„Superluminal" scalar fields & cosmology

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- General formalism
- Inflation with large $h_{\mu\nu}$
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• Motivation

- Non-standard (non-quadr.) kinetic terms are rather common in EFT. (e.g. Born-Infeld)

- Non-standrd kinetic terms → rich dynamic, interesting cosmology e.g. :

  - Inflation without $V(\phi)$, but with **sound speed** $c_s \ll 1$

    „K-Inflation" (Armendariz-Picon, Damour, Mukhanov 99)

  - Quintessence with $c_s \ll 1$, „solution" of coincidence problem

    „K-essence" (Armendariz-Picon, Steinhardt, Mukhanov 2000)

What if $c_s > 1$?

General formalism

Scalar field ϕ :

Action: $S_\phi = \int d^4x \sqrt{-g}\, P(\phi, X)$

where $X = \frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$ $\quad \overset{\text{sign}}{(+,---)}$

EMT

$$T_{\mu\nu} = P_{,X} \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} P$$

NEC: $\underset{\uparrow \text{null}}{T_{\mu\nu} n^\mu n^\nu} \geq 0 \leftrightarrow \boxed{P_{,X} \geq 0}$

Hydrodynamics: (if $X > 0$)

$$\underset{\text{pressure}}{P = P(\phi, X)}, \quad \underset{\text{energy dens.}}{\varepsilon = 2X P_{,X} - P}$$

$$\boxed{\underset{\text{4 velocity}}{U_\mu = \frac{\nabla_\mu \phi}{\sqrt{2X}}}, \quad \underset{\text{sound speed}}{C_s^2 = \frac{P_{,X}}{\varepsilon_{,X}}}}$$

[Mukhanov, Garriga 99]

EoM:

$$\mathcal{G}^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{\mathcal{E}_{,\phi}}{P_{,X}} = 0$$

where

$$\mathcal{G}^{\mu\nu} = g^{\mu\nu} + \frac{P_{,XX}}{P_{,X}}\nabla^\mu\phi\nabla^\nu\phi$$

EoM *is hyperbolic if*

$$\boxed{1 + \frac{P_{,XX}}{P_{,X}}2X > 0}$$

$$c_s^2 > 0$$

Mukhanov, Garriga 99

$$\left[\, c_s^2 = \left[1 + \frac{P_{,XX}}{P_{,X}}2X\right]^{-1} \right]$$

Aharonov, Komar, Susskind, 69

Armendaric-Picon, Lim 05

Rendall 05

Characteristics cone

$$\mathbb{G}^{-1}_{\mu\nu}\mathcal{N}^{\mu}\mathcal{N}^{\nu}=0$$

Where

$$\mathbb{G}^{-1}_{\mu\nu}=g_{\mu\nu}-c_s^2\frac{P_{,XX}}{P_{,X}}\nabla_\mu\phi\nabla_\nu\phi$$

induced eff. metric

if n^μ – „null" for $g_{\mu\nu}$:

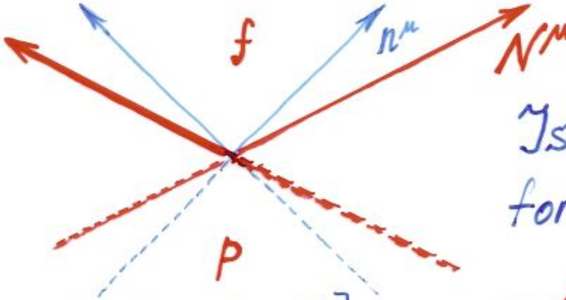
$$\mathbb{G}^{-1}_{\mu\nu}n^\mu n^\nu=-c_s^2\frac{P_{,XX}}{P_{,X}}\left(\nabla_\mu\phi\, n^\mu\right)^2$$

if $\boxed{\frac{P_{,XX}}{P_{,X}}<0}$ n^μ is timelike in $\mathbb{G}^{-1}_{\mu\nu}$ →

$\mathcal{N}^\mu$ spacelike in $g_{\mu\nu}$ →

superluminality

$c_s>1$ $(X>0)$



[see Bruneton 06]

Is this a problem for causality?

„No!" / Global properties

Example: Friedmann universe

ϕ - cosmological scalar : $\phi_0(t)$

$$ds^2 = dt^2 - a^2(t)\, d\vec{x}^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$d\ell^2 = G^{-1}_{\mu\nu} dx^\mu dx^\nu = ds^2 -$$

$$- c_s^2 \frac{P_{,XX}}{P_{,X}} 2X\, dt^2 \rightarrow$$

$$d\ell^2 = c_s^2(t)\, dt^2 - a^2(t)\, d\vec{x}^2$$

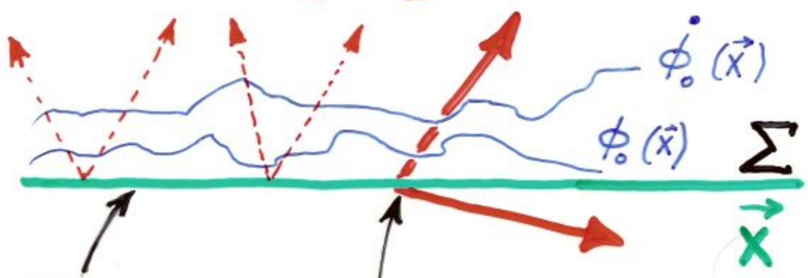
$d\tau = c_s dt \longrightarrow d\ell^2$ - „Friedmann"

$$d\ell^2 = d\tau^2 - a^2(\tau)\, d\vec{x}^2$$

Not all initial data are allowed for well posed Cauchy problem.

① $1+2X\dfrac{P_{,XX}}{P_{,X}}>0$ Hyperbolicity

② $G^{-1}_{\mu\nu}\left[\phi\big|_{\Sigma},\nabla_\mu\phi\big|_{\Sigma}\right]$, Σ space like in $g_{\mu\nu}$



Σ space-like in $G^{-1}_{\mu\nu}$

Σ time-like in $G^{-1}_{\mu\nu}$ **ill posed Cauchy problem**

$\vec{X}$ - null in $G^{-1}_{\mu\nu}$

if $\vec{X}^2=-C_s^2\dfrac{P_{,XX}}{P_{,X}}\left(\vec\nabla\phi\cdot\vec{X}\right)^2$

$$1+C_s^2\frac{P_{,XX}}{P_{,X}}\left(\vec\nabla\phi\right)^2>0$$

• „Simple" Inflation With $C_s > 1$.

Friedmann universe with small perturbations

• $$dS^2 = (1+2\Phi)dt^2 - a^2(t)\left[(1-2\Phi)\delta_{ik} + \overset{T}{h}_{ik}\right]dx^i dx^k$$

$\left\{ \begin{array}{c} \cancel{\text{Vectors}} \sim \frac{1}{a^2}, \quad \partial_i \overset{T}{h}_{ik} = 0, \ \overset{T}{h}_{ii} = 0 \\ \text{Longitudinal, conform-Newtonian} \\ \text{gauge}, \ \boxed{K=0}, \ \delta T^i_{j\,i\neq j} = 0 \end{array} \right\}$

— Background —

Consider Lagrangians

★ $P(X,\phi) = K(X) - V(\phi)$,

→ Energy density

★ $\mathcal{E}(X,\phi) = 2XK' - K + V$,

$X = \frac{1}{2}(\nabla_\mu \phi)^2 = \frac{1}{2}\dot{\phi}^2$

EOM: $\boxed{\ddot{\phi} + 3C_s^2 H\dot{\phi} + \frac{V_{,\phi}}{\mathcal{E}_{,X}} = 0}$

(for $\phi(t)$)

+

Friedmann: $H^2 = \frac{8\pi}{3}\mathcal{E}$, $C_s^2 \equiv \frac{P_{,X}}{\mathcal{E}_{,X}}$ speed of sound. (Garriga, Mukhanov 99)

if $XK' \ll V$, $K \ll V$, $|\ddot{\phi}| \ll V_{,\phi}/\mathcal{E}_{,X}$

⬇ for $N > 75$

SLOW-ROLL inflation

If we allow „all" $K(X)$

➔ „all" c_s !

Ex: $K(X) = \alpha X^{\beta}$

$$\Downarrow$$

$$c_s^2 = \frac{1}{2\beta - 1} \xrightarrow[\beta \to 1/2]{} \infty$$

c_s Becomes an additional free parameter of the Inflationary theory !

Amplitude δ_ϕ depends on c_s.

- **Concrete example**

$$p(X,\phi) = \alpha^2\left[\sqrt{1+\frac{2X}{\alpha^2}}-1\right]-\frac{1}{2}m^2\phi^2$$

Lagrangian (pressure)

$$p_{,X} = \left(1+\frac{2X}{\alpha^2}\right)^{-1/2} > 0 \quad (\mathbf{NEC})$$

$$\mathcal{E} = \alpha^2\left[1-\left(1+\frac{2X}{\alpha^2}\right)^{-1/2}\right]+\frac{1}{2}m^2\phi^2 \underset{(X>0)}{>} 0$$

Energy density

$$c_s^2 = 1+\frac{2X}{\alpha^2} \quad \left[>1,\ X=\frac{1}{2}\dot{\phi}^2\right]$$

Slow-roll regime $(\phi>0)$

$$H \simeq \sqrt{\frac{4\pi}{3}}\, m\phi \ , \quad 3c_s^{-1}H\dot{\phi}+m^2\phi \simeq 0$$

Solution
Attractor

$$\dot{\phi}_{sr} \simeq -\frac{m c_*}{\sqrt{12\pi}} \quad (\phi\to\infty)$$

$$c_* = c_s(\dot{\phi}_{sr}) = \left[1-\frac{m^2}{12\pi\alpha^2}\right]^{-1/2}$$

only if $12\pi\alpha^2 > m^2$

On slow-roll solution :

$$p \simeq m^2\left[\frac{1}{12\pi}\frac{C_\star^2}{1+C_\star} - \frac{\phi^2}{2}\right]$$

$$\mathcal{E} \simeq m^2\left[\frac{1}{12\pi}\frac{C_\star}{1+C_\star} + \frac{\phi^2}{2}\right]$$

$$\frac{\mathcal{E}+P}{\mathcal{E}} \simeq \frac{C_\star}{6\pi\phi^2}$$

Inflation is over if $\frac{\mathcal{E}+P}{\mathcal{E}} \simeq 1$

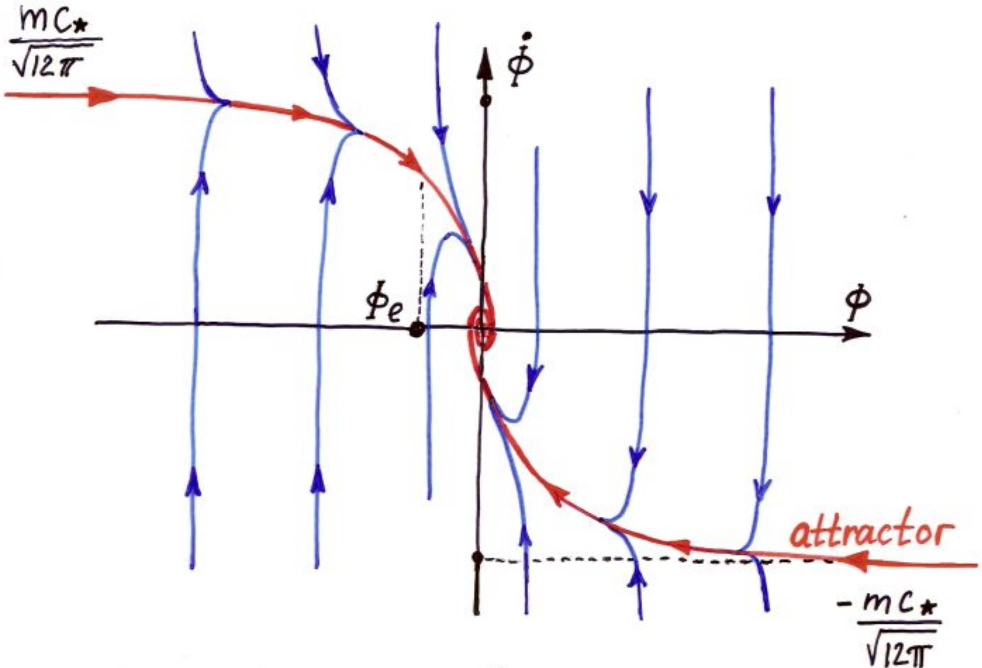
$$\boxed{\phi_e \simeq \sqrt{\frac{C_\star}{6\pi}}}$$

$$a(\phi) \simeq a_e \exp\left(\frac{2\pi}{C_\star}\left(\phi_e^2 - \phi^2\right)\right)$$

N_e – number of e-folds before the End of Inflation

$$\phi^2(N_e) \simeq \frac{C_\star}{2\pi} N_e$$

and, hence $\quad \frac{\mathcal{E}+p}{\mathcal{E}} \simeq \frac{1}{3N_e}$



$$m = 1{,}5 \times 10^{-7}$$

$$C_\star = 3{,}67$$

$$\phi_e \simeq 0.32 \qquad w(\phi_e) = -\frac{1}{3}$$

$$p(X,\phi) = \alpha^2\left[\sqrt{1+\frac{2X}{\alpha^2}} - 1\right] - \frac{m^2\phi^2}{2}$$

• Perturbations

h_{ik} , Φ , $\delta\varphi$ new variable v

are not independend due to Bianchi (Mukhanov)

$$v \equiv a\sqrt{\varepsilon_{,x}}\left(\delta\varphi + \frac{\varphi_0'\Phi}{\mathcal{H}}\right), \quad \varphi_0' = \frac{d}{d\eta}\phi_0$$

$$\mathcal{H} = \frac{a'}{a}$$

$$z \equiv \frac{a^2(\varepsilon + p)^{1/2}}{c_s \mathcal{H}}$$

$$S_{pert} = \frac{1}{2}\int d\eta\, d^3x \left(v'^2 + c_s^2 v \Delta v + \frac{z''}{z} v^2\right)$$

$$v'' - c_s^2 \Delta v - \frac{z''}{z} v = 0$$

v is canonical for quantization.

$$\langle 0|\, \hat{\Phi}(\eta, x)\, \hat{\Phi}(\eta, y)|0\rangle = \int \delta_\Phi^2(k,\eta)\, \frac{\sin kr}{kr} \frac{dk}{k}$$

$$(\star)\quad \delta^2_{\Phi_{rad}} \simeq \frac{64}{81}\left[\frac{\varepsilon}{c_s(1 + p/\varepsilon)}\right]_{c_s k \simeq Ha}$$

post-inflationary in rad. dom. epoch.

$$\langle 0|\, \hat{h}^i_j(\eta, x)\, \hat{h}_i^{\,j}(\eta, y)|0\rangle = \int \delta_h^2(k,\eta)\, \frac{\sin kr}{kr} \frac{dk}{k}$$

$$(\star)(\star)\quad \delta_h^2 \simeq \frac{8 H^2_{k=Ha}}{\pi} = \frac{64}{3}\,\varepsilon\Big|_{k=Ha}$$

$$n_T \equiv \frac{d\ln\delta_h^2}{d\ln k} \simeq -3\left(1+\frac{p}{\varepsilon}\right)$$

[Mukhanov Book 2005]

$$n_s - 1 \equiv \frac{d\ln\delta_\phi^2}{d\ln k} \simeq -3\left(1+\frac{p}{\varepsilon}\right) - \frac{1}{H}\left[\ln\left(1+\frac{p}{\varepsilon}\right)\right]^{\bullet} - \frac{(\ln c_s)^{\bullet}}{H}$$

($\dot{c}_s$ neglig.) in our case

General Results

$$r = \frac{\delta_h^2}{\delta_\phi^2} \simeq -9\, c_s\, n_T$$

δ_ϕ is observed to be 10^{-5}.

$$n_T \simeq -\frac{1}{N_e}$$

$$n_s - 1 \simeq -\frac{2}{N_e}$$

the same tilts as in usual $\frac{1}{2}m^2\phi^2$

$$m \simeq \frac{3\sqrt{3\pi}\,\delta_\Phi}{4N_e}$$

$N_e \sim 60 \quad m \sim 10^{-7}$

the same as in $\frac{1}{2}m^2\phi^2$

$$r \simeq -\frac{9C_*}{N_e}$$ arbitrary > Lyth bound.

$$\alpha^2 > \frac{1}{12\pi}\left(\frac{m}{M_{pe}}\right)^2 M_{pe}^4$$

Fine tuning? Scale of cut-off?

$$12\pi\alpha^2 > m^2 \rightarrow \frac{\alpha^2}{12\pi M_{pe}^4} > \frac{m^2}{M_{pe}^2}$$

What happens
wiht Black Holes,
if information
can propagate
faster than Light?

• Model

$$p(X,\phi) = \alpha^2\left[\sqrt{1+\frac{2X}{\alpha^2}} - 1\right]$$

Stationary, spherically symmetric
Accretion onto a Schwarzschild BH
No Backreaction „test fluid"

$$ds^2 = \left(1-\frac{r_g}{r}\right)dV^2 - 2dVdr - r^2 d\Omega^2$$

$$r_g = 2GM \qquad d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

Eddington - Finkelstein inf. coordinates

$$u^r = u^r(r), \quad \varepsilon = \varepsilon(r), \quad p = p(r)$$

$$C_s^2 = 1 + \frac{2X}{\alpha^2} > 1$$

$\phi = \phi(V,r)$ solution of EoM

$$G^{\mu\nu}\nabla_\mu \nabla_\nu \phi = 0$$

Model with $p(X)$ is equiv. to hydrodynamics.

$u^r(r) \to$ Ansatz

$$\boxed{\phi = \dot{\phi}_\infty \left(V + F(r)\right)}$$

$$V = t + r^*, \quad r^* = r + r_g \ln\left(\frac{r}{r_g} - 1\right)$$

$F(r)$ is solution $F_{\underbrace{c_\infty^2, A}_{\text{const.}}}(r)$

$$F(r)_{r\to\infty} \to -r, \qquad \phi_\infty = \dot{\phi}_\infty t$$

and "no singularities" $\qquad c_\infty^2 = 1 + \frac{\dot{\phi}_\infty^2}{\alpha^2}$

$\Rightarrow$ for a given c_∞ there exists? $A(c_\infty)$.

— Solution —

$$u^r = -\frac{A}{\sqrt{x}}\sqrt{\frac{c_\infty^2 x - 1}{c_\infty^2 x^4 - A^2}}$$

$$A = -u\left(\frac{r}{r_g}\right)^2 \frac{n}{n_\infty}$$

$$u_{Hor} = -\frac{A}{4}\frac{n_\infty}{n_H}$$

$$dn/n = \frac{d\varepsilon}{\varepsilon + p}$$

$$c_s^2 = c_\infty^2 \frac{x^3(c_\infty^2 x - 1)}{A^2(c_\infty^2 - 1) + c_\infty^2 x^3 (x-1)}$$

$$x = r/r_g$$

Solutions

$$u^r = -\frac{A}{\sqrt{x}}\sqrt{\frac{C_\infty^2 x - 1}{C_\infty^2 x^4 - A^2}}, \qquad \boxed{x = \frac{r}{r_g} \quad A>0}$$

$$C_s^2 = C_\infty^2 \frac{x^3 (C_\infty^2 x - 1)}{A^2 (C_\infty^2 - 1) + C_\infty^2 (x-1) x^3}$$

For propagation „Vektors" $\boxed{f(x) = 1 - x^{-1}}$

$$\eta^{\pm} \equiv \frac{dV}{dr}\Big|_{\pm} = \frac{1}{f(x)} \pm \frac{x(x-1)C_\infty \mp A(C_\infty^2 - 1)}{f(x)\sqrt{C_\infty^2 - x^{-1}}\sqrt{A^2(C_\infty^2-1) + C_\infty^2 x^3 (x-1)}}$$

Event horizon $x = 1$

$$\eta^+\Big|_{eH} = \frac{(A + C_\infty)^2}{2A^2(C_\infty^2 - 1)} \neq \infty$$

$$\eta^-\Big|_{eH} = \frac{(A - C_\infty)^2}{2A^2(C_\infty^2 - 1)} \neq \infty$$

No divirgenses in Event Horizon, $u^r \xrightarrow[r\to\infty]{} 0$

? How to find unique solution for C_∞? $A(C_\infty)$

For every A there are x where $C_s^2 < 0$ and $C_s = \infty$. We require that these points are inside of the sound Horizon

$\eta = \infty$ ≈ „Cosmic censorship"

This is only possible if

$\boxed{C_\infty^2 < 4/3}$ and $\boxed{A = 1/C_\infty^3}$ $\quad r_{SH} = \frac{r_g}{C_\infty^2}$

Then on the sound Horizon $\boxed{x_{SH} = 1/C_\infty^2}$

$$y_{+SH} = \infty \qquad y_{-SH} = -\frac{C_\infty\sqrt{4-3C_\infty^2} + C_\infty^2 - 2}{C_\infty(C_\infty^2-1)\sqrt{4-3C_\infty^2}}$$

$$C_s\Big|_{SH} = \frac{C_\infty}{\sqrt{4-3C_\infty^2}}$$

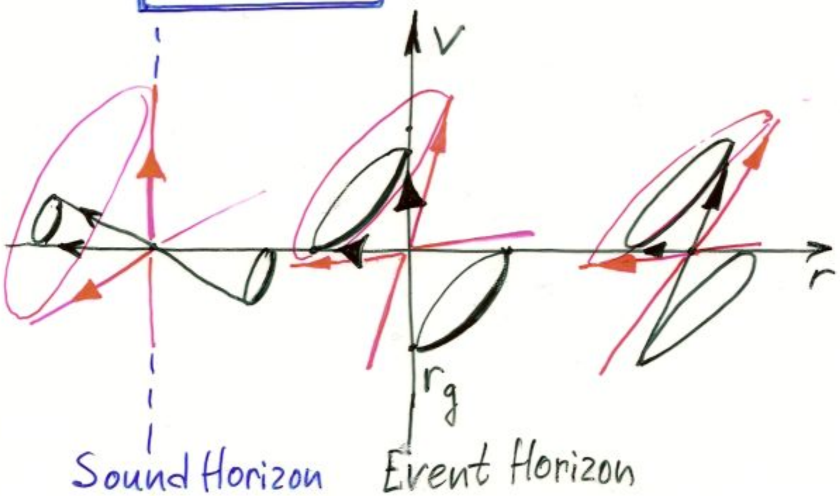
Accretion rate

! $\dot{M} = 4\pi M^2 \alpha^2 (C_\infty^2 - 1) C_\infty^{-4}$

can be very small by C_∞

Thus one can obtain information from

$\boxed{r > \frac{3}{4} r_g}$



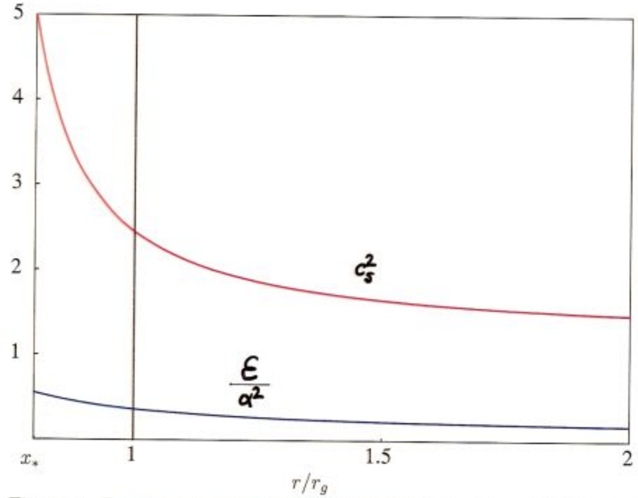


Figure 1: For the background solution in the case $c_\infty^2 = 5/4$ the squared sound speed (red) and the normalized energy density, (, (blue) are shown as functions of radial coordinate $x \equiv r/r_g$. The sound horizon $r_*/r_g = 4/5$ is located inside the Schwarzschild horizon.

$$\omega_\infty = \omega_0 \frac{C_\infty^8 x^3(x-1) + C_\infty^2 - 1}{x^2 C_\infty^4 (x^2 C_\infty^4 + 1)}$$

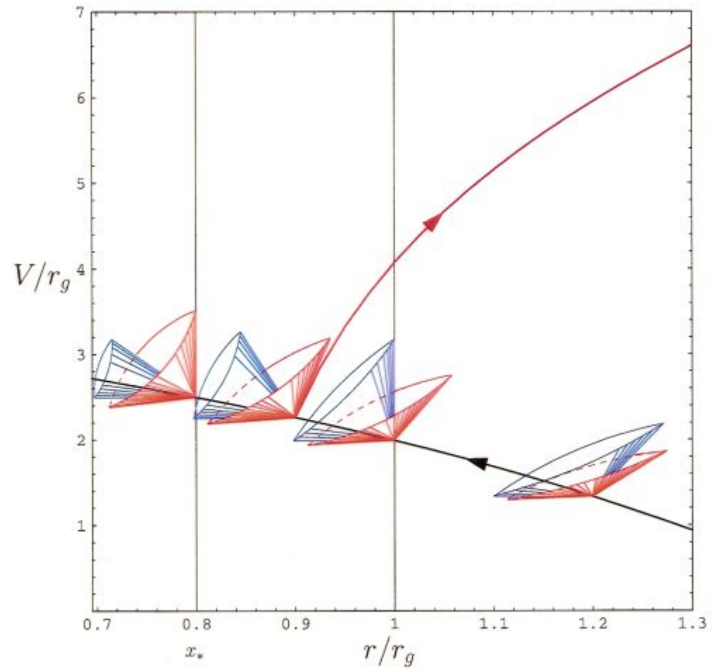
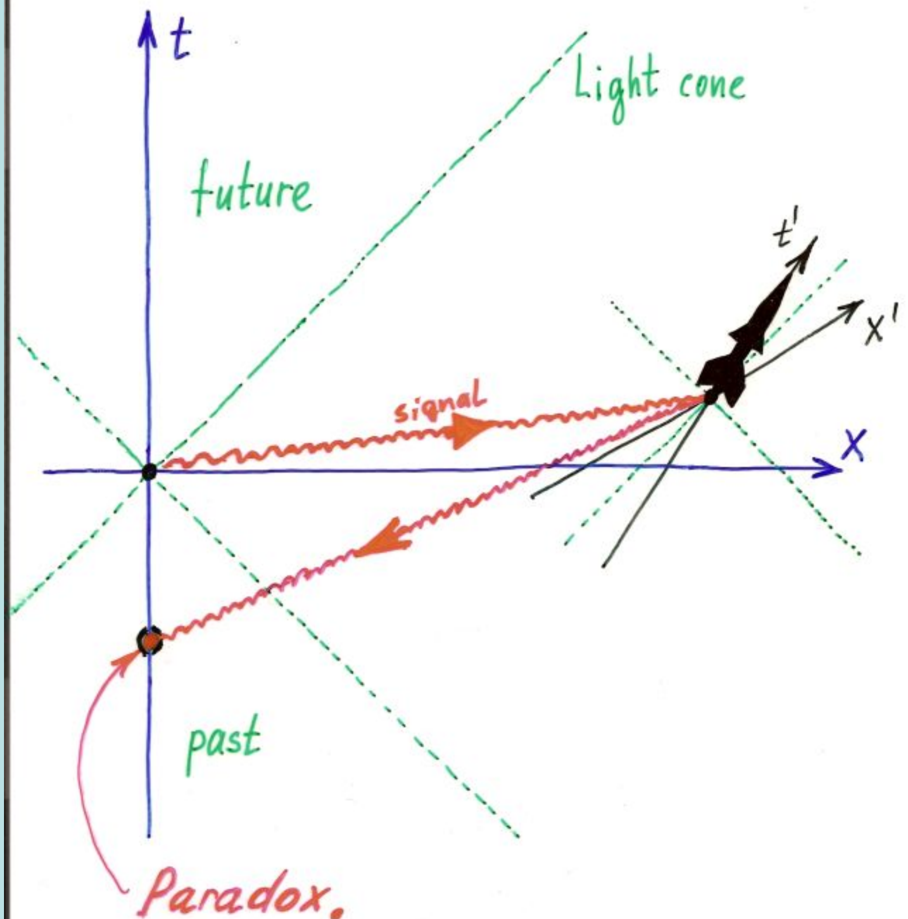
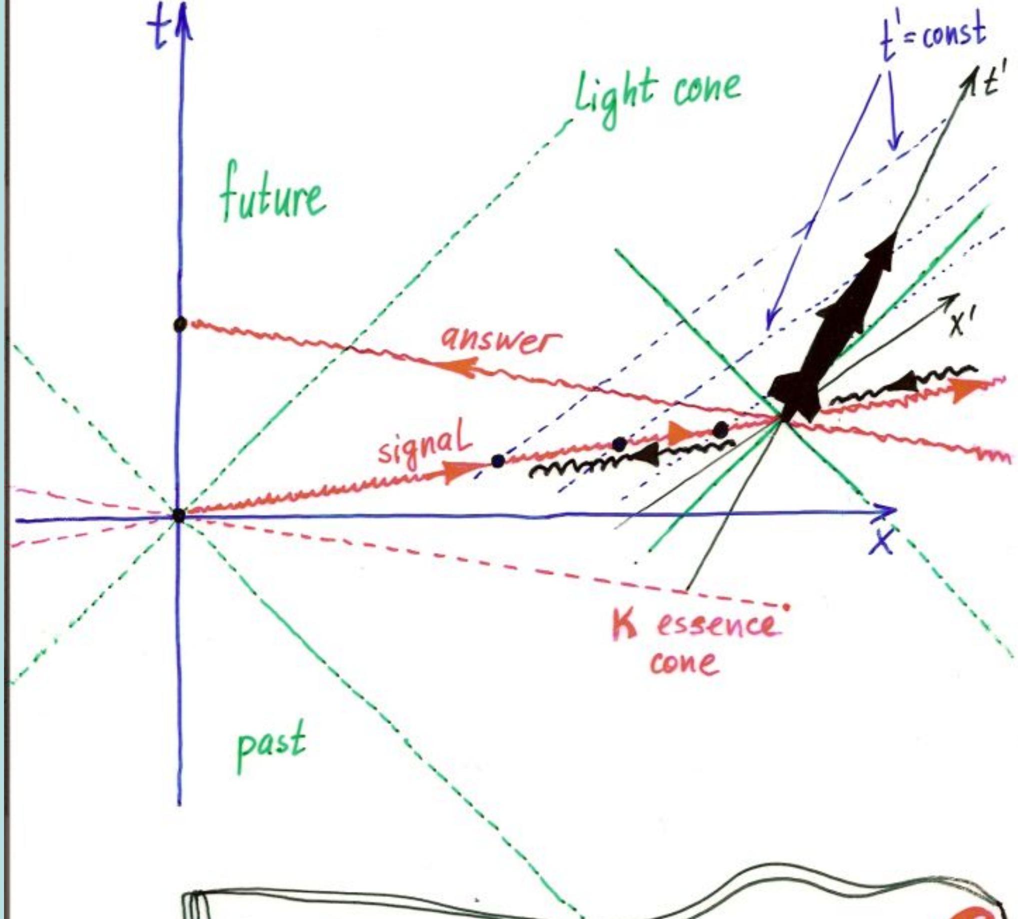


Figure 2: In the Eddington-Finkelstein coordinates the emission of a sound signal from the falling spacecraft is shown. The blue cones correspond to the future light cones and the red cones are the future sonic cones. The black curve represents the world line obtained numerically from for the spacecraft which moves together with a falling background field ϕ. Being between the Schwarzschild ($r = r_g$) and sound ($r = r_*$) horizons the spacecraft emits an acoustic signal (shown by red) which reaches the distant observer in finite time. The trajectory of the signal is obtained by the numerical integration.

- Is there causality violation?

  Well known paradox with spacelike signals.





?No? violation of causality?

! Causality is preserved in the both cases under consideration: Cosmology and BH accretion.

Sound Horizon

$$\Downarrow$$

$$T_{Hawking} = \frac{\kappa}{2\pi}$$

κ – surface gravity

$$T_H^{ac} = T_H \, c_\infty^3 \sqrt{4 - 3c_\infty^2}$$

$$T_H^{a} = T_H \left(\frac{c^4}{c_H}\right) < T_H$$

No violation of the $\underline{\mathrm{II}}$ Law!

system is not closed:

energy flux from ∞.

compare with „Ghost Condensate":

[Dubovsky, Sibiryakov 2006]

• Conclusion

- If $\exists$:
  - ★ One can send information faster than light
  - ★ Inflation with large GW $(h_{\mu\nu}, r)$ and the same n_s, n_T, m_ϕ
  - ★ Schwarzschild Horizon $r = 2GM$ changes its universal meaning
  - ★ Different T_H
  - ★ Causality is preserved in both cases: of accretion and Inflation

  But: there are backgrounds where causality is violated $\approx$ GR: chronology protection conjecture

- QM/QFT problems...?

  [Adams et.al 2006]