DARK ENERGY IN THE UNIVERSE

Modern paradigm of the Universe evolution

$$\cdots\to DS \to FRW\,RD \to FRW\,MD \xrightarrow{\;t_0\;} \widetilde{DS} \cdots\to$$

$a(t)$: $e^{\int H dt}$ ($|\dot H| \ll H^2$), $t^{1/2}$, $t^{2/3}$, $e^{\int H dt}$ $\qquad H \equiv \frac{\dot a}{a}$

Existense of dark energy - - kinematical statement assuming the "Einsteinian interpretation"

$$R_i^k - \frac{1}{2}\delta_i^k R = -8\pi G\left(T_{i\,(m)}^k + \tilde T_{i\,(DE)}^k\right) \qquad 4D$$

$T_{i\,(m)}^k$ — matter seen through its active gravitational mass (effect on motion of stars, galaxies and light)

$$T^k_{i(DE);k} = 0$$

$$\frac{\Delta\Phi}{a^2} = 4\pi G\,\delta\rho_m \Leftarrow$$

Remarkably $\tilde T_{i\,(DE)}^k \approx \varepsilon_{DE}\,\delta_i^k$

FRW symmetry : $\varepsilon_{DE}(z)$
$p_{DE}(z)$

$$\frac{G^2\hbar\,\varepsilon_{DE}}{c^7} = 1.25\cdot 10^{-123}\cdot\frac{\Omega_{DE}}{0.7}\left(\frac{H_0}{70}\right)^2$$

$$\rho_{DE} = \varepsilon_{DE}c^{-2} = 6.44\cdot 10^{-30}\ \mathrm{g\,cm^{-3}}\ (\ldots\ldots)$$

Investigation of dark energy

I. From observations to theory

Reconstruction of
(1998)

1) $H(z)$, $\varepsilon_{DE}(z)$
2) $q(z)$, $p_{DE}(z)$, $w_{DE}(z)$
3) $r(z)$, $\frac{dw_{DE}}{dz}$

1. Inversion of classical cosmological tests
2. CMB (acoustic peaks spacing, ISW)
3. $\left(\frac{\delta\rho}{\rho}\right)_m(z)$, $\Phi(z)$ from gravitational lensing, correlation of $\frac{\delta\rho}{\rho}$ with IS[W]

II. From theory to observations

Models (many of them!)
(qualitatively - the same as for inflation)

1. Fundamental constant
2. Scalar field (with $m \sim 10^{-33}$ eV)
3. Geometrical dark energy
(e.g., dark energy in scalar-tensor gravity)

$$
\begin{tikzcd}
\mathcal{D}_L(z) \arrow[dr] & & \\
& H(z) \arrow[r, "\Omega_m"] & V(\varphi) \\
\left(\frac{\delta\rho}{\rho}\right)_{CDM}(z) \arrow[ur, "H_0"'] & &
\end{tikzcd}
$$

$$q_0 = -1 + \left.\frac{d\ln H}{d\ln(1+z)}\right|_{z=0}$$

$$\Lambda = \text{const} \rightarrow H^2(z) = H_0^2\left(1-\Omega_m+\Omega_m(1+z)^3\right)$$

$$\rightarrow \quad q_0 = \frac{3}{2}\Omega_m - 1$$

II. Reconstruction of $V(\varphi)$ from $H(a)$

$$\begin{cases} 8\pi G V = aH\dfrac{dH}{da} + 3H^2 - \dfrac{3}{2}\Omega_m H_0^2 \cdot \left(\dfrac{a_0}{a}\right)^3 \\ 4\pi G\, a^2 H^2 \left(\dfrac{d\varphi}{da}\right)^2 = -aH\dfrac{dH}{da} - \dfrac{3}{2}\Omega_m H_0^2\left(\dfrac{a_0}{a}\right)^3 \end{cases}$$

Necessary condition $(\varepsilon_{DE} + p_{DE} \geq 0)$

$$w_{DE} \equiv \frac{p_{DE}}{\varepsilon_{DE}} \geq -1$$

$$\frac{dH^2}{dz} \geq 3\Omega_m H_0^2 (1+z)^2$$

$$H^2 \geq H_0^2\left(1 + \Omega_m(1+z)^3 - \Omega_m\right)$$

In particular: $\quad q_0 \geq \frac{3}{2}\Omega_m - 1$

No such a condition in case of scalar–tensor gravity

S. Nesseris, L. Perivolaropoulos, astro-ph/0610092

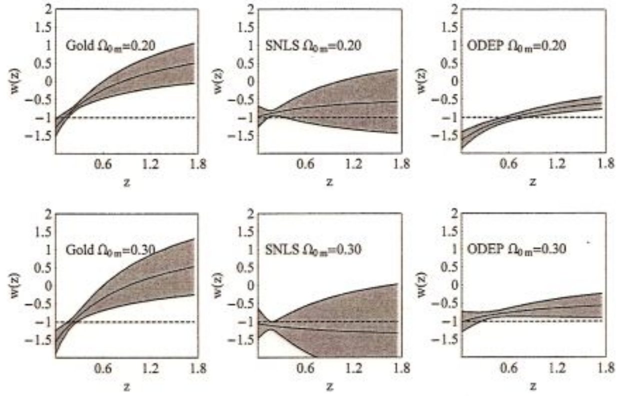

FIG. 7: The best fit form of $w(z)$ for each dataset category for both $\Omega_{0m} = 0.2$ and $\Omega_{0m} = 0.3$ along with the 1σ errors (shaded region). The categories are: Gold dataset (column 1), SNLS (column 2) and Other Dark Energy Probes (ODEP column 3).

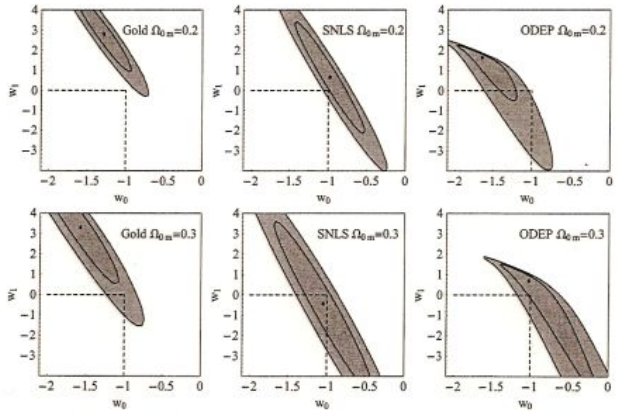

FIG. 8: The 68% and 95% χ^2 confidence contours in the $w_0 - w_1$ parameter space for each dataset category for both $\Omega_{0m} = 0.2$ and $\Omega_{0m} = 0.3$. Notice that for the SNLS dataset ΛCDM is within the 1σ region.

$$w(z) = w_0 + w_1 \frac{z}{1+z}$$

Models of dynamical dark energy

Practical use of the remarkable similarity between primordial DE (supporting inflation) and present DE:
the same models may be used for description of both inflation and present dark energy

Single inflation	Quintessence
$R+R^2$ model	$F(R)$ model
Extended inflation	Scalar-tensor DE
k-inflation	k-essence
Brane inflation	Brane DE
String inflation	String DE

The most critical problem:

"Graceful exit" "Graceful entrance"

Model requirements

1. Stability of the Minkowski space-time with respect to perturbations with $\omega^2 \gg H_0^2$
   a) absence of tachyons,
   b) absence of ghosts
2. Solar System tests
3. Stability of the MD-stage

Example: $\mathcal{L} = F(R) \Rightarrow \begin{cases} \frac{dF}{dR} > 0 \\ \frac{d^2F}{dR^2} \geq 0 \end{cases}$

4. MD- and RD-stages should be generic

What if recent phantom behaviour of dark energy will be confirmed by observations ?

Ghost phantom models of dark energy are bad.

1. Quantum instability

2. At the classical level :

does not explain homogeneity and isotropy of the Universe

E.g. : for a given $\bar{H} = \frac{1}{3}\frac{d}{dt} \ln abc$, it is much more probable to have very different $\frac{\dot{a}}{a}, \frac{\dot{b}}{b}, \frac{\dot{c}}{c}$ compensated by the negative energy density of the ghost field.

Scalar-tensor models of dark energy do not have this problem.

Geometrical $F(R)$ model of dark energy

$$S = \frac{1}{16\pi G}\int F(R)\sqrt{-g}\, d^4x + S_m$$

$$F(R) = R + f(R) \qquad R \equiv R_i^i$$

$$\frac{F}{2}\delta_i^k - F' R_i^k - \left(\Box \delta_i^k - \nabla_i \nabla^k\right) F' = -8\pi G\, T_i^k$$

$$-R_i^k + \frac{1}{2}\delta_i^k R = 8\pi G\, T_i^k + \underbrace{- f' R_i^k + \frac{f}{2}\delta_i^k - \left(\Box \delta_i^k - \nabla_i \nabla^k\right) f'}_{8\pi G\, T_{i,\,DE}^k}$$

Particle content: graviton + massive scalar particle (called "scalaron" in A.S., 1980)

No ghosts if

① $F' > 0 \qquad f' > -1$

② $F'' \geqslant 0 \qquad f'' \geqslant 0$

Classically for FRW model
$$\begin{cases} F'(R_0) = 0 & \text{– loss of homogeneity} \\ F''(R_0) = 0 & \text{– non-analytical behaviour of } R(t) \ \left(R(t) = R_0 + R_1 t \ln^{1/2} t + \ldots\right) \end{cases}$$

Equivalence between $\mathcal{L} = \frac{1}{16\pi G}[R + f(R)]$

and $\mathcal{L} = \frac{R}{16\pi G} + \frac{1}{2}\varphi_{;i}\varphi^{;i} - V(\varphi)$ theories

1^{st} class includes the (symplified) Starobinsky's inflationary model (1980); 2^{nd} class includes the original Guth's "old" inflation (1981), "new" inflation (1982), Linde's chaotic inflation (1983), power-law inflation (1984).

Theorem. There is one to one correspondence between solutions of the both theories for which $(1 + \frac{df}{dR}) \neq 0$:

$$(g_{ik}, R(g_{ik})) \longleftrightarrow (\tilde{g}_{ik}, R(\tilde{g}_{ik}), \varphi)$$

where
$$\tilde{g}_{ik} = \left|1 + \frac{df}{dR}\right| g_{ik}$$
$$\Phi = \sqrt{\frac{3}{16\pi G}}\, \ln\left|1 + \frac{df}{dR}\right| \quad \Longrightarrow R = R(\Phi)$$
$$V(\Phi) = \frac{1}{16\pi G}\, \frac{\left(R\frac{df}{dR} - f\right)}{\left(1 + \frac{df}{dR}\right)^2}\, \mathrm{sign}\left(1 + \frac{df}{dR}\right)$$

Examples.

1. $f(R) = \frac{R^2}{6M^2}$ $\qquad V(\Phi) = \frac{3M^2}{32\pi G}\left[1 - \exp\left(-\sqrt{\frac{16\pi G}{3}}\,\Phi\right)\right]^2$

$\boxed{f(R) = R^2/6M^2}$

Internally self-consistent inflationary model with a graceful exit to the subsequent FRW stage (first, MD; second, after scalaron decay, RD)

$\tau \sim \frac{M_{Pl}^2}{M^3}$ $(\rightarrow N \sim 53)$ A. S., PLB (1980)

$M = 2.7 \times 10^{-6} M_{Pl} \cdot \frac{53}{N}$ $(A_s = 2.2 \times 10^{-9})$

$n_s = 1 - \frac{2}{N} \simeq 0.96$, $r = \frac{12}{N^2} \simeq 4 \cdot 10^{-3}$

$\longleftrightarrow$

Recent idea:

$F(R)$ model with $F(R) \rightarrow \infty$ for $R \rightarrow 0$ as a model of dark energy at present time

Capozziello 2002
Capozziello et al. 2003
Carroll et al. 2003 ....

Main problem: effective coupling to usual matter in the Einstein frame

STABILITY OF THE MD STAGE IN THE $f(R)$ THEORY

Requires $F''(R) \geqslant 0$ for $|R| > M_0^2$

The model $F(R) = R - R_0\left(\frac{R}{R_0}\right)^\alpha$,

$$R_0 \sim M_0^2, \quad 0 < \alpha < 1$$

satisfies this condition. For oscillations:

$$\overline{w}_{osc} = 1 - \frac{\alpha}{2} \quad \text{during MD stage}$$

It also has the stable late-time DS attractor with

$$R = R_0 \cdot (2-\alpha)^{1/(1-\alpha)}$$

and $F'(R) > 0$.

$\alpha < 0$: the stable model is

$$F(R) = R + R_0\left(\frac{R}{R_0}\right)^\alpha$$

sign opposite to that used in Carroll et al.

No dS attractor

General theory

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + f(R)\right)$$

$$|f| \ll |R|, \quad \left|\frac{df}{dR}\right| \ll 1, \quad \left|R\frac{d^2f}{dR^2}\right| \ll 1$$

$$R = \underbrace{8\pi G T}_{R^{(0)}} + R^{(1)} + \delta$$

↑ non-oscillating term

$$R^{(1)} \sim f(R^{(0)})$$

$$\delta \propto a^{-3/2}(f'')^{-3/4} \exp\left(i\int \frac{dt}{\sqrt{3f''}}\right)$$

Stability: $f'' \equiv \frac{d^2f}{dR^2}(R^{(0)}) \geq 0$

Effective EMT of these oscillations ('scalarons'): dust-like but with variable mass (so $p \neq 0$)

$$\varepsilon \sim \frac{f''}{G}\delta^2 \sim \frac{m_{eff}}{a^3} \qquad m_{eff}^2 \propto \frac{1}{f''}$$

If $R^{(0)}$ is produced by massive particles

$$R^{(0)} \propto a^{-3} \longrightarrow m_{eff} \propto a^{\frac{3}{2}\alpha - 3} \text{ for } f \propto |R|^{\alpha}$$

$$\overline{w}_{osc} = 1 - \frac{\alpha}{2}$$

Valid during the RD stage, too

Solar System tests

The main problem:

$F(R)$ theory is equivalent to the $\omega = 0$ scalar-tensor theory

$$S = \int \left(A(\varphi) R + B(\varphi) \right) \sqrt{-g}\, d^4x$$

If scalaron is very light, $\gamma = 1/2$ (though $\beta = 1$)

The way out: make the scalaron massive, i.e. by taking

$$f(R) = \frac{R^2}{6M^2} \quad \text{for sufficiently large } R$$

But then the problem is how to combine

$m_{eff} \propto \frac{1}{\sqrt{f''(R)}}$ large in the Solar System

$\left(m_{eff} R_{\odot} \gg 1 \text{ for } \rho \sim (10^{-24} - 10^{-19}_{?})\ \text{g/cm}^3 \right)$

with $m_{eff} \sim 10^{-33}\ \text{eV}$ for $\rho \sim 10^{-29}\ \text{g/cm}^3$ at cosmological scales keeping

$f''(R) > 0$ for all $R \gg R_0 \sim H_0^2$

Conclusion:

Difficult to have anything significantly different from $\Lambda = -\frac{1}{2} f(0) = \text{const}$ at cosmological scales

Reconstruction of dark energy in scalar-tensor gravity

B. Boisseau, G. Esposito-Farese,
D. Polarski, A. S.

Phys. Rev. Lett. 85, 2236 (2000)

$\varepsilon_{DE} + p_{DE} < 0$ is permitted

$$\mathcal{L} = \frac{1}{2}\left(F(\varphi)R + Z(\varphi)\varphi_{,\mu}\varphi^{,\mu}\right) - V(\varphi) + \mathcal{L}_m$$

Includes $R + f(R)$ theory for $Z(\varphi) = 0$.

$Z(\varphi) = 1$ $\qquad \omega^{-1}(\varphi) = F^{-1}\left(\frac{dF}{d\varphi}\right)^2$

Two independent observable cosmological functions are required for reconstruction of $F(\varphi)$ and $V(\varphi)$

$$D_L(z), \quad \delta(z)$$

$$\downarrow \qquad\qquad \downarrow$$

$$H(z) \longrightarrow F(z)$$

Properties of scalar-tensor models of dark energy

R. Gannouji, D. Polarski, A. Ranquet, A. S.
JCAP 09, 016 (2006) [astro-ph/0606287]

1. Temporary phantom behaviour and crossing of the phantom boundary $w=-1$ are possible for an open set of $F(\varphi)$ and non-zero and non-constant $V(\varphi)$.
   "Curvature induced phantomness"
2. In the absence of dust-like matter ($\Omega_m = 0$), power-law solutions leading to the Big Rip singularity in future and to $w < -1$ exist if

$$F = \alpha \varphi^2, \quad \varphi \to \infty$$

$$V = V_0 |\varphi|^n, \quad 2 < n < 4$$

(Barrow & Maeda 1990)

Then $a(t) \propto (t_0 - t)^q$, $\quad q = \dfrac{2\left(n+2+\frac{1}{\alpha}\right)}{(n-2)(n-4)} < 0$

$\varphi(t) \propto (t_0 - t)^r$, $\quad r = \dfrac{2}{2-n} < 0$

However, for these solutions $|w+1| \leq \frac{\alpha}{3} \sim \frac{1}{\omega_{BD}}$ and very small.

Present observational bounds:

$$\gamma_{PN} - 1 = (2.1 \pm 2.3)\cdot 10^{-5}$$

Bertotti et al., 2003 → Cassini mission

$$\beta_{PN} - 1 = (0 \pm 1)\cdot 10^{-4}$$

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = (-0.2 \pm 0.5)\cdot 10^{-13}\, y^{-1}$$

Pitjeva, 2005 → ephemerides of planets

$$\beta_{PN} - 1 = (1.2 \pm 1.1)\cdot 10^{-4}$$

Williams et al, 2005 → lunar laser ranging

$$\omega_{BD,0} \equiv \left(\frac{F}{\left(\frac{dF}{d\varphi}\right)^2}\right)_0 > 4\cdot 10^4$$

3. Small z expansion.

$$\frac{F(z)}{F_0} = 1 + F_1 z + F_2 z^2 + \ldots$$

$$\frac{V(z)}{3F_0 H_0^2} = \Omega_{V,0} + u_1 z + u_2 z^2 + \ldots$$

$$\frac{H^2(z)}{H_0^2} = 1 + h_1 z + h_2 z^2 + \ldots$$

$$F_0^{-1/2}\varphi'(z) = \varphi_0' + \varphi_1' z + \varphi_2' z^2 + \ldots$$

$$w_{DE}(z) = w_0 + w_1 z + w_2 z^2 + \ldots$$

$$|F_1| < 10^{-2}$$

$$\varphi_0'^2 = 6\,(1 - \Omega_{m,0} - \Omega_{V,0} - F_1) \geq 0$$

What is required to get significant phantomness ($|w+1| \gg \frac{1}{\omega_{BD,0}}$) ?

$$F_2 < 0 \,, \quad |F_2| \sim 1 \gg |F_1| \qquad (|F_1| < 10^{-2})$$

$$|F_2| > 3\,(\underbrace{\Omega_{DE,0}}_{\simeq\, 1-\Omega_{m,0}} - \Omega_{V,0}) > 0$$

$$w_0 + 1 = \frac{2F_2 + 6(\Omega_{DE,0} - \Omega_{V,0})}{3\Omega_{DE,0}} < 0$$

k_1 can be negative, too

4. Connection with post-Newtonian parameters in the significantly phantom case.

$$\gamma_{PN} - 1 = -\frac{F_1^2}{6(\Omega_{DE,0} - \Omega_{V,0})} < 0$$

$$\beta_{PN} - 1 = -\frac{F_1^2 F_2}{72(\Omega_{DE,0} - \Omega_{V,0})} > 0$$

$$-4 < \frac{\gamma_{PN} - 1}{\beta_{PN} - 1} = \frac{12(\Omega_{DE,0} - \Omega_{V,0})}{F_2} < 0$$

However, $|\gamma_{PN} - 1|$ and $|\beta_{PN} - 1|$ may be much smaller than $|1+w|$ if F_1 is very small

$$\frac{\dot{G}_{eff,0}}{G_{eff,0}} = H_0 F_1 \left(1 - \frac{F_2}{3(\Omega_{DE,0} - \Omega_{V,0})}\right)$$

Positive detection of $\gamma_{PN} < 1$, $\beta_{PN} > 1$ may be a strong argument for significant phantomness of present dark energy. Negative detection tell us nothing.

5. Correct asymptotic behaviour for large z ($\varphi'^2 \geq 0$, $w_{DE} \leq 0$) requires non-zero and non-constant $V(\varphi)$.

Example: $F(\varphi) \to F_\infty < F_0$, $V(\varphi) \propto \exp\left(\sqrt{\frac{3}{2F_0\Omega_{m,\infty}}}\,\varphi\right)$ for $z, \varphi \to \infty$

CONCLUSIONS

1. Expected deviations ($\Delta G_{eff}/G_{eff}$, $\max\overline{|w+1|}$) less than 10%. At least 1% accuracy level is needed for further progress.
2. Still it is not guaranteed that any deviation of dark energy from the cosmological constant will be discovered (that, of course, does not mean that the present dark energy is stable and eternal)
3. More possibilities to find deviation from Λ for $\delta(z)$