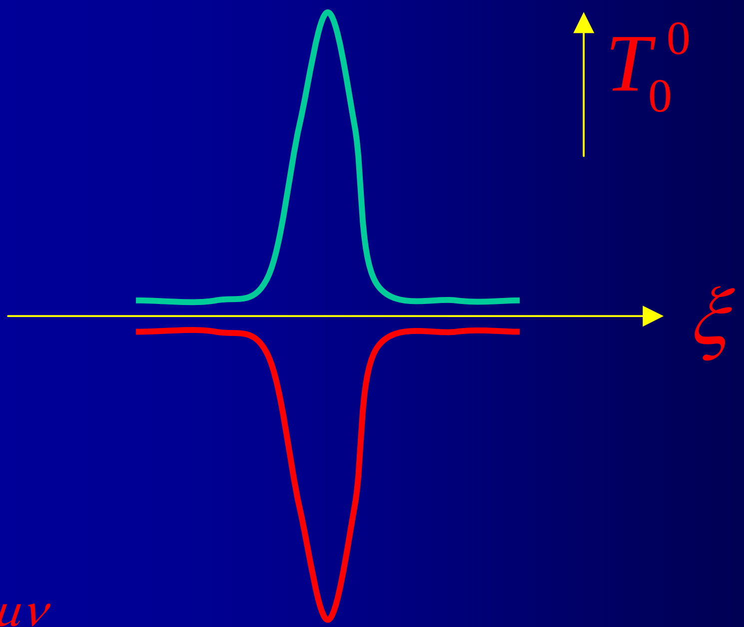
Absolute 4D screening

# Domain Walls in DGP: Exact Solutions

Conventional Branch

Subcritical Wall $\sigma < 2\pi M_*^3$

Domain wall




'Mirror'
Domain wall
produced by $K_{\mu\nu}$

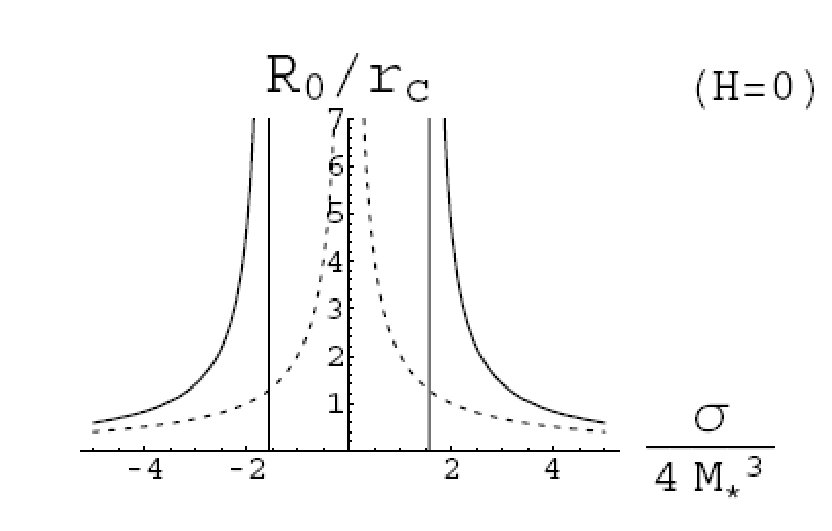
# Domain Walls in DGP: Exact Solutions

Conventional Branch

Super-critical Wall $\sigma > 2\pi M_*^3$

$\kappa = 1$

$$\frac{1}{R_0} + \frac{\pi}{4r_c} = \frac{\sigma}{4m_P^2}$$



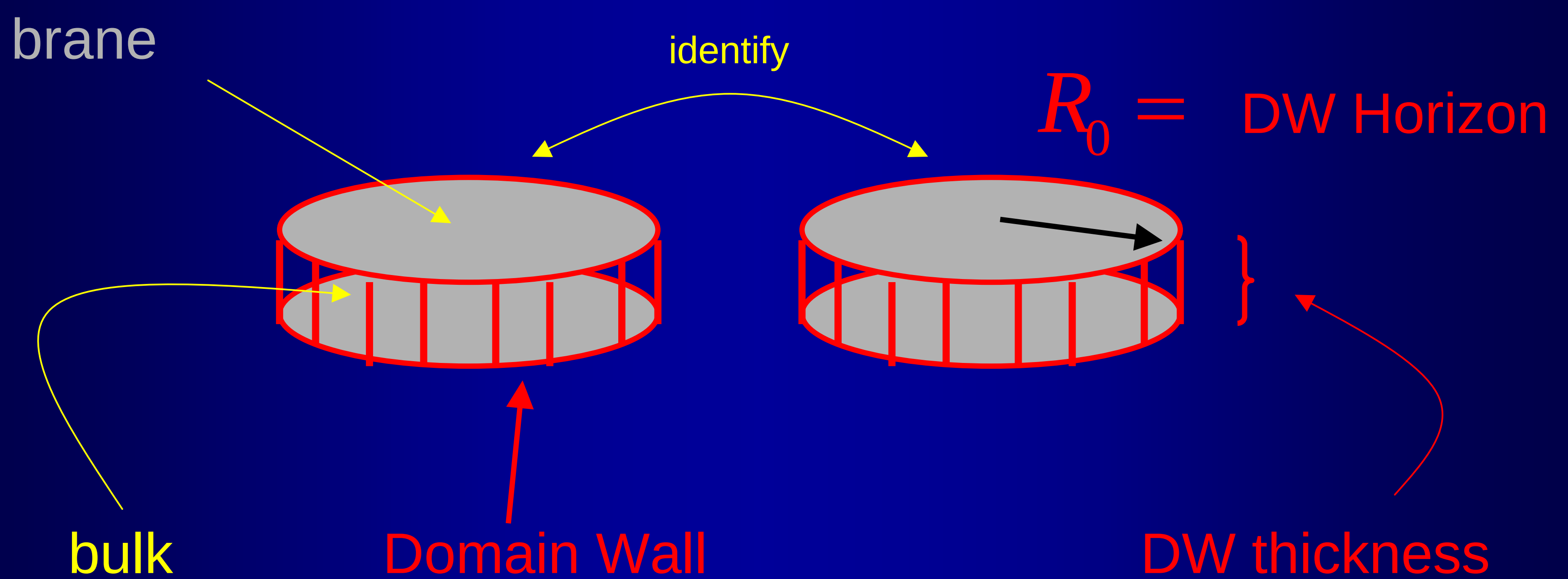

Partial 4D Screening

# Domain Walls in DGP: Exact Solutions

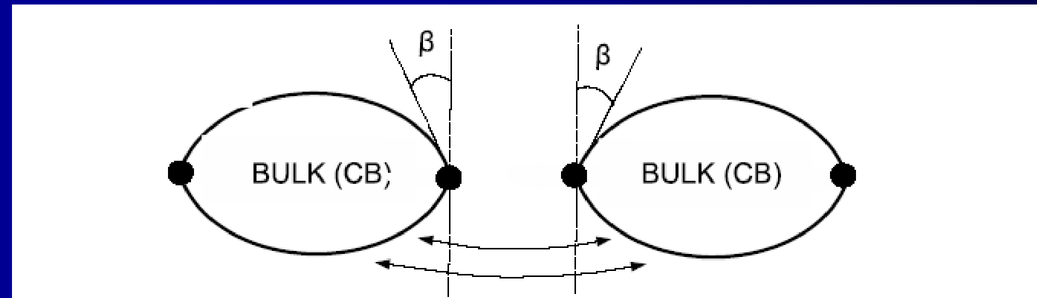
Conventional Branch

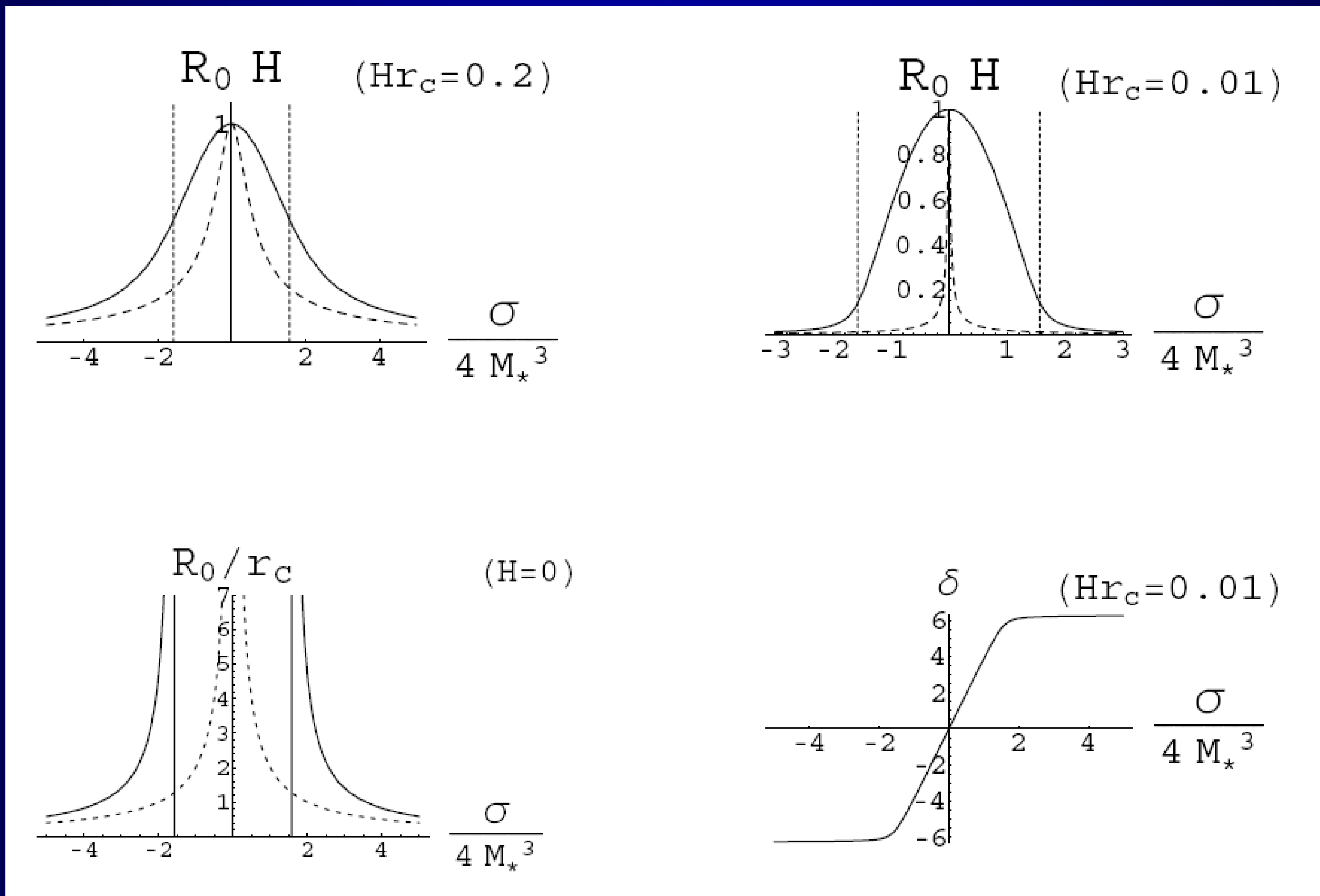
geometry

Super-critical Wall $\sigma > 2\pi M_*^3$



# Non-zero brane tension $H \neq 0$



# Domain Walls in DGP: Exact Solutions

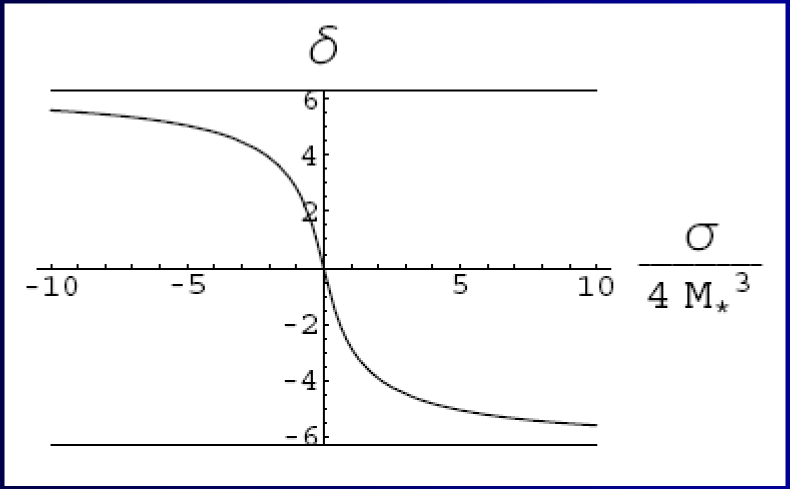
Self-Accelerated Branch $(\kappa = 1)$

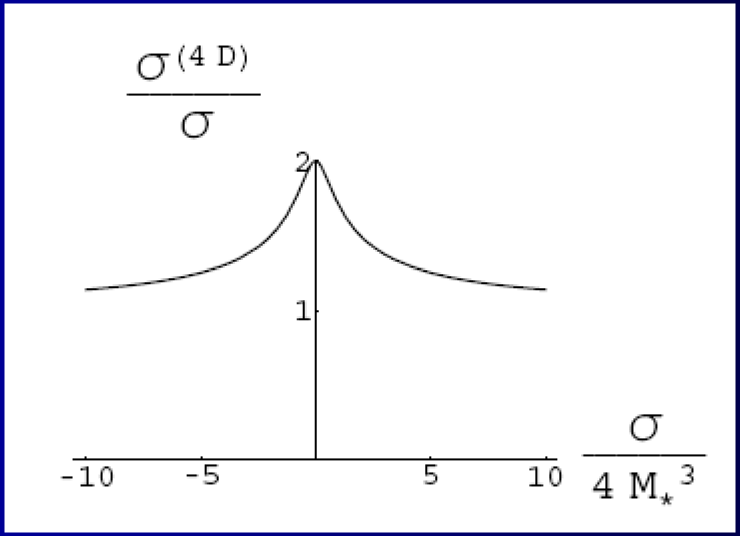
$\delta < 0!!$

5D 'over-screening'

$\sigma_{eff}^{(4D)} > \sigma !!$

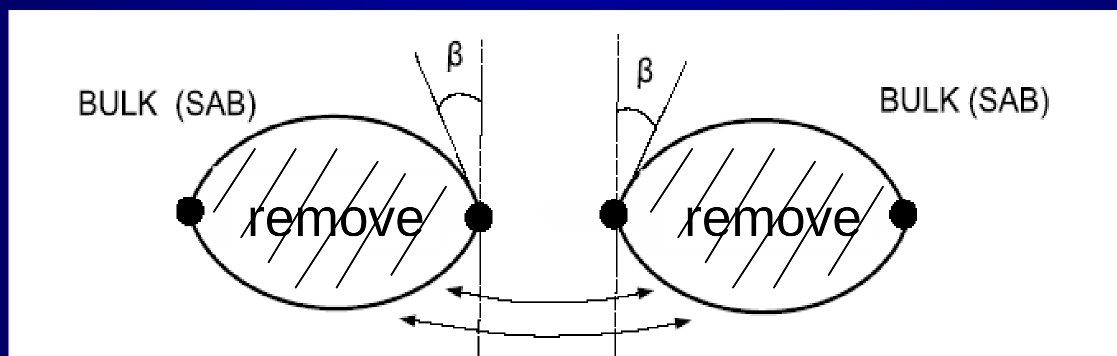
4D 'anti-screening'



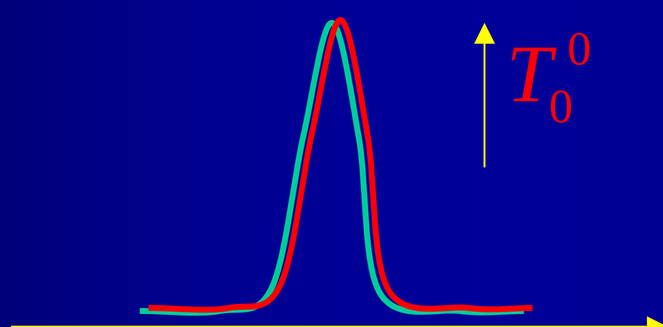



# Domain Walls in DGP: Exact Solutions

Self-Accelerated Branch geometry




Domain wall




'Replicant' Domain wall produced by $K_{\mu\nu}$

# summary

| branch | DW tension | $\sigma_{\text{eff}}^{(4D)}/\sigma$ | $\sigma_{\text{eff}}^{(5D)}/\sigma$ |
|---|---|---|---|
| CB | $\sigma < 2\pi M_*^3$ | $0$ | $1$ |
| | $\sigma > 2\pi M_*^3$ | $1 - \frac{2\pi M_*^3}{\sigma}$ | ?? |
| SAB | $\sigma \ll 2\pi M_*^3$ | $2\left(1 - \left(\sigma/M_*^3\right)^2/48 + \dots\right)$ | $-\left(1 - \left(\sigma/M_*^3\right)^2/24 + \dots\right)$ |
| | $\sigma \gg 2\pi M_*^3$ | $1 + \frac{2\pi M_*^3}{\sigma} + \dots$ | ?? |

Schwarzchild-DGP

| branch | $M^{(4D)}/M$ | $M^{(5D)}/M$ |
|---|---|---|
| CB | $0$ | $0.56\,\frac{r_*}{r_c}$ |
| SAB | $0$ | $-0.45\,\frac{r_*}{r_c}$ |

# Super-critical walls and 5D screening

With No DGP 'induced gravity' term
(supermassive codimension-2 brane)

$$\sigma > 2\pi M_*^3 \quad \Rightarrow \quad \begin{cases} D \neq 0 & \text{thickness} \\ P \neq 0 & \text{pressure along normal direction} \end{cases}$$

$$T_\mu^\nu = \mathrm{diag}(-\rho, -\rho, -\rho, P)$$

$$2\pi M_*^3 = \int d\xi \, (\rho + 2P/3) \quad \Rightarrow \quad -\frac{2}{3} P_0 \, d = \sigma - 2\pi M_*^3$$

$$-P_0 = \frac{6M_*^3}{R_0} \quad \Rightarrow \quad \frac{1}{R_0} = \frac{1}{4dM_*^3} \left(\sigma - 2\pi M_*^3\right)$$

# Super-critical walls and 5D screening

With No DGP 'induced gravity' term

(supermassive codimension-2 brane)

$$\sigma \;>\; 2\pi M_*^3 \quad \Rightarrow \quad \frac{1}{R_0} = \frac{1}{4dM_*^3}\left(\sigma - 2\pi M_*^3\right)$$

With No DGP 'induced gravity' term,

$$\sigma \;>\; 2\pi M_*^3 \quad \Rightarrow \quad \frac{1}{R_0} + \frac{\pi}{4r_c} = \frac{\sigma}{4m_P^2}$$

# Super-critical walls and 5D screening

With No DGP 'induced gravity' term
(supermassive codimension-2 brane)

$$\sigma \;>\; 2\pi M_*^3 \quad\Rightarrow\quad \frac{1}{R_0} = \frac{1}{4dM_*^3}\left(\sigma - 2\pi M_*^3\right)$$

With No DGP 'induced gravity' term,

$$\sigma \;>\; 2\pi M_*^3 \quad\Rightarrow\quad \frac{1}{R_0} = \frac{1}{4m_P^2}\left(\sigma - 2\pi M_*^3\right)$$

$$\frac{1/R_0^{(DGP)}}{1/R_0^{(5D)}} = \frac{d}{2r_c}$$

The SAME suppression factor!!

$$r_* = d$$

## DW

| branch | DW tension | $\sigma_{\text{eff}}^{(4D)}/\sigma$ | $\sigma_{\text{eff}}^{(5D)}/\sigma$ |
|---|---|---|---|
| CB | $\sigma < 2\pi M_*^3$ | $0$ | $1$ |
| | $\sigma > 2\pi M_*^3$ | $1 - \frac{2\pi M_*^3}{\sigma}$ | $\frac{r_*}{2r_c}$ |
| SAB | $\sigma \ll 2\pi M_*^3$ | $2\left(1 - (\sigma/M_*^3)^2/48 + \ldots\right)$ | $-\left(1 - (\sigma/M_*^3)^2/24 + \ldots\right)$ |
| | $\sigma \gg 2\pi M_*^3$ | $1 + \frac{2\pi M_*^3}{\sigma} + \ldots$ | $-\frac{r_*}{2r_c}$ |

## Schwarzchild-DGP

| branch | $M^{(4D)}/M$ | $M^{(5D)}/M$ |
|---|---|---|
| CB | $0$ | $0.56\,\frac{r_*}{r_c}$ |
| SAB | $0$ | $-0.45\,\frac{r_*}{r_c}$ |

# Generic IR-modified gravity

Linearized Einstein's eqs:

$$\mathcal{E}_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} \; - \; m^2(\Box)\,(h_{\mu\nu} \; - \; \eta_{\mu\nu} h) \; = \; -16\pi G_N \, T_{\mu\nu}$$

$$m^2(\Box) = r_c^{-2(1-\alpha)} \Box^{\alpha}$$

$$0 \leq \alpha < 1$$

$\alpha = 1/2$ DGP

$\alpha = 0$ Fierz–Pauli Massive gravity

# Generic IR-modified gravity

Couples to conserved source as

$$h_{\mu\nu} = -16\pi G_N \frac{1}{\Box - m^2(\Box)} \left\{ T_{\mu\nu} - \frac{1}{3} \left( \eta_{\mu\nu} - \frac{1}{m^2(\Box)} \partial_\mu \partial_\nu \right) T \right\}$$

'pure gauge' form

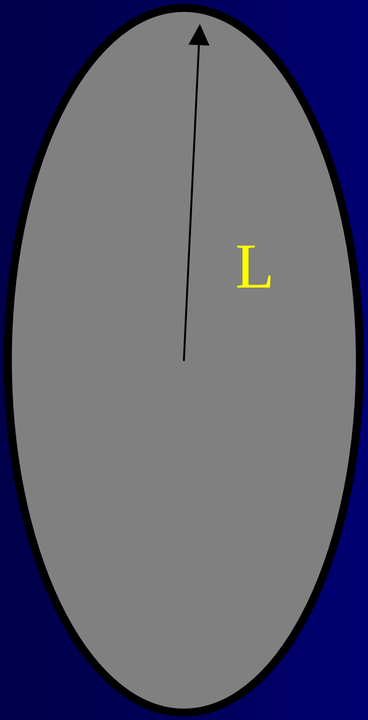
For DWs, $T_{\mu\nu} - (1/3) T \eta_{\mu\nu} \propto \delta_\mu^z \delta_\nu^z$ $\propto \mathrm{diag}(1, \quad 0, \quad 0, \quad 0)$

Lowest order

Tree-level amplitude

$$\mathcal{A} \propto G_N \int d^4x \, \frac{T_{\mu\nu} T'^{\mu\nu} - \frac{1}{3} T T'}{\Box - m^2(\Box)} = 0$$

# Finite size DWs

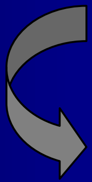



$$M = \pi L^2 \sigma$$

$$r_* \sim \left(\delta L^2 r_c\right)^{1/3}$$

$$\delta \equiv \sigma / M_*^3$$

Non-linearities do not appear if $r_* \ll L$



$$L \gg \delta r_c$$

DW probes large distances in longitudinal directions

# CONCLUSIONS

- In DGP, sub-critical DWs do not gravitate
  => short distance probe of gravity
- expect the same in other IR-modified gravity theories
  (non-linearities do not contribute for DWs)
- Screening mechanism in DGP -> Extrinsic curvature
- Almost the same as for Schwarzchild-like case
  (Gabadadze&Iglesias)
- Super-massive codimension-2 branes
- Screening of the 5D-tension

# Domain Walls

| branch | DW tension | $\sigma_{\text{eff}}^{(4D)}/\sigma$ | $\sigma_{\text{eff}}^{(5D)}/\sigma$ |
|---|---|---|---|
| CB | $\sigma < 2\pi M_*^3$ | $0$ | $1$ |
| | $\sigma > 2\pi M_*^3$ | $1 - \frac{2\pi M_*^3}{\sigma}$ | $\frac{d}{2r_c}$ |
| SAB | $\sigma \ll 2\pi M_*^3$ | $2\left(1 - \left(\sigma/M_*^3\right)^2/48 + \dots\right)$ | $-\left(1 - \left(\sigma/M_*^3\right)^2/24 + \dots\right)$ |
| | $\sigma \gg 2\pi M_*^3$ | $1 + \frac{2\pi M_*^3}{\sigma} + \dots$ | $-\frac{d}{2r_c}$ |

# Schwarzchild-DGP

| branch | $M^{(4D)}/M$ | $M^{(5D)}/M$ |
|---|---|---|
| CB | $0$ | $0.56\,\frac{r_*}{r_c}$ |
| SAB | $0$ | $-0.45\,\frac{r_*}{r_c}$ |

# FUTURE

- notion of $r_*$ for dilute walls $(d \geq r_c)$
- less symmetric configurations
- general solutions (curved bulk)
- field theoretical analog??