Towards the understanding of gravity on conical branes

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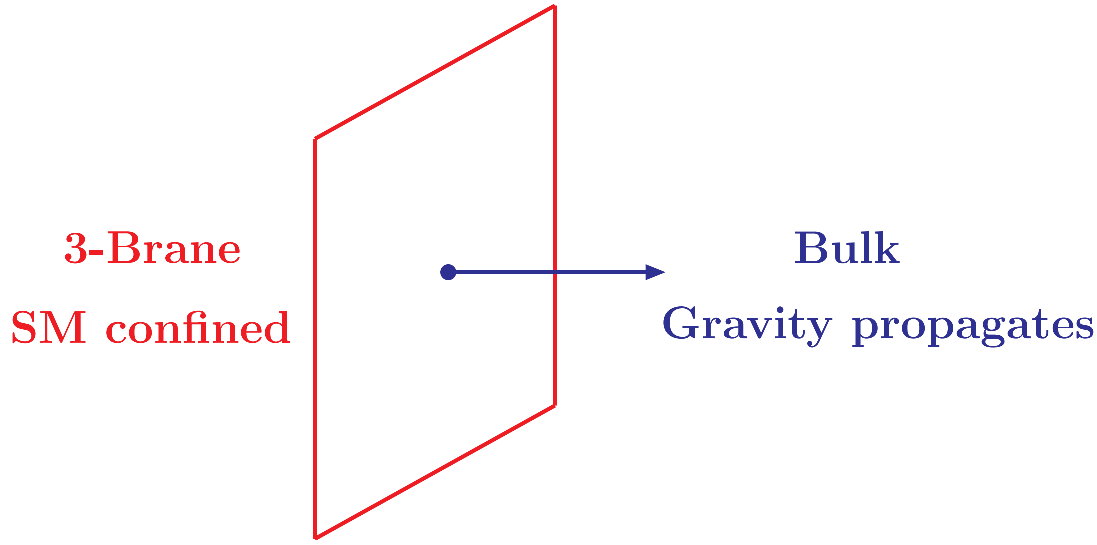
Outline

- Properties of codimension-2 branes
- 6D models vs selftuning
- Gravity problem on conical branes
  - Gauss-Bonnet term inclusion and constraints
  - Regularization by lowering the codimension

On several works with:
Nilles, Tasinato, Lee, Papantonopoulos, Zamarias

Brane world universes



- Interesting for providing alternative explanations to long standing problems in physics
  - electroweak hierarchy, Yukawa hierarchies, cosmological constant
- Most thoroughly studied in 5D

  One dim. $\perp$ to the brane $\equiv$ Codimension-1 brane
- Einstein equation projected on the brane

$$E^{(4)}_{\mu\nu} = \frac{1}{M_{Pl}^2} T^{(br)}_{\mu\nu} + \{T^2_{(br)}\}_{\mu\nu} + \{C\}_{\mu\nu} + \Lambda_4 g_{\mu\nu}$$

  obtained by the junction condition $K_{\mu\nu} \equiv g'_{\mu\nu} \sim T^{(br)}_{\mu\nu}$
- Example of cosmology on the brane

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{1}{3M_{Pl}^2}\left[\rho + \frac{\rho^2}{2T} + \frac{C}{a^4}\right]$$

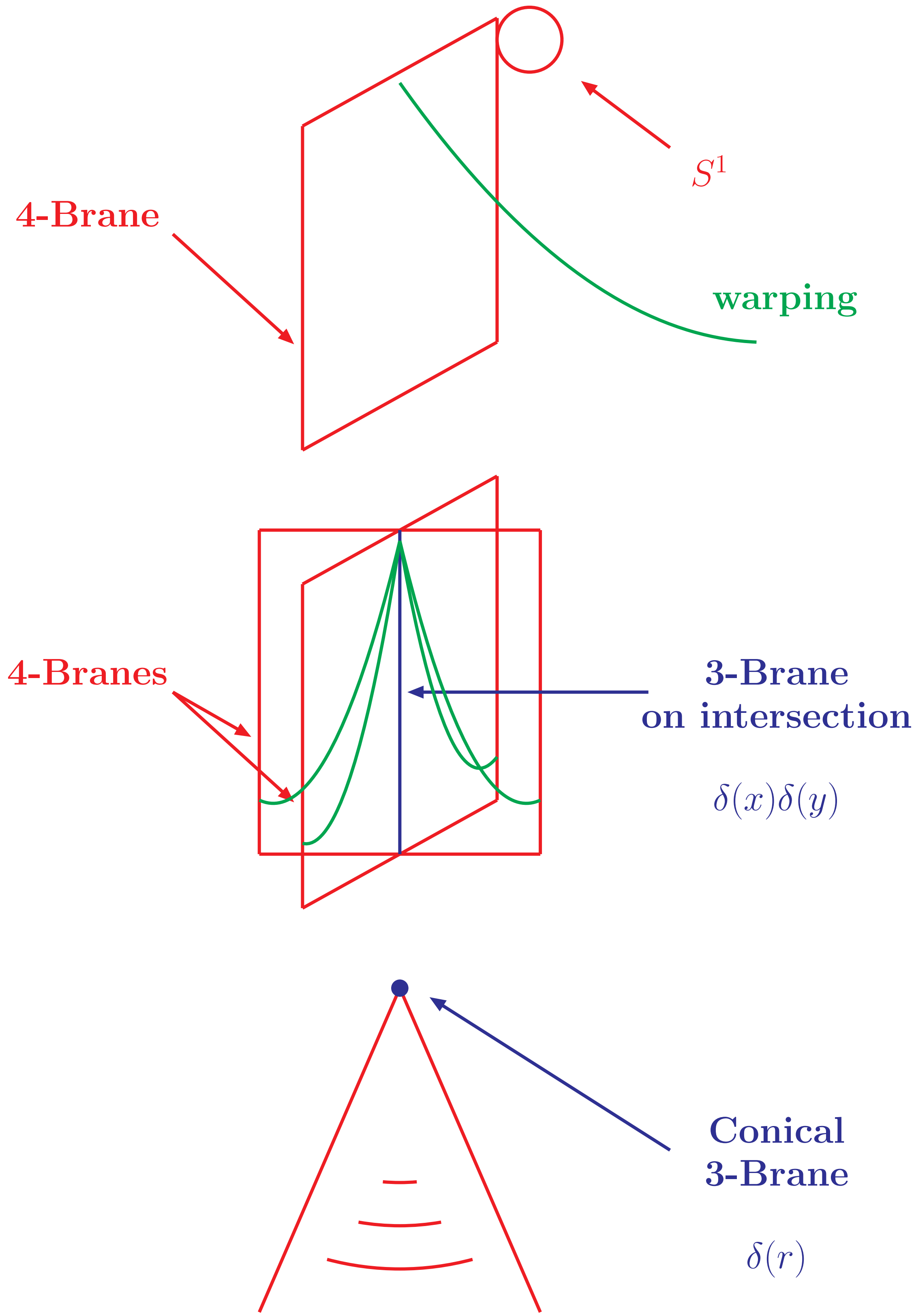
  Early time cosmology (for $\rho \gg T \sim M_{Pl}^4$) is 5D
- Interesting late time modifications also possible

6D theories

- **Less understood are 6D brane worlds**

  Two dim. $\perp$ to the brane $\equiv$ Codimension-2 brane

- **In 6D there are two extra dimensions to hide**



Conical branes

[H-P.Nilles,A.P.,G.Tasinato, hep-th/0309042]

- **Suppose a p-brane in D dimensions ($D = p + 1 + d$) with tension T_p. Then**

$$R = R^{(reg)} + R^{(sing)}\delta^{(d)}(r)$$

- **The singular part of the Einstein equations gives**

$$R^{(sing)} = \frac{p+1}{D-2}\,T_p$$

- **On shell value of the action**

$$S = \int d^D x(R^{(reg)} + \mathcal{L}^{(bulk)}) + \int d^{p+1}x(R^{(sing)} - T_p)$$

- **Cancellation of $R^{(sing)}$ and T_p happens automatically if**

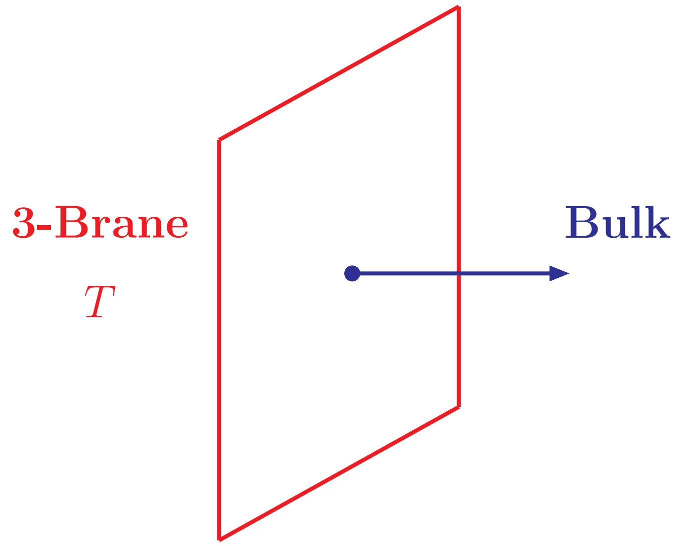
$$\frac{p+1}{D-2} = 1 \qquad \Rightarrow \qquad D = (p+1) + 2$$

  i.e. **for a codimension-2 brane.**

- **For a ($p = 3$)-brane, $D = 6$**

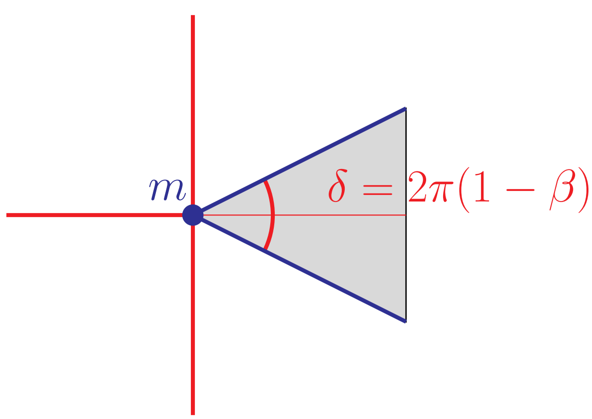
- **This automatic cancellation of the brane tension, makes selftuning possible**

Selftuning



- Selftuning model $\equiv$ Brane world model where
  1. The brane can be flat for any T
  2. No fine-tuning between T and other bulk quantities
- Solution to the cosmological constant problem, if in addition there are no fine-tuning between the bulk parameters
- Selftuning attempts in 5D models with codimension-1 branes failed (singularities, hidden finetuning)
- 6D attempts more promising (codimension-2 property)

  - Not guaranteed, relation of T with other bulk parameters has to be checked
- Origin of mechanism: in 1+2 dimensions, sources do not curve the space, but only introduce a deficit angle δ



$$ds_2^2 = dr^2 + r^2 d\phi^2, \ \phi \in [0, 2\pi\beta)$$

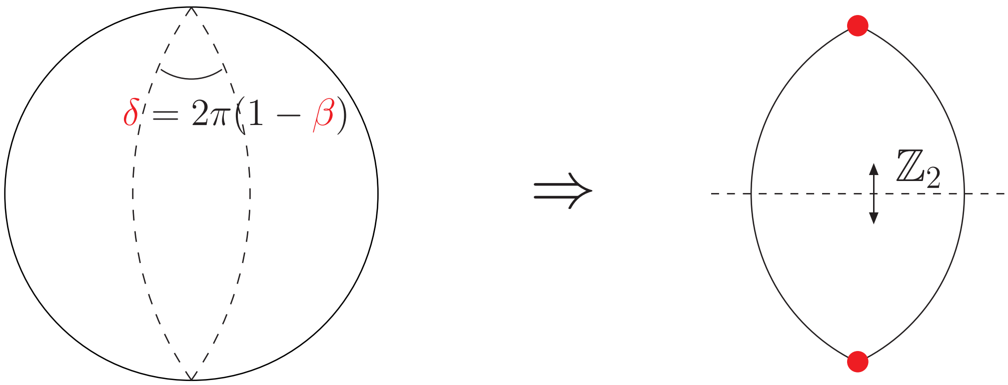
or

$$ds_2^2 = dr^2 + \beta^2 r^2 d\varphi^2, \ \varphi \in [0, 2\pi)$$

with $\beta = 4Gm$

Flux compactification model

[Z.Horvath,L.Palla,E.Cremmer,J.Scherk, NPB127 (1977) 57]
[S.M.Carroll, M.M.Guica, hep-th/0302067]
[I.Navarro, hep-th/0302129]



- **Gravity + gauge field** $F_{MN} = \partial_{[M}A_{N]}$ **+ bulk c.c.** Λ

$$S = \int d^6x\sqrt{-g_6}\left[\frac{1}{2}R_6 - \Lambda - \frac{1}{4}F^2_{MN}\right] - T\int d^4x\sqrt{-g_4}$$

- **Turning on the flux** $F_{\theta\varphi} = f\,\epsilon_{\theta\varphi}$**, the internal space is spontaneously compactified**

$$ds_6^2 = \eta_{\mu\nu}dx^\mu dx^\nu + R_0^2\left(d\theta^2 + \beta^2\sin^2\theta\,d\varphi^2\right)$$

with

$$\frac{1}{R_0^2} = f^2 \quad , \quad \Lambda = \frac{f^2}{2} \quad , \quad \beta = 1 - \frac{T}{2\pi}$$

- T **has no apparent relation to** Λ **or** f

  Selftuning ???

- **However, there is a flux quantization condition**

[Z.Horvath,L.Palla,E.Cremmer,J.Scherk, NPB127 (1977) 57]
[I.Navarro, hep-th/0305014]

$$f = \frac{N}{2eR_0^2\beta}$$

$$\Rightarrow \qquad N = \frac{2e}{\sqrt{2\Lambda}}\left(1 - \frac{T}{2\pi}\right)$$

- **This quantization condition ruins selftuning**

OTHER TYPE OF COMPACTIFICATION ???

[S.Radjbar-Daemi,V.Rubakov, hep-th/0407176]
[H.M.Lee,A.P., hep-th/0407208]

- **Instead of a gauge field + bulk c.c., one can use a 2d σ-model to compactify the internal space**

$$S = \int d^6x\sqrt{-g_6}\left[\frac{1}{2}R_6 - \frac{2\ \partial_M\Phi\partial^M\bar{\Phi}}{(1+|\Phi|^2)^2}\right] - T\int d^4x\sqrt{-g_4}$$

- **Obtain identical solution**

$$ds_6^2 = \eta_{\mu\nu}dx^\mu dx^\nu + R_0^2\left(d\theta^2 + \beta^2\sin^2\theta\, d\varphi^2\right)$$

$$\Phi = \left(\tan\frac{\theta}{2}\right)e^{i\beta\varphi} \qquad , \qquad \beta = 1 - \frac{T}{2\pi}$$

- **Now, the solution *is* selftuning**

Problem with gravity

[J.M.Cline,J.Descheneau,M.Giovannini,J.Vinet, hep-th/0304147]

- **For the static models we assumed** $T_{\mu\nu} = -T g_{\mu\nu} \delta^{(2)}(\vec{r})$
- **What if we have matter on the conical branes ???**

  e.g. cosmological fluid $T_{\mu\nu} = \mathrm{diag}(-\rho, P, P, P)\delta^{(2)}(\vec{r})$

- **Assumption: There is no singularity worse than conical**
- **Consequences:**
  - **For the metric ansatz**

$$ds^2 = -N^2(t,r)dt^2 + A^2(t,r)d\vec{x}^2 + dr^2 + L^2(t,r)d\theta^2$$

  with $L \sim \beta r + \dots$ **and** $R_{00} \sim \frac{N'}{r} + \dots$, $R_{ij} \sim \frac{A'}{r}\delta_{ij} + \dots$
  - **Thus** $A' = N' = 0 \Rightarrow$ **no singular part in** A'', N''
  - **Equations of motion**

$$\begin{aligned} 3\frac{A''}{A} + \frac{L''}{L} + \dots &= -\rho\ \delta^{(2)}(\vec{r}) \qquad (00) \\ 2\frac{A''}{A} + \frac{N''}{N} + \frac{L''}{L} + \dots &= P\ \delta^{(2)}(\vec{r}) \qquad (ij) \end{aligned}$$

  - **Only tension allowed** $\rho = -P$

- **Ways out:**

  ★ **Keep assumption and complicate gravity dynamics**

  ★ **Abandon assumption and regularize the brane around** $r = 0$

Modifying gravity dynamics

[P.Bostock,R.Gregory,I.Navarro,J.Santiago, hep-th/0311074]

- **Modify the singularity structure of the equations of motion**

  $\Rightarrow$ **Add a bulk Gauss-Bonnet term**

$$\mathcal{S} = \frac{M_6^4}{2}\int d^6x\sqrt{G}\left[R^{(6)} + \alpha(R^{(6)\ 2} - 4R_{MN}^{(6)\ 2} + R_{MNK\Lambda}^{(6)\ 2})\right] + \int d^6x\mathcal{L}_{Bulk} + \int d^4x\mathcal{L}_{brane}\,\frac{\delta(r)}{2\pi L}$$

- **Metric ansatz**

$$ds^2 = g_{\mu\nu}(x,r)dx^\mu dx^\nu + dr^2 + L^2(x,r)d\theta^2$$

with $L = \beta(x)r + \mathcal{O}(r^3)$

- **Conical singularity conditions**

$$K_{\mu\nu} \equiv g'_{\mu\nu}\big|_{r=0} = 0 \quad \text{and} \quad \beta = const.$$

- **The δ-function part of the $(\mu\nu)$ Einstein equations gives**

$$R_{\mu\nu}^{(4)} - \frac{1}{2}R^{(4)}g_{\mu\nu} = \frac{1}{M_{Pl}^2}\left[T_{\mu\nu}^{(br)} - \Lambda_4 g_{\mu\nu}\right]$$

with, $M_{Pl}^2 = 8\pi(1-\beta)\alpha M_6^4$ **and** $\Lambda_4 = -2\pi(1-\beta)M_6^4$

4D EQUATION WITH AN INDUCED Λ_4

Bulk & brane matter relations

[E.Papantonopoulos,A.P., hep-th/0501112]
[E.Papantonopoulos,A.P., hep-th/0507278]

- **There is more information coming from the (rr) equation evaluated at $r = 0$**

$$R^{(4)} + \alpha[R^{(4)\ 2} - 4R^{(4)\ 2}_{\mu\nu} + R^{(4)\ 2}_{\mu\nu\kappa\lambda}] = -\frac{2}{M_6^2} T_r^{(B)r}$$

  - $R^{(4)}_{\mu\nu}$ **and** $R^{(4)}$ **given by** $T^{(br)}_{\mu\nu}$
  - $R^{(4)}_{\mu\nu\kappa\lambda}$ **is arbitrary in general**

- **In several interesting cases $R^{(4)}_{\mu\nu\kappa\lambda}$ is also related to $T^{(br)}_{\mu\nu}$**

  e.g. **cosmological isotropic metric**

$$ds^2 = -N^2(t,r)dt^2 + A^2(t,r)d\vec{x}^2 + dr^2 + L^2(t,r)^2 d\theta^2$$

  Then, brane matter is tuned to bulk matter

- **Different from the $5D$ brane cosmology**

  In $5D$ $K_{\mu\nu} \neq 0$ on the brane

  $\Rightarrow$ **Independence of brane matter from bulk matter**

- **Example: isotropic cosmology for $T^{(B)}_{MN} = -\Lambda_B G_{MN}$,**

  $\rho = -\Lambda_4 + \rho_m$ **and** $P = \Lambda_4 + w\rho_m$

  - **Brane matter tuning:**

$$-\frac{\Lambda_B}{M_6^4} = \frac{\rho_m}{M_{Pl}^2}\left[\frac{1}{2}(3w-1) + \frac{2}{3}(3w+1)\alpha\frac{\rho_m}{M_{Pl}^2}\right]$$

  - $w = 1/3$ **attractor for** $\Lambda_B = 0$
  - $w = -1$ **attractor for** $\Lambda_B > 0$ **with** $\frac{\alpha\rho_f}{M_{Pl}^2} = -\frac{3}{4} + \frac{3}{4}\sqrt{1 + \frac{4}{3}\frac{\Lambda_B}{M_6^4}}$

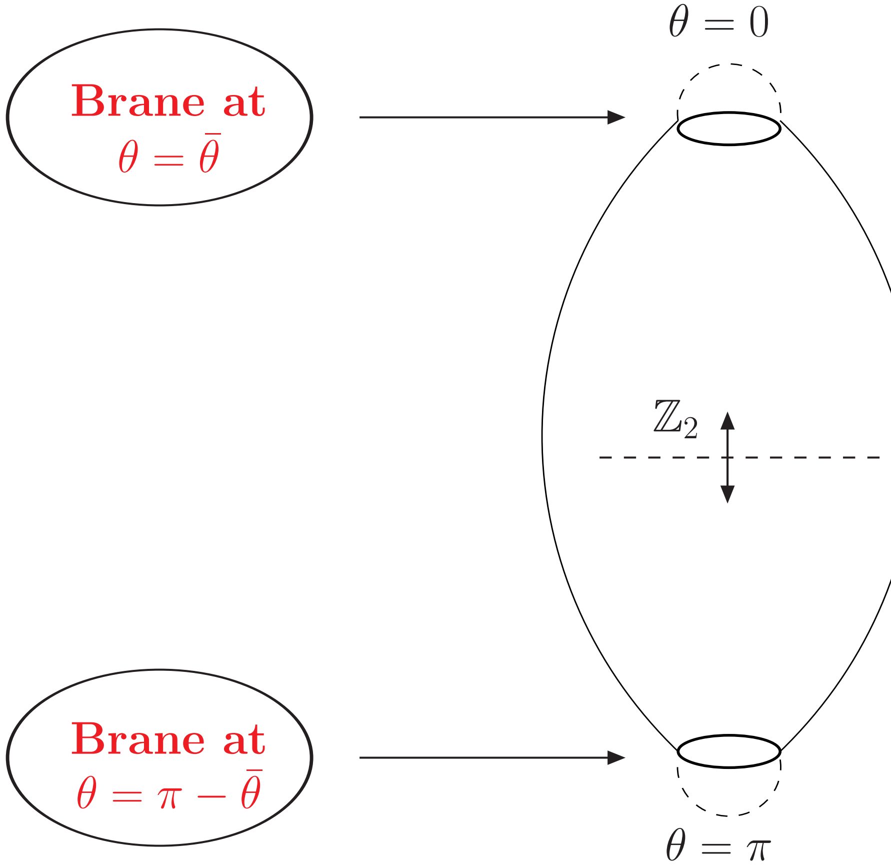
Brane regularization

[M.Peloso,L.Sorbo,G.Tasinato, hep-th/0603026]

- Consider instead a thick defect
- Simplest possibility: lowering codimension

  Codimension-2 $\Rightarrow$ Codimension-1
- Example: flux compactification model

  Cut space and replace with ring + smooth cap



Cap: $R_c = M_6^4/(2\Lambda_c)$, $c_c = 1$

Bulk: $R_0 = M_6^4/(2\Lambda_0)$, c_0

Cap: $R_c = M_6^4/(2\Lambda_c)$, $c_c = 1$

$$
\begin{aligned}
ds^2 &= \eta_{\mu\nu}dx^\mu dx^\nu + R_\#^2(d\theta^2 + c_\#^2 \sin^2\theta \; d\varphi^2) \\
F_{\theta\varphi} &= c_\# \; R_\# \; M^2 \cos\theta
\end{aligned}
$$

- continuity fixes $R_c = c_0 R_0$

- quantization condition $N = 2c_0 R_0 M_6^2 e$, $N \in \mathbf{Z}$

What kind of ring ?

- **Junction conditions for this particular geometry dictate $T^{(br)}_{\varphi\varphi} = 0$ and $T^{(br)}_{\mu\nu} \sim \eta_{\mu\nu}$**
- **Brane action proposal**

$$S_{br} = -\int d^5x\sqrt{-\gamma}\left(\lambda + \frac{v^2}{2}(\tilde{D}_{\hat{\mu}}\sigma)^2\right)$$

with $\tilde{D}_{\hat{\mu}}\sigma = \partial_{\hat{\mu}}\sigma - eA_{\hat{\mu}}$, $\quad \hat{\mu} = (\mu, \varphi)$

- **Origin: Higgs phase $H = v\ e^{i\sigma}$, when the radial (heavy) part is integrated out**
- **Scalar field solution**

$$\sigma = n_{\pm}\varphi \quad , \quad n_{\pm} \in \mathbf{Z}$$

- **Furthermore the junction conditions determine v, λ as functions of $\bar{\theta}$ and relate the quantum numbers**

$$n_{\pm} = \pm\frac{N}{2}$$

Gravity for matter perturbations

- **Suppose that $T_{\mu\nu} \to T_{\mu\nu} + \delta T_{\mu\nu}$ (also for $T_{\varphi\varphi}$)**
- **The theory is Brans-Dicke with heavy scalar, decoupling in the conical limit ($\bar{\theta} \to 0$)**

$$R^{(4)}_{\mu\nu} = \frac{1}{M^2_{Pl}}\left[\delta T_{\mu\nu} - \frac{1}{2}\delta T\eta_{\mu\nu}\right] + (1-\cos\bar{\theta})F(\beta,\bar{\theta},\partial_\mu)\left(\frac{1}{3}\delta T - \delta T^{\varphi}_{\varphi}\right)$$

- **The brane bending mode diverges in the conical limit (strong couping)**

Can we do more than that ?

- **If we wish to check selftuning, we should have in mind that the quantum contributions of the brane fields are of the order of the tension of the 4-brane**

- **One should be able to discuss brane motion**

- **Simplest case: mirage cosmology**

  ★ **Use the static bulk sections we know and see what cosmology the brane motion induces on the brane**

  [A.Kehagias,E.Kiritsis,hep-th/9910174]

- **Suppose a static bulk metric**

$$ds^2_{(d)} = A^2(r)\eta_{\mu\nu}dx^\mu dx^\nu + dr^2 + B_{mn}(r)dx^m dx^n$$

A brane moving as $r = \mathcal{R}(t)$ induces a cosmology

$$ds^2_{(d-1)} = -[A^2(\mathcal{R}(t)) - \dot{\mathcal{R}}^2(t)]dt^2 + A^2(\mathcal{R}(t))d\vec{x}^2 + B_{mn}(\mathcal{R}(t))dx^m dx^n$$

$$\Rightarrow \quad ds^2_{(d-1)} = -d\tau^2 + A^2(\mathcal{R}(\tau))d\vec{x}^2 + B_{mn}(\mathcal{R}(\tau))dx^m dx^n$$

- **Need warping to have induced cosmology**

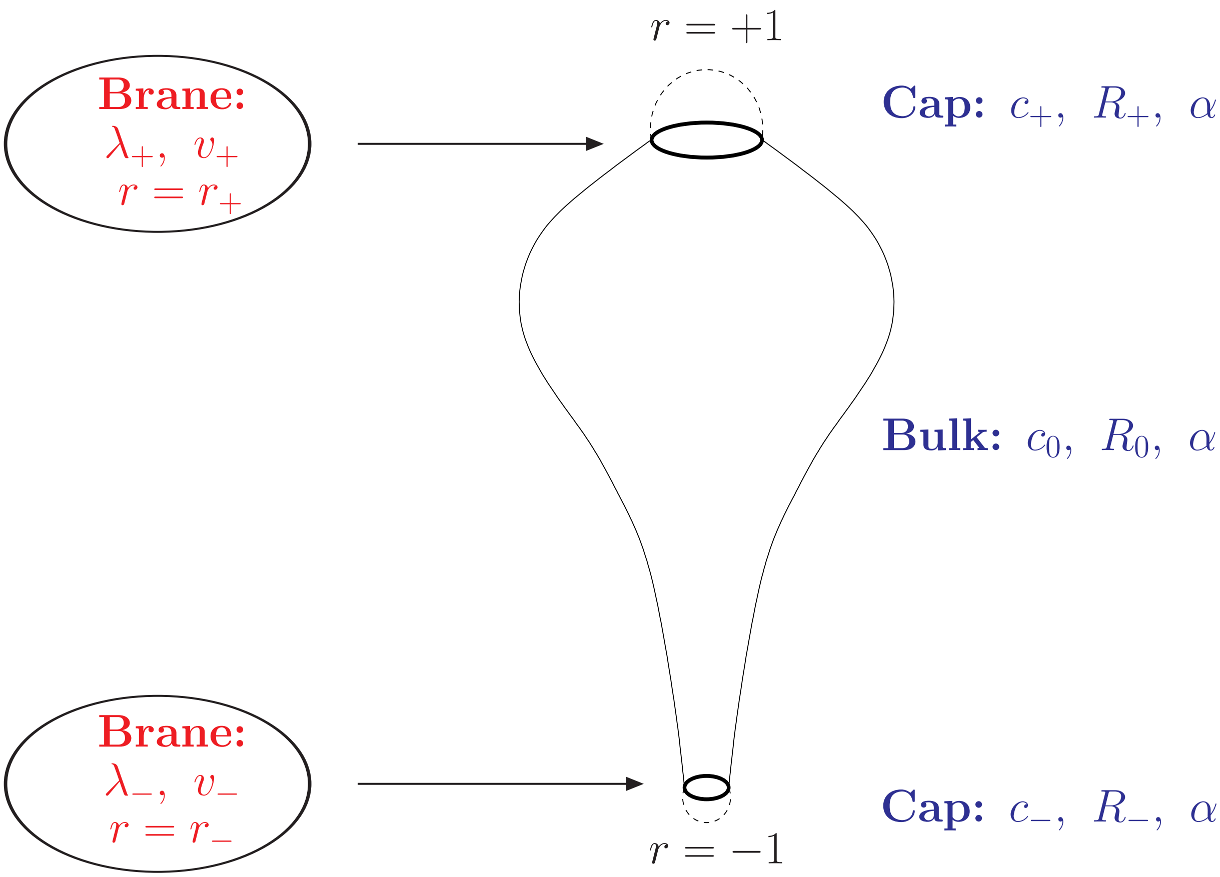
- **To do list**

  ★ **Find regularization for the warped analogues of the "football" solutions (√)**

  ★ **Study mirage cosmology in these backgrounds (work in progress)**

Warped model regularization

[E.Papantonopoulos,A.P.,V.Zamarias, hep-th/0611311]



- **Warped solution with codimension-2 known (Wick-rotated 6d Reissner-Nordström BH)**

[H.Yoshiguchi,S.Mukohyama,Y.Sendouda,S.Kinoshita, hep-th/0512212]

- **Use this to build the regular solution**

$$ds_6^2 = z^2 \eta_{\mu\nu} dx^\mu dx^\nu + R_\#^2 \left[\frac{dr^2}{f} + c_\#^2 f \ d\varphi^2 \right]$$

$$\mathcal{F}_{r\varphi} = -c_\# R_\# M^2 S(\alpha) \cdot \frac{1}{z^4}$$

with $R_\# = M^4/(2\Lambda_\#)$, $c_\pm = 1/X_\pm(\alpha)$, $R_\pm = c_0 R_0 X_\pm$ **and**

$$f = \frac{1}{5(1-\alpha)^2} \left[-z^2 + \frac{1-\alpha^8}{1-\alpha^3} \cdot \frac{1}{z^3} - \alpha^3 \frac{1-\alpha^5}{1-\alpha^3} \cdot \frac{1}{z^6} \right]$$

$$z = [(1-\alpha) r + 1 + \alpha]/2$$

Ring dynamics

- **Use again a scalar (Goldstone-like) field**

$$S_{br} = -\int d^5x\sqrt{-\gamma}\left(\lambda + \frac{v^2}{2}(\tilde{D}_{\hat{\mu}}\sigma)^2\right)$$

with $\tilde{D}_{\hat{\mu}}\sigma = \partial_{\hat{\mu}}\sigma - eA_{\hat{\mu}}$, $\quad \hat{\mu} = (\mu, \varphi)$

- **Scalar field solution**

$$\sigma = n_{\pm}\varphi \quad , \quad n_{\pm} \in \mathbf{Z}$$

- **From the junction conditions we determine** $v_{\pm}$, $\lambda_{\pm}$ **as functions of** $r_{\pm}$ **and relate the quantum numbers**

$$n_{\pm} = \pm\frac{N}{2}w_{\pm}(\alpha) \quad , \quad n_+ - n_- = N$$

and

$$w_+(\alpha) = \frac{2}{(1-\alpha^3)}\left[\frac{5(1-\alpha^8)}{8(1-\alpha^5)} - \alpha^3\right]$$

- **Restriction of warping** α**, quantum number** N
  - **Cannot have warped solutions for** $N \leq 4$
  - **First warped solution for** $N = 5$, $n_+ = 3$, $\alpha \approx .44$

- **Regularization scheme for** N**'s,** α**'s other than permitted breaks down**

Supersymmetric model (Salam-Sezgin)

[G.W.Gibbons,R.Guven,C.N.Pope, hep-th/0307238]
[C.P.Burgess,F.Quevedo,G.Tasinato,I.Zavala,hep-th/0408109]

- **Repeating the regularization procedure for the known warped solutions we find**

$$n_{\pm} = \pm\frac{N}{2}$$

- **No restriction of the warping** α

Conclusions

- 6D models with codimension-2 branes interesting because of their potential selftuning property

- General problem with gravity on them
  - either conical singularities + modified bulk gravity
  - or general singularities + regularization

- Including a bulk Gauss-Bonnet term we can get a 4D Einstein equation on the brane, but rather restricted matter

- Regularization of singularities, *e.g.* by lowering the codimension more promising way forward

- However, we expect in general a dependence on the regularization scheme